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Influences of Initial States on Entanglement Dynamics of Two Central Spins in a Spin Environment

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Abstract We investigate the entanglement dynamics of two electronic spins coupled to a bath of nuclear spins for two special cases, one is that two central spins both interact with a common bath, and the other is that one of two spins interacts with a bath. We consider three types of initial states with different correlations between the system and the bath, i.e., quantum correlation, classical correlation, and no-correlation. We show that the initial correlations (no matter quantum correlations or classical correlations) can effectively avoid the occurrence of entanglement sudden death. Irrespective of whether both two spins or only one of the two spins interacts with the bath, the system can gain more entanglement in the process of the time evolution for initial quantum correlations. In addition, we find that the effects of the distribution of coupling constants on entanglement dynamics crucially depend on the initial state of the spin bath.

Keywords Central two spins · Initial correlations · Concurrence

1 Introduction

Entanglement is viewed as a basic resource for quantum information processing (QIP) [1], quantum computation [2] and quantum communication [3]. It is worth mentioning that over the last few years, many proposals have been proposed for the implementation of quantum information processing [4, 5]. However, a quantum system used in quantum information processing inevitably interacts with the surrounding environment, which turns the quantum

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world into the classical world [6–8]. This is referred to as the decoherence process, which is the main obstacle to quantum information processing [9, 10]. The environment destroys quantum interferences of the system, and the unwanted influences of environment reduce the advantages of the quantum computing methods. As we know that the central spin model has been widely used for experimental and theoretical studies in the areas of quantum information processing and quantum decoherence. In this regard, the decoherence behavior and the entanglement dynamics of the central spin model have been extensively studied in recent years [11–16].

The exact entanglement dynamics of two coupled spins in a quantum Heisenberg XY spin star environment in the thermodynamic limit at arbitrarily finite temperatures is investigated in Ref. [17]. It was shown that the entanglement dynamics depends on the initial state of the system, the coupling strength between the two central spins, the temperature of the environment, and the interaction between the constituents of the spin environment. The dynamics of two spins coupled to separate spin star environments via Heisenberg XY interactions have been investigated [18]. The analysis of the concurrence and the purity have shown that decoherence can be minimized by allowing the central spins to strongly interact with each other. The decay of entanglement in a high temperature nuclear spin bath at large external magnetic field was discussed in Ref. [19]. In Refs. [20–22], the magnetic field was assumed to be zero, and the focus was on two coupled spins with exchange interaction. In recent works [23, 24], more general quantum correlations were considered for two uncoupled spins freely evolving at rather small magnetic field, and interacting with thermal nuclear baths. Apart from that, the entanglement dynamics of two electron spins in two quantum dots, in which each electron is interacting with its nuclear spin environment was studied in Ref. [25]. For low and moderately-low magnetic fields, it provides the analytical calculations of the entanglement dynamics of two uncoupled spins initialized in a Bell-diagonal state.

Most of previous studies concentrate on the entanglement dynamics with system and environment being separable initially. However, it is often unavoidable to have some correlations between system and environment which may play significant roles in the time evolution of the system. Therefore, one should not neglect the initial system-environment correlations [26–28]. In recent years, the influences of initial correlations on open-system dynamics have been intensively studied. The effects of initial correlations on the dynamics of system have been observed experimentally by means of trace distance [29]. The study [30] has demonstrated that the initial correlations have nontrivial features in the quantum tomography process. The effect of the initial correlations on the entanglement between two spins in a very simple model of two spins interacting with a family baths with homogenous couplings has also been discussed [31].

In this paper, we mainly study the effects of initial system-bath correlations and the role of the initial bath states on the entanglement dynamics of two central spins coupled to a bath of nuclear spins. Two special cases were considered: one is that two central spins both interact with a common bath, and the other is that one of two spins interacts with a bath. We discuss the time evolution of the system associated with three different initial system-bath states, i.e., quantum correlation, classical correlation, and no-correlation. It is shown that the initial correlations have remarkable influences on the entanglement dynamics of two central spins. The initial correlations between the system and the bath can effectively avoid the concurrence of the entanglement sudden death (ESD). And the system can gain more entanglement in the process of the time evolution for initial quantum correlations. For initial product states between the system and bath, we find that the decoherence of the system can be suppressed greatly when the initial bath state is the maximally entangled state. Nevertheless, the decay of the concurrence occurs when the initial bath state is the completely mixed state, especially for the low polarization of the bath. In addition, we find that the influences of the distribution of hyperfine interaction on the entanglement dynamics of the system crucially depend on the initial state of the spin bath.

The paper is organized as follows: In Section 2 we introduce the Hamiltonian of the model. In Section 3 we study the entanglement dynamics of system for three different initial correlations between the system and the bath. In Section 4 the entanglement dynamics with two different initial bath states are investigated. Finally, we summarize our findings in Section 5.

2 Model and Formalism

We consider two coupled electron spins interacting with a common spin bath composed of N noninteracting spin -$\frac{1}{2}$ particles, and the Hamiltonian can be written as

$$\hat{H} = \mathbf{S}_1 \cdot \sum_k^N A_k^1 \mathbf{I}_k + \mathbf{S}_2 \cdot \sum_k^N A_k^2 \mathbf{I}_k + J_{ex} \mathbf{S}_1 \cdot \mathbf{S}_2, \tag{1}$$

where J_{ex} denotes the exchange coupling between the two electron spins $\mathbf{S}_1$, $\mathbf{S}_2$, and $\mathbf{I}_k$ is the nuclear spins of the kth nucleus with $\mathbf{S}_j = (\sigma_x, \sigma_y, \sigma_z)$ $(j = 1, 2)$, $\mathbf{I}_k = (\sigma_x^k, \sigma_y^k, \sigma_z^k)$. A_k^1 and A_k^2 are the coupling parameters for hyperfine interaction of electron spins with the surrounding nuclear spins of the kth nucleus. Generally, A_k^1 and A_k^2 are proportional to the square modulus of the respective electronic wave function at the site of the nuclear spins and is therefore spatially dependent,

$$A_k^j = A_k \upsilon |\Psi^j(\mathbf{r}_k)|^2, \tag{2}$$

where υ is the volume of the unit cell containing one nuclear spin and $\Psi^j(\mathbf{r}_k)$ is the electronic wave function of electron $j = 1, 2$ at the site of kth nucleus. The quantity A_k denotes the hyperfine coupling strength which depends on the respective nuclear species through the nuclear gyromagnetic ratio.

In this paper, we discuss the effects of different degree of inhomogeneity by numerical simulation. We assume a Gaussian distribution with the site index k [32, 33],

$$A_k^j = \frac{x_1 N e^{-(\frac{kB}{N})^2}}{\sum_{k=1}^N e^{-(\frac{kB}{N})^2}}, \tag{3}$$

which allows an easy control over the two relevant characteristics of the distribution of A_k^j, namely, the mean value $x_1 = \frac{\sum_k A_k}{N}$ and the degree of inhomogeneity B. We choose $B = 2$ as a generic value for inhomogeneous couplings, $B = 0.8$ as a value for nearly homogeneous couplings, and $B = 0$ for homogeneous couplings. It is noted that the central spin model with inhomogeneous couplings has been studied in Ref. [32], focusing on the spectral properties and static correlation functions in the ground state and excited states. Here we are interested in the influences of the initial system-environment correlations and different bath states on the entanglement dynamics of the system.

We use the Wootters concurrence [34], ranging from 0 for separable states to 1 for maximally entangled states, to quantify the amount of entanglement encoded in the two central spins. It is defined as [34]

$$C = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\}, \tag{4}$$

where λ_i are the square roots of the eigenvalues of the product matrix $\rho_s(\sigma^y \otimes \sigma^y)\rho_s^*(\sigma^y \otimes \sigma^y)$ in decreasing order and ρ_s^* is the complex conjugation of ρ_s.

3 The Effects of Initial States with System-Bath Correlation

In this section, in order to investigate the effects of different initial system-bath correlations on the entanglement dynamics of two central spins, we consider three types of initial states,

$$\rho_{SE}^1 = (\alpha \left|eg\widetilde{G}\right\rangle + \beta \left|ge\widetilde{G}\right\rangle + \gamma \left|gg\widetilde{W}\right\rangle) \\ (\alpha^* \left\langle eg\widetilde{G}\right| + \beta^* \left\langle ge\widetilde{G}\right| + \gamma^* \left\langle gg\widetilde{W}\right|), \tag{5}$$

$$\rho_{SE}^2 = (\alpha \left|eg\right\rangle + \beta \left|ge\right\rangle)(\alpha^* \left\langle eg\right| + \beta^* \left\langle ge\right|) \otimes \left|\widetilde{G}\right\rangle\left\langle\widetilde{G}\right| + \\ |\gamma|^2 \left|gg\right\rangle \left\langle gg\right| \otimes \left|\widetilde{W}\right\rangle\left\langle\widetilde{W}\right|, \tag{6}$$

$$\rho_{SE}^3 = \rho_S \otimes \rho_E. \tag{7}$$

We denote the ground state of nuclear spins as $|\widetilde{G}\rangle = |00...0\rangle$ representing all spins in the down state and the W state $|\widetilde{W}\rangle = (C_{M_b}^N)^{-\frac{1}{2}}(|100...0\rangle + |010..0\rangle + ..|000..1\rangle)$ for $M_b = 1$ in the bath. M_b is the number of flipped spins in the bath, and $C_{M_b}^N = \frac{N!}{M_b!(N-M_b)!}$. And here $|\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1$. For simplicity, we choose the parameters $\alpha = \sin\theta\sin\varphi$, $\beta = \sin\theta\cos\varphi$ and $\gamma = \cos\theta$ ($\theta \in [0, \pi]$ and $\varphi \in [0, 2\pi]$). $|e\rangle$ and $|g\rangle$ are the excited and ground states of two central spins. ρ_S and ρ_E in (7) are given by

$$\rho_S = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & |\alpha|^2 & \alpha\beta^* & 0 \\ 0 & \alpha^*\beta & |\beta|^2 & 0 \\ 0 & 0 & 0 & |\gamma|^2 \end{pmatrix}, \tag{8}$$

$$\rho_E = \begin{pmatrix} |\gamma|^2 & 0 \\ 0 & |\alpha|^2 + |\beta|^2 \end{pmatrix}. \tag{9}$$

The above states ρ_{SE}^1, ρ_{SE}^2, and ρ_{SE}^3 represent three different initial states, quantum correlations, classical correlations, and no any correlations respectively, and they have the same reduced density matrices for both the system and the bath, i.e., (8) and (9).

For these choices of initial states, the density matrix $\rho_{SE}(t)$ can be obtained and therefore the reduced density matrix $\rho_S(t) = Tr_E[\rho_{SE}(t)]$ can be calculated numerically, where the bath degree of freedoms are traced out. In the following, we consider two special cases: one is that two central spins both interact with a common bath, and the other is that one of two spins interacts with a bath.

3.1 Both Two Central Spins Interact with the Bath

We consider the first case in this section, i.e., two central spins both interact with a common bath. In Fig. 1, we plot the concurrence of the two central spins for initial states ρ_{SE}^1, ρ_{SE}^2 and ρ_{SE}^3 for $A_k^1 = A_k^2 \neq 0$ and $B = 2$, (a) $\theta = \frac{\pi}{6}$ and $\gamma = \frac{\pi}{3}$, (b) $\theta = \frac{9\pi}{10}$ and $\gamma = \frac{\pi}{6}$, (c) $\theta = \frac{2\pi}{3}$ and $\gamma = \frac{\pi}{3}$. From Fig. 1a, we can see that the concurrence with initial states ρ_{SE}^1 and ρ_{SE}^2 displays periodic oscillatory behaviors. The amplitude of the oscillation for the initial state ρ_{SE}^1 is larger than that of ρ_{SE}^2, while the average value of the concurrence for ρ_{SE}^2 is larger than that of ρ_{SE}^1. For ρ_{SE}^3, the entanglement of two central spins shows the phenomenon of ESD (see Figs. 1a and b) and the quantum beats (see Fig. 1c). The initial

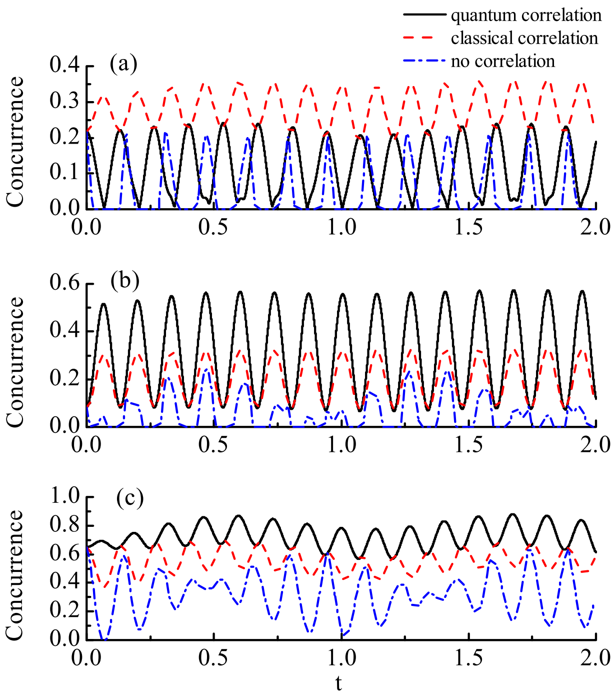

Fig. 1 (*Color online*) The concurrence of the two central spins for three types of initial correlations. **a** $\theta = \frac{\pi}{6}$ and $\gamma = \frac{\pi}{3}$, **b** $\theta = \frac{9\pi}{10}$ and $\gamma = \frac{\pi}{6}$, **c** $\theta = \frac{2\pi}{3}$ and $\gamma = \frac{\pi}{3}$. (i) ρ^1_{SE} (*black solid line*), (ii) ρ^2_{SE} (*red dash line*), (iii) ρ^3_{SE} (*blue dash dot line*). The number of nuclear spins in the bath $N = 20$, $J_{ex} = 1$, $x_1 = 1$ and $B = 2$

value of the concurrence is the maximum in the process of evolution for the initial states ρ^1_{SE} and ρ^3_{SE} shown in Fig. 1a. From Fig. 1b, we can see that for ρ^1_{SE} and ρ^2_{SE} the concurrence present the similar behavior, and the initial value of the concurrence is the minimal in the process of time evolution. However, the average value of the concurrence for ρ^1_{SE} is larger than that of ρ^2_{SE}. From Fig. 1c, we can see that the phenomenon of ESD has disappeared for ρ^3_{SE}. Apart from that, we find that for the initial state ρ^3_{SE} the average value of the concurrence is always minimal comparing the cases of initial states ρ^1_{SE} and ρ^2_{SE} in Fig. 1.

From our numerical simulations, we find that the difference of the concurrence between the initial quantum correlations and the classical correlations strongly depends on the value of θ. From (5) we can see that when $\theta \in [\frac{\pi}{2}, \pi]$ there is a relative phase π between the basis vectors of bath $|\widetilde{G}\rangle$ and $|\widetilde{W}\rangle$. For the initial quantum correlations, the average values of the concurrence is larger than classical correlations (see Figs. 1b and c). However, for $\theta \in [0, \frac{\pi}{2}]$, there is no relative phase, and in this case the the average values of the concurrence for initial classical correlations is larger than that of quantum correlations (see Fig. 1a).

In order to further explain the above phenomenon, as an example we take $\varphi = \frac{\pi}{4}$, $N = 1$, $J_{ex} = 1$, and $A^1_1 = A^2_1 = 1$. We obtain analytically the concurrence for the initial quantum correlations and the classical correlations,

$$C_1 = \frac{1}{18}[9 - (1 + 8\cos 6t)\cos 2\theta], \tag{10}$$

$$C_2 = C_1 + \frac{2\sqrt{2}}{9} \sin^2 3t \sin 2\theta, \tag{11}$$

where C_λ ($\lambda = 1, 2$, corresponding to the initial states ρ_{SE}^1 and ρ_{SE}^2 respectively) indicate the concurrence of two central spins. From (10) and (11), we can see that the relative value between C_1 and C_2 depends on the term $\sin 2\theta$, i.e., for $\theta \in [0, \frac{\pi}{2}]$, the average value of the concurrence for initial classical correlations is larger than that of quantum correlations, and for $\theta \in [\frac{\pi}{2}, \pi]$, the average value of the concurrence for initial quantum correlations is larger than that of classical correlations.

In Fig. 2, we plot the time evolution of the concurrence for the same initial states as shown in Fig. 1, and we have assumed $B = 0$ which indicates that $A_k = 1\ \forall\ k$. It can be seen that the time evolution of the concurrence also displays the periodic oscillations for three initial states, and the phenomenon of the quantum beats for ρ_{SE}^3 (see Fig. 2c). Comparing Figs. 1 and 2, we find that the influences of the three different initial correlations on the concurrence are similar for inhomogenous couplings ($B = 2$) and the homogeneous couplings ($B = 0$). In one word, our numerical simulations show that the concurrence of the system in the cases of the inhomogenous and homogenous couplings are similar for these three initial states. It is noted that for homogeneous couplings the entanglement dynamics in a system of two spins has been investigated analytically while the system and the environment being initially statistically independent in Refs. [20, 22]. It was shown that the exchange interaction between two coupled spins and the polarization of the bath had the significant influences on the entanglement dynamics and the decoherence time scale of the

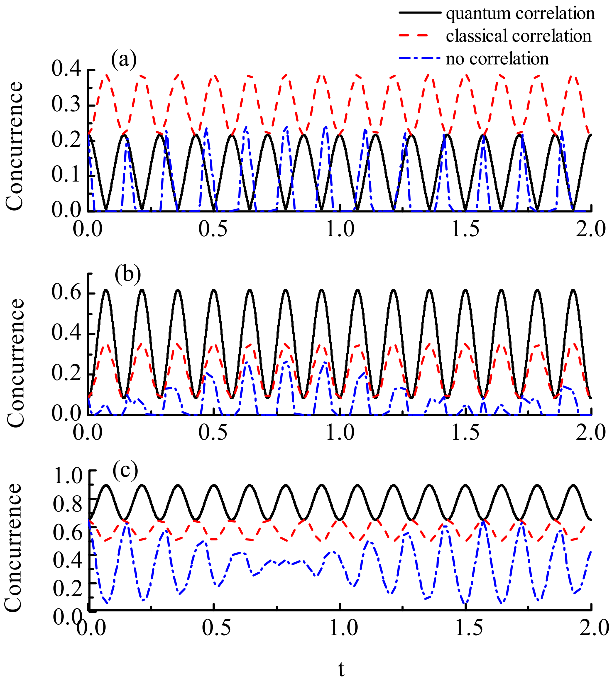

Fig. 2 (*Color online*) The concurrence of the two central spins for three types of initial correlations. **a** $\theta = \frac{\pi}{6}$ and $\gamma = \frac{\pi}{3}$, **b** $\theta = \frac{9\pi}{10}$ and $\gamma = \frac{\pi}{6}$, **c** $\theta = \frac{2\pi}{3}$ and $\gamma = \frac{\pi}{3}$. (i) ρ_{SE}^1 (*black solid line*), (ii) ρ_{SE}^2 (*red dash line*), (iii) ρ_{SE}^3 (*blue dash dot line*). The number of nuclear spins in the bath $N = 20$, $J_{ex} = 1$, $x_1 = 1$ and $B = 2$

system. In this paper, we study the influences of the initial system-environment correlations on the entanglement dynamics of the system.

Let us now investigate the effect of the exchange coupling J_{ex} on the entanglement dynamics of the system. In Fig. 3 the entanglement dynamics for initial quantum correlation and the different values of J_{ex} is plotted, (a) $\theta = \frac{\pi}{6}$ and $\gamma = \frac{\pi}{3}$, (b) $\theta = \frac{5\pi}{12}$ and $\gamma = \frac{\pi}{10}$. We can see that the different exchange coupling has almost no influences on the entanglement dynamics of the system shown in Fig. 3a. The reason is that, from (5-7) the system states are the eigenstates of $\mathbf{S}_1 \cdot \mathbf{S}_2$ with $|\alpha| = |\beta|$ for ρ^1_{SE}, ρ^2_{SE} and ρ^3_{SE}. The initial system states are close to the eigenstates of $\mathbf{S}_1 \cdot \mathbf{S}_2$ in Fig.3a. However, the exchange coupling has obvious influences on the entanglement dynamics of the system in Fig. 3b, and the concurrence can reach the maximal value rapidly in initial short time for larger J_{ex}. So that the influences of the exchange coupling J_{ex} on the entanglement dynamics depend on the initial states of the system.

3.2 Only One of the Two Spins Interacts with the Bath

In this subsection, we consider $A^1_k \neq 0$, $A^2_k = 0$ which indicates that only one of two central spins interacts with the bath. Fig. 4 shows the concurrence dynamics of the system for initial states ρ^1_{SE}, ρ^2_{SE} and ρ^3_{SE}, with the same parameters as given in Fig. 1. We can see that the concurrence of the system also displays periodic oscillatory behaviors. However, the phenomenon of the quantum beats has disappeared (see Fig. 4c). Our numerical simulations show that the difference of the average values of the concurrence between the initial states ρ^1_{SE} and ρ^2_{SE} also depends on the parameter θ which can be seen in Fig. 4. Compared with the initial quantum correlations and the classical correlations, the average values of the concurrence for no-correlation are always minimal. Apart from that, we also find that the initial system-bath correlations would avoid the occurrence of the ESD.

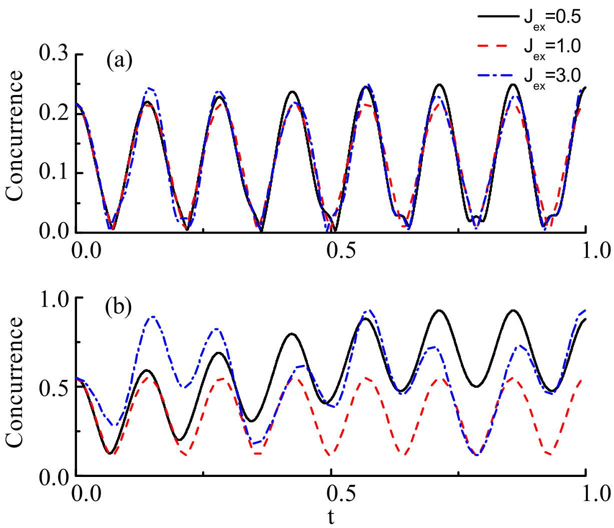

Fig. 3 (*Color online*) The concurrence of the two central spins for initial quantum correlation. **a** $\theta = \frac{\pi}{6}$ and $\gamma = \frac{\pi}{3}$, **b** $\theta = \frac{5\pi}{12}$ and $\gamma = \frac{\pi}{10}$. (i) $J_{ex} = 0.5$ (*black solid line*), (ii) $J_{ex} = 1$ (*red dash line*), (iii) $J_{ex} = 3$ (*blue dash dot line*). The number of nuclear spins in the bath $N = 20$, $J_{ex} = 1$, $x_1 = 1$ and $B = 2$

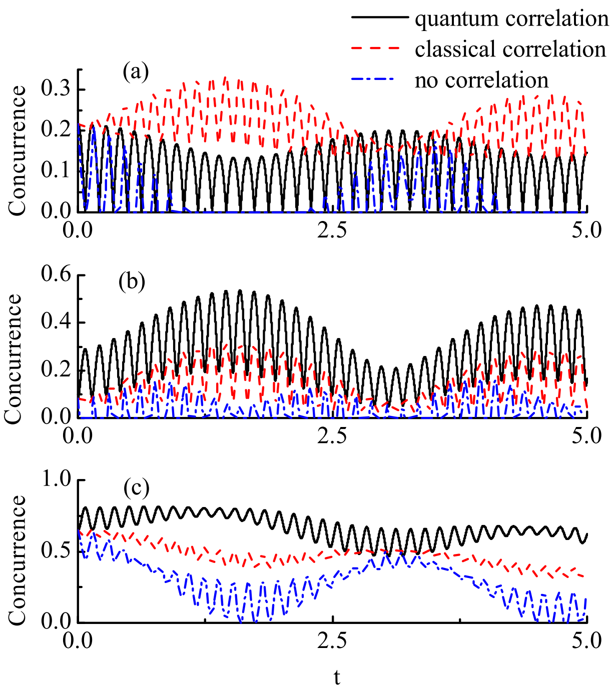

Fig. 4 (*Color online*) The concurrence of the two central spins for three types of initial correlations. **a** $\theta = \frac{\pi}{6}$ and $\gamma = \frac{\pi}{3}$, **b** $\theta = \frac{9\pi}{10}$ and $\gamma = \frac{\pi}{6}$, **c** $\theta = \frac{2\pi}{3}$ and $\gamma = \frac{\pi}{3}$. (i) ρ_{SE}^1 (*black solid line*), (ii) ρ_{SE}^2 (*red dash line*), (iii) ρ_{SE}^3 (*blue dash dot line*). The number of nuclear spins in the bath $N = 20$ and $J_{ex} = 1$. The parameter $B = 2$, $x_1 = 1$ and $A_k^1 \neq 0$, $A_k^2 = 0$

In Fig. 5, we plot the concurrence of the two central spins for three types of initial correlations and the exchange coupling $J_{ex} = 0$. Numerical calculations show that for initial classical correlations, the initial values of the concurrence are maximum in the process of the time evolution, i.e., the system can not gain more entanglement. It is unlike the case for both spins interacting with the bath (see Figs. 1a and b). In the case of only one of the two spins interacting with the bath, our numerical simulations show that the system can gain more entanglement in the process of the time evolution only for initial quantum correlations. Physically, the two central spins and the bath can be considered as a tripartite system. As time evolves, the quantum correlations are redistributed among the three parties for initial state ρ_{SE}^1, so that the entanglement between two central spins can be larger than its initial value. Similar to the case of both two cental spins interacting with the bath, we take $N = 1$, $J_{ex} = 0$, $A_1^1 = 1$, and $A_1^2 = 0$ to further understand the results of numerical simulations. The concurrence for initial quantum correlations and classical correlations can be obtained analytically:

$$C_1 = 2|\cos\varphi \sin\theta|\sqrt{(\sin^2\theta \sin^2\varphi - \cos^2\theta)\cos^2 2t + \cos^2\theta}, \tag{12}$$

$$C_2 = |\cos 2t \sin 2\varphi| \sin^2\theta. \tag{13}$$

Equation (13) shows that the initial values of C_2 are maximum in the process of the time evolution, however, it depends on the parameters θ and φ for C_1 from (12). So that we can

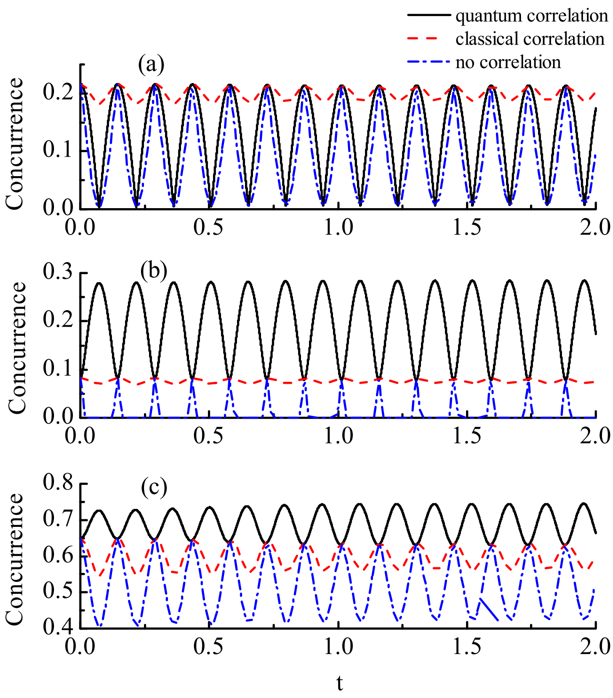

Fig. 5 (*Color online*) The concurrence of the two central spins for three types of initial correlations. **a** $\theta = \frac{\pi}{6}$ and $\gamma = \frac{\pi}{3}$, **b** $\theta = \frac{9\pi}{10}$ and $\gamma = \frac{\pi}{6}$, **c** $\theta = \frac{2\pi}{3}$ and $\gamma = \frac{\pi}{3}$. (i) ρ_{SE}^1 (*black solid line*), (ii) ρ_{SE}^2 (*red dash line*), (iii) ρ_{SE}^3 (*blue dash dot line*). The number of nuclear spins in the bath $N = 20$ and $J_{ex} = 0$. The parameter $B = 2$, $x_1 = 1$ and $A_k^1 \neq 0$, $A_k^2 = 0$

conclude that initial quantum correlations can lead to an increase in the entanglement of the two central spins, and it depends on the θ and φ.

It is clear that although the reduced density matrices of the two spins and the bath are the same for these three initial states, the dynamical behavior of the concurrence are quite different. The presence of the system-bath initial correlations affects considerably the dynamical behavior of the entanglement of the two central spins. Compared with the initial quantum correlations and the classical correlations, the average values of the concurrence for no-correlation are always minimal. The initial system-bath correlations can effectively avoid the occurrence of the ESD. In addition, irrespective of whether both two central spins or only one of the two spins interacts with the bath, the system can gain more entanglement in the process of the time evolution for initial quantum correlations.

4 The Effect of Different Initial Bath States on the Entanglement Dynamics of System

In the following, in order to clarify solely the role of initial bath state in the entanglement dynamics of the system, we will discuss two types initial states of the spin bath. One is the maximally entangled states, and the states of whole system are as follows:

$$\rho_{SE}^4 = |\mu\rangle\langle\mu| \otimes |\widetilde{W}\rangle\langle\widetilde{W}|, \tag{14}$$

$$\rho_{SE}^5 = |\mu\rangle\langle\mu| \otimes |\widetilde{4}\rangle\langle\widetilde{4}|. \tag{15}$$

where we take the system to be in the state $|\mu\rangle = \sin\theta\,|eg\rangle + \cos\theta\,|ge\rangle$, and the whole state of the total system is assumed to be separable initially. The state $|\widetilde{W}\rangle$ is the same as previously mentioned in (5). We denote bath state $|\widetilde{4}\rangle = (C_{M_b}^N)^{-\frac{1}{2}}(|111100...\rangle + |111010...\rangle + ...|...001111\rangle)$ being the maximally entangled state for $M_b = 4$. The other initial states of the spin bath are the completely mixed states. And the state of the whole system is given by the density matrix

$$\rho_{SE}^6 = (C_{M_b}^N)^{-1}|\mu\rangle\langle\mu| \otimes \mathcal{I}_{M_b}, \tag{16}$$

where $\mathcal{I}_{M_b}$ are the completely mixed states of the bath and $M_b = 1, 4$.

4.1 Both Two Central Spins Interact with the Bath

In Fig. 6, the time evolution of the entanglement dynamics of the two cental spins is plotted for $\theta = \frac{\pi}{6}$, $A_k^1 = A_k^2$ and different B. We choose the initial states ρ_{SE}^4 in (a), ρ_{SE}^5 in (c), ρ_{SE}^6 ($M_b = 1$) in (b) and ρ_{SE}^6 ($M_b = 4$) in (d). Comparing the left two panels and right two panels, we can see that for the initial states ρ_{SE}^6, the concurrence will decay obviously for inhomogeneous couplings ($B = 2$, blue dash dot line) and nearly homogeneous couplings ($B = 0.8$, red dash line). However, the concurrence for the initial states ρ_{SE}^4 and ρ_{SE}^5 almost do not decay (see Fig. 6a and c). The reason is that the entanglement in the bath protects the system from decohering for initial states ρ_{SE}^4 and ρ_{SE}^5. In other words, the decoherence of the system is suppressed in the presence of entanglement between the bath spins in initial states ρ_{SE}^4 and ρ_{SE}^5 which are maximally entangled. The previous studies [36, 37] have established a relation between decoherence and entanglement inside the bath which was illustrated by using Ising model, and our results are consistent with theirs. Apart from that, comparing Figs. 6b and d, we find that the ESD time for low polarization is longer than high polarization for initial state ρ_{SE}^6. Here, we consider the polarization $p_b = 20\ \%$ ($p_b = (N - 2M_b)/N$) as the low polarization ($M_b = 4$, $N = 10$), and $p_b = 80\ \%$ as the high polarization ($M_b = 1$). From Fig. 6a, we can see that there is no ESD for the initial

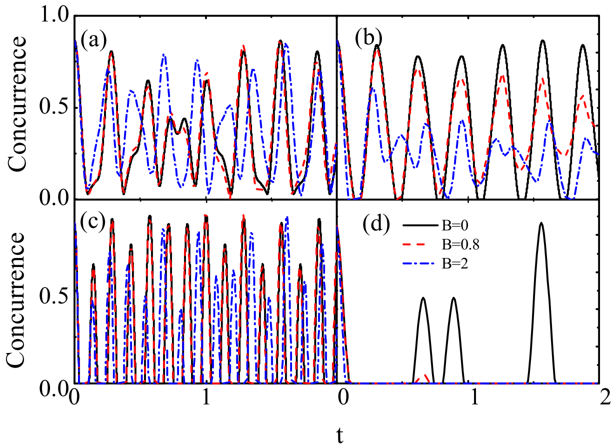

Fig. 6 (*Color online*) Time evolution of the concurrence for $N = 10$, $\theta = \frac{\pi}{6}$, $x_1 = 1$, and $J_{ex} = 1$. $M_b = 1$ [in (**a**) and (**b**)] and $M_b = 4$ [in (**c**) and (**d**)]. The different initial states ρ_{SE}^4 in (**a**) and (**c**), ρ_{SE}^5 in (**b**) and ρ_{SE}^6 in (**d**). The homogeneous coupling $B = 0$ (*black solid line*), the nearly homogeneous coupling $B = 0.8$ (*red dash line*) and the inhomogeneous coupling $B = 2$ (*blue dash dot line*)

state ρ_{SE}^4, and there is ESD for initial state ρ_{SE}^5 (see Fig. 6c). It is due to the fact that the decay of the entanglement dynamics is more pronounced with the decreasing of the bath polarization.

Furthermore, it is noted that for the initial state ρ_{SE}^6 there is no obvious decay in the case of homogeneous coupling ($B = 0$, black solid line, see Figs. 6b and d). And compared with the case of nearly homogeneous coupling, we can see that the decay of the concurrence becomes more pronounced in the case of inhomogeneous coupling, and with the increasing of B, the average values of the concurrence become smaller. However, Figs. 6a and c show that the variation of B has almost no effect on the entanglement dynamics for the initial states ρ_{SE}^4 and ρ_{SE}^5. We conclude that the effects of the inhomogeneous coupling on the entanglement dynamics crucially depend on the state of the spin bath, i.e., the inhomogeneous coupling can lead to the decay of concurrence of the system and the decrease of the average value for the initial completely mixed bath states, while for the initial maximally entangled states the entanglement can constrain the decoherence, no matter it is inhomogenous or homogeneous coupling.

In the following, we consider the initial state of the system is $|eg\rangle$ which indicates the two central spins to be separable initially. The possibility of entanglement creation via the common spin bath for a pair of initially disentangled spins without the direct interaction have been investigated in Refs. [11, 12, 14]. We have known that it is possible to create entanglement via the spin bath without a direct interaction ($J_{ex} = 0$), due to the fact that the two spins can have an indirect interaction between them for the common bath. In Fig. 7, we plot the concurrence as a function of time for $\theta = \frac{\pi}{2}$ and different initial spin bath states. It is shown that the induced entanglement can reach the maximal value in initial short time. And one observes damped periodic oscillations of entanglement for ρ_{SE}^6 as shown in Fig. 7a. Physically, the induced indirect exchange interaction due to a spin bath which also introduces quantum noise. In the beginning short time, the indirect interaction plays a

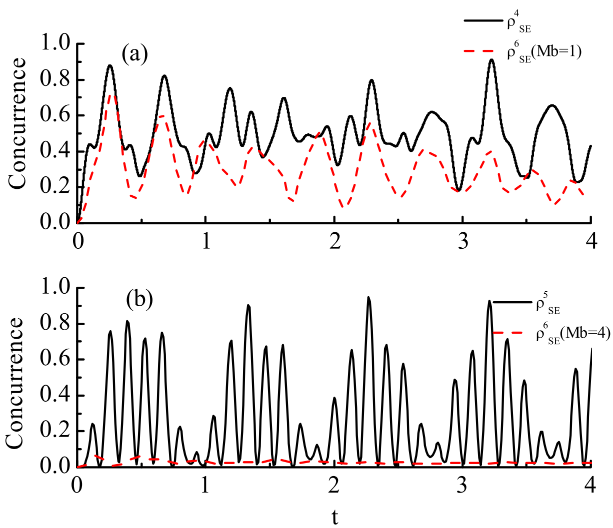

Fig. 7 (*Color online*) Time evolution of the concurrence for different initial states, and $\theta = \frac{\pi}{2}$, $J_{ex} = 0$. **a** The initial state ρ_{SE}^4 (*black solid line*), ρ_{SE}^6 with $M_b = 1$ (*red dash line*). **b** The initial state ρ_{SE}^5 (*black solid line*), ρ_{SE}^6 with $M_b = 4$ (*red dash line*). The number of nuclear spins in the bath $N = 10$, $x_1 = 1$ and $B = 2$

leading role. However, for longer times, the quantum noise effects will eventually dominate and the entanglement will be erased [13]. Furthermore, for initial maximally entangled bath states the induced entanglement is larger than that of initial completely mixed states. In the case of low polarization, the entanglement is much smaller for ρ_{SE}^6 than for ρ_{SE}^5 as shown in Fig. 7b. So that we can conclude that the induced entanglement depends on the initial spin bath states, the maximally entangled bath states can lead to more entanglement generations comparing the completely mixed states. In addition, it is interesting that the induced entanglement is periodic oscillating for the initial state ρ_{SE}^5.

4.2 Only One of the Two Spins Interacts with the Bath

In the case of only one of the two central spins interacting with the bath, i.e. $A_k^1 \neq 0$, $A_k^2 = 0$, we plot the time evolution of the entanglement dynamics of the system in Fig. 8 with the same parameters as given in Fig. 6. Irrespective of whether the initial bath state is maximally entangled state or completely mixed state, we find that the concurrence will decay in the cases of inhomogeneous couplings ($B = 2$, blue dash dot line), and the average values of the concurrence become smaller with the increasing of B. For nearly homogeneous couplings ($B = 0.8$, red dash line), the concurrence only decays in the case of initial completely mixed states. In the case of homogeneous coupling, there is no decay. Compared with the initial states ρ_{SE}^4 and ρ_{SE}^5, it is shown that the decay of the concurrence becomes more pronounced for initial states ρ_{SE}^6 in Fig. 8. Compared with the case of both two central spins interacting with the bath ($A_k^1 = A_k^2 \neq 0$) in Fig. 6, for inhomogeneous coupling with $M_b = 1$, the decay of the entanglement is more obvious even for the initial maximally entangled bath states (see Figs. 8a and b). It is due to the fact that for initial states ρ_{SE}^4 and ρ_{SE}^5 the protection from the entanglement in the bath becomes weak enough in the case of only one of the two central spins interacting with the bath. Apart from that, for $M_b = 4$

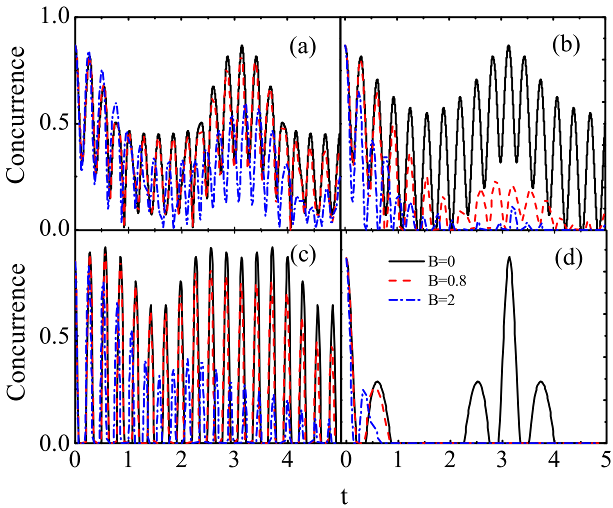

Fig. 8 (*Color online*) Time evolution of the concurrence for $N = 10$, $\theta = \frac{\pi}{6}$, $x_1 = 1$, and $J_{ex} = 1$, $A_k^2 = 0$. $M_b = 1$, $p_b = 80$ % [in (**a**) and (**b**)] and $M_b = 4$, $p_b = 20$ % [in (**c**) and (**d**)]. The different initial states ρ_{SE}^4 in (**a**), ρ_{SE}^5 in (**c**), ρ_{SE}^6 in (**b**) and (**d**). The homogeneous coupling $B = 0$ (*black solid line*), the nearly homogeneous coupling $B = 0.8$ (*red dash line*) and the inhomogeneous coupling $B = 2$ (*blue dash dot line*)

there is a obvious entanglement sudden birth in the cases of inhomogeneous couplings and nearly homogeneous couplings (see Fig. 8d).

5 Conclusion

In this paper, we have investigated the entanglement dynamics of two electronic spins coupled to a bath of nuclear spins with different hyperfine coupling constants. We have considered three types of initial states with different system-bath correlations: quantum correlation, classical correlation and no-correlation. We find that the influences of initial correlations on the entanglement dynamics of the system are remarkable. Significantly, the initial system-bath correlations can avoid the occurrence of ESD. And the system can gain more entanglement in the process of the time evolution for initial quantum correlations irrespective of whether both two central spins or only one of the two spins interacts with the bath. Compared with the initial quantum correlations and the classical correlations, the average values of the concurrence for no-correlation are always minimal. For these three different initial correlations, the entanglement dynamics of system are similar with different distributions of hyperfine coupling constants for the initial maximally entangled spin bath. Apart from that, we have studied the influences of different initial bath states on the time evolution of the concurrence. Our numerical simulations show that the decoherence of the system can be suppressed greatly for the bath being initially in the maximally entangled states. However, the decay of the concurrence occurs obviously for the initial completely mixed bath states especially for the low polarization of the bath and in the case of one of two spins interacting with the bath. Interestingly, we find that the effects of inhomogeneous couplings on entanglement dynamics of the two central spins crucially depend on the state of the spin bath. The inhomogeneous couplings can lead to the decoherence of system for the initial completely mixed bath states, while for the initial bath state is in the maximally entangled state the entanglement can constrain the effect of decoherence, no matter the coupling is inhomogenous or homogeneous couplings. Furthermore, the maximally entangled bath states can lead to more entanglement generations comparing the completely mixed states.

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