Int J Theor Phys (2016) 55:1882–1889
DOI 10.1007/s10773-015-2827-4

Charged Fermions Tunnel from the Kerr-Newman Black Hole Influenced by Quantum Gravity Effects

Ruyi Ren[1] · Deyou Chen[1] · Jin Pu[1]

Received: 24 July 2015 / Accepted: 25 September 2015 / Published online: 8 October 2015

Abstract Taking into account quantum gravity effects, we investigate the tunnelling radiation of charged fermions in the Kerr-Newman black hole. The result shows that the corrected Hawking temperature is determined not only by the parameters of the black hole, but also by the energy, angular momentum and mass of the emitted fermion. Meanwhile, an interesting found is that the temperature is affected by the angle θ. The quantum gravity correction slows down the evaporation.

Keywords Charged fermions · Generalized uncertainty principle · Tunnelling radiation · Remnants

1 Introduction

Hawking radiation is described as a quantum tunnelling effect near the horizons of black holes [1]. When Hawking first found it, the background spacetime of the black hole was seen as a fixed one. Thus the pure thermal spectrum was gotten. This implied that the standard Hawking radiation formula would lead to the complete evaporation of the black hole. The problem of information loss appeared. Many attempts was made to solve this problem.

The semi-classical tunnelling method is an effective method to research on the Hawking radiation . This method was first put forward by Kruas, Parikh and Wilczek [2, 3]. In this

✉ Deyou Chen
dchen@cwnu.edu.cn

Ruyi Ren
renruyinc@163.com

Jin Pu
red6011@126.com

1 Institute of Theoretical Physics, China West Normal University, Nanchong, Sichan 637009, China

method, the energy conservation and the self-gravity interaction were taken into account in the scalar particle's tunnelling radiation. The result showed that the tunnelling rate satisfied the unitary theory, which provided the possibility to solve the information loss problem. The corrected Hawking temperature was higher than the standard one, which implied that the varied spacetime accelerated the evaporation of the black holes. Subsequently, Zhang and Zhao extended this work to the tunnelling radiation of charged and massive particles [4, 5]. The same result was recovered in [6]. Using the Hamilton-Jacobi method [7], Kerner and Mann investigated the tunnelling radiation of the fermions in [8, 9]. The self-gravity interaction wasn't considered in their investigation, therefore, the standard Hawking temperature was recovered [10–12].

All of the above results show the complete evaporation. However, there is a minimal observable length in various theories of quantum gravity [13–15]. A simple and effective way to to derive this length is the generalized uncertainty principle (GUP)

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left[1+\beta \Delta p^2\right], \tag{1}$$

which is gotten by modifying the fundamental commutation relations [16], where $\beta = \beta_0 \frac{l_p^2}{\hbar^2}$, β_0 is a dimensionless constant expressed quantum gravity parameter, and l_p is the Planck length. The commutation relations were first modified as $\left[x_i, p_j\right] = i\hbar\delta_{ij}\left[1+\beta p^2\right]$ by Kempf et al, where $x_i = x_{0i}$ and $p_i = p_{0i}(1+\beta p^2)$ are position and momentum operators, respectively. x_{0i} and p_{0j} satisfy the canonical commutation relations $\left[x_{0i}, p_{0j}\right] = i\hbar\delta_{ij}$.

A black hole is the matter body. So it also owns itself minimal observable length. Incorporating GUP into the black hole physics, many interesting result were found [17–23]. In [17], the minimal scale was found and the thermodynamics was studied. In [21], the corrected tunneling rate of the scalar particle was found when GUP was introduced to modify the relation between the radial coordinate and its conjugate momentum. Recently, taking into account quantum gravity effects, the author adopted the modified Dirac equation to investigate the fermion's tunnelling radiation and the scale of the remnant was found [23]. The modified Dirac equation is given by [23]

$$-i\gamma^0\partial_0\psi = \left(i\gamma^i\partial_i + i\gamma^\mu\Omega_\mu + i\gamma^\mu\frac{i}{\hbar}eA_\mu + \frac{m}{\hbar}\right)\left(1+\beta\hbar^2\partial_j\partial^j - \beta m^2\right)\psi. \tag{2}$$

However, in their work, they only considered the special case that a fermion tunnelling from the charged spacetime (or the rotating spacetime).

In this paper, considering effects of quantum gravity, we investigate the tunnelling radiation of a charged fermion in the Kerr-Newman spacetime. The modified Dirac (2) is adopted. The corrected Hawking temperature is gotten and lower than the standard temperature. The correction is not only determined by the parameters of the black hole, but also affected by the energy, angular and mass of the fermion. The result shows that the quantum gravity correction slows down the evaporation of the black hole.

The rest is organized as follow. For the convenience of discussion, we investigate the radiation in the dragging coordinate system. Therefore, the dragging coordinate transformation is performed in the next section. Then we discuss the tunnelling radiation of the fermion in this coordinate system. Section 3 is devoted to our discussion and conclusion.

2 Effects of Quantum Gravity on the Tunneling Radiation of a Charged Fermion in the Charged and Rotating Spacetime

In this section, we discuss the tunnelling radiation of a charged particle in the Kerr-Newman spacetime. The Kerr-Newman metric is given by

$$\begin{aligned} ds^2 =& -\left(1-\frac{2Mr-Q^2}{\rho^2}\right)dt^2+\frac{\rho^2}{\Delta}dr^2-\frac{2\left(2Mr-Q^2\right)a\sin^2\theta}{\rho^2}dtd\varphi+\rho^2d\theta^2 \\ &+\left[(r^2+a^2)+\frac{\left(2Mr-Q^2\right)a^2\sin^2\theta}{\rho^2}\right]\sin^2\theta d\varphi^2, \end{aligned} \tag{3}$$

with the electromagnetic potential

$$\tilde{A_\mu}=\tilde{A_t}dt+\tilde{A_\varphi}d\varphi=\frac{Qr}{\rho^2}dt-\frac{Qra\sin^2\theta}{\rho^2}d\varphi, \tag{4}$$

where

$$\rho^2=r^2+a^2\cos^2\theta,\quad \Delta=(r-r_+)(r-r_-), \tag{5}$$

$r_+=M+\sqrt{M^2-Q^2-a^2}$ and $r_-=M-\sqrt{M^2-Q^2-a^2}$ are the locations of the outer and inner horizons, respectively. To describe the behavior of the fermion, we should first choose a tetrad and construct gamma matrices. We can use the metric (3) to get the matrices. However, for the convenience, we discuss the radiation in the dragging coordinate system. Thus, performing the dragging coordinate transformation

$$d\phi=d\varphi-\Omega dt=d\varphi-\frac{(r^2+a^2-\Delta)a}{\left(r^2+a^2\right)^2-\Delta a^2\sin^2\theta}dt, \tag{6}$$

on the metric (3), we get

$$\begin{aligned} ds^2 =& -\frac{\Delta\rho^2}{(r^2+a^2)^2-\Delta a^2\sin^2\theta}dt^2+\frac{\rho^2}{\Delta}dr^2+\rho^2d\theta^2 \\ &+\frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)^2-\Delta a^2\sin^2\theta\right]d\phi^2. \\ \equiv& -F(r)dt^2+\frac{1}{G(r)}dr^2+g_{\theta\theta}d\theta^2+g_{\phi\phi}d\phi^2. \end{aligned} \tag{7}$$

Now, the electromagnetic potential in the dragging coordinate system takes on the form

$$A_\mu=A_tdt+A_\phi d\phi=\frac{(r^2+a^2)Qr}{(r^2+a^2)^2-\Delta a^2\sin^2\theta}dt-\frac{Qra\sin^2\theta}{\rho^2}d\phi. \tag{8}$$

There are many choices to construct gamma matrices. We choose a tetrad as $e_\mu{}^a=\mathrm{diag}\left(\sqrt{\mathrm{F}},1/\sqrt{\mathrm{G}},\sqrt{\mathrm{g}^{\theta\theta}},\sqrt{\mathrm{g}^{\phi\phi}}\right)$. Then the gamma matrices are given by

$$\begin{aligned} \gamma^t &= \frac{1}{\sqrt{F(r)}}\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \quad \gamma^\theta=\sqrt{g^{\theta\theta}}\begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix}, \\ \gamma^r &= \sqrt{G(r)}\begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix}, \quad \gamma^\phi=\sqrt{g^{\phi\phi}}\begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}. \end{aligned} \tag{9}$$

To research on the tunnelling radiation, we should know the wave function of the particle. For a spin-1/2 fermion, there are two states corresponding respectively to the spin up and spin down. In this paper, we only investigate the radiation of the spin up state. The discussion of the spin down state is parallel. Following the usual procedure, we assume the wave function to be

$$\Psi = \begin{pmatrix} A \\ 0 \\ B \\ 0 \end{pmatrix} \exp\left(\frac{i}{\hbar} I\left(t, r, \theta, \phi\right)\right), \tag{10}$$

where A, B are the functions of t, r, θ and ϕ, and I is the action of the fermion and its expression needs to be solved. Inserting the wave function and the gamma matrices into the generalized Dirac equation and using the WKB approximation, we neglect the contributions from ∂A, ∂B and high orders of $\hbar$ and get four decoupled equations

$$\begin{aligned} -iA\frac{1}{\sqrt{F}}\partial_t I - B\left(1-\beta m^2\right)\sqrt{G}\partial_r I - Am\beta\left[g^{rr}\left(\partial_r I\right)^2 + g^{\theta\theta}\left(\partial_\theta I\right)^2 + g^{\phi\phi}\left(\partial_\phi I\right)^2\right] \\ +B\beta\sqrt{G}\partial_r I\left[g^{rr}\left(\partial_r I\right)^2 + g^{\theta\theta}\left(\partial_\theta I\right)^2 + g^{\phi\phi}\left(\partial_\phi I\right)^2\right] + Am\left(1-\beta m^2\right) \\ -iA\frac{eA_t}{\sqrt{F}}\left[1-\beta m^2 - \left(g^{rr}\left(\partial_r I\right)^2 + g^{\theta\theta}\left(\partial_\theta I\right)^2 + g^{\phi\phi}\left(\partial_\phi I\right)^2\right)\right] = 0, \end{aligned} \tag{11}$$

$$\begin{aligned} iB\frac{1}{\sqrt{F}}\partial_t I - A\left(1-\beta m^2\right)\sqrt{G}\partial_r I - Bm\beta\left[g^{rr}\left(\partial_r I\right)^2 + g^{\theta\theta}\left(\partial_\theta I\right)^2 + g^{\phi\phi}\left(\partial_\phi I\right)^2\right] \\ +A\beta\sqrt{G}\partial_r I\left[g^{rr}\left(\partial_r I\right)^2 + g^{\theta\theta}\left(\partial_\theta I\right)^2 + g^{\phi\phi}\left(\partial_\phi I\right)^2\right] + Bm\left(1-\beta m^2\right) \\ +iB\frac{eA_t}{\sqrt{F}}\left[1-\beta m^2 - \left(g^{rr}\left(\partial_r I\right)^2 + g^{\theta\theta}\left(\partial_\theta I\right)^2 + g^{\phi\phi}\left(\partial_\phi I\right)^2\right)\right] = 0, \end{aligned} \tag{12}$$

$$\begin{aligned} A\Big\{-\left(1-\beta m^2\right)\sqrt{g^{\theta\theta}}\partial_\theta I + \beta\sqrt{g^{\theta\theta}}\partial_\theta I\left[g^{rr}(\partial_r I)^2 + g^{\theta\theta}(\partial_\theta I)^2 + g^{\phi\phi}(\partial_\phi I)^2\right] \\ -i\left(1-\beta m^2\right)\sqrt{g^{\phi\phi}}\partial_\phi I + i\beta\sqrt{g^{\phi\phi}}\partial_\phi I\left[g^{rr}(\partial_r I)^2 + g^{\theta\theta}(\partial_\theta I)^2 + g^{\phi\phi}(\partial_\phi I)^2\right] \\ -i\sqrt{g^{\phi\phi}}eA_\phi\left[1-\beta m^2 - g^{rr}(\partial_r I)^2 - g^{\theta\theta}(\partial_\theta I)^2 - g^{\phi\phi}(\partial_\phi I)^2\right]\Big\} = 0. \end{aligned} \tag{13}$$

$$\begin{aligned} B\Big\{-\left(1-\beta m^2\right)\sqrt{g^{\theta\theta}}\partial_\theta I + \beta\sqrt{g^{\theta\theta}}\partial_\theta I\left[g^{rr}(\partial_r I)^2 + g^{\theta\theta}(\partial_\theta I)^2 + g^{\phi\phi}(\partial_\phi I)^2\right] \\ -i\left(1-\beta m^2\right)\sqrt{g^{\phi\phi}}\partial_\phi I + i\beta\sqrt{g^{\phi\phi}}\partial_\phi I\left[g^{rr}(\partial_r I)^2 + g^{\theta\theta}(\partial_\theta I)^2 + g^{\phi\phi}(\partial_\phi I)^2\right] \\ -i\sqrt{g^{\phi\phi}}eA_\phi\left[1-\beta m^2 - g^{rr}(\partial_r I)^2 - g^{\theta\theta}(\partial_\theta I)^2 - g^{\phi\phi}(\partial_\phi I)^2\right]\Big\} = 0. \end{aligned} \tag{14}$$

Equations (13) and (14) are the same equation and are reduced into an equation

$$\begin{aligned} \left(\sqrt{g^{\theta\theta}}\partial_\theta I + i\sqrt{g^{\phi\phi}}\partial_\phi I + i\sqrt{g^{\phi\phi}}eA_\phi\right) \\ \times\left[\beta g^{rr}(\partial_r I)^2 + \beta g^{\theta\theta}(\partial_\theta I)^2 + \beta g^{\phi\phi}(\partial_\phi I)^2 + \beta m^2 - 1\right] = 0, \end{aligned} \tag{15}$$

which means

$$\sqrt{g^{\theta\theta}}\partial_\theta I + i\sqrt{g^{\phi\phi}}\partial_\phi I + i\sqrt{g^{\phi\phi}}eA_\phi = 0. \tag{16}$$

Obviously, the above equation is different from that gotten in [22, 23]. In their discussion, the contribution of the angular part of the action to the tunnelling rate is zero. However, this case does not exist here. In [23], they discussed the emission of the uncharged particle and found $\sqrt{g^{\theta\theta}}\partial_\theta I + i\sqrt{g^{\phi\phi}}\partial_\phi I = 0$. This case also existed in [23] even though the particle is charged in the charged spacetime. This phenomenon appeared in (16) is caused by the interaction of the charged fermion and the charged and rotating spacetime. It is very difficult to solve (11) and (12) by using (16). Therefore, considering a toy model, the emitted particle is assumed to be uncharged. To solve (11) and (12), we carry out separation of variables as

$$I = -(\omega - j\Omega) + W(r) + \Theta(\theta) + j\phi, \tag{17}$$

where ω and j is the energy and angular momentum of the emitted fermion, respectively. Inserting (17) into (11) and (12) and canceling A and B, we get

$$A_6(\partial_r W)^6 + A_4(\partial_r W)^4 + A_2(\partial_r W)^2 + A_0 = 0, \tag{18}$$

where

$$\begin{aligned} A_6 &= \beta^2 G^3 F, \\ A_4 &= \beta G^2 F\left(m^2\beta - 2\right), \\ A_2 &= GF\left[\left(1-\beta m^2\right)^2 + 2\beta m^2\left(1 - m^2\beta\right)\right], \\ A_0 &= -m^2\left(1-\beta m^2\right)^2 F - (\omega - j\Omega)^2 . \end{aligned} \tag{19}$$

The expression of F and G is given by (7). Neglecting the higher order of β and solving the above equation at the outer horizon yield the solution. The tunnelling rate is related to the imaginary part. Therefore, we choose the imaginary part

$$\begin{aligned} ImW_\pm &= \pm Im\int dr\sqrt{\frac{(\omega-j\Omega)^2+m^2F}{FG}}\left[1+\beta\left(m^2+\frac{(\omega-j\Omega)^2}{F}\right)\right] \\ &= \pm\pi\left(\omega - j\Omega_+\right)\frac{r_+^2+a^2}{r_+-r_-}\left(1+\beta\Xi\right), \end{aligned} \tag{20}$$

where $+(-)$ are the solutions of the outgoing (ingoing) waves, and $\Omega_+ = \frac{a}{r_+^2+a^2}$ is the angular velocity at the outer horizon. The expression of Ξ is very complex. So we write it as $\Xi = \Xi(M, Q, J, a, \theta, \omega, j, m)$. Using the expressions of r_+ and r_- and handling it with the special way, we can prove $\Xi > 0$. In Ref. [24, 25], the authors calculated the tunnelling rate from the invariance under canonical transformations. The tunnelling rate is expressed as

$$\begin{aligned} \Gamma &\propto exp[-Im\oint p_r dr] = exp\left[-Im\left(\int p_r^{out}dr - \int p_r^{in}dr\right)\right] \\ &= exp\left[\mp 2Im\int p_r^{out,in}dr\right], \end{aligned} \tag{21}$$

where $p_r^{out} = -p_r^{in} = \partial_r W_+$ in the above equation. Using (20), we get the tunnelling rate as follow

$$\Gamma \propto exp[-2\pi\,(\omega - j\Omega_+)\,\frac{r_+^2 + a^2}{r_+ - r_-}\,(1 + \beta\Xi)], \tag{22}$$

Clearly, the above tunnelling rate is not right. This problem was found and resolved by Akhmedova et al [26–28]. They found that the contribution of the spatial part of the action was only taken into account, while that coming from the temporal part was ignored. Using the method in Ref. ([26, 27]), we will find the temporal contribution in the following discussion. We first perform the coordinate transformation and solve the problem in the Kruskal coordinates (T, R). In this coordinates system, the spacetime is separated into two regions. The exterior region is defined by $r > r_+$ and described by

$$T = e^{\kappa_+ r_*} sinh(\kappa_+ t), \quad R = e^{\kappa_+ r_*} cosh(\kappa_+ t). \tag{23}$$

In the above equations, we introduce the tortoise coordinate $r_* = r + \frac{1}{2\kappa_+} ln\frac{r - r_+}{r_+} - \frac{1}{2\kappa_-} ln\frac{r - r_-}{r_-}$. $\kappa_\pm = \frac{r_+ - r_-}{2(r_\pm^2 + a^2)}$ are the surface gravities at the outer/inner horizons. The interior region is described by

$$T = e^{\kappa_+ r_*} cosh(\kappa_+ t), \quad R = e^{\kappa_+ r_*} sinh(\kappa_+ t). \tag{24}$$

These two regions are connected by the horizon. We rotate the time $t \to t - i\frac{\pi\kappa_+}{2}$ to connect them. This operation would produce the imaginary part of the temporal contribution [26]. Choosing the imaginary part, we get $Im(E\Delta t^{out,in}) = \frac{1}{2}\pi E\kappa_+$ with $E = \omega - j\Omega_+$. Therefore, the total contribution of the temporal part is $Im(E\Delta t) = \pi E\kappa_+$. Then the tunnelling rate contains the temporal contribution and the spatial contribution, which is

$$\begin{aligned}\Gamma &\propto exp\left(-Im(E\Delta t) - Im\oint p_r dr\right)\\ &= -4\pi\,\frac{(\omega - j\Omega_+)(r_+^2 + a^2)}{(r_+ - r_-)}\left(1 + \frac{1}{2}\beta\Xi\right).\end{aligned} \tag{25}$$

Thus the Hawking temperature is

$$T = \frac{(r_+ - r_-)}{4\pi(r_+^2 + a^2)\left(1 + \frac{1}{2}\beta\Xi\right)} = T_0\left(1 - \frac{1}{2}\beta\Xi\right), \tag{26}$$

where $T_0 = \frac{(r_+ - r_-)}{4\pi(r_+^2 + a^2)}$ is the original Hawking temperature. Our result shows that when the quantum gravity effects are taken into account, the corrected temperature is lower than the original one. The correction is not only determined by the parameter of the black hole, but also affected by the energy, angular momentum and mass of the emitted fermion. An interesting result is that the angular θ affects the temperature. The quantum gravity correction slows down the increase of the Hawking temperature. Finally, there is a balance state appeared. At this state, the black hole's mass is the remnant.

3 Discussion and Conclusion

In the above section, we have discussed the tunneling radiation of the fermion. The corrected Hawking temperature was gotten. Due to the effects of electromagnetic field and dragging system, the calculation is very complex. Therefore, we only discuss the un-charged fermion. This complexity can also be seen in the tunneling radiation of a scalar particle with quantum gravity effects.

The motion of a scalar particle obeys the Klein-Gordon equation, namely, $(P^{\mu}P_{\mu} + m^2)\Phi = 0$. When quantum gravity effects were considered, the modified Klein-Gordon equation was derived in [29]. Here, we take into account the electromagnetic field and quantum gravity effects. Thus the Klein-Gordon equation is modified as

$$-g^{tt}\left(i\hbar\partial_t - qA_t\right)^2\Psi = \left[g^{ii}\left(i\hbar\partial_i - qA_i\right)^2 + m^2\right] \times \left[1 - 2\beta\left((i\hbar)^2\partial^i\partial_i + m^2\right)\right]\Psi. \tag{27}$$

Similarly, we assume the expression of the wave function to be $\Phi = \exp\left[\frac{i}{\hbar}I(t, r, \theta, \phi)\right]$. Inserting the function and the inverse metrics of (7) into the modified equation yields a very complex expression, which is

$$-g^{tt}\left[(\partial_t I)^2 + 2qA_t\partial_t I + q^2A_t^2\right] = \left\{1 - 2\beta\left[g^{rr}(\partial_r I)^2 + g^{\theta\theta}(\partial_\theta I)^2 + g^{\phi\phi}\left(\partial_\phi I\right)^2 + m^2\right]\right\} \times \left\{g^{rr}(\partial_r I)^2 + g^{\theta\theta}(\partial_\theta I)^2 + g^{\phi\phi}\left[\left(\partial_\phi I\right)^2 + 2qA_\phi\partial_\phi I + q^2A_\phi^2\right] + m^2\right\}. \tag{28}$$

Here we can not simply let the contribution of the angular part of the action be zero. Even if it is zero, it is also difficult to solve above equation. This phenomenon shows that the quantum gravity effects on the charged particle tunnels from the charged and rotating spacetime is very complex. This is the reason that we only discussed the tunnelling behavior of the uncharged particle in the Kerr-Newman spacetime. When the particle's charge is zero, we can neglect the contribution of the angular part and carry out the separation of variables to solve the above equation. Here we do not calculate it and review the tunneling radiation. In recent research [30–32], the authors adopted other ways to discuss the tunnelling radiation. The interesting results were gotten.

Acknowledgments This work is supported by the National Natural Science Foundation of China (Grant No. 11205125), by Sichuan Province Science Foundation for Youths (Grant No. 2014JQ0040) and by the Innovative Research Team in China West Normal University (Grant No. 438061).

References

1. Hawking, S.W.: Particle creation by black holes. Math. Phys. **43**, 199 (1975)
2. Kraus, P., Wilczek, F.: Self-interaction correction to black hole radiance. Nucl. Phys. B **433**, 403 (1995)
3. Parikh, M.K., Wilczek, F.: Hawking radiation as tunneling. Phys. Rev. Lett. **85**, 5042 (2000)
4. Zhang, J.Y., Zhao, Z.: Hawking radiation of charged particles via tunneling from the Reissner-Nordstrom black hole. JHEP **10**, 055 (2005)
5. Zhang, J.Y., Zhao, Z.: Charged particles' tunnelling from the Kerr-Newman black hole. Phys. Lett. B **638**, 110 (2006)
6. Jiang, Q.Q., Wu, S.Q., Cai, X.: Hawking radiation as tunneling from the Kerr and Kerr-Newman black holes. Phys. Rev. D **73**, 064–003 (2006). Erratum-ibid. **D 73** (2006) 069902
7. Angheben, M., Nadalini, M., Vanzo, L., Zerbini, S.: Hawking radiation as tunneling for extremal and rotating black holes. JHEP **05**, 014 (2005)
8. Kerner, R., Mann, R.B.: Fermions tunnelling from black holes. Class. Quant. Grav. **25**, 095014 (2008)
9. Kerner, R., Mann, R.B.: Charged fermions tunnelling from Kerr-Newman black holes. Phys. Lett. B **665**, 277 (2008)
10. Li, R., Ren, J.R.: Dirac particles tunneling from BTZ black hole. Phys. Lett. B **661**, 370 (2008)
11. Chen, D.Y., Jiang, Q.Q., Zu, X.T.: Fermions tunnelling from the charged dilatonic black holes. Class. Quantum Grav. **25**, 205022 (2008)

12. Di Criscienzo, R., Vanzo, L.: Fermion tunneling from dynamical horizons. Eur. Phys. Lett. **82**, 60001 (2008)
13. Konishi, K., Paffuti, G., Provero, P.: Minimum physical length and the generalized uncertainty principle in string theory. Phys. Lett. B **234**, 276 (1990)
14. Amati, D., Ciafaloni, M., Veneziano, G.: Can spacetime be probed below the string size? Phys. Lett. B **216**, 41 (1989)
15. Townsend, P.K.: Small-scale structure of spacetime as the origin of the gravitational constant. Phys. Rev. D **15**, 2795 (1977)
16. Kempf, A., Mangan, G., Mann, R.B.: Hilbert space representation of the minimal length uncertainty relation. Phys. Rev. D **52**, 1108 (1995)
17. Adler, R.J., Chen, P., Santiago, D.I.: The generalized uncertainty principle and black hole remnants. Gen. Rel. Grav. **33**, 2101 (2001). arXiv:gr-qc/0106080
18. Kim, Y.M., Park, Y.J.: Entropy of the Schwarzschild black hole to all orders in the Planck length. Phys. Lett. B **655**, 172 (2007). arXiv:0707.2128[qr-qc]
19. Bina, A., Jalalzadeh, S., Moslehi, A.: Quantum black hole in the generalized uncertainty principle framework. Phys. Rev. D **81**, 023–528 (2010). arXiv:1001.0861[[qr-qc]]
20. Xiang, L., Wen, X.Q.: Black hole thermodynamics with generalized uncertainty principle. JHEP **0910**, 046 (2009)
21. Nozari, K., Saghafi, S.: Natural cutoffs and quantum tunneling from black hole horizon. JHEP **1211**, 005 (2012)
22. Chen, D.Y., Jiang, Q.Q., Wang, P., Yang, H.: Remnants, fermions' tunnelling and effects of quantum gravity. JHEP **1311**, 176 (2013)
23. Chen, D.Y., Wu, W.H., Yang, H.: Observing remnants by fermions' tunneling. JCAP **03**, 036 (2014)
24. Akhmedov, E.T., Akhmedova, V., Pilling, T., Singleton, D.: Thermal radiation of various gravitational backgrounds. Int. J. Mod. Phys. A **22**, 1705 (2007)
25. Akhmedov, E.T., Akhmedova, V., Singleton, D.: Hawking temperature in the tunneling picture. Phys. Lett. B **642**, 124 (2006)
26. Akhmedova, V., Pilling, T., de Gill, A., Singleton, D.: Temporal contribution to gravitational WKB-like calculations. Phys. Lett. B **666**, 269 (2008)
27. Akhmedov, E.T., Pilling, T., Singleton, D.: Subtleties in the quasi-classical calculation of Hawking radiation. Int. J. Mod. Phys. D **17**, 2453 (2008)
28. Chowdhury, B.D.: Problems with tunneling of thin shells from black holes. Pramana **70**, 593 (2008)
29. Chen, D.Y., Wu, H.W., Yang, H., Yang, S.Z.: Effects of quantum gravity on black holes. Int. J. Mod. Phys. A **29**, 1430054 (2014)
30. Wang, P., Yang, H., Ying, S.X.: Black hole radiation with modified dispersion relation in tunneling paradigm: free-fall frame. arXiv:1505.04568[gr-qc]
31. Wang, P., Yang, H.: Black hole radiation with modified dispersion relation in tunneling paradigm: static frame. arXiv:1505.03045[gr-qc]
32. Mu, B.R., Wang, P., Yang, H.: Minimal length effects on tunnelling from wpherically wymmetric black holes. Adv. High Energy Phys. **2015**, 898–916 (2015)