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Structure Group and Fermion-Mass-Term in General Nonlocality

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Abstract In our previous work (Wang J. Math. Phys. **49**, 033513 (2008)) two problems remain to be resolved. One is that we lack a minimal group to replace GL(4,C), the other is that the Equation of Motion (EoM) for fermion has no mass term. After careful investigation we find these two problems are linked by conformal group, a subgroup of GL(4,C). The Weyl group, a subgroup of conformal group, can bring about the running of mass, charge etc. while making it responsible for the transformation of interaction vertex. However, once concerning the generation of the mass term in EoM, we have to resort to the whole conformal group, in which the generators K_μ play a crucial role in making vacuum vary from space-like (or light-cone-like)to time-like. Physically the starting points are our previous conclusion, $\vec{E}^2 - \vec{B}^2 \neq 0$ for massive bosons, and the two-photon process yielding e^+e^- pair. Finally we get to the conclusion that the mass term of strong interaction is linearly relevant to (chromo-)magnetic flux as well as angular momentum.

Keywords Nonlocal interaction · Conformal group · Fermion mass

1 Introduction

In our previous paper [1], a complex-geometry approach to quantum field theory was proposed, aiming at describing nonlocal interaction (in the low-energy limit) through curvature effects in certain kinds of complex space. In such approach, the field equations for interacting fermions and bosons are obtained based on the assumption that there always exists a complex reference frame for a fermion in which it behaves as a plane wave, i.e. as a free

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(non-interacting) particle. This assumption is regarded as "principle of nonlocality"——but actually it is a "complexification" of the Einstein's Equivalence Principle (EEP). The equation of motion (EoM) for fermions is derived by demanding that the fermions move just along a "geodesic line" in the complex space. And by a transformation of observers, we conclude that it is just the quadratic form of the standard Dirac equation, yet without the mass term. The field equation for bosons is also derived, in which the mass of boson occurs automatically and is proportional to $\vec{E}^2 - \vec{B}^2$. The conclusion holds apparently for photon since $\vec{E}^2 - \vec{B}^2 = 0$ there. The theory might be useful in studying non-perturbative dynamical systems, such as hadron spectra or scattering between hadrons. In such a theory, a few problems remain to resolve, one of which is that the structure group GL(4,C) is too large. Another is that a mass term is missing while deriving EoM for fermions. In this paper we try to analyze and resolve these two problems consistently.

In our model, there are two kinds of curving——hyperbolic curving and elliptic curving. They are represented by two different structure groups GL(n,C) and U(n,C) [1], and in this paper we are mainly concerned about the group GL(n,C). The 4-dimension Conformal Group is a promising mini-group for our model, in which the scaling transformation is a typical one in generators, nevertheless is seldom used independent of other generators [2]. The algebra structure of conformal group has been investigated thoroughly from different aspects [3–10], and its applications to quantum fields have also been widely considered. However none of the applications is satisfactory because hitherto no other perfect quantum system than photon field [11, 12] has been found so that the corresponding Lagrangian is conformally invariant, unless the masses of involved particles are null [2, 13–18]. Furthermore, according to Noether's theorem if a Lagrangian is conformally invariant, then the trace of the energy-momentum tensor should be null [2, 15]. Other efforts were also experienced to search for invariant fermion equation or scattering amplitude [14, 16, 19], and even to apply it to nonlocal action [20, 21]. In recent years there have been surges using conformal invariance accompanied with chiral symmetry breaking in studying hadron-level strong interaction [22, 23]. So far it can almost be concluded that the conformal invariance is unreachable to known material fields, by which we may suspect the applicability of conformal group. However from another point of view, the perturbative renormalization results [24–28] shed much light on the relationship between the physical constants and scale transformation, that is, some of the constants vary regularly with the energy scale. It follows that the scale transformation can cause the variation of coupling constants and masses. That suggests that scaling transformation might be helpful in generating mass term in EoM mathematically. Whereas after delicate analysis, we find out that in our model, without the help of other generators of conformal group, the mass term cannot be generated solely by scaling transformation.

We analyze the mass term problem from two respects, the physical respect and math respect. On physical side, the mass generation ascribes to dynamical chiral symmetry breaking (DCSB) and Higgs mechanism. The effect of these two mechanisms is to turn vacuum or a massless particle from space-like (or light-cone like) to time-like, and thus makes particle massive. In view of our former result that a massive boson meets the condition $\vec{E}^2 - \vec{B}^2 \neq 0$, and of the Higgs mechanism that demands bosons occur first and then decay into fermions, we employ a shortcut process $\gamma + \gamma \longrightarrow e^+e^-$ [29–40] to analyze the transition from space-like (or light-cone like) to time-like. In the process we perceive, at least equivalently, one photon is stimulated by the other then becomes time-like and succeedingly decays into electron pairs. The stimulation might lead to $\vec{E}^2 - \vec{B}^2 \neq 0$. Such processes are truly relevant to our consideration due to the fact that Higgs boson was first evidenced by decaying into two-photon resonance. Such seemingly rough physics consideration is helpful in our treating on math side, where we will note only particular generators of conformal group are responsible for shifting space-time or energy-momentum from space-like to time-like [41], which is not able to be realized solely by the scaling transformation of conformal group. With the above cognition we find out that the answer to mass term problem points to one conjecture in our original paper, i.e an integral which we had not carried out.

The structure of this paper is as follows. In Section 2 we introduce two representations of 4-dimensional conformal group. The focus is on how to derive the spinor representation, which is actually employed as our nonlocal structure group. In Section 3 we link the two representations of conformal group by physical interaction vertices in the conventional sense, especially we elucidate the function of generators K_μ on chiral-like vertex $\gamma_\mu(1-\gamma_5)$, which is useful for our further discussion on mass term in EoM. Section 4 is dedicated to elaborating the mathematical processes of massless particles (bosons and fermions) getting massive and to deriving the mass term. Then follows the conclusions and discussions. In appendix A. we explain why a massive fermion spinor cannot be generated solely by the scaling transformation. The scaling transformation just transforms a spinor from helicity representation to spinor representation.

2 The Spatial and Spinor Representations for Conformal Group

In this section we separately present the spatial/operator representation and unitary/spinor representation of the 4-dimensional Conformal group [2, 42], including their generators and commutations among them. The spatial representation is mainly referencing to that of Ref. [42, 43] and the unitary representation is derived by applying Cartan method [3] to $SO(6) - SU(4)$ transform. The unitary representation is the focus of this section.

We start with the null vector space (Euclidean space, with the first and the sixth imaginary),

$$\sum_{a=1}^{6} \eta_a^2 = 0. \tag{1}$$

reserving which gives the popular definition of conformal group [3]. A special expression of the differential forms in 4-dimension spatial representation can be derived directly from the above equation. In derivation we need to apply the following variables [42]

$$x_\mu = \frac{\eta_\mu}{K}\text{ , where } K = \eta_5 + i\,\eta_6\text{ , where } \mu = 1, 2, 3, 4 \tag{2}$$

together with the differential form

$$\frac{\partial}{\partial \eta_a} = \frac{1}{K}\left\{[\delta_{a\mu} - (\delta_{a5} + i\delta_{a6})x_\mu]\frac{\partial}{\partial x_\mu} + (\delta_{a5} + i\delta_{a6})K\frac{\partial}{\partial K}\right\}\text{ , where } a = 1, 2, \cdots, 6 \tag{3}$$

to the definition of 6-dimensional angular-momentum

$$M_{ab} = i\left(\eta_a\frac{\partial}{\partial \eta_b} - \eta_b\frac{\partial}{\partial \eta_a}\right)\text{, where } a, b = 1, 2, \cdots, 6. \tag{4}$$

Then one gets the following generators for conformal group [42] [of which in eq. (56)]

$$\begin{aligned}
D &= iM_{56} = -\left(\eta_5\frac{\partial}{\partial\eta_6} - \eta_6\frac{\partial}{\partial\eta_5}\right) = i\left(x_\mu\frac{\partial}{\partial x_\mu} - K\frac{\partial}{\partial K}\right),\\
P_\mu &= M_{5\mu} + iM_{6\mu} = i\frac{\partial}{\partial x_\mu}, K_\mu = M_{5\mu} - iM_{6\mu}\\
&= i\left\{-x^2\frac{\partial}{\partial x_\mu} + 2x_\mu x_\nu\frac{\partial}{\partial x_\nu} - 2K\,x_\mu\frac{\partial}{\partial K}\right\},
\end{aligned} \tag{5}$$

The projected form (making K as constant boundary of Minkowski space[4]) shifting to Minkowski convention then is

$$\begin{aligned}
D &= i\,x_\mu\frac{\partial}{\partial x_\mu}, M_{\mu\nu} = i\left(x_\mu\frac{\partial}{\partial x^\nu} - x_\nu\frac{\partial}{\partial x^\mu}\right),\\
P_\mu &= i\frac{\partial}{\partial x^\mu}\,, K_\mu = -i\left(x^2\frac{\partial}{\partial x^\mu} - 2x_\mu x^\nu\frac{\partial}{\partial x^\nu}\right),
\end{aligned} \tag{6}$$

where $M_{\mu\nu}$ represent the components of conventional angular momentum in 4-dimension. The corresponding commutation relation can be obtained by direct computation,

$$\begin{aligned}
[M_{\mu\nu}, M_{\rho\sigma}] &= i(g_{\nu\rho}M_{\mu\sigma} + g_{\mu\sigma}M_{\nu\rho} - g_{\mu\rho}M_{\nu\sigma} - g_{\nu\sigma}M_{\mu\rho}),\\
[M_{\mu\nu}, P_\rho] &= i(g_{\nu\rho}P_\mu - g_{\mu\rho}P_\nu),\\
[D, P_\mu] &= -iP_\mu, [D, K_\mu] = iK_\mu,\\
[D, M_{\mu\nu}] &= 0\\
[M_{\mu\nu}, K_\rho] &= i(g_{\nu\rho}K_\mu - g_{\mu\rho}K_\nu) \qquad (7)\\
[P_\mu, K_\rho] &= -2i\,(g_{\mu\rho}\,D + M_{\mu\rho}) \qquad (8)
\end{aligned}$$

Before using Cartan method to achieve its unitary representation, let's review first the steps of Cartan method with $SO(3) - SU(2)$ mapping as an example [3][of which in pp. 41–48]. To keep the invariance of $x_1^2 + x_2^2 + x_3^2 = 0$, one defines the matrix

$$X = \begin{pmatrix} x_3 & x_1 - i\,x_2 \\ x_1 + i\,x_2 & -x_3 \end{pmatrix}. \tag{9}$$

The trace $\mathrm{Tr}(X^\dagger X)$ is $x_1^2 + x_2^2 + x_3^2$. With U as an element of $SU(2)$ group, we define

$$X' = U^{-1}XU\,, \tag{10}$$

immediately we have

$$Tr(X'^\dagger X') = Tr(X^\dagger X)\,, \tag{11}$$

thus $SU(2)$ group keeps the trace invariant, and by this way the group also keeps the metric $x_1^2 + x_2^2 + x_3^2$. With the knowledge that the $SO(3)$ group directly reserves the metric $x_1^2 + x_2^2 + x_3^2$, we conclude that Cartan matrix X acts as a mapping between $SO(3)$ and $SU(2)$. By the Cartan Matrix X, one can define spinor $\begin{pmatrix}\xi_0\\ \xi_1\end{pmatrix}$ by

$$X\begin{pmatrix}\xi_0\\ \xi_1\end{pmatrix} = 0\,, \tag{12}$$

with the solution $\xi_0 = \pm\sqrt{\frac{x_1 - i\,x_2}{2}}$ and $\xi_1 = \pm\sqrt{\frac{-x_1 - i\,x_2}{2}}$, and the reverse yields

$$\begin{aligned} x_1 &= \xi_0^2 - \xi_1^2 \\ x_2 &= i\left(\xi_0^2 + \xi_1^2\right) \\ x_3 &= -2\xi_0\xi_1 \,, \end{aligned} \tag{13}$$

which automatically satisfies $x_1^2 + x_2^2 + x_3^2 = 0$ from which we can define the spinor reversely.

From the above Cartan matrix X we can extract the Pauli matrices $\sigma_1, \sigma_2, \sigma_3$ separately from the coefficients of x_1, x_2, x_3. Meanwhile Pauli matrices $\sigma_1, \sigma_2, \sigma_3$ act as the generators of $SU(2)$ group mentioned above. Furthermore it is easy to test that $SU(2)$ group reserves the metric

$$\mid \xi_0 \mid^2 + \mid \xi_1 \mid^2 = \Xi^\dagger \Xi \,. \tag{14}$$

And coincidentally the $n-$vectors form (defined in (21)) based on Pauli matrices don't generate new matrices, neither the multiplications nor the commutations among them, because they themselves are closed. Now in what follows we would find the corresponding Cartan matrix from $SO(6)$ to $SU(4)/SU(2,2)$, namely the spinor representation for 4-dimension Conformal group.

To achieve its unitary/spinor representation in 4-dimension, mimicking the relationship between the metric $x_1^2 + x_2^2 + x_3^2$ and that in (14), we shall associate the metric in (1) with the invariant quadratic form

$$\mid z_1 \mid^2 + \mid z_2 \mid^2 + \mid z_3 \mid^2 + \mid z_4 \mid^2 = Z^\dagger Z \,, \tag{15}$$

by the following matrix [44],

$$A = \begin{pmatrix} 0 & x_1 + i\,x_2 & x_3 + i\,x_4 & x_5 + i\,x_6 \\ -(x_1 + i\,x_2) & 0 & x_5 - i\,x_6 & -x_3 + i\,x_4 \\ -(x_3 + i\,x_4) & -x_5 + i\,x_6 & 0 & x_1 - i\,x_2 \\ -(x_5 + i\,x_6) & x_3 - i\,x_4 & -x_1 + i\,x_2 & 0 \end{pmatrix} . \tag{16}$$

Count the degrees of freedom of the groups conserving separately (1) and (15), one finds they are both 15. Next we only need to extract the coefficients before x_i's to get the unitary matrices as generators of $SU(4)$, just like the method used in three dimension example (7–13). If we want to get the generators of $SU(2,2)$ we need only to change the signs before x_1 and x_2 and those ahead of corresponding matrices, which would change the (1) and (15) to

$$-x_1^2 - x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2 = 0 \ . \tag{17}$$

and

$$-\mid z_1 \mid^2 - \mid z_2 \mid^2 + \mid z_3 \mid^2 + \mid z_4 \mid^2 = Z^\dagger Z \,. \tag{18}$$

the latter falls into Dirac spinor like

$$\tilde{\psi} = (z_1, z_2, z_3, z_4) \,.$$

It can be examined that the matrix A in (16) meets the invariant expression

$$Tr(A^\dagger A) = 4\left(x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2\right) \tag{19}$$

just like the above 3-dimension example, while the $SU(4)$ group keeps the above trace $x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2 = constant$, it simultaneously reserves the metric (15). The above method of linking real metric to a matrix is closely analogous to the Cartan method of constructing a spinor representation in any real space. Actually, the true spinor space for 4-d conformal group following Cartan method should be of 8-dimension instead of 4-dimension [3][of which in pp. 88–89]. In what follows we would take over the process of deriving all of the n-vectors along the Cartan method [3][of which in pp. 81–83], though we work in 4-dimension rather than 8-dimension. First we extract the matrices before x_i's in (16) , $i.e.$1−vectors,

$$\begin{aligned} B_1 &= \begin{pmatrix} i\,\sigma_2 & 0 \\ 0 & i\,\sigma_2 \end{pmatrix}, B_2 = \begin{pmatrix} -\,\sigma_2 & 0 \\ 0 & \sigma_2 \end{pmatrix}, \\ B_3 &= \begin{pmatrix} 0 & \sigma_3 \\ -\,\sigma_3 & 0 \end{pmatrix}, B_4 = \begin{pmatrix} 0 & i\,I \\ -i\,I & 0 \end{pmatrix}, \\ B_5 &= \begin{pmatrix} 0 & \sigma_1 \\ -\,\sigma_1 & 0 \end{pmatrix}, B_6 = \begin{pmatrix} 0 & -\sigma_2 \\ -\,\sigma_2 & 0 \end{pmatrix}. \end{aligned} \tag{20}$$

where σ_i's are Pauli matrices. The definition of k-vector is

$$B_{k-vector} = \sum_P (-1)^P B_{n_1} B_{n_2} \cdots B_{n_k}\ , \tag{21}$$

where P denotes different permutations. Apply the above formula to 2-vector, and use the corresponding subscripts to denote the 1-vectors involved, then

$$B_{12} = B_1 B_2 - B_2 B_1 = \begin{pmatrix} i\,\sigma_2 & 0 \\ 0 & i\,\sigma_2 \end{pmatrix} \begin{pmatrix} -\,\sigma_2 & 0 \\ 0 & \sigma_2 \end{pmatrix} - \begin{pmatrix} -\,\sigma_2 & 0 \\ 0 & \sigma_2 \end{pmatrix} \begin{pmatrix} i\,\sigma_2 & 0 \\ 0 & i\,\sigma_2 \end{pmatrix} = 0\,.$$

Similarly, let's exhaust all possibilities, then obtain other nontrivial 2-vectors

$$\begin{aligned} B_{13} &= 2\begin{pmatrix} 0 & -\,\sigma_1 \\ \sigma_1 & 0 \end{pmatrix}, B_{15} = 2\begin{pmatrix} 0 & \sigma_3 \\ -\,\sigma_3 & 0 \end{pmatrix}, \\ B_{35} &= 2i\begin{pmatrix} \sigma_2 & 0 \\ 0 & \sigma_2 \end{pmatrix}, B_{36} = 2i\begin{pmatrix} \sigma_1 & 0 \\ 0 & -\sigma_1 \end{pmatrix}, \\ B_{46} &= -2i\begin{pmatrix} \sigma_2 & 0 \\ 0 & -\sigma_2 \end{pmatrix}, B_{24} = 2i\begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix}, \\ B_{23} &= -2i\begin{pmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{pmatrix}. \end{aligned} \tag{22}$$

We note that the new ones which are independent of B_i's are just B_{23} and B_{36}. The same line can be followed to carry out the 3-vectors. Ignoring the repeating ones, we find the new 3-vectors independent of both 1-vectors and 2-vectors are

$$\begin{aligned} B_{123} &\sim \begin{pmatrix} 0 & \sigma_3 \\ \sigma_3 & 0 \end{pmatrix}, B_{134} \sim \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_1 \end{pmatrix}, \\ B_{145} &\sim \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}, B_{245} \sim \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\,\sigma_3 \end{pmatrix}, \\ B_{345} &\sim \begin{pmatrix} 0 & \sigma_2 \\ -\,\sigma_2 & 0 \end{pmatrix}, B_{146} \sim \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \\ B_{124} &\sim \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}. \end{aligned} \tag{23}$$

Computing the 4-vectors and the higher ones would not give new independent matrices. Finally, we can rearrange all above k-vector-produced matrices as follows [44],

$$
\begin{aligned}
U_i &= \frac{1}{2}\begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix} \\
V_\mu &= -\frac{1}{2}\begin{pmatrix} \sigma_\mu & 0 \\ 0 & -\sigma_\mu \end{pmatrix} \\
W_\mu &= \frac{i}{2}\begin{pmatrix} 0 & \sigma_\mu \\ \sigma_\mu & 0 \end{pmatrix} \\
Y_\mu &= \frac{1}{2}\begin{pmatrix} 0 & \sigma_\mu \\ -\sigma_\mu & 0 \end{pmatrix}
\end{aligned} \tag{24}
$$

where σ_i, $i = 1, 2, 3$ are normal Pauli matrices and $\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. The convention can be changed from Minkowski to Euclidean spaces while alternatively requiring $\sigma_\mu^2 = -1$, i.e. making $\sigma_0 = i$ and replacing definition of σ_i by those in [44]. The route of inquiring the concrete matrices following Cartan method as above could be a shortcut that rarely mentioned in literature. It is can be checked that the commutations among U_i, V_μ, W_μ, Y_μ are just those for conformal group [42, 43], accordingly the mapping from these matrices to differential forms turns out to be

$$
\begin{aligned}
U_i &\leftrightarrow \gamma_i\gamma_j \longrightarrow i\left(x_j\frac{\partial}{\partial x^k} - x_k\frac{\partial}{\partial x^j}\right) \longrightarrow M_{jk} \\
W_i &\leftrightarrow \gamma_0\gamma_i \longrightarrow i\left(x_i\frac{\partial}{\partial x^0} - x_0\frac{\partial}{\partial x^i}\right) \longrightarrow M_{0k} \\
W_0 &\leftrightarrow \gamma_5 \longrightarrow i\, x_\mu\frac{\partial}{\partial x_\mu} \longrightarrow D \\
V_\mu + Y_\mu &\leftrightarrow \gamma_\mu(1-\gamma_5) \longrightarrow i\frac{\partial}{\partial x^\mu} \longrightarrow P_\mu \\
V_\mu - Y_\mu &\leftrightarrow \gamma_\mu(1+\gamma_5) \longrightarrow -i\left(\frac{1}{2}x_\nu x^\nu\frac{\partial}{\partial x_\mu} - x_\mu x_\nu\frac{\partial}{\partial x_\nu}\right) \longrightarrow K_\mu.
\end{aligned} \tag{25}
$$

We use $\longrightarrow$ to represent the accurate mappings and $\leftrightarrow$ the equivalence. After examining the commutations by computer, we have to extend the matrix of operator D to be $1 + \gamma_5$, due to the commutation between P_μ and K_μ.

3 The Physical Relationship Between the Two Representations of Conformal Group

In this section we use scaling transformation as paradigm to investigate the affection of conformal group on interaction vertex. Enlightened by applying Lorentz transformation to Dirac equation, we first try to link physically the spatial form of scaling transformation with its spinor/unitary form, the former representing the realistic expansions and contractions (dilatation and shrinkage) of space-time, the latter representing the intrinsic freedom closely analogous to spin angular momentum. Then we will generalize the results from scaling transformation to other generators of conformal group. By this way we analyze the structure of conformal group and its relationship with the vertices of conventional quantum field theory, and judge the concrete physical significance of every block of the conformal group.

As for Lorentz transformation, the transformation matrix $\left(\Lambda^{\nu}_{\ \mu}\right)$ for $j^{\mu}(y) = \bar{\psi}(y)\gamma^{\mu}\psi(y)$ corresponds to a complex transformation S for $\psi(y)$ so that the effect of the transformed result $\bar{\psi}(y)S^{-1}\gamma^{\mu}S\psi(y)$ is equivalent to $\bar{\psi}(y)\Lambda^{\mu}_{\ \nu}\gamma^{\nu}\psi(y)$. Referencing the case of Lorentz transformation, our goal in this section is to search for the corresponding vertex-form Γ^{μ} so that it links with transformation S' by $S'^{-1}\Gamma^{\mu}S' = \Lambda'^{\mu}_{\ \nu}\Gamma^{\nu}$, where $S' = e^{\frac{u}{2}(1+\gamma_5)}$, $1+\gamma_5$ is the spinor representation of the scaling operator D, and Λ'^{μ}_{ν} represent tensor's components of scaling transformation. The similar method was used in a previous paper [45], but for totally different motivation.

Usually we perform the spatial Lorentz transformation on the vectors A_{μ} and γ^{μ}. Obviously this combination brings about invariant formalism like $A^{\nu}(q^2)\bar{\psi}(p)\gamma_{\nu}\psi(p')$. We follow the convention that the same set of $\{\gamma^{\mu}\}$ is used in different coordinate systems, which naturally yields an equivalence transformation S satisfying [46, 47]

$$S^{-1}\gamma^{\mu}S = \Lambda^{\mu}_{\nu}\gamma^{\nu} = \gamma'^{\mu}, \tag{26}$$

where Λ^{μ}_{ν} stand for the tensors' components of the Lorentz transformation. Substituting (26) into $A_{\mu}(x)\bar{\psi}(x)\gamma^{\mu}\psi(x)$ yields

$$A'_{\mu}(y)\bar{\psi}'(y)S^{-1}\gamma^{\mu}S\psi'(y) = A'_{\mu}(y)\bar{\psi}(y)\gamma'^{\mu}\psi(y). \tag{27}$$

While looking for Γ^{μ} we would follow the same convention as that in the above paragraph, i.e., in different coordinate system we use the same set of $\{\Gamma^{\mu}\}$. Then analogously, we use the form of the above formula (26) for scaling transformation as

$$S'^{-1}\Gamma^{\mu}S' = \Lambda'^{\mu}_{\nu}\Gamma^{\nu}, \tag{28}$$

where formally we have used Λ'^{μ}_{ν} to represent the scaling transformation to every coordinate component [14, 16, 19][of which (2)] instead of using the usual form $e^{-\alpha}$ [43]. Different from the operator $\mu\frac{\mathrm{d}}{\mathrm{d}\mu}$ appearing in renormalization group equation [25, 27, 28]

$$\mu\frac{\mathrm{d}\Lambda_R}{\mathrm{d}\mu} + \gamma_F\,\Lambda_R = 0\,, \tag{29}$$

where γ_F is the anomalous scaling dimension defined by

$$\gamma_F = \mu\frac{\mathrm{d}}{\mathrm{d}\mu}\ln Z_F\ ,$$

here the operator D has the usual form $D = i\,x^{\nu}\partial_{\nu}$, being a hermit one. With the relation $e^{-i\,\alpha\,D}p_{\mu}e^{i\,\alpha\,D} = e^{-\alpha}p_{\mu}$, i.e. $[D,\ p_{\mu}] = -i\ p_{\mu}$ [43], we have

$$(\Gamma^{\mu}p_{\mu})'_{scaling\,transform} = S'^{-1}\Gamma^{\mu}S'\Lambda'^{\nu}_{\ \mu}\,p_{\nu} = S'^{-1}\Gamma^{\mu}S'\ e^{-i\,\alpha\,D}p_{\mu}e^{i\,\alpha\,D}. \tag{30}$$

Now let's submit $S' = e^{\frac{u}{2}\gamma_5}$ obtained from the last section (henceforth we use $e^{\frac{u}{2}\gamma_5}$ instead of $e^{\frac{u}{2}(1+\gamma_5)}$ as scaling transformation while no confusion occurs), where u is the infinitesimal parameter. Formally we get

$$\begin{aligned} S'^{-1}\Gamma^{\mu}S'\Lambda'^{\nu}_{\ \mu}\,p_{\nu} &= e^{-\frac{u}{2}\gamma_5}\Gamma^{\mu}e^{\frac{u}{2}\gamma_5}(p_{\mu})'_{scaling\,transform} \\ &= e^{-\frac{u}{2}\gamma_5}\Gamma^{\mu}e^{\frac{u}{2}\gamma_5}e^{-i\,\alpha\,D}p_{\mu}e^{i\,\alpha\,D} \\ &= e^{-\frac{u}{2}\gamma_5}\Gamma^{\mu}e^{\frac{u}{2}\gamma_5}e^{-\alpha}p_{\mu} \\ &\doteq e^{-\frac{u}{2}\gamma_5}\Gamma^{\mu}e^{\frac{u}{2}\gamma_5}p_{\mu}(1-\alpha)\,. \end{aligned} \tag{31}$$

From the experience of calculating $\gamma-$matrix and the following relations

$$e^{-\frac{u}{2}\gamma_5}\gamma^\mu e^{\frac{u}{2}\gamma_5} \simeq \left(1-\frac{u}{2}\gamma_5\right)\gamma^\mu\left(1+\frac{u}{2}\gamma_5\right) \simeq \gamma^\mu + u\,\gamma^\mu\gamma_5\,, \tag{32}$$

$$e^{-\frac{u}{2}\gamma_5}\gamma^\mu\gamma_5 e^{\frac{u}{2}\gamma_5} \simeq \left(1-\frac{u}{2}\gamma_5\right)\gamma^\mu\gamma_5\left(1+\frac{u}{2}\gamma_5\right) \simeq \gamma^\mu\gamma_5 + u\,\gamma^\mu\,, \tag{33}$$

$$e^{-\frac{u}{2}\gamma_5}\gamma^\mu(1\pm\gamma_5)e^{\frac{u}{2}\gamma_5} \simeq \left(1-\frac{u}{2}\gamma_5\right)\gamma^\mu(1\pm\gamma_5)\left(1+\frac{u}{2}\gamma_5\right) \simeq (1\pm u)\gamma^\mu(1\pm\gamma_5)\,, \tag{34}$$

we find out a possible form of Γ^μ

$$\Gamma^\mu = \gamma^\mu(1\pm\gamma_5) \text{ while } \alpha \sim u\,. \tag{35}$$

The transformation in (32, 33, 34) can be replaced by $e^{-\frac{u}{2}(1+\gamma_5)}$ and $e^{\frac{u}{2}(1+\gamma_5)}$, the results would be the same obviously. The coefficients $(1\pm u)$ of (34) can be contracted now to be 1 with coefficients $(1\mp u)$ that come from the transformation of p_μ. And we note that the infinitesimal parameters u and α are not independent. By this way we set up the relationship between the operator D and $S' = e^{\frac{u}{2}(1+\gamma_5)}$ directly.

And we know $S' = e^{\frac{u}{2}(1+\gamma_5)}$ is acting on Dirac spinor as expected. The same transformation holds for vertex $\Gamma^\mu A_\mu$, as well as $\Gamma^\mu p_\mu$. The resultant vertex is different from that of Ref. [19] due to the choice of γ_5, since we have followed the convention of Quantum Field Theory. In fact we have extended the transformation, the interaction vertex and spinor space simultaneously, and these elements can be extended further while we involving more generators of conformal group.

What if we perform the scaling transformation S' succeedingly N times upon the vector vertex-form γ^μ. Different from (32, 33, 34), now we employ the following formulism without approximation

$$\left(e^{-\frac{u}{2}(1+\gamma_5)}\right)^N \gamma^\mu \left(e^{\frac{u}{2}(1+\gamma_5)}\right)^N = \gamma^\mu\cosh Nu + \gamma^\mu\gamma_5\sinh Nu\;, \tag{36}$$

from which one notes that the vector vertex arrives at its limits $\gamma^\mu(1\pm\gamma_5)$ only if $\frac{\cosh Nu}{\sinh Nu} \to \pm 1$, i.e. $Nu \to \pm\infty$. $Nu \to \pm\infty$ means one carrying out enough steps of inflating or shrinking transformation. We call such states involving interaction vertices $\gamma^\mu(1\pm\gamma_5)$ as extreme states, which evolve from the interaction vertex γ^μ after the scale constantly changing. And the variation of coupling constant is assumed to be absorbed into coupling constant. It turns out that such scaling transformation doesn't conserve the vector-dominant interaction, or alternatively, the transformation tends to transform the relating spinor from a normal one to a chiral one.

We have derived the above limits with the relation $e^{-i\alpha D}p_\mu e^{i\alpha D} = e^{-\alpha}p_\mu$, i.e. $[D,\,p_\mu] = -i\,p_\mu$ [43], The P_μ plays the role of the production operator or annihilation operator of D from the point of view of quantum mechanics. Analogously, we may notice another similar relationship among conformal group by the commutation $[M_{\mu\nu}, K_\rho] = i(g_{\nu\rho}K_\mu - g_{\mu\rho}K_\nu)$, however we fail to get the variation of vertex $\gamma^\rho(1\pm\gamma_5)$ with the transformation $e^{\frac{u}{2}\gamma_\mu\gamma_\nu}$ (operation of $M_{\mu\nu}$ in spinor space). So the analog cannot be followed by any other commutations of conformal group. The details are as follows. While the angular momentum transformation is written to be $S = e^{\frac{u}{2}\gamma_\mu\gamma_\nu}$, its effect on vector vertex γ^μ [we label it by Γ^μ]

$$\begin{aligned}\Gamma^{\mu\prime} &= e^{-\frac{u}{2}\gamma_\rho\gamma_\sigma}\Gamma^\mu e^{\frac{u}{2}\gamma_\rho\gamma_\sigma} \approx \left(1-\frac{u}{2}\gamma_\rho\gamma_\sigma\right)\Gamma^\mu\left(1+\frac{u}{2}\gamma_\rho\gamma_\sigma\right) \approx \gamma^\mu + u\left(g^\mu_\rho\gamma_\sigma - g^\mu_\sigma\gamma_\rho\right)\\ P'_\mu &= e^{-iaM_{\rho\sigma}}P_\mu e^{iaM_{\rho\sigma}} = P_\mu + ia[P_\mu, M_{\rho\sigma}] = P_\mu - a(g_{\mu\rho}P_\sigma - g_{\mu\sigma}P_\rho).\end{aligned} \tag{37}$$

Combining the above transformations yields

$$\begin{aligned}\left(\Gamma^\mu P_\mu\right)' &= \left[\gamma^\mu + u\left(g^\mu_\rho\gamma_\sigma - g^\mu_\sigma\gamma_\rho\right)\right][P_\mu - a(g_{\mu\rho}P_\sigma - g_{\mu\sigma}P_\rho)]\\ &= \Gamma^\mu P_\mu + u\left(g^\mu_\rho\gamma_\sigma - g^\mu_\sigma\gamma_\rho\right)P_\mu - a\gamma^\mu(g_{\mu\rho}P_\sigma - g_{\mu\sigma}P_\rho)\\ &= \Gamma^\mu P_\mu + (u+a)(\gamma_\sigma P_\rho - \gamma_\rho P_\sigma)\,,\end{aligned} \tag{38}$$

when $u \sim -a$, one obtains $\left(\Gamma^\mu P_\mu\right)' = \Gamma^\mu P_\mu$. If replacing γ^μ axil-vector vertex yeilds $\gamma^\mu(1\pm\gamma_5)$, we would get the same results

$$\begin{aligned}\Gamma^{\mu\prime} &= e^{-\frac{u}{2}\gamma_\rho\gamma_\sigma}\Gamma^\mu e^{\frac{u}{2}\gamma_\rho\gamma_\sigma} \approx \left(1-\frac{u}{2}\gamma_\rho\gamma_\sigma\right)\Gamma^\mu\left(1+\frac{u}{2}\gamma_\rho\gamma_\sigma\right) \approx \Gamma^\mu + u\left(g^\mu_\rho\Gamma_\sigma - g^\mu_\sigma\Gamma_\rho\right)\\ P'_\mu &= e^{-iaM_{\rho\sigma}}P_\mu e^{iaM_{\rho\sigma}} = P_\mu + ia[P_\mu, M_{\rho\sigma}] = P_\mu - a(g_{\mu\rho}P_\sigma - g_{\mu\sigma}P_\rho)\,.\end{aligned} \tag{39}$$

Combining the above transformations yields

$$\left(\Gamma^\mu P_\mu\right)' = \Gamma^\mu P_\mu + (u+a)(\Gamma_\sigma P_\rho - \Gamma_\rho P_\sigma)\,. \tag{40}$$

As for the kinetic term of an extended particle in the extreme state where the scale transformation is repeated infinite times, the momentum becomes light-cone like and the kinetic mass tends to zero since,

$$m^2_{kinetic} = (\Gamma_\mu p^\mu)(\Gamma_\nu p^\nu) = \gamma_\mu(1-\gamma_5)p^\mu\gamma_\nu(1-\gamma_5)p^\nu = p^2(1+\gamma_5)(1-\gamma_5) = 0\,, \tag{41}$$

here the $m^2 = 0$ may just have comparable meaning while its momentum is very large and its mass can be ignored approximately, we call such mass as kinematic mass. So we conclude the extreme vertices $\gamma^\mu(1\pm\gamma_5)$ give rise to no mass, and to make a fermion massive, we have to use the group method to make the interaction vertex leave the form $\gamma^\mu(1\pm\gamma_5)$. However since that the limits $\gamma^\mu(1\pm\gamma_5)$ of interaction form are invariant under transformation $e^{\frac{u}{2}(1+\gamma_5)}$, it is impossible for $e^{\frac{u}{2}(1+\gamma_5)}$ to make the extreme states $\gamma^\mu(1\pm\gamma_5)$ leave its form. To make a fermion leave this state to get mass, it is necessary to consider further the other generators of conformal group. Whereas according to our knowledge of commutations among the generators of conformal group, there is no much room to choose another generator. For the vertex $\gamma^\mu(1-\gamma_5)$ we may choose the generators K_μ, otherwise for $\gamma^\mu(1+\gamma_5)$ we may choose P_μ. For example, for $\Gamma^\mu = \gamma^\mu(1-\gamma_5)$ we have,

$$\begin{aligned}\Gamma^{\mu\prime} &= e^{\frac{u}{2}\gamma_\rho(1+\gamma_5)}\gamma^\mu(1-\gamma_5)e^{\frac{u}{2}\gamma_\rho(1+\gamma_5)}\\ &= [1+\frac{u}{2}\gamma_\rho(1+\gamma_5)]\gamma^\mu(1-\gamma_5)[1-\frac{u}{2}\gamma_\rho(1+\gamma_5)]\\ &= \gamma^\mu(1-\gamma_5) + \frac{u}{2}\left[\gamma_\rho(1+\gamma_5), \gamma^\mu(1-\gamma_5)\right] - \frac{u^2}{4}\gamma_\rho(1+\gamma_5)\gamma^\mu(1-\gamma_5)\gamma_\rho(1+\gamma_5)\\ &= \gamma^\mu(1-\gamma_5) + 2u\left[g^\mu_\rho(1-\gamma_5) - \gamma^\mu\gamma_\rho\right] - u^2\gamma_\rho\gamma^\mu\gamma_\rho(1+\gamma_5)\,,\end{aligned} \tag{42}$$

when $u \longrightarrow 0$, $\Gamma^{\mu\prime} \longrightarrow \left(\gamma^\mu + 2ug^\mu_\rho\right)(1-\gamma_5) - 2u\gamma^\mu\gamma_\rho$ and when $u \longrightarrow \infty$, $\Gamma^{\mu\prime} \longrightarrow -u^2\gamma_\rho\gamma^\mu\gamma_\rho(1+\gamma_5)$. Then combine the above equation with the spatial transformation $P'_\mu = e^{-iaK_\rho}P_\mu e^{iaK_\rho} \approx [1-iaK_\rho]P_\mu[1+iaK_\rho] = P_\mu + ia[P_\mu, K_\rho] = P_\mu + 2a(g_{\mu\rho}D - M_{\mu\rho})$, we have

$$\begin{aligned}\left(\Gamma^\mu P_\mu\right)' &\approx \left[\left(\gamma^\mu + 2ug^\mu_\rho\right)(1-\gamma_5) - 2u\gamma^\mu\gamma_\rho\right]\left[P_\mu + 2a(g_{\mu\rho}D - M_{\mu\rho})\right] \\ &\approx \Gamma^\mu P_\mu + 2u\left[g^\mu_\rho(1-\gamma_5) - \gamma^\mu\gamma_\rho\right]P_\mu + 2a\gamma^\mu(1-\gamma_5)(g_{\mu\rho}D - M_{\mu\rho}) \quad (43)\end{aligned}$$

The generator P_μ would have similar effect on vertex $\gamma^\mu(1+\gamma_5)$. The generator $K_\mu(P_\mu)$ first makes the bosons getting mass by transmitting space-time from space-like to time-like, then it makes fermions (with chiral interaction vertex, meaning the fermions have no dynamical mass) leave the vertex, i.e. makes fermions massive.

4 Generating Fermion Mass Term with the Help of Conformal Group

Conventionally only linearly flat unitary-complex-space is used in quantum theory. In our model we have instead tried to express some of the effects, such as interaction or symmetries breaking etc., by curved complex-space in the strong interaction or strong correlation regime. Once a space is curved, particles become living in a larger space with larger symmetry group. Here we find that the space-curving can also be helpful in understanding the mass generation. Mathematically, we explain the mass generation in two steps. The first is how a boson becomes massive with the help of conformal transformation, the second is how a massless fermion gets massive from the massive boson. Coincidentally, we find the two steps could be ascribed to the same generators of conformal group. Furthermore, we will discuss the first step by photons pair production [29–40], $\gamma + \gamma \longrightarrow e^+e^-$. The previous works on photons pair production mainly focus on calculating the amplitude in semi-classical level. In this paper we don't get involved in the concrete calculation of the amplitude of the process, but instead focus on how the process could be in accordance with the inequality $\vec{E}^2 - \vec{B}^2 \neq 0$.

To interpret the first step, let's carry some simple calculation of energy-momentum of classical electromagnetic field. It is well known that the energy density of electromagnetic field is

$$\varpi = \frac{1}{2}(\vec{E}^2 + \vec{B}^2)\,,$$

and the momentum of electromagnetic fields is Poynting vector

$$\vec{S} = \vec{E} \times \vec{B}\,,$$

using the formula

$$(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a}\,,$$

we obtain

$$\begin{aligned}\varpi^2 - \vec{S}^2 &= \frac{1}{4}(\vec{E}^2 + \vec{B}^2)^2 - (\vec{E} \times \vec{B})^2 \\ &= \frac{1}{4}(\vec{E}^2 + \vec{B}^2)^2 - (\vec{E} \times \vec{B})\cdot(\vec{E} \times \vec{B})\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{4}(\vec{E}^2+\vec{B}^2)^2-(\vec{E}\times\vec{B}\times\vec{E})\cdot\vec{B}\\
&= \frac{1}{4}(\vec{E}^2+\vec{B}^2)^2-[(\vec{E}\cdot\vec{E})\vec{B}-(\vec{E}\cdot\vec{B})\vec{E}]\cdot\vec{B}\\
&= \frac{1}{4}(\vec{E}^4+2\vec{E}^2\vec{B}^2+\vec{B}^4)-(\vec{E}\cdot\vec{E})(\vec{B}\cdot\vec{B})\\
&= \frac{1}{4}(\vec{E}^4-2\vec{E}^2\vec{B}^2+\vec{B}^4)\\
&= \frac{1}{4}(\vec{E}^2-\vec{B}^2)^2\,,
\end{aligned}
$$

which to some extent mimics the general relationship of energy and momentum $p_0^2-\vec{p}^2=m^2$. And the result $\vec{E}^2-\vec{B}^2\neq 0$ corresponds to boson mass according to our conclusion of previous paper. We note that the free electromagnetic field satisfies $\varpi^2-\vec{S}^2=\frac{1}{4}(\vec{E}^2-\vec{B}^2)^2=0$ since $\mid\vec{E}\mid=\mid\vec{B}\mid$ in nature unit. And if the photons participate in the reaction $\gamma+\gamma\longrightarrow e^+e^-$, then maybe a photon meets $\vec{E}^2-\vec{B}^2\neq 0$, subsequently $(\vec{E}^2-\vec{B}^2)^2>0$ and thus $\varpi^2-\vec{S}^2>0$, which is the typical time-like relationship of energy-momentum. This suggests that somehow a photon has gained its mass by the stimulation of another photon. In view of the above analysis, we regard one ultra-high-energy (laser) photon stimulates another light-cone photon to make it transit to time-like [36–40] . We find the conformal group can be responsible for such transition from space-(light-cone-)like to time-like .

The second step is to transform a massless state "like neutrino" to a massive one "like electron" [48]. Since even for a massive boson it can decay into many massless energetic fermions, at least from mathematics it is possible. With the analysis of the last section, we note that the massless state seems to occur effectively at the limits of very high energy while the mass of fermion is very small comparing with the kinetic energy. At the limits, the fermions interact with each other by the vertex $\gamma^\mu(1\pm\gamma_5)$, coincidentally the neutrinos pick the $\gamma^\mu(1-\gamma_5)$. So the interaction vertex deviating from the form $\gamma^\mu(1-\gamma_5)$ is a sign of "neutrino" getting its mass. With the analysis of the previous section, we know that the dilatation transformation cannot play the role in varying the interaction vertex, since $e^{\frac{u}{2}(1+\gamma_5)}$ reserves the vertex $\gamma^\mu(1-\gamma_5)A_\mu$ [$e^{\frac{u}{2}(1+\gamma_5)}$ can only ex-change the spinor between helicity representation and spin representation. See Appendix A.]. Neither can the momentum transformation $M_{\mu\nu}=e^{\frac{u}{2}\gamma_\mu\gamma_\nu}$ change the vertex $\gamma^\mu(1-\gamma_5)$, as evidenced in the last section. So let's turn to the transformation $e^{\frac{u}{2}K_\nu}$, while the $e^{\frac{u}{2}K_\nu}$ keeps directly the vertex $\gamma^\mu(1-\gamma_5)$, it does not keep the whole form $\gamma^\mu(1-\gamma_5)A_\mu$ invariant, nor does the generator P_μ keep the whole form $\gamma^\mu(1+\gamma_5)A_\mu$. Thus we choose K_μ as a candidate to make $\gamma^\mu(1-\gamma_5)A_\mu$ leave $\gamma^\mu(1-\gamma_5)$ and begin to run. It runs to normal interaction vertex γ^μ, as shown in (43), which is for massive fermions.

Now let's come back to the equation of our model. In our previous paper [1], we have laid two ways of generating fermion mass term in EoM, which are just the possibilities from the viewpoint of mathematics. Now with the deep insight of physics, we recognize that the second one must be the candidate for the mass term, i.e. calculating a remaining integral in the equation. Now we will treat the candidate term $-i\int A_\nu\partial_\lambda\,\partial^\nu\psi\,dx^\lambda$ in the

EoM. Considering the hermit of the operators $i\partial_\lambda,\ i\partial^\nu$, and apart from the coefficients, we have

$$
\begin{aligned}
&\int A_\nu \partial_\lambda\, \partial^\nu \psi\, dx^\lambda \\
&= \int dx^\lambda \partial_\lambda\, (\partial^\nu A_\nu)\psi \\
&= \int (dt\frac{\partial}{\partial t} - d\vec{x}\cdot\vec{\nabla})\,(\frac{\partial}{\partial t}A_0 - \vec{\nabla}\cdot\vec{A})\psi\ .
\end{aligned}
\tag{44}
$$

If we omit the term $\vec{\nabla}\cdot\vec{A}$ by considering the Columb gauge as used previously [1], then the above equation yeilds

$$
\begin{aligned}
&\int (dt\frac{\partial}{\partial t} - d\vec{x}\cdot\vec{\nabla})\,(\frac{\partial}{\partial t}A_0)\psi \\
&= \int [dt\frac{\partial^2}{\partial t^2}A_0 - \frac{\partial}{\partial t}(d\vec{x}\cdot\vec{\nabla}A_0)]\psi \\
&= \int [dt\frac{\partial^2}{\partial t^2}A_0 - d\vec{x}\cdot\frac{\partial}{\partial t}(\vec{E} + \frac{\partial}{\partial t}\vec{A})]\psi \\
&= \int [dt\frac{\partial^2}{\partial t^2}A_0 - d\vec{x}\cdot\frac{\partial^2}{\partial t^2}\vec{A} - d\vec{x}\cdot(\vec{\nabla}\times\vec{B} - \vec{J})]\psi \\
&= \int [dt\frac{\partial^2}{\partial t^2}A_0 - d\vec{x}\cdot\frac{\partial^2}{\partial t^2}\vec{A} - d\vec{x}\cdot(\vec{\nabla}\times\vec{\nabla}\times\vec{A} - \vec{J})]\psi \\
&= \int [dt\frac{\partial^2}{\partial t^2}A_0 - d\vec{x}\cdot\frac{\partial^2}{\partial t^2}\vec{A} - d\vec{x}\cdot(-\vec{\nabla}^2\vec{A} - \vec{J})]\psi \\
&= \int [dt\frac{\partial^2}{\partial t^2}A_0 - d\vec{x}\cdot(\frac{\partial^2}{\partial t^2}\vec{A} - \vec{\nabla}^2\vec{A}) + d\vec{x}\cdot\vec{J}]\psi \\
&= \int [dt\frac{\partial^2}{\partial t^2}A_0 - d\vec{x}\cdot\Box\vec{A} + d\vec{x}\cdot\vec{J}]\psi \\
&= \int [dt\frac{\partial^2}{\partial t^2}A_0 + m^2 d\vec{x}\cdot\vec{A} + d\vec{x}\cdot\vec{J}]\psi \\
&= (\lambda_1 m^2\phi + \lambda_2 L)\psi\ ,
\end{aligned}
\tag{45}
$$

where we propose ψ is almost flat (thus unchanged) under perturbative condition, ϕ is a flux relating to concrete situation, and L is an angular moment. The imaginary number i is absorbed by $d\vec{x}$ since we have carried the conformal transformation operated by K_μ, which makes space-time varying from space-like to time-like, i.e. isotropically interpreted by $x_\mu \to ix_\mu$ [20]. In the last two equalities of (45) we have used the equation referring to (9.27) of previous paper [1]

$$
\vec{A}\cdot\Box\vec{A} = \vec{B}^2 - \vec{E} \overset{d}{=} -m^2\vec{A}^2\ ,
\tag{46}
$$

where the equation holds only while the considered transverse filed (Coulomb gauge) has no source, thus no scalar component [47] (of the first chapter). Now we discuss the mass term problem under same situ, so follows the same conditions. The conditions seem not so general, however at least they reflect certain aspects of physics, actually the Coulomb gauge $\nabla \cdot \vec{A} = 0$ plus radiation fields method can resolve a wide class of pratical problems.

To understand the λ_1 and λ_2 physically, let's analyze the first term and the second term in the last equality of (45). The first term is more like for hadrons, or for another kind of composite particle. At first sight, the square of fermion mass being proportional to the square of boson mass looks strange. However, this might become possible if we look into Higgs-mechanism [47] (eqs. (19.8) and (19.4)), with relations $m_l = \upsilon g_l/\sqrt{2}$, and $m_W = \frac{1}{2}\upsilon g$, where g and g_l being adjusting parameters and υ a part of a scalar fields. The two relations suggest the reasonability of the formula like $m^2_{fermion} \propto m^2_{boson}$. On the other hand, we can analyze the dimension of the first term. Concerning the geometrical phase factor like $\exp[i\frac{e}{\hbar}\oint \vec{A}\cdot d\vec{r}]$, the dimension of magnetic flux is $\left[\frac{\hbar}{e}\right]$. And concerning the definition of magnetic moment $\vec{\mu} = \frac{e}{m_f}\vec{S}$ ($\vec{S}$ is the spin, m_f is the fermion mass), the dimension of magnetic moment is $\left[\frac{\hbar e}{m_f}\right]$. Therefore the ratio of the flux and magnetic moment has dimension like $\left[\frac{\hbar}{e}/\left(\frac{\hbar e}{m_f}\right)\right] = \left[\frac{m_f}{e^2}\right]$, i. e. if alternatively we write the first term as $\lambda_1'\frac{\phi}{\mu}$ (where μ is the absolute value of magnetic moment) then the $[\lambda_1' = \lambda_1 m\hbar e]$ is the proportional coeficient of flux and magnetic moment, which is coincident with the view that this mass term is for a composite particle. We also observe that in natural unit λ_1' differ from λ_1 in a mass factor, where λ_1 itself has the dimension reciprocal of magnetic flux. Please caution that although we constantly mention the electric or magnetic fields, they are not the true electromagnetic fields. We just employ the analorgy between Abelian field and non-Abelian field, our interest is non-Abelian field.

The second term may be for an elementary particle. The term mimics the result $M^2 \propto \mu^2 L + \alpha$ (μ^2, α constants) from Regge poles [49]. From this analogy the parameter λ_2 has the dimension of mass square, since angular momentum L is dimensionless. At present stage we may further surmise that the first term, which represents the mass of a hadron fermion and thus is dependent on interaction, may be related to DCSB [50, 51]. And the second term may possibly link with our suggested process $\gamma + \gamma \longrightarrow e^+e^-$, in which the photon in curved complex space is split into elementary particles. At the same time the electro-field and magnetic field are wrapped: the electro-field wrapping to form charges, and the magnetic field wrapping to form spins relating to masses. Thus both the first and second terms of the equation correspond to similar pictures, the picture of curved (wrapped) complex-space. This conclusion may help us make the mass problem calculable, not just as legitimate parameters confirmed by DCSB and Higgs mechanism. And also we should caution that here the obtained mass may be just "effective mass", since classically there is no system mimicking the strong interaction and strong correlation, and all that we have at hand are just the experiences of treating strong interaction in previous literature [52–54], which must be relevant to topology. And mostly topology is relating to magnetic flux.

5 Conclusions and Discussions

In this paper we have analyzed the effect of conformal group on interaction vertices, especially on the vector vertex and the chiral vertex, and then pointed out the relationship between conformal group and generation of mass term. If only concerning mass-term problem in EoM, the structure group would not be smaller than conformal group. Based on our analyses, the scaling transformation allows a special chiral-like vertex $\gamma^\mu(1 \pm \gamma_5)$ to be invariant. And normal vector vertex becomes running under such scaling transformation.

So with the scaling transformation only, one cannot endow a massless fermion like ”neutrino” with mass, since scaling transformation happens to keep the chiral interaction form, and cannot make fermions like ”neutrino” deviate from the vertex form [48]. So we have to involve other generators of conformal group to transform the interaction vertex, for example, the generator K_μ. And coincidentally, we find K_μ can also be responsible for transforming space-time from space-like to time-like, which in physics should occur before transforming fermions from massless to massive. With the above cognition, we get the mass term by directly calculating the integral occurred in original paper, which falls into one of the possibilities of our original surmise.

According to the formula we get, we present a plausible understanding of the generation of mass term. Since we have obtained the mass by directly calculating the integral in original equation, we believe the mass term is generated by the curvature (wrapped) of complex space. The wrapping of complex space shows its effect by deforming electro-field and magnetic field if we use the shortcut process $\gamma + \gamma \longrightarrow e^+e^-$ as paradigm. The progress implies that the photon in curved complex space is split into particle pairs, and at the same time the electro-field and magnetic-field of the photon are wrapped: the electro-field forms charge, and magnetic field forms spin. This conclusion may help us make the mass problem calculable, not just as legitimate parameters in DCSB and Higgs mechanism. Of course it is necessary to confirm physically whether spin or magnetic flux actually holds mass. And we should caution that here the obtained mass may be just ”effective mass”, which roots obviously in interaction. We conceive that this result might be helpful in understanding spectra problem of hadrons, whose mass may be proportional to (chromo-)magnetic flux. The further work along this line is in progress.

We have achieved the conclusion that in our theory the minimal group should be the entire conformal group. On mathematical side the conformal group has been investigated thoroughly from different aspects [9], and its application to physics especially to quantum field once was also widely considered. However the application is not so satisfactory because hitherto no other perfect quantum system than photon field [11, 12] has been found so that the corresponding Lagrangian is conformally invariant, unless the mass of the involved particles are null [2, 13–18]. In our treatment we turn from searching for invariant dynamics to how the conformal group makes physics quantities running, so as to match the renormalization results that some of the physics constants vary regularly with the energy scale, for instance charge, mass, and Green functions. According to our analysis, the scaling transformation can somehow cause variation of coupling constants and masses. And in this paper, we have also discussed the function of generators K_μ and P_μ which conformally change the interaction vertex from a chiral one to a non-chiral one. The action performed by generators P_μ or K_μ may be caused by the external non-hermite stimulation. Basically it is with such stimulations that the value of $\vec{E}^2 - \vec{B}^2$ changes from 0 to $\neq 0$. Then by certain decaying process, fermion mass is generated. In summary, we get to the conclusion that the running of interaction vertex corresponds to certain kinds of curving of complex space, which is certainly governed by generators of conformal group. Conformal group might live for curving, but not for invariance.

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Appendix A. How $e^{\frac{u}{2}\gamma_5}$ Exchange the Fermion Spinors Between Helicity Representation and Spin Representation

We recognize here that the performance of $e^{\frac{u}{2}\gamma_5}$ can only exchange the fermion spinors between helicity representation and spin representation, or vice versa. Accordingly such transformation cannot yield the generation of spinor, or the mass of fermions. In what follows we present the example of how $e^{\frac{u}{2}\gamma_5}$ changes the representation of spinors.

Despite of the algebric structure, while $m = 0$ the spinor could be any "vector-form" homogenously because the term $\begin{pmatrix} a & b & c & d \end{pmatrix} m \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$ is equal to null. Nevertheless, when $m \neq 0$, the term spinor(vector) $\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$ cannot be selected arbitrarily, since the condition $\begin{pmatrix} a & b & c & d \end{pmatrix} m \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = m$ should be satisfied. The most simple option is $\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, or some analog like $\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ can be used. Subsequently we will focus on the transformation from the special spinor $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ to its helicity form in arbitrary reference coordinates.

Firstly, we refer to the Dirac equation for a rest particle

$$\gamma_0 \psi = m\psi \ , \tag{47}$$

whose solution is $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ for normal particles, and $\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ for anti-particles. One also notes that $\begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} \gamma_0 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = m$ from (47). This knowledge legitimates the writing from the rhs to lhs of (47). That means we can generate spinor via mass term

$$m = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} m \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} \gamma_0 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} . \tag{48}$$

Assume that the motion of fermion is along z direction, one solution of Dirac equation $(\not p - m)\psi = 0$ in helicity representation is $\psi = \begin{pmatrix} sth \\ 0 \\ sth \\ 0 \end{pmatrix}$. Further the transformation responsible for the variation from $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ to $\begin{pmatrix} sth \\ 0 \\ sth \\ 0 \end{pmatrix}$ is $e^{\frac{u}{2}\gamma_5}$, explicitly

$$e^{\frac{u}{2}\gamma_5}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \cosh\frac{u}{2} \\ 0 \\ \sinh\frac{u}{2} \\ 0 \end{pmatrix}, \tag{49}$$

succeeding transformation $e^{\frac{u}{2}\gamma_5}$ could be employed if necessary. Then the resultant spinor doesn't satisfy the (47) any longer, which matches our common knowledge of quantum field theory.

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