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Entangled Particles Tunneling From a Schwarzschild Black Hole immersed in an Electromagnetic Universe with GUP

A. Övgün[1]

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Abstract Quantum gravity has exciting peculiarities on the Planck scale.The effect of generalized uncertainty principle (GUP) to the entangled scalar/fermion particles' tunneling from a Schwarzschild black hole immersed in an electromagnetic Universe is investigated by the help of semi-classical tunneling method. The quantum corrected Hawking temperature of this black hole with an external parameter "a" modifies the Hawking temperature for the entangled particles.

Keywords Hawking radiation · Generalized uncertainty principle · Entangled particles

1 Introduction

A breathtaking process like white rabbit and black magic, first theorized by Stephen Hawking, by which a black hole can emit particles [1]. Hawking originally have used Bogoliubov's method [2], however, after that several methods of deriving Hawking radiation appeared. Understanding the Hawking radiation is a subject of long interest and the tunneling phenomenon has been extensively studied and it is applied on various black holes and also wormholes [3–21] . Today, one of the main challenges in physics is to merge quantum theory and the theory of general relativity into a unified framework, which should be modified with a minimum length scale of the order of the Planck length. So that the minimal length is developed on a very strong background as a quantum gravity such as string theory, loop quantum gravity and a non-commutative geometry [22–26]. This length derived from some different ways such as a generalized uncertainty principle (GUP), an

✉ A. Övgün
ali.ovgun@emu.edu.tr

[1] Physics Department, Eastern Mediterranean University, Famagusta, Northern Cyprus, Mersin 10, Turkey

extended uncertainty principle (EUP) and a generalized EUP (GEUP) [27–31]. There are many applications of the GUP to physics such as compact stars, Newtonian gravity, inflationary cosmology, violation of Lorentz invariance and measurable maximum energy and minimum time interval [32–40]. Furthermore, after Dvali and Gomez [41–43] proposed the idea of quantum black holes as modeled in Bose-Einstein condensation (BEC) of marginally bound, self-interacting gravitons, recently one shows that quantization of gravity is possible by using the Horizon Wave Function (HWF) formalism [44]. Halilsoy et. al. introduced a new metric of Schwarzschild black hole which is coupled to an external, stationary electrostatic field by using the interpolation of two exact well-known solutions of Einstein's equations such as the Schwarzschild (S) metric and a uniform electromagnetic (em) field solution of Bertotti and Robinson (BR) [45, 46]. We will here address the Hawking radiation of entangled particles as an emission of quanta by using this metric. Furthermore, we investigate the tunneling effect of entangled particles from such a black hole with the effect of GUP.

Entanglement which plays a frontier role on quantum information, is an important resource for different computational tasks such as quantum communication and teleportation. By understanding the entanglement in the frame of black holes, will give us an important resolution of information paradox of black holes. On this regard, we propose that two observers, Alice and Bob, share a generically entangled state at the same initial point. Behind the coincidence point where the particles tunnel, one of them tunnels from event horizon of black hole, while the other one stays in the extremal black hole (BR space-time). We focus our attention on the result of one of the entangled state which is also equal to another one which may be lost in singularity of black hole or can tunnel through another universe by a wormhole. The semi-classical Hawking temperature is derived by applying the WKB approximation and the Hamilton-Jacobi method to solve the Klein-Gordon (K-G) and Dirac Equations for the entangled states.

The structure of this paper is as follows. In Section 2, we briefly give the black hole solution which will be phrased hereafter as the Schwarzschild-Electromagnetic black hole, and in Section 3, by using the Hamilton-Jacobi method and suitable entangled ansatz, we derive the corrected Hawking temperature of entangled scalar particles from the new black hole. Last but not least, in Section 4 we compute the tunneling rate of entangled fermion particles from the same black hole. Finally, we will conclude with some comments in Section 5.

2 Schwarzschild-Electromagnetic Black Hole (SEBH)

The metric for the SEBH in an external electrostatic field in four dimensions is presented by Halilsoy et al. as [45, 46] [1]

$$ds^2 = -f dt^2 + \frac{1}{f} dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{1}$$

where

$$f = 1 - \frac{2M}{r} + \frac{M^2(1-a^2)}{r^2}, \tag{2}$$

[1] The erroneous metric in eq. (15) of ref.[45] is corrected here.

with the external parameter a ($0 < a \leq 1$), where the mass M is coupled to an external em-field. Note that $a = 0$ is the extremal Reissner–Nordstrom (RN) case which is transformable to the BR metric. The horizon is located for the above metric at

$$r_h = M(1 + a). \tag{3}$$

Let us note that the radial coordinate r is related to the vacuum (say $\tilde{r}$) coordinate by $r = a\tilde{r} + M(1 - a)$. in which $a = 0$ is excluded. Clearly r and $\tilde{r}$ are related by a translation and scaling transformation. Since $a \leq 1$ the horizon radius is $r_h \leq 2m$, which implies that the em field shrinks the horizon of the Schwarzschild black hole.

Ricci components are

$$R_t^t = R_r^r = \frac{M^2(a^2 - 1)}{r^4}, \tag{4}$$

$$R_\theta^\theta = R_\varphi^\varphi = \frac{-M^2(a^2 - 1)}{r^4}. \tag{5}$$

The Kretschmann scalar which is a quadratic scalar invariant is calculated as

$$K = R_{abcd}R^{abcd} = \frac{56M^2\left[(a^2 - 1)^2M^2 + \frac{12}{7}Mr\left(a^2 - 1\right) + \frac{6r^2}{7}\right]}{r^8} \tag{6}$$

The Ricci scalar of the SEBH is calculated as zero (since the source is pure)

$$R = 0. \tag{7}$$

Hence, it is clear that the Einstein tensors are equal to Ricci tensors ($G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$) as given

$$G_t^t = G_r^r = \frac{M^2(a^2 - 1)}{r^4}, \tag{8}$$

$$G_\theta^\theta = G_\varphi^\varphi = \frac{-M^2(a^2 - 1)}{r^4}. \tag{9}$$

The following energy-momentum tensor according to Einstein's field equations can be easily obtained by using Einstein tensors ($G_{\mu\nu} = 8\pi T_{\mu\nu}$). Furthermore, the action is

$$S = \int d^4x\sqrt{-g}L \tag{10}$$

where $L = \frac{R}{16\pi} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$, and constants of G and c are 1. The energy-momentum tensors for the vector potential

$$A_\mu = (\pm\frac{1}{2}\frac{M\sqrt{1 - a^2}}{ar}, 0, 0, 0), \tag{11}$$

are defined as

$$T_{\mu\nu} = F_{\mu\alpha}F_\nu^\alpha - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}. \tag{12}$$

The corresponding Hawking temperature is found as

$$T_H = \left(\frac{-g'_{tt}}{4\pi\sqrt{-g_{tt}g_{rr}}}\right)_{r=r_h} = \frac{1}{4\pi}\frac{2a}{M(a + 1)^2}. \tag{13}$$

One easily observes that for $a = 1$ we recover the Schwarzschild result. For $a = 0$ which we have already excluded we obtain $T_H = 0$, which is analogous to the extremal RN geometry.

3 Entangled Scalar Particles Tunneling from SEBH with GUP

The modified commutation relation

$$[x_i, p_j] = i\hbar(1+\alpha p^2)\delta_{ij} \tag{14}$$

is used to derive GUP [31, 47–49] which is given by

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left\{1+\alpha(\Delta p)^2\right\}, \tag{15}$$

where $\alpha = \alpha_0/(m_p^2) = \alpha_0 l_p^2/\hbar^2$ is a small value, m_p is the Planck mass, l_p is the Planck length ($\sim 10^{-35}m$) and $\alpha_0 < 10^{34}$ is a dimensionless parameter.

By using the effect of quantum gravity, we define the generalized commutation relation to modify KG equation for scalar particles, so to account for the effects from quantum gravity. The position, momentum, energy and frequency operators are modified respectively as [31, 48]

$$x_i = x_{oi}, \tag{16}$$

$$p_i = p_{0i}(1+\alpha p^2), \tag{17}$$

$$\varepsilon = E(1+\alpha^2 E^2), \tag{18}$$

and

$$\bar{\omega} = E(1-\alpha E^2), \tag{19}$$

with the energy operator $E = i\hbar\partial_0$.

The square of momentum operators up to order α^2 is calculated by

$$\begin{aligned} p^2 &= -\hbar^2[1-\alpha^2\hbar^2\partial_j\partial^j]\partial_i[1-\alpha^2\hbar^2\partial_j\partial^j]\partial^i \\ &= -\hbar^2[\partial_i\partial^i - 2\alpha^2\hbar^2(\partial_j\partial^j)(\partial_k\partial^k)] + (\alpha^4). \end{aligned} \tag{20}$$

where in the last step, we only keep the leading order term of α.

Using the generalized Klein-Gordon equation for scalar field in Planck scale, the generalized K-G equation under the effect of minimum length having the wave function Ψ can be written as

$$-(i\hbar)^2\partial^t\partial_t\Psi = \left[(i\hbar)^2\partial^i\partial_i + m_p^2\right]\left[1-2\alpha\left((i\hbar)^2\partial^i\partial_i + m_p^2\right)\right]\Psi. \tag{21}$$

Herein, using the SEBH metric (1) as a background we determine the entangled scalar particle motion. An important point in this paper is the entanglement between the infalling and outgoing Hawking particles. The difficulty of this entanglement is to observe inner and outer sides of the black hole horizon. Entangled states can be described by Ψ. After substituting the chosen entangled ansatz of Ψ [53–56],

$$\Psi = \kappa e^{\frac{i}{\hbar}S_A(t,r,\theta,\varphi)} + \sqrt{1-\kappa^2}e^{\frac{i}{\hbar}S_B(t,r,\theta,\varphi)} \tag{22}$$

where S_A is for Alice and S_B is for Bob. Note that κ is some real number which satisfies $|\kappa| \in (0,1)$, so that κ and $\sqrt{1-\kappa^2}$ are normalized partners. The fate of the Alice and Bob depends on the value of κ. The discussion of Alice state is the same as that of the Bob state. Here, the entanglement can then be observed with only one observation for each member of the ensemble so that we just consider the Alice case. To illustrate the idea, we consider the Alice and Bob shares entangled state at the same point in the BR spacetime.

Whenever, Alice falls in toward a SEBH and locate near the event horizon of SEBH, as noted here particle state is unentangled when the Hawking temperature is zero and approaches a maximally entangled Bell state as known the black hole evaporates completely [57, 58]. We investigate the fate of this particle by choosing the specific case of $\kappa = 1$, which provides us to calculate by using the Alice of entangled ansatz of Ψ inside the K-G solutions for the SEBH, which is obtained by [49] as follows

$$\frac{1}{f}(\partial_t S_A)^2 = f(\partial_r S_A)^2 + \frac{1}{r^2}(\partial_\theta S_A)^2 + \frac{1}{r^2\sin^2\theta}(\partial_\varphi S_A)^2 + m_p^2 \times \left[1 - 2\alpha\left(f(\partial_r S_A)^2 + \frac{1}{r^2}(\partial_\theta S_A)^2 + \frac{1}{r^2\sin^2\theta}(\partial_\varphi S_A)^2 + m_p^2\right)\right] \quad (23)$$

After it is expanded into the lowest order of $\hbar$ to find the solution of K-G equation we use the method of separation of variables as follows:

$$S_A(t, r, \theta, \varphi) = -Et + W(r) + j(\theta, \phi) + C, \quad (24)$$

where C, E and j are the complex constant, energy and angular momentum of the scalar particles, respectively.

After substituting (24) into (23), taking only the radial part yields

$$\frac{1}{f}E^2 = f(\partial_r W)^2 + m_p^2 \times \left[1 - 2\alpha\left(f(\partial_r W)^2 + m_p^2\right)\right] \quad (25)$$

and solving for the $W(r)$, it is found that

$$W_\pm = \pm\int dr \frac{1}{f}\frac{\sqrt{E^2 - m_p^2(1 - 2\alpha m_p^2)f}}{\sqrt{1 - 2\alpha m_p^2}}. \quad (26)$$

The positive "+" signature is for outgoing entangled Alice scalar particles and the solution with negative "−" signature for the ingoing Alice scalar particles. Calculating the above integral around the pole at the horizon by expanding the metric function f about r_h ; $f(r_h) \approx f'(r_h)(r - r_h)$, where prime "′" denotes a derivative respect to r, gives

$$W_\pm = \pm\frac{i\pi E M(1 + a)^2}{2a\sqrt{1 - 2\alpha m_p^2}} \quad (27)$$

While computing the imaginary part of the action, we note that it is same for both the incoming and outgoing solutions. Herein, a factor two problem is arisen when calculating the tunneling rate [50], however this problem can be resolved by different method found by Akhmedova et al. [51, 52]. In this paper we use the most common solution that if we set the probability of ingoing particles to 100 % (i.e., $P_- \simeq e^{-2ImW_-} = 1$)

Thus, the ingoing and outgoing imaginary action solution is derived as

$$ImS_- = ImW_- + ImC = 0, \quad (28)$$

and

$$ImS_+ = ImW_+ + ImC \quad (29)$$

which result in $ImC = -ImW_-$. Contrariwise $W_+ = -W_-$ in order that the probabilities of radiating particles are obtained as

$$P_+ = e^{-2ImS} \simeq e^{-4ImW_+}. \quad (30)$$

Now, the probability of particles tunneling from inside to outside the horizon is given by

$$\Gamma = \frac{P_+}{P_-} \simeq e^{(-4ImW_+)} \tag{31}$$

$$\Gamma = e^{-\frac{4i\pi EM(1+a)^2}{2a\sqrt{1-2\alpha m_p^2}}} \tag{32}$$

and the Hawking temperature for the scalar particles with the effect of minimum length is obtained as

$$T_H = \frac{2a\sqrt{1-2\alpha m_p^2}}{4\pi EM(1+a)^2}. \tag{33}$$

Since $(0 < a \leq 1)$ it is observed that the electromagnetic field increases both the tunneling rate (32) and the Hawking temperature (33). When the minimum length effect parameter $\alpha = 0$, it is equal to the original result of Hawking Temperature. The effect of the external parameter is shown in Fig. 1.

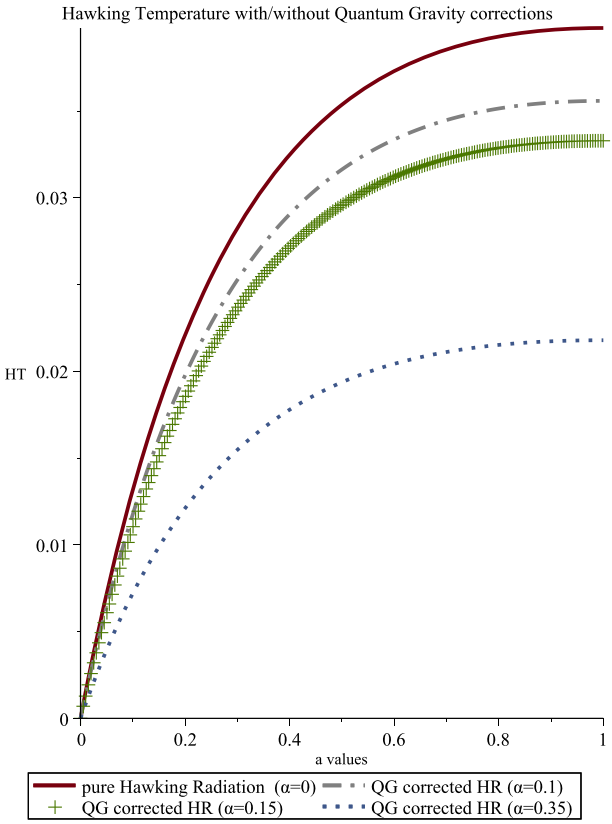

Fig. 1 The variation of Hawking Temperature against the constant a for $M = E = m_p = 1$

4 Entangled Fermion Particles Tunneling from SEBH with GUP

This section makes use of the generalized Dirac equation for fermions [40, 59–61] on the background of SEBH [45]. The generalized Dirac equation under the effect of minimum length can be written as

$$-i\gamma^t\partial_t\psi = (i\gamma^i\partial_i - \frac{1}{2}\gamma^\mu g^{\alpha\beta}\Gamma^\alpha_{\mu\nu}J_{\alpha\alpha} + \frac{m_p}{\hbar})\left[1+\alpha\left(\hbar^2\partial^i\partial_i - m_p^2\right)\right]\psi \;; i = r,\theta,\varphi \quad (34)$$

where the γ matrices are expressed in terms of the Pauli matrices σ^i as follows

$$\begin{aligned}\gamma^t &= \frac{1}{\sqrt{f}}\begin{pmatrix} i & 0 \\ 0 & -i\end{pmatrix}, \quad \gamma^r = \sqrt{g}\begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0\end{pmatrix}, \\ \gamma^\theta &= \frac{1}{r}\begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0\end{pmatrix}, \quad \gamma^\varphi = \frac{1}{r\sin\theta}\begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0\end{pmatrix},\end{aligned} \quad (35)$$

and

$$J_{\alpha\alpha} = \frac{i}{4}\left[\gamma^\alpha,\gamma^\beta\right], \{\gamma^\mu,\gamma^\nu\} = 2g^{\mu\nu}, \quad (36)$$

$$\Gamma^r_{tt} = \frac{f'f}{2}, \Gamma^t_{tr} = \frac{f'}{2f}. \quad (37)$$

Consequently, the generalized Dirac equation can be written as [61]

$$\begin{aligned} i\gamma^t\partial_t &+ i\gamma^i\partial_i\left(1-\alpha m_p^2\right) + i\gamma^i\alpha\hbar^2\left(\partial_j\partial^j\right)\partial_i + \frac{m_p}{\hbar}\left(1+\alpha\hbar^2\partial_j\partial^j - \alpha m_p^2\right) \\ &+ i\gamma^\mu\Omega_\mu\left(1+\alpha\hbar^2\partial_j\partial^j - \alpha m_p^2\right)\psi = 0. \end{aligned} \quad (38)$$

By using the ansatz for the entangled spin-up ψ and only the r direction, one obtains

$$\psi_\uparrow = \kappa\begin{pmatrix} c_0 \\ c_2\end{pmatrix}e^{\frac{i}{\hbar}S_A(t,r)} + \sqrt{1-\kappa^2}\begin{pmatrix} c_4 \\ c_6\end{pmatrix}e^{\frac{i}{\hbar}S_B(t,r,\theta,\varphi)}, \quad (39)$$

with constant spinor components c_i, $i = 0, 2, 4, 6$.

Upon choosing the specific case of $\kappa = 1$ (the Alice case) of entangled ansatz of Ψ and solve the generalized Dirac equation on the background of SEBH, taking the lowest order of $\hbar$, it is obtained that the set of equations for the form depending only on the radial part of $S_A(t,r) = Et + W(r)$ are [49]

$$\begin{aligned}\alpha^2 f^4(\partial_r W)^6 &+ \alpha f^3(3m_p^2\alpha - 2)(\partial_r W)^4 + f^2[(1-\alpha m_p^2)^2 - \alpha(2m_p^2 - 2m_p^4\alpha)](\partial_r W)^2 \\ &+ m_p^2 f(1-\alpha m_p^2)^2 - E^2 = 0\end{aligned} \quad (40)$$

To analyze the solution of $W(r)$, by neglecting the higher order terms of α, the solution for $W(r)$ is calculated as

$$W_\pm = \pm\int dr\frac{1}{f}\frac{\sqrt{E^2 - m_p^2(1-2\alpha m_p^2)f}}{\sqrt{1-2\alpha m_p^2}}. \quad (41)$$

$$W_\pm = \pm\frac{i\pi EM(1+a)^2}{2a\sqrt{1-2\alpha m_p^2}} \quad (42)$$

Now, using the same procedure in (28)–(30), the probability of the particles going out of horizon from inside is written as

$$\Gamma = \frac{P_+}{P_-} \simeq e^{(-4ImW_+)} \tag{43}$$

$$\Gamma = e^{-\frac{4i\pi EM(1+a)^2}{2a\sqrt{1-2\alpha m_p^2}}} \tag{44}$$

and the corresponding Hawking temperature is

$$T_H = \frac{2a\sqrt{1-2\alpha m_p^2}}{4\pi EM(1+a)^2}. \tag{45}$$

5 Conclusion

In summary, by using the modified Klein-Gordon and Dirac equations under the effect of quantum gravity, we have examined the entangled scalar/fermion particle's tunneling from SEBH. The generalized uncertainty principle and application on the fields are used to derive corrected Hawking radiation with the help of Hamilton-Jacobi method. Entangled particles such as Alice and Bob particles can tunnel from the black hole with an equivalent energy. Charge, mass and energy of the tunneled entangled particles are only properties. Furthermore, it is easy to conclude that during the evaporation, temperature increase is decelerated by the effect of GUP. Hence, it is understood that the two effects will be canceled at some point in the radiation and remnants are left. In addition, the external parameter of SEBH "a" can be chosen as the value of $(0 < a \leq 1)$ to change the Hawking temperature. This is how the Hawking temperature modifies when the Schwarzschild black hole is immersed in a uniform external electromagnetic field specified by the parameter "a". All our results reduce to that of Schwarzschild case whenever we set $a = 1$.

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