

Entanglement in Mixed-Spin (1/2, 3/2) Heisenberg XXZ Model with Dzyaloshinskii-Moriya Interaction

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Abstract In this paper, the entanglement in a mixed-spin (1/2, 3/2) Heisenberg XXZ model with Dzyaloshinskii-Moriya (DM) interaction in an inhomogeneous external magnetic field is studied. We not only calculate the ground-state entanglement but also investigate the behaviors of quantum phase transition following the changes of DM interaction and nonuniform magnetic field. More importantly, we note that the DM interaction improves the critical magnetic field B_c , the critical temperature T_c and broadens the region of entanglement.

Keywords Entanglement · Quantum phase transition · Mixed-spin (1/2, 3/2) · Dzyaloshinskii-Moriya interaction

1 Introduction

In recent years, the quantum entanglement has become a crucial physical resource in many fields of quantum information processing such as quantum communication [1–3] and quantum computation [4, 5] due to its nonlocal correlation. The Heisenberg chain as the simplest but an operable model has been used for quantum dots [6], cavity QED [7] and so on.

In particular, the thermal entanglement is a natural entanglement, which is different from the other kinds of entanglements by its advantages of stability for the reduction in entanglement of an entangled state due to various sources of decoherence [8, 9]. Recently, lots of studies have devoted to the thermal entanglement in various spin Heisenberg models with or without external magnetic field [8–21]. As we all know, Dzyaloshinsky and Moriya initially

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pointed out the Dzyaloshinskii-Moriya interaction [22, 23], which stems from the spin-orbit coupling. By adding DM interaction, many researchers have investigated the thermal entanglement and teleportation [24–31]. In addition, the quantum discord [32] that a more general measure of the quantum correlation with DM interaction also has been studied in Ref. [33, 34]. The above mentioned investigations suggest the thermal entanglement is an important source of mixed entanglement, and the magnetic field and DM interaction play a significant role in various quantum tasks.

It is noteworthy that, in Refs. [19, 20], by using the concept of negativity, Guo et al. have studied the entanglement of mixed-spin (1/2, 3/2) Heisenberg XXY and XX models with the nonuniform magnetic field, and noted that the mixed-spin (1/2, 3/2) Heisenberg chain can generate more entanglement and higher critical temperature T_c than those in spin 1/2 and mixed-spin (1/2, 1) Heisenberg models for the same parameters. It indicates that the mixed-spin (1/2, 3/2) spin chain is a promising system. However, we notice the entanglement in mixed-spin (1/2, 3/2) Heisenberg XXZ chain with DM interaction in an inhomogeneous external magnetic field has not been investigated, and through the above discussions, we realize that this study is essential. In this work, by adding the DM interaction, we not only calculate the ground-state entanglement, but also discuss the behaviors of quantum phase transition (QPT) [35], which takes place in the ground state at zero temperature. In addition, we note that DM interaction not only improves the critical values of the uniform magnetic field and temperature, but also broadens the region of the entanglement.

2 The Model and Calculation

In this paper, the Hamiltonian for the mixed-spin (1/2, 3/2) Heisenberg XXZ model with z -component DM interaction and inhomogeneous magnetic field is given by

$$\begin{aligned}
 H = & J (S_1^x S_2^x + S_1^y S_2^y + \Delta S_1^z S_2^z) + (B - b)S_1^z + (B + b)S_2^z \\
 & + D_z (S_1^x S_2^y - S_1^y S_2^x), \tag{1}
 \end{aligned}$$

where J denotes the exchange constant, for $J > 0$ corresponds to the anti-ferromagnetic case, and $J < 0$ the ferromagnetic case, Δ is the anisotropy parameter, D_z is DM coupling parameter in the z direction, B describes uniform external magnetic field, and b controls the inhomogeneity of external magnetic field, S_1^l and S_2^l ($l = x, y, z$) are the spin operators of spin 1/2 and spin 3/2, respectively.

In order to study the entanglement, we need to obtain the eigenvalues and eigenstates for the mixed-spin (1/2, 3/2) system from Eq. (1). In this work, we choose the standard basis $\left\{ \left| \frac{1}{2}, \frac{3}{2} \right\rangle, \left| \frac{1}{2}, \frac{1}{2} \right\rangle, \left| \frac{1}{2}, -\frac{1}{2} \right\rangle, \left| \frac{1}{2}, -\frac{3}{2} \right\rangle, \left| -\frac{1}{2}, \frac{3}{2} \right\rangle, \left| -\frac{1}{2}, \frac{1}{2} \right\rangle, \left| -\frac{1}{2}, -\frac{1}{2} \right\rangle, \left| -\frac{1}{2}, -\frac{3}{2} \right\rangle \right\}$, here $|n, m\rangle$ is the eigenstate of S_1^z and S_2^z with the corresponding eigenvalues given by n and m , respectively. Thus the Hamiltonian (1) can be calculated in the matrix form as

$$H = \begin{pmatrix} a_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 & a_{25} & 0 & 0 & 0 \\ 0 & 0 & a_{33} & 0 & 0 & a_{36} & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 & 0 & a_{47} & 0 \\ 0 & a_{52} & 0 & 0 & a_{55} & 0 & 0 & 0 \\ 0 & 0 & a_{63} & 0 & 0 & a_{66} & 0 & 0 \\ 0 & 0 & 0 & a_{74} & 0 & 0 & a_{77} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{88} \end{pmatrix}, \tag{2}$$

where $a_{11} = \frac{3}{4}J\Delta + 2B + b$, $a_{22} = \frac{1}{4}J\Delta + B$, $a_{25} = a_{52}^* = a_{47} = a_{74}^* = \frac{\sqrt{3}}{2}J + \frac{\sqrt{3}}{2}Di$, $a_{33} = -\frac{1}{4}J\Delta - b$, $a_{36} = a_{63}^* = J + Di$, $a_{44} = -\frac{3}{4}J\Delta - B - 2b$, $a_{55} = -\frac{3}{4}J\Delta + B + 2b$, $a_{66} = -\frac{1}{4}J\Delta + b$, $a_{77} = \frac{1}{4}J\Delta - B$ and $a_{88} = \frac{3}{4}J\Delta - 2B - b$. After a mathematical calculation, the eigenvalues are given by

$$\begin{aligned} E_1 &= -3\zeta + 2B + b, \\ E_2 &= -3\zeta - 2B - b, \\ E_3 &= b + B + \zeta + \tau_1, \\ E_4 &= b + B + \zeta - \tau_1, \\ E_5 &= -b - B + \zeta + \tau_2, \\ E_6 &= -b - B + \zeta - \tau_2, \\ E_7 &= \zeta + \vartheta, \\ E_8 &= \zeta - \vartheta, \end{aligned} \tag{3}$$

where $\zeta = -\frac{1}{4}J\Delta$, $\vartheta = \sqrt{J^2 + b^2 + D_z^2}$, $\tau_1 = \frac{1}{2}\sqrt{4b^2 - 4J\Delta b + J^2\Delta^2 + 3J^2 + 3D_z^2}$, $\tau_2 = \frac{1}{2}\sqrt{4b^2 + 4J\Delta b + J^2\Delta^2 + 3J^2 + 3D_z^2}$, and the corresponding eigenstates are expressed as

$$\begin{aligned} |\Psi_1\rangle &= \left| \frac{1}{2}, \frac{3}{2} \right\rangle, \\ |\Psi_2\rangle &= \left| -\frac{1}{2}, -\frac{3}{2} \right\rangle, \\ |\Psi_3\rangle &= \frac{1}{\sqrt{g_1g_1^* + 1}} \left(g_1 \left| -\frac{1}{2}, \frac{3}{2} \right\rangle + \left| \frac{1}{2}, \frac{1}{2} \right\rangle \right), \\ |\Psi_4\rangle &= \frac{1}{\sqrt{g_2g_2^* + 1}} \left(g_2 \left| -\frac{1}{2}, \frac{3}{2} \right\rangle + \left| \frac{1}{2}, \frac{1}{2} \right\rangle \right), \\ |\Psi_5\rangle &= \frac{1}{\sqrt{g_3g_3^* + 1}} \left(g_3 \left| \frac{1}{2}, -\frac{3}{2} \right\rangle + \left| -\frac{1}{2}, -\frac{1}{2} \right\rangle \right), \\ |\Psi_6\rangle &= \frac{1}{\sqrt{g_4g_4^* + 1}} \left(g_4 \left| \frac{1}{2}, -\frac{3}{2} \right\rangle + \left| -\frac{1}{2}, -\frac{1}{2} \right\rangle \right), \\ |\Psi_7\rangle &= \frac{1}{\sqrt{g_5g_5^* + 1}} \left(g_5 \left| -\frac{1}{2}, \frac{1}{2} \right\rangle + \left| \frac{1}{2}, -\frac{1}{2} \right\rangle \right), \\ |\Psi_8\rangle &= \frac{1}{\sqrt{g_6g_6^* + 1}} \left(g_6 \left| -\frac{1}{2}, \frac{1}{2} \right\rangle + \left| \frac{1}{2}, -\frac{1}{2} \right\rangle \right), \end{aligned} \tag{4}$$

where $g_1 = \frac{2}{\sqrt{3}} \frac{(b+2\zeta+\tau_1)(J-D_z i)}{J^2+D_z^2}$, $g_2 = \frac{2}{\sqrt{3}} \frac{(b+2\zeta-\tau_1)(J-D_z i)}{J^2+D_z^2}$, $g_3 = \frac{2}{\sqrt{3}} \frac{(-b+2\zeta+\tau_2)(J-D_z i)}{J^2+D_z^2}$, $g_4 = \frac{2}{\sqrt{3}} \frac{(-b+2\zeta-\tau_2)(J-D_z i)}{J^2+D_z^2}$, $g_5 = \frac{(b+\vartheta)(J-D_z i)}{J^2+D_z^2}$ and $g_6 = \frac{(b-\vartheta)(J-D_z i)}{J^2+D_z^2}$.

For a state of system in thermal equilibrium (temperature T), which is described by the density operator $\rho(T) = (1/Z) \exp(-H/k_B T)$, where H is the Hamiltonian, $Z = \text{tr}[\exp(-H/k_B T)]$ is the partition function, and k_B is Boltzmann constant, for simplicity, we write $k_B = 1$. The entanglement of states of the system at thermal equilibrium is called

thermal entanglement [8, 9]. In the standard basis above-mentioned, the density operator of the system can be given by

$$\rho(T) = \frac{1}{Z} \begin{pmatrix} \rho_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{22} & 0 & 0 & \rho_{25} & 0 & 0 & 0 \\ 0 & 0 & \rho_{33} & 0 & 0 & \rho_{36} & 0 & 0 \\ 0 & 0 & 0 & \rho_{44} & 0 & 0 & \rho_{47} & 0 \\ 0 & \rho_{52} & 0 & 0 & \rho_{55} & 0 & 0 & 0 \\ 0 & 0 & \rho_{63} & 0 & 0 & \rho_{66} & 0 & 0 \\ 0 & 0 & 0 & \rho_{74} & 0 & 0 & \rho_{77} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{88} \end{pmatrix}, \tag{5}$$

where $\rho_{11} = \zeta_1, \rho_{22} = \frac{\zeta_3}{g_1 g_1^* + 1} + \frac{\zeta_4}{g_2 g_2^* + 1}, \rho_{25} = \frac{\zeta_3 g_1^*}{g_1 g_1^* + 1} + \frac{\zeta_4 g_2^*}{g_2 g_2^* + 1}, \rho_{33} = \frac{\zeta_7}{g_5 g_5^* + 1} + \frac{\zeta_8}{g_6 g_6^* + 1}, \rho_{36} = \frac{\zeta_7 g_5^*}{g_5 g_5^* + 1} + \frac{\zeta_8 g_6^*}{g_6 g_6^* + 1}, \rho_{44} = \frac{\zeta_5 g_3 g_3^*}{g_3 g_3^* + 1} + \frac{\zeta_6 g_4 g_4^*}{g_4 g_4^* + 1}, \rho_{47} = \frac{\zeta_5 g_3}{g_3 g_3^* + 1} + \frac{\zeta_6 g_4}{g_4 g_4^* + 1}, \rho_{52} = \frac{\zeta_3 g_1}{g_1 g_1^* + 1} + \frac{\zeta_4 g_2}{g_2 g_2^* + 1}, \rho_{55} = \frac{\zeta_3 g_1 g_1^*}{g_1 g_1^* + 1} + \frac{\zeta_4 g_2 g_2^*}{g_2 g_2^* + 1}, \rho_{63} = \frac{\zeta_7 g_5}{g_5 g_5^* + 1} + \frac{\zeta_8 g_6}{g_6 g_6^* + 1}, \rho_{66} = \frac{\zeta_7 g_5 g_5^*}{g_5 g_5^* + 1} + \frac{\zeta_8 g_6 g_6^*}{g_6 g_6^* + 1}, \rho_{74} = \frac{\zeta_5 g_3}{g_3 g_3^* + 1} + \frac{\zeta_6 g_4}{g_4 g_4^* + 1}, \rho_{77} = \frac{\zeta_5}{g_3 g_3^* + 1} + \frac{\zeta_6}{g_4 g_4^* + 1}, \rho_{88} = \zeta_2, \zeta_i = \exp\left(-\frac{E_i}{T}\right), (i = 1, 2, 3...8), \text{ and } Z = \sum_1^8 \zeta_i = 2 \left[\exp\left(\frac{3\zeta}{T}\right) \cosh\left(\frac{2B+b}{T}\right) + \exp\left(-\frac{b+B+\zeta}{T}\right) \cosh\left(\frac{\tau_1}{T}\right) + \exp\left(\frac{b+B-\zeta}{T}\right) \cosh\left(\frac{\tau_2}{T}\right) + \exp\left(-\frac{\vartheta}{T}\right) \cosh\left(\frac{\vartheta}{T}\right) \right].$

As is known to all, according to the Peres-Horodecki Criterion [36, 37] which can give a qualitative way for judging whether the state of high dimensional bipartite systems is entangled or not, the negativity was proposed by Vidal and Werner [38] and can be used to effectively compute the entanglement for any mixed state of an arbitrary bipartite system, and it does not increase under local manipulations of the system, thus we use it to study the entanglement in our mixed-spin (1/2, 3/2) system. The negativity of a state ρ is defined as

$$N(\rho) = \frac{\|\rho^{T_1}\| - 1}{2} = \frac{\|\rho^{T_2}\| - 1}{2} = \sum_i |u_i|, \tag{6}$$

where $\rho^{T_1} (\rho^{T_2})$ denotes the partial transpose of the total state ρ respect to the first (second) subsystem, and u_i is the negative eigenvalue of $\rho^{T_1} (\rho^{T_2})$. The corresponding partial transpose ρ^{T_1} can be given by

$$\rho^{T_1} = \frac{1}{Z} \begin{pmatrix} \rho_{11} & 0 & 0 & 0 & 0 & \rho_{52} & 0 & 0 \\ 0 & \rho_{22} & 0 & 0 & 0 & 0 & \rho_{63} & 0 \\ 0 & 0 & \rho_{33} & 0 & 0 & 0 & 0 & \rho_{74} \\ 0 & 0 & 0 & \rho_{44} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_{55} & 0 & 0 & 0 \\ \rho_{25} & 0 & 0 & 0 & 0 & \rho_{66} & 0 & 0 \\ 0 & \rho_{36} & 0 & 0 & 0 & 0 & \rho_{77} & 0 \\ 0 & 0 & \rho_{47} & 0 & 0 & 0 & 0 & \rho_{88} \end{pmatrix}. \tag{7}$$

Hence, from Eqs. (6) and (7), one can obtain the expression of negativity as follows

$$\begin{aligned}
 N = \frac{1}{2Z} \left\{ \max \left\{ 0, \sqrt{(\rho_{77} - \rho_{22})^2 + 4\rho_{36}\rho_{63}} - \rho_{77} - \rho_{22} \right\} \right. \\
 + \max \left\{ 0, \sqrt{(\rho_{66} - \rho_{11})^2 + 4\rho_{25}\rho_{52}} - \rho_{66} - \rho_{11} \right\} \\
 \left. + \max \left\{ 0, \sqrt{(\rho_{88} - \rho_{33})^2 + 4\rho_{47}\rho_{74}} - \rho_{88} - \rho_{33} \right\} \right\}, \tag{8}
 \end{aligned}$$

which is a function of parameters T, J, Δ, B, b and D_z .

3 Ground-State Entanglement and Quantum Phase Transitions (QPT)

In order to investigate the entanglement of ground states and QPT, we analyze the energy eigenvalues behaviour following the change of different parameters. In Fig. 1, we plot the eigenvalues as the functions of the z -component of DM interaction by fixing the rest. Obviously, which eigenstate is the ground state depends on the parameter D_z in this case. It is clear that, when $D < D_{zc1} \approx -2.57$, the ground state is the entangled state $|\Psi_6\rangle$ with the eigenvalue E_6 , and the corresponding negativity is given by $N = \frac{|\xi|}{|\xi|^2+1}$, where $\xi = \frac{(2\sqrt{D_z^2+21}+3)(D_z i-1)}{4\sqrt{3}(D_z^2+1)}$; when $D = D_{zc1}$, the ground-state eigenvalue E_6 and the eigenvalue E_2 with the first excited state $|\Psi_2\rangle$ have an intersection, namely, the ground states are doubly degenerate at the intersection, and this suggests that quantum phase transition occurs [35], which is induced by the discontinuous changes in a property of the ground state $|\Psi_6\rangle$ and the structure of the first excited state $|\Psi_2\rangle$ when the external parameter D_z traverses a critical point D_{zc1} , and in the QPT point, the quantum fluctuations play a dominant role and the thermal fluctuations become frozen. Moreover, when $D_{zc1} < D < D_{zc2} \approx -2.57$, the ground-state is the unentangled stat $|\Psi_2\rangle$, so the negativity equals to zero; when $D_z = D_{zc2}$, the ground-state energy eigenvalue E_2 and the eigenvalue E_6 with the first excited state $|\Psi_6\rangle$ show a crossing point, and the QPT also takes place at $D_z = D_{zc2}$; when $D_z > D_{zc2}$, the ground state becomes $|\Psi_6\rangle$ again. It is observed that the positions of two QPT points ($D_{zc1} \approx -2.57$ and $D_{zc2} \approx 2.57$) are symmetric about $D_z = 0$. In addition, from Fig. 1, we can clearly see the phenomenon that the other energy eigenvalues also display the intersection points, which is caused by the degeneracy of the excited states with the change of DM interaction.

Fig. 1 The eigenvalues E versus the z -coponent of DM interaction D_z , where we assume $J = 1, \Delta = 0.5, B = 3$ and $b = 0.5$

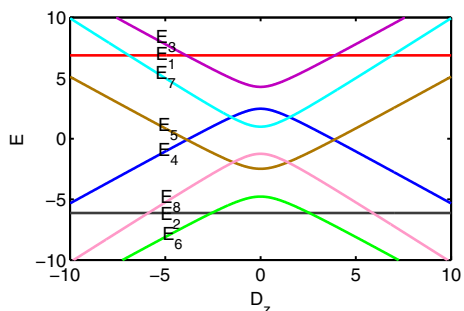
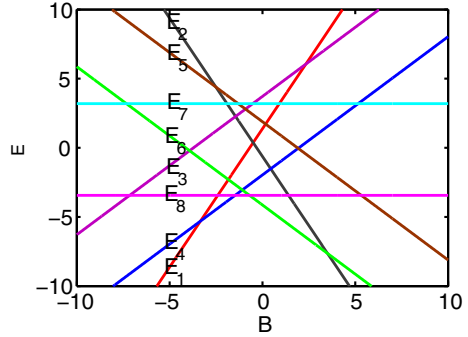


Fig. 2 The eigenvalues E versus uniform magnetic field B , where we assume $J = 1, \Delta = 0.5, b = 1$ and $D_z = 3$



The eigenvalues versus magnetic field B are plotted in Fig. 2. It is easy to find that there are four QPT points ($B_{c1} \approx -3.34, B_{c2} \approx -1.48, B_{c3} \approx -0.69$ and $B_{c4} \approx 3.51$), and the ground states can be described by

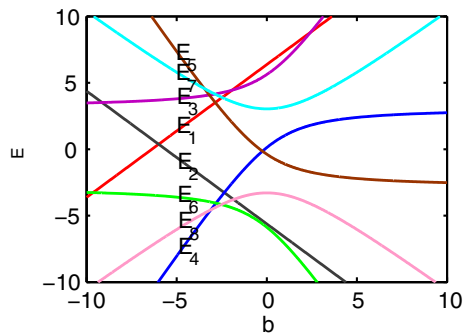
$$\Gamma_{GS} = \begin{cases} |\Psi_1\rangle\langle\Psi_1| & B < B_{c1}, \\ \frac{1}{2}|\Psi_1\rangle\langle\Psi_1| + \frac{1}{2}|\Psi_4\rangle\langle\Psi_4| & B = B_{c1}, \\ |\Psi_4\rangle\langle\Psi_4| & B_{c1} < B < B_{c2}, \\ \frac{1}{2}|\Psi_4\rangle\langle\Psi_4| + \frac{1}{2}|\Psi_8\rangle\langle\Psi_8| & B = B_{c2}, \\ |\Psi_8\rangle\langle\Psi_8| & B_{c2} < B < B_{c3}, \\ \frac{1}{2}|\Psi_8\rangle\langle\Psi_8| + \frac{1}{2}|\Psi_6\rangle\langle\Psi_6| & B = B_{c3}, \\ |\Psi_6\rangle\langle\Psi_6| & B_{c3} < B < B_{c4}, \\ \frac{1}{2}|\Psi_6\rangle\langle\Psi_6| + \frac{1}{2}|\Psi_2\rangle\langle\Psi_2| & B = B_{c4}, \\ |\Psi_2\rangle\langle\Psi_2| & B_{c4} < B, \end{cases} \quad (9)$$

and the corresponding negativities are obtained as

$$N = \begin{cases} 0 & B < B_{c1}, \\ 0.0973 & B = B_{c1}, \\ 0.4979 & B_{c1} < B < B_{c2}, \\ 0.2971 & B = B_{c2}, \\ 0.4939 & B_{c2} < B < B_{c3}, \\ 0.2931 & B = B_{c3}, \\ 0.4822 & B_{c3} < B < B_{c4}, \\ 0.0880 & B = B_{c4}, \\ 0 & B_{c4} < B. \end{cases} \quad (10)$$

By observing Eq. (10), we can find the entanglement of ground state is independent of the uniform magnetic field B when B is in a certain range.

Fig. 3 The eigenvalues E versus inhomogeneous magnetic field b , where we assume $J=1, \Delta = 0.5, B = 3$ and $D_z=3$



Finally, in Fig. 3, we study the influence of the inhomogeneous magnetic parameter b and note that there are two QPT points ($b_{c1} \approx -3.10$ and $b_{c2} \approx -2.51$). It is observed that, with the increasing of b , E_6 has an abrupt change near $b = 0$, and the ground state initially changes from the entangled state $|\Psi_4\rangle$ ($b < b_{c1}$) to the entangled state $|\Psi_8\rangle$ ($b_{c1} < b < b_{c2}$), then jumps to the entangled $|\Psi_6\rangle$ ($b > b_{c2}$) at the positions of QPT ($b = b_{c1}$ and $b = b_{c2}$), respectively. The corresponding negativities can be obtained as $N_{|\Psi_4\rangle} = \frac{|\sigma_1|}{|\sigma_1|^2+1}$, $N_{|\Psi_8\rangle} = \frac{|\sigma_2|}{|\sigma_2|^2+1}$, and $N_{|\Psi_6\rangle} = \frac{|\sigma_3|}{|\sigma_3|^2+1}$, where $\sigma_1 = -\frac{\sqrt{3}}{60} \left[-4b + 1 + \sqrt{16b^2 - 8b + 121} + (12b - 3 - 3\sqrt{16b^2 - 8b + 121})i \right]$, $\sigma_2 = \frac{1}{10} \left[b - \sqrt{b^2 + 10} - (3 + 3\sqrt{b^2 + 10})i \right]$, and $\sigma_3 = -\frac{\sqrt{3}}{60} \left[4b + 1 + \sqrt{16b^2 - 8b + 121} - (12b + 3 + 3\sqrt{16b^2 - 8b + 121})i \right]$. From the above discussions, not only can we change the positions of QPT points, but also obtain the optimal entanglement of ground state by coordinating the parameters D_z , B and b in our model.

4 Thermal Entanglement

Firstly, we would like to study the effect of the uniform field B on thermal entanglement when DM interaction is included. The N as the functions of T and B are plotted by fixing the parameters J , Δ , b and D_z in Fig. 4. Looking at Fig. 4a, we find that, at zero-temperature limit, the N maintains the maximal value until B arrives at a critical value B_c , above which the N disappears. At the same time, we note that the entanglement also decreases to zero at critical temperature T_c , for the reason that as temperature goes up the mixing of ground state with more excited states acts as a destructive noise that leads to the entanglement decays. In Fig. 4b, six contour lines of $N = 0, 0.05, 0.1, 0.2, 0.3, 0.4$ are shown, respectively. Beyond the contour line $N = 0$, there is no entanglement, which also can be seen Eq. (9) the entanglement goes to zero when $\rho_{77}\rho_{22} \geq \rho_{36}^2$, $\rho_{66}\rho_{11} \geq \rho_{25}^2$, and $\rho_{88}\rho_{33} \geq \rho_{47}^2$. Comparing with Ref. [19], where the DM interaction is absent, D_z can enhance the maximal value of N and the critical temperature T_c . For example, by controlling the rest parameters,

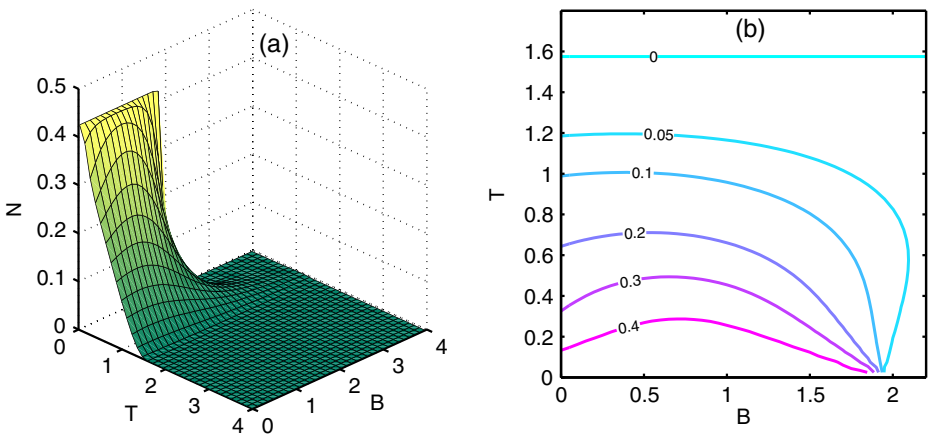
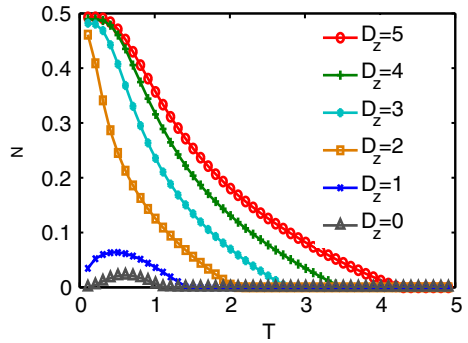


Fig. 4 The negativity N is plotted as functions of magnetic field B and temperature T when $J = 1$, $\Delta = 0.5$, $b = 0.5$ and $D_z = 1$. **(a)** The curve of N as functions of B and T . **(b)** The contour lines of N versus B and T

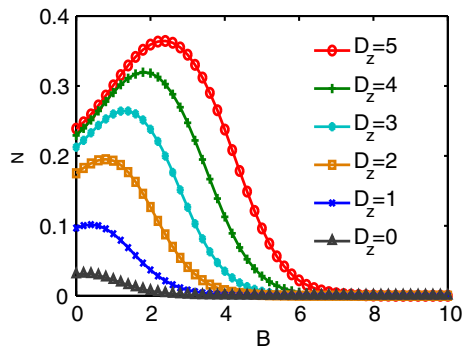
Fig. 5 The negativity N versus temperature T for different z -component of DM interaction D_z , where we assume $J = 1$, $\Delta = 0.5$, $B = 2$ and $b = 0.5$



$D_z = 0$, $N_{\max} \approx 0.38$, $T_c \approx 1.13$ [19]; $D_z = 1$, $N_{\max} \approx 0.43$, $T_c \approx 1.58$. However, we have to note the D_z can't change the intrinsic nature of the entanglement: when D_z is excluded, observed from the Figures 1 and 3 of Ref. [19] and the Figure 1 of Ref. [20], and when D_z is included, seen from Fig. 4 in our work, we obtain the same result that the critical temperature T_c is completely independent of the uniform magnetic field B .

Secondly, we discuss the role played by the z -component DM interaction in adjusting thermal entanglement. The N as a function of temperature T is plotted for different D_z in Fig. 5, from which one can easily find that the value of N from zero goes up to a maximum with the increases of D_z at zero-temperature limit. Furthermore, when $T < T_c$ and $D_z > 1$, the higher the temperature is, the smaller the N is, and the value of N decreases due to the destruction of the quantum entanglement by classical thermal fluctuations. However, when $0 < D_z < 1$, with the increasing of T , the N goes up gradually to a maximum, then drops slowly to zero. More importantly, it is observed that the larger D_z can improve critical temperature T_c , which means that we can gain the same conclusion with Refs. [25, 28, 29] that the entanglement can be obtained at higher temperature as D_z is raised. In the following, the N versus magnetic field B for different values of D_z is plotted in Fig. 6. When $D_z = 0$, the N is a small value, and it monotonically decreases to zero with B increasing; when $D_z > 1$, we find that, as B increases, N goes to a maximum and then decays monotonously to zero at critical uniform magnetic field B_c , and it is easy to see that B_c is broadened by the increasing of D_z . One can also find, for a finite magnetic field $B < B_c$, the larger D_z can enhance the maximal value of entanglement. Thus one can obtain the conclusion that D_z plays a considerably important role in improving entanglement.

Fig. 6 The negativity N versus magnetic B for different z -component of DM interaction D_z , where we assume $T = 1$, $J = 1$, $\Delta = 0.5$ and $b = 0.5$



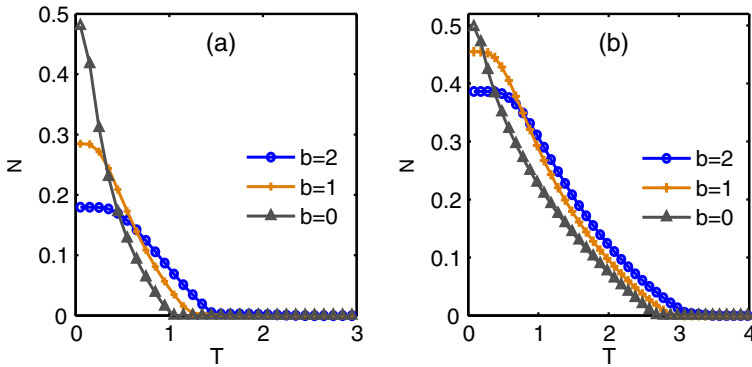


Fig. 7 The negativity N versus temperature T for different values of inhomogeneous magnetic b , where we assume $J = 1, \Delta = 0.5$ and $B = 1$. **(a)** $D_z = 0$; **(b)** $D_z = 3$

In the next step, we discuss how the thermal entanglement behaves as we change the inhomogeneous magnetic field b for various values of D_z . Looking at Fig. 7a and 7b, for $T \rightarrow 0$, it is quite obvious that the parameter b plays a negative role, namely, the higher the b is, the smaller the N is. And as temperature T increases, the N monotonously decreases, the reason is that the maximally entangled state mixes with other excited states. When T is large enough, the all entanglement will be destroyed by classical thermal fluctuations. Comparing Fig. 7a with 7b, the same results can be discovered with Fig. 5 that the larger D_z not only broadens the region of entanglement but also improves the critical temperature T_c . For example, $D_z = 0, b = 0, T_c \approx 1.08$, and the same result can be seen in Figure 1(b) of Ref. [19]; $D_z = 3, b = 0, T_c \approx 2.72$; $D_z = 0, b = 2, T_c \approx 1.82$; $D_z = 3, b = 2, T_c \approx 3.45$. At the same time, it is found that the maximal value of N doesn't have an obvious change when D_z increases for $b = 0$. Besides, whether or not D_z is considered, we clearly observe that T_c depends on the inhomogeneity of the magnetic field b , and can be improved by the increasing of b . It could be a significant supplement to Refs. [19, 20], in which the same result is obtained in the case that DM interaction is absent.

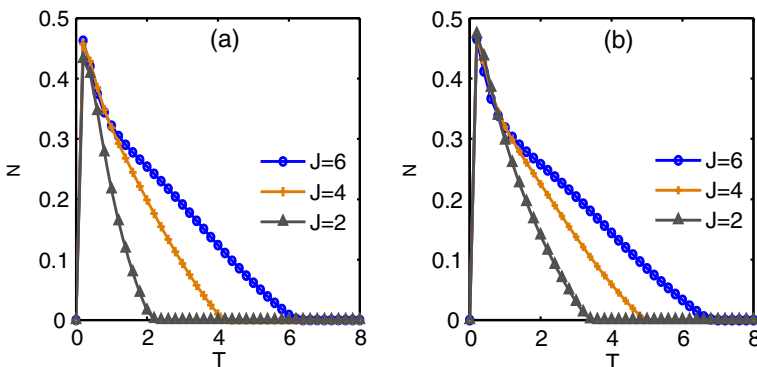


Fig. 8 The negativity N versus temperature T for different values of coupling constant J , where we assume $\Delta = 0.5, B = 1$ and $b = 0.5$. **(a)** $D_z = 0$; **(b)** $D_z = 3$

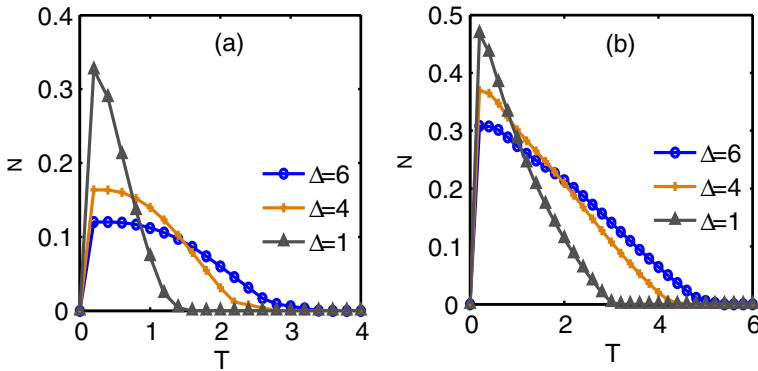


Fig. 9 The negativity N versus temperature T for different values of anisotropy parameter Δ , where we assume $J = 1$, $B = 1$ and $b = 0.5$. (a) $D_z = 0$; (b) $D_z = 3$

Ultimately, in Figs. 8 and 9, the N versus temperature for different values of coupling constant J or anisotropy parameter Δ with different D_z values. Looking at Fig. 8a, where $D_z = 0$, as the temperature increases, the N increases sharply from zero to a peak value, and after the peak value, it decreases monotonously in that the thermal fluctuation gradually dominates the system. It is clear that the maximum value of the N and the critical temperature T_c are enhanced by the increasing of J . In Fig. 8b, where $D_z = 3$, contrasting with the $D_z = 0$ case, we find that the larger D_z leads to a result that the maximal value of negativity does not show significant variations as J increases. For the same parameter case, we can obtain more entanglement under higher temperature because of the existence of DM interaction. Moreover, In Fig. 8a and 8b, for different J values, we clearly see that, before N arriving at a maximum, the behaviors of the N are always consistent, which suggests that, in the special region, the temperature is the main factor to affect the entanglement, and the influences of J and D_z can be almost ignored. Looking at Fig. 9a and 9b, we can find, even if the DM interaction is added, the anisotropy parameter Δ still distinctly suppresses the maximal value of N , but it can improve the critical temperature T_c and broaden the region of entanglement for the same Δ . It is also clear that the T_c increases with the increasing of Δ , which agrees with the conclusion of Ref. [19]. In addition, for various Δ values, before the N increasing to a peak value, the behaviors of N are inconsistent, but D_z gradually eliminates this difference.

5 Conclusions

In this article, by using the concept of negativity, we study the effects of the z -component of DM interaction and an inhomogeneous external magnetic field on the entanglement and QPT in mixed-spin $(1/2, 3/2)$ Heisenberg XXZ model. Firstly, we calculate the ground-state entanglement, and note that not only the positions of QPT points can be changed but also the optimal ground-state entanglement can be obtained by coordinating the parameters D_z , B and b in our model. It is noteworthy that, even if the D_z is included, the critical temperature T_c is still completely independent of uniform magnetic B , while D_z can enhance T_c , which means that we can gain more entanglement under higher temperature. It is also observed that D_z improves the critical magnetic field B_c . At the same time, we find that the inhomogeneity

of magnetic field b , coupling constant J and the anisotropy parameter Δ play substantially important roles in improving the critical temperature T_c and enhancing the entanglement in our system.

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