Int J Theor Phys (2015) 54:4410–4422
DOI 10.1007/s10773-015-2618-y

# A Survey of the ESR Model for an Objective Reinterpretation of Quantum Mechanics

**Claudio Garola**

Received: 13 October 2014 / Accepted: 14 March 2015 / Published online: 28 March 2015


**Abstract** *Contextuality* and *nonlocality* (hence *nonobjectivity* of physical properties) are usually maintained to be unavoidable features of quantum mechanics (QM), following from its mathematical apparatus. Moreover they are considered as basic in quantum information processing. Nevertheless they raise still unsolved problems, as the *objectification problem* in the quantum theory of measurement. The *extended semantic realism* (*ESR*) *model* offers a way out from these difficulties by reinterpreting quantum probabilities as conditional rather than absolute and embedding the mathematical formalism of QM into a broader mathematical framework. A noncontextual hidden variables theory can then be constructed which justifies the assumptions introduced in the ESR model and proves its objectivity. Both linear and nonlinear time evolution occur in this model, depending on the physical environment, as in QM. In addition, the ESR model implies modified Bell's inequalities that do not necessarily conflict with QM, supplies different mathematical representations of proper and improper mixtures, provides a general framework in which the local interpretations of the GHZ experiment obtained by other authors are recovered, and supports an interpretation of quantum logic which avoids the introduction of the problematic notion of quantum truth.

**Keywords** Contextuality · Nonlocality · Objectification problem · Nonobjectivity · ESR model · "no-go" theorems · GHZ experiment · Quantum logic

## 1 Introduction

Since its birth quantum mechanics (QM) proved to be a theory of outstanding empirical success, but also a source of problems and paradoxes. These mainly follow from the proposed

✉ Claudio Garola
garola@le.infn.it

Department of Mathematics and Physics, University of Salento,
73100 Lecce, Italy

interpretations of the theory, which multiplied in time and are still debated. According to Busch et al. [1] these interpretation can be divided in two classes.

*Statistical interpretations*: QM refers to frequencies of *measurements outcomes* only. No reference to microscopic objects should enter its language.

"*Ontic"*, or "*realistic"*, *interpretations*:[1] QM deals with items of physical systems, or *individual objects*, and their (*physical*) *properties*.

The statistical interpretations avoid many problems but can be criticized from several viewpoints. They imply indeed an instrumentalist view and lack explanatory power. Moreover, nowadays experimental physicists often claim that they can deal with individual objects, not only with statistical ensembles.

The realistic interpretations can be reformulated avoiding ontological commitments if "individual object" is considered as a term of the theoretical language of QM, interpreted (via observational language) as a click in a preparing device. But in these interpretations, however reformulated, a crucial problem occurs: for every individual object in a given state, there exist properties that are *nonobjective* (roughly speaking, that do not preexist to a measurement). This feature of QM follows from "no-go" theorems as Bell-Kochen-Specker's [2, 3], which proves the *contextuality* of QM, and Bell's [4], which proves the *contextuality at a distance*, or *nonlocality*, of QM. Indeed, nonobjectivity has some well known intriguing consequences. First of all, the *objectification problem* in the quantum theory of measurement: if QM is a universal theory, nonobjectivity extends to the properties of macroscopic objects, against evidence (this problem is illustrated by famous paradoxes, as Schrödinger's cat, Wigner's friend, etc). Secondly, no intuitive model for QM can be provided (*wave-particle duality*), as every such model would imply objectivity of all properties. Thirdly, quantum probabilities are *nonepistemic*, that is, they do not bear an ignorance interpretation, for the values of nonobjective properties cannot be assigned independently of a measurement context, that is, independently of observation (hence some interpretative problems occur when classical and quantum probabilities are mixed: in particular, proper and improper mixtures have the same mathematical representation but different physical interpretations). Finally, no truth value can be assigned to a sentence attributing a property to an individual object in a given state of $\Omega$ if the property is nonobjective: hence, a non-classical notion of truth (*quantum truth*) seems to be required.

Notwithstanding the problematic consequences summarized above, contextuality and nonlocality are usually maintained to be distinguishing features of QM, independently of the foundational approach that is adopted (e.g., the quantum logical, the operational and the algebraic approach, Bohm's theory, etc.). Moreover, nowdays quantum information theory considers contextual and nonlocal correlations as basic resources for quantum information processing and inspires new foundational approaches, as Zeilinger's [5], Clifton-Bub-Halvorson's [6], etc. The acceptance of contextuality and nonlocality in all these approaches is based not only on theoretical reasons, as the "no-go" theorems, but also on a series of experimental results that started with the famous Aspect's experiments [7–9].

Philosophers of science know, however, that no set of experimental results may determine in a unique way a theory that explains them. Moreover, every "no-go" theorem follows from assumptions (some of which are often left implicit) that can be questioned. Several years ago a research was therefore started by the author, together with some collaborators,

[1]It must be noted that the term "realistic" is used here in a very weak sense. Well known interpretations or modifications of QM, as Bohm's theory, multi-world interpretations and GRW theory, are "realistic" in a much stronger sense and will not be considered in this paper.

with the aim of inquiring whether it was possible to recover objectivity by embedding the mathematical apparatus of QM into a broader mathematical framework and reinterpreting it in such a way to circumvent the "no-go" theorems. Of course, this new framework had to satisfy a basic requirement, that is, it had to explain the experimental results mentioned above and, more generally, the empirical success of QM. This research has been recently completed with the proposal of a new theory called *ESR* (*extended semantic realism*) *model* [10–19].

Let us resume some basic features and results of the ESR model that are discussed in this paper.

(i) The *fundamental equation* of the ESR model formalizes the intuitive idea that an essential nondetection can occur in every measurement, which depends on the set of all properties that are objective for the individual object on which the measurement is performed (such a set must be determined by the ESR model itself, as in QM) and is not a consequence of flaws or lack of efficiency of the real measuring apparatuses. Hence nondetection may occur also in the case of *idealized* (efficiency 1) measurements, and only measurements of this kind are considered in the ESR model, for the sake of simplicity. Based on the fundamental equation, the standard quantum probabilities are reinterpreted as referring only to the set of all individual objects that are detected when idealized measurements are performed (*conditional on detection* probabilities). This *fundamental assumption* of the ESR model allows us to recover the standard mathematical apparatus of QM within a broader mathematical framework, in which two new probabilities are introduced, a *detection probability* and an *overall probability*, the latter referring to the set of all individual objects that are produced (Section 3).

It must be stressed that we have no theory, at present, to justify the fundamental equation and assumption of the ESR model, hence to predict the values of the detection probabilities. Therefore these occur as parameters in the model (each parameter depending on the state of the individual object and on the property that is measured) to be determined experimentally. If this determination is done in a specific case, then the ESR model yields complete prediction of the remaining probabilities. The fundamental equation and assumption must be considered as a priori hypotheses, to be justified by their physical consequences, explanatory power and predictions.

(ii) The reinterpretation of quantum probabilities as conditional on detection has deep consequences. Indeed, it allows to construct a hidden variables model (Section 4) that provides a set-theoretical picture at a microscopic level from which the fundamental equation of the ESR model can be deduced, showing that all properties can be considered objective in this model (the exact meaning of the terms "property" and "objective" is specified in Section 2). It follows that the ESR model is a noncontextual, hence a local, theory, even if its formalism embodies the formalism of QM. The "no-go" theorems are thus circumvented. The deep reason of this result is that these theorems rest on the standard interpretation of quantum probabilities as absolute, and are no more valid if the foregoing reinterpretation is adopted. In particular, objectivity of properties implies *modified Bell's inequalities* that do not conflict with the (reinterpreted) quantum probabilities if the values of the detection probabilities do not exceed some upper values (Section 6).[2] Also in the case of the proofs of nonlocality that do not resort to inequalities, as Greenberger-Horne-Zeilinger's [30] or Mermin's [31], the aforesaid reinterpretation invalidates the proofs and leads to an explanation of the experimental results predicted by QM that does not imply nonlocality and coincides with similar explanations supplied by other authors [32] (see again Section 6).

(iii) The term "individual object" in the ESR model does not imply that individual objects can be pictured as point-like classical particles with trajectories (nor objectivity of properties implies such a classical model). Hence, the interference occurring in two slits experiments is predicted by the ESR model, which yields the same predictions of QM because of its fundamental assumption whenever only detected objects (which could be all produced objects in this case, see (v)) are taken into account.

(iv) Objectivity of physical properties implies that no "actualization" of physical properties that are only potential occurs when a measurement is performed. The Lüders postulate of QM is modified and recovered in the ESR model (Section 5) but it must not be interpreted as implying a "collapse of the wave function", as in QM. Rather, it describes the transformation of the state that occurs whenever an individual object interacts with a macroscopic measuring device. This description is then used as a starting point for hypothesizing the general laws of time evolution in the ESR model, that basically reproduce standard quantum laws (see again Section 5).

(v) Detection probabilities, which depend on the state of the individual objects and on the properties that are measured, are different whenever different physical systems are considered. We expect that they are very close to 1, or just 1, for massive particles, as heavy ions (which does not prohibit interference, see (iii)), while they should have lower upper limits for lighter particles. In the case of the modified Bell's inequalities rather restrictive assumptions lead to an upper value of 0.8165 for the efficiency of the electron's spin measuring devices in the Bohm's variant of the Einstein-Podolski-Rosen thought experiment [13]. We have not yet an estimate for different particles and experiments (such an estimate is not immediate if one does not want to introduce too restrictive assumptions) as several different detection probabilities enter the equations.

(vi) Finally, we anticipate that the predictions of the ESR model can be different from the predictions of QM whenever overall probabilities are considered, but may be very close to the latter and undistinguishable from them in real experiments if the values of the parameters (detection probabilities) are close to 1. If only individual objects that are detected are taken into account, as in Aspect's and following experiments, the predictions of the ESR model coincide with the predictions of QM. One can, however, conceive experiments that distinguish the two theories by considering proper mixtures, whose mathematical representation in the ESR model is different from the mathematical representation provided by QM (Section 5).

[2] An intuitive explanation of this result can be given as follows: the detection probabilities change with the measurement that is performed, and the subset of individual objects that is detected generally is not a fair sample of the set of all objects that are produced. This argument resembles the argument that has been often raised to question the results of Aspect's and similar experiments, in which *fair sampling* is usually assumed [20, 21]. But unfair sampling depends on the features of real measuring devices in the latter argument: hence it would not occur in the case of idealized measurements. It depends instead on the physical properties of the individual objects, hence on different physical variables, in the ESR model, so that it may occur also in the case of idealized measurements [22].

We add that the conventional interpretation of the "no-go" theorems has been questioned in particular by several authors in the framework of a statistical interpretation of QM that maintains the standard interpretation of quantum probabilities [23–27]. The criticism of this "statistical opposition" leads to avoid nonlocality of QM, while contextuality is preserved and explained, in the case of photons, by taking into account the

## 2 On the Notion of Nonobjectivity

To make the notion of nonobjectivity adopted in this paper more precise, let us firstly recall that a physical system $\Omega$ is usually associated in QM with a set $\mathcal{S}$ of states and a set $\mathcal{O}$ of *observables*. The set $\mathcal{S}$ is partitioned into a subset $\mathcal{P}$ of *pure* states and a subset $\mathcal{M}$ of *mixtures*. Furthermore, a (physical) property is defined as a pair $F = (A, \Sigma)$, with $A \in \mathcal{O}$ and $\Sigma$ a Borel subset of the set $\Xi$ of all possible values of $A$ [1, 33]. The physical system $\Omega$ can then be characterized by a triple $(\mathcal{S}, \mathcal{F}, p)$ [1, 34]), where $\mathcal{F}$ is the set of all physical properties of $\Omega$ and $p$ is a probability function

$$p : (S, F) \in \mathcal{S} \times \mathcal{F} \longrightarrow [0, 1]. \tag{1}$$

Because of the characterization above, properties play a basic role in the foundations of QM. For every $F = (A, \Sigma) \in \mathcal{F}$, one says that $F$ has truth value *true* (*false*) if and only if the value of $A$ belongs (does not belong) to $\Sigma$. If one adopts the realistic interpretation of QM (Section 1), every $F \in \mathcal{F}$ is in principle measurable (but different properties may be not simultaneously measurable) on an individual object $\alpha$, that is, an item of $\Omega$. The standard formulations of QM usually consider only idealized measurements. These measurements are dichotomic, have efficiency 1, and their outcomes are labeled *yes* and *no*, the former corresponding to the value *true* of $F$ and the latter to the value *false*.

The notion of objectivity can now be defined referring to an item of the physical system that is considered, as follows.

*A property $F$ of $\Omega$ is objective for an individual object $\alpha$ if and only if its value* (true/false) *is not only assigned for every measurement context* (value definiteness) *but also independent of this context.*

This definition implies that a property $F$ is *nonobjective* whenever its value is not assigned for every measurement context or, if assigned, depends on the context. Hence the realistic interpretations of QM imply that QM is a nonobjective theory, in the sense that, for every individual object $\alpha$ in a given state $S$ of a physical system $\Omega$, there are both properties that can be considered objective and properties that must be considered nonobjective. To be precise, if an individual object $\alpha$ is in the state $S$, then the property $F$ can be considered objective for $\alpha$ if $p(S, F) \in \{0, 1\}$, but it is necessarily nonobjective if $p(S, F) \notin \{0, 1\}$ (note that this conclusion implies that $F$ is objective for $\alpha$ if and only if it is objective for every individual object in the state $S$).

## 3 The Fundamentals of the ESR Model

As anticipated in Section 1, the ESR model stems from the intuitive idea that the set of all properties which are objective for an individual object $\alpha$ in a state $S$ (to be determined by the model itself, as in QM) may be such that $\alpha$ has nonzero probability of remaining undetected when a property $F$ is measured on it. This "no-detection" probability may vary with F and $S$ but does not depend on the device that is used to perform the measurement: hence, it may be different from 0 also in the case of idealized measurements. The lack of efficiency of real measurements superimposes to it, usually hiding it.

thresholds that occur in real detectors [28, 29]. Our present view is obviously different, for it refers to a "realistic" interpretation of QM and circumvents the "no-go" theorems by modifying the standard interpretation of quantum probabilities, thus avoiding contextuality, hence nonlocality.

To formalize the intuitive idea expounded above, the ESR model starts from the quantum description of a physical system $\Omega$ in terms of states and observables, but adds a "no-registration outcome" $a_0$ to the set $\Xi$ of all possible values of any quantum observable $A$ [12–19]. The outcome $a_0$ is considered as a possible result of an idealized measurement of $A$ and not only as the initial position of a pointer that is abandoned when the measurement is performed. Hence the introduction of $a_0$ transforms the quantum observable $A$ into a *generalized observable* $A_0$. This generalized observable is then associated with a family of properties of the form $(A_0, \Sigma)$, where $\Sigma$ is a Borel subset of the set $\Xi_0 = \Xi \cup \{a_0\}$ of all possible values of $A_0$. When $a_0$ does not belong to $\Sigma$, the property $F = (A_0, \Sigma)$ coincides with the property $(A, \Sigma)$ of QM. Therefore the subset $\{(A_0, \Sigma) \mid a_0 \notin \Sigma\}$ corresponds bijectively to the set of all properties of $\Omega$ in QM and can be identified with it (hence it is denoted by $\mathcal{F}$ in the following). Our intuitive idea can then be formally expressed by the fundamental equation of the ESR model

$$p^t(S, F) = p^d(S, F)p(S, F). \quad (2)$$

In this equation $S$ is a state and $F = (A_0, \Sigma) \in \mathcal{F}$. Then, $p^t(S, F)$ is the probability that an idealized measurement of $F$ performed on an individual object $\alpha$ in the state $S$ yields outcome *yes* (*overall probability*), $p^d(S, F)$ is the probability that $\alpha$ is detected in the measurement (*detection probability*), and $p(S, F)$ is the probability that the measurement yields outcome *yes* when $\alpha$ is detected (*conditional on detection probability*).

The fundamental assumption of the ESR model can now be stated as follows.

AX. *Let $S \in \mathcal{P}$ and $F \in \mathcal{F}$. Then, the probability $p(S, F)$ coincides with the probability of the property $F$ in the state $S$ supplied by QM via Born's rule.*

We stress that assumption AX concerns pure states only (mixtures require a separate treatment, see Section 4). Furthermore, it has two relevant consequences.

(i) *Conservative*. The ESR model embodies the formalism of QM.
(ii) *Innovative*. The ESR model modifies the interpretation of the formalism of QM. According to QM, Born's rule supplies an absolute probability (physically interpreted as the large number limit of the ratio $n/N$, where $n$ is the number of individual objects in the state $S$ that display the property $F \in \mathcal{F}$ when $F$ is measured, and $N$ is the number of individual objects in the state $S$ that are produced). According to the ESR model, if $S$ is pure the same rule supplies a conditional probability (physically interpreted as the large number limit of the ratio $n/N^d$, where $N^d \le N$ is the number of all individual objects in the state $S$ that are detected when $F$ is measured).

Let us come to the mathematical apparatus of the ESR model. For every $S \in \mathcal{P}$ and $F = (A_0, \Sigma) \in \mathcal{F}$, the introduction of the three probabilities $p^t(S, F)$, $p^d(S, F)$ and $p(S, F)$ in place of the standard quantum probability implies that the mathematical apparatus of QM must be extended to take into account these probabilities. Such an extension leads to new representations of states, observables and properties.

*The detection probability $p^d(S, F)$*. No theory is available at present to predict $p^d(S, F)$. Hence $p^d(S, F)$ is considered as a parameter in the ESR model, to be determined empirically. It is only required that $p^d(S, F)$ satisfies a mathematical condition to be stated in the following.

*The conditional on detection probability $p(S, F)$*. Assumption AX implies that this probability can be obtained by using standard quantum rules. Hence, as far as $p(S, F)$ is concerned, the physical system $\Omega$ can be associated with a Hilbert space $\mathcal{H}$. Moreover, a pure state $S$ can be represented by a one-dimensional orthogonal projection operator $\rho_S$ on $\mathcal{H}$, the generalized observable $A_0$ can be represented by the same self-adjoint operator $\hat{A}$

that represents the observable $A$ of QM from which $A_0$ is obtained, and the property $F$ can be represented by an orthogonal projection operator $P^{\hat{A}}(\Sigma)$ on $\mathcal{H}$. Furthermore, the standard quantum equation holds

$$p(S, F) = Tr[\rho_S P^{\hat{A}}(\Sigma)]. \tag{3}$$

*The overall probability* $p^t(S, F)$. Bearing in mind the fundamental equation of the ESR model and the mathematical representation of $p(S, F)$, one obtains

$$p^t(S, F) = Tr[p^d(S, F)\rho_S P^{\hat{A}}(\Sigma)]. \tag{4}$$

Hence one puts

$$p^t(S, F) = Tr[\rho_S T_{S,A_0}(\Sigma)]. \tag{5}$$

The linear operator $T_{S,A_0}(\Sigma) = p^d(S, F)P^{\hat{A}}(\Sigma)$ is positive and bounded by 0 and 1 (*effect*). One then assumes that a mapping $p^d_{S,A_0}(\lambda)$ of the set $\Xi$ of all possible values of $A$ into [0, 1] exists such that

$$T_{S,A_0}(\Sigma) = \int_\Sigma p^d_{S,A_0}(\lambda)P^{\hat{A}}(d\lambda). \tag{6}$$

Equation (6) states the mathematical condition on $p^d(S, F)$ mentioned above.

Equations (4)–(6) imply that, as far as $p^t(S, F)$ is concerned, the pure state $S$ can still be represented by $\rho_S$. The property $(A_0, \Sigma)$ is represented instead by a family $\{T_{S,A_0}(\Sigma)\}_{S\in\mathcal{P}}$ of effects. Moreover, let $B(\Xi)$ be the set of all Borel subsets of $\Xi$ and $\mathfrak{B}(\mathcal{H})$ the set of all bounded positive operators on $\mathcal{H}$. Then, the generalized observable $A_0$ is represented by the family of *commutative operator valued measures*

$$\mathcal{T}_{A_0} = \{T_{S,A_0} : \Sigma \in B(\Xi) \longrightarrow T_{S,A_0}(\Sigma) \in \mathfrak{B}(\mathcal{H})\}_{S\in\mathcal{P}}. \tag{7}$$

Putting together the representations of properties to be used to evaluate the probabilities $p_S(F)$ and $p^t(S, F)$ in the case of pure states, one obtains that a complete mathematical representation of a property $F = (A_0, \Sigma) \in \mathcal{F}$ is provided in the ESR model by the pair

$$\left(P^{\hat{A}}(\Sigma), \{T_{S,A_0}(\Sigma)\}_{S\in\mathcal{P}}\right). \tag{8}$$

Analogously, a complete representation of the generalized observable $A_0$ is provided by the pair

$$(\hat{A}, \mathcal{T}_{A_0}) = \left(\hat{A}, \{T_{S,A_0} : \Sigma \in B(\Xi) \longrightarrow T_{S,A_0}(\Sigma) \in \mathfrak{B}(\mathcal{H})\}_{S\in\mathcal{P}}\right). \tag{9}$$

The following remarks are then important.

(i) In the representation of $F$ the first element of the pair coincides with the standard representation of $F$ in QM. In the representation of $A_0$ the first element of the pair coincides with the standard representation of the quantum observable $A$ from which $A_0$ is obtained.

(ii) In both representations the second element is a family, parametrized by the set of pure states. Hence, as far as $p^t(S, F)$ is concerned, the representation of a property, or of an observable, is not given once for all, because it depends on the state of the individual object on which the property, or the observable, is measured.

## 4 H.V. Models and Objectivity

We have already explained in Section 1 that the main aim of the ESR model is supplying an objective theory, embodying from one side the basic formalism of QM and avoiding, on the other side, the problems following from nonobjectivity. Let us therefore discuss the crucial issue of nonobjectivity.

The proof of the objectivity of the ESR model is obtained by showing that this model admits noncontextual (hence local) *hidden variables* (*h.v.*) *models* (at variance with earlier formulations [10–18], the latest version of the ESR model [19] does not introduce h.v. from the beginning). To this end a set $\mathcal{F}_\mu$ of *microscopic properties* of the physical system $\Omega$ is introduced which is in one-to-one correspondence with the set $\mathcal{F}$ of (macroscopic) properties. For every individual object $\alpha$, the set $\mathcal{F}_\mu$ is then partitioned into two subsets, the subset $s$ of all the microscopic properties that are *possessed* by $\alpha$ and the subset $\mathcal{F}_\mu \setminus s$ of all the microscopic properties that are *not possessed* by $\alpha$. The subset $s$ is called *the microscopic state* of $\alpha$. Then, an overall probability, a detection probability and a conditional on detection probability are introduced referring to the microscopic state $s$ of $\alpha$ rather than to its (macroscopic) state $S$. By introducing the further probability $p(S \mid s)$ that an individual object $\alpha$ in the state $S$ is in the microscopic state $s$, one can deduce the fundamental equation of the ESR model, thus obtaining the desired noncontextual h.v. model.

Because of the above result and of the one-to-one correspondence between $\mathcal{F}_\mu$ and $\mathcal{F}$, one concludes that all properties in $\mathcal{F}$ can be considered objective in the sense specified in Section 2. Hence the ESR model is an objective theory. It follows that all probabilities can be considered epistemic, so that no objectification problem occurs. Of course, this result finds its roots in the reinterpretation of quantum probabilities as conditional on detection rather than absolute (Section 3), which allows to circumvent the "no-go" theorems of QM (Sections 11 and 6).

## 5 Mixtures, Generalized Lüders Postulate and Time Evolution

To complete our presentation of the ESR model three main issues must still be discussed. Indeed we have not yet considered mixtures, changes of state induced by measurements and time evolution. Let us briefly deal with these topics.

According to many authors [1, 34–36] there are in QM both *proper* and *improper* mixtures, which are mathematically represented in the same way (density operators) but have different operational definitions. In short, the preparation of an individual object $\alpha$ in a state $M$ that is a proper mixture of pure states can be described as follows: first, prepare a set $\mathcal{Q}$ of ensembles of individual objects such that each ensemble consists of individual objects in the same pure state, different ensembles corresponding to different pure states; second, prepare a new ensemble by mixing the ensembles of $\mathcal{Q}$ without registering the state of each individual object; third, choose an individual object $\alpha$ in the final ensemble. One can then introduce the probability that $\alpha$ is in a prefixed pure state $S$ belonging to the set of all states associated with the ensembles of $\mathcal{Q}$. This probability is epistemic, for it has an ignorance interpretation (the state of $\alpha$ would be known if it had been registered when preparing the mixture $M$). Now, $M$ is represented in QM by a density operator, say $\rho_M$, which can be described in an infinite number of ways as a convex combination of projection operators representing pure states. If one chooses to decompose $\rho_M$ in terms of the pure states associated with the ensembles of $\mathcal{Q}$, then the coefficient of $S$ in this decomposition coincides with the foregoing epistemic probability.

The preparation of an individual object $\alpha$ in a state $N$ that is an improper mixture is obtained instead by considering a composite physical system, preparing an ensemble of items of such a system in a given state (take a pure state for the sake of simplicity), selecting one of the component subsystems and considering the ensemble of all items of this subsystem. Also the mixture $N$ is represented in QM by a density operator, say $\rho_N$, but in this case it does not exist any decomposition of $\rho_N$ in terms of pure states whose coefficients can be interpreted as epistemic probabilities.

At variance with QM, proper and improper mixtures have different mathematical representations, corresponding to their different operational definitions, in the ESR model [12, 14, 15, 19].

*Proper mixtures*. Each proper mixture has a rather complicated representation as a family of pairs parametrized by the set $\mathcal{F}$ of properties. Each pair in the family consists of a density operator and a detection probability. The explicit form of these mathematical entities is given in [12, 14, 15] and will not be reported here for the sake of brevity.

*Improper mixtures*. These mixtures can be represented by the same density operators that represent them in QM. Assumption AX can be extended to improper mixtures by substituting the subset $\mathcal{P}$ of all pure states with the subset $\mathcal{P} \cup \mathcal{N}$, where $\mathcal{N}$ is the subset of all improper mixtures. The representations of properties and observables can then be extended to improper mixtures by introducing the same substitution. Hence improper mixtures are considered as *generalized pure states* in the ESR model.

Let us come to the change of state induced by a measurement. In QM the Lüders postulate selects a subset of *ideal first kind measurements* that change a state according to a prefixed rule [33]. This postulate is generalized in the ESR model by considering a dichotomic measurement $\mathfrak{M}$ of a property $F = (A_0, \Sigma) \in \mathcal{F}$ on an individual object $\alpha$ in the state $S$, with $S \in \mathcal{P} \cup \mathcal{N}$. Then $\mathfrak{M}$ is an *idealized* measurement of $F$ if, whenever the *yes* outcome is obtained, the state $S_F$ after the measurement is represented by the density operator

$$\rho_{S_F} = \frac{T_{S,A_0}(\Sigma)\rho_S T^{\dagger}_{S,A_0}(\Sigma)}{Tr\left[T_{S,A_0}(\Sigma)\rho_S T^{\dagger}_{S,A_0}(\Sigma)\right]}. \tag{10}$$

By analogy with QM, the rule expressed by the equation above is called *the generalized Lüders postulate*. It must be stressed that it does not apply to proper mixtures. However, the representation of the final state in the case of proper mixtures can be deduced from (10). Its mathematical form is rather complicated [12, 14, 15] and will not be reported here.

It is important to recall that the objectivity of the ESR model implies that no "collapse of the wave function" occurs in this model, as we have seen in Section 1. Rather, the generalized Lüders postulate supplies an example of the change of state of an individual object interacting with another individual object (the measuring apparatus). This example provides some suggestions for the dynamics of the composite system of two individual objects. In particular, a crucial difference from time evolution in QM occurs because the generalized Lüders postulate introduces a change of state also in the case of individual objects that are not detected by the measurement.

Bearing in mind the above example we have recently discussed time evolution in the framework of the ESR model [19]. Also the details of this treatment will not be reported here, and we limit ourselves to resume our conclusions.

Firstly, let us consider a physical system in a state $S \in \mathcal{P} \cup \mathcal{N}$.

(i) If the physical system is closed, one can assume that it undergoes linear Hamiltonian evolution, as in QM.
(ii) If the physical system is open, its time evolution may be linear or not, depending on its interaction with the environment, as in QM.
(iii) The time evolution induced by an idealized measurement is necessarily nonlinear.

The above results show that time evolution in the ESR model matches time evolution in QM, but for the distinguishing feature in item (iii).

Secondly, let us consider time evolution of a physical system in a state $S \in \mathcal{T}$, where $\mathcal{T}$ is the set of all proper mixtures. One can then prove that time evolution can be deduced in this case from the time evolution discussed above.

## 6 Empirical Consequences and Applications

The empirical success of QM imposes a fundamental constraint on every attempt at modifying QM to avoid the problems following from nonobjectivity. The predictions of QM that have been experimentally verified must in fact be reproduced by the new theory within the limits of the experimental errors. On the other side, the new theory should also provide some testable predictions that make it empirically different from QM, allowing one to check which theory is correct. The ESR model satisfies both these conditions. Indeed, the predictions of the ESR model in experiments on overall probabilities are formally different from the predictions of QM, but, if the state $S$ of the individual objects that are considered is a pure state or an improper mixture, they may be close to the quantum predictions whenever the detection probabilities are close to 1. Moreover the predictions of the ESR model in experiments on conditional on detection probabilities (as Aspect's experiments, in which non-detected individual objects are not taken into account [7, 8]) are identical to the predictions of QM. The predictions of the ESR model in experiments on overall probabilities in which the state $S$ of the individual objects that are considered is a proper mixture may be instead very different from the predictions of QM and single out a class of experiments that can distinguish the two theories.

We add that the ESR model has been used to deal with some well known problematical issues in QM, to check its predictions. The obtained results can be resumed as follows.

(i) *The "no-go" theorems*. Because of assumption AX (Section 3) the "no-go" theorems of QM do not hold in the ESR model [11–13, 18]. We illustrate this result by summarily discussing some examples.

Let us firstly consider the proofs of the Bell theorem that resort to inequalities. We have shown in this case that Bell's inequalities do not hold in the ESR model at a macroscopic level, notwithstanding objectivity (they hold instead in the h.v. models discussed in Section 4, at a purely theoretical microscopic level). Indeed, dichotomic observables must be substituted by thrichotomic observables, because a no-registration outcome must be added in the set of all possible values of each observable (Section 3) Hence the original Bell's inequality must be replaced by the *modified Bell's inequlity*

$$|E(A_0(a), B_0(b)) - E(A_0(a), B_0(c))| \leq 1 + E(A_0(b), B_0(c)). \qquad (11)$$

The symbols $E(A_0(a), B_0(b))$, $E(A_0(a), B_0(c))$ and $E(A_0(b), B_0(c))$ in (11) denote the expectation values of products of the trichotomic observables $A_0(a)$, $A_0(b)$, $B_0(b)$ and $B_0(c)$ depending on the parameters $a$, $b$ and $c$. Let us then consider a composite physical system $\Omega$ made up of two spin-1/2 systems $\Omega_1$ and $\Omega_2$, and let the state $S$ of $\Omega$ be the singlet spin state. Let the quantum observables $A(x)$ and $B(y)$ ($x = a, b$; $y = b, c$) from which the generalized observables $A_0(x)$ and $B_0(y)$ are obtained be the observables "spin of $\Omega_1$ along the $x$ direction" and "spin of $\Omega_2$ along the $y$ direction", respectively. By introducing the simplifying assumption that $A_0(x)$ and $B_0(y)$ are such that the detection probability of any spin property in the singlet state depends on the generalized observable but not on its value, we obtain from (11)

$$
\begin{aligned}
| - & p^d(S, A_0(a))p^d(S, B_0(b))a \cdot b + p^d(S, A_0(a))p^d(S, B_0(c))a \cdot c \,| \\
& \leq 1 + p^d(S, A_0(b))p^d(S, B_0(c))b \cdot c
\end{aligned} \tag{12}
$$

Inequality (12) does not imply, a priori, any contradiction. It establishes instead a condition that must be fulfilled by the detection probabilities that occur in it. An estimate under rather restrictive assumptions (in particular, the four detection probabilities in (12) are supposed to be identical) leads to an upper value of 0.8165 for the detection efficiency of spin measurements on items of $\Omega_1$ and $\Omega_2$ [13], as reported in Section 1.

Secondly, let us consider the proofs of the Bell-KS and Bell theorems that do not resort to inequalities. All these proofs proceed ab absurdo (see, e.g., [30, 31]). They select some different quantum laws linking together physical properties of an item $\alpha$ of a composite physical system, assume that they must hold simultaneously, and show that a contradiction occurs if all properties of $\alpha$ are supposed to be objective. Contextuality or nonlocality then follow, depending on the physical system that is considered. It is important to note, however, that the laws that are chosen cannot be checked simultaneously. Indeed each of them contains some observables that are incompatible with some of the observables that occur in the other laws. Let us come now to the ESR model. Each of the aforesaid laws holds in this model for every individual object that is detected when the law is checked, because of assumption AX. But the sets of objects that are detected are generally different when different laws are checked. Therefore if a measurement is performed on $\alpha$ to check one of the laws and $\alpha$ is detected, then the law will be confirmed, but one cannot simultaneously check all the remaining laws and cannot exclude that the objective properties of $\alpha$ are such that $\alpha$ would have not been detected in some of these checks. In fact, the contradiction mentioned above implies that $\alpha$ must necessarily remain undetected in at least one check of this kind. Thus, the assumption that all laws must simultaneously be valid for $\alpha$ does not hold in the ESR model, which invalidates the aforesaid proofs of contextuality and nonlocality, hence of nonobjectivity.

(ii) *The GHZ experiment*. The general h.v. models for the ESR model can be used to produce h.v. models for specific physical situations and experiments. In particular, it has been recently proved that the finite "toy models" contrived by Szabó and Fine [32] to provide a local explanation of the Greenberger-Horne-Zeilinger (GHZ) experiment can be obtained as special cases of the general h.v. models [16].

(iii) *Quantum logic and quantum truth*. It has also been recently shown that quantum logic can be embedded into a suitable extended classical logic, the embedding preserving the logical order but not the algebraic structure [17]. This result must be considered as purely formal if one accepts the standard interpretation of QM, but acquires a physical interpretation in the ESR model because of objectivity of properties. Objectivity indeed allows one to consider the set of individual objects formally associated with every $F \in \mathcal{F}$ as the set of all objects that *possess* the property $F$. It follows that no notion of quantum truth, different from classical truth and incompatible with it is needed in the ESR model. Rather, quantum logic can be seen as a mathematical structure formalizing the metalinguistic notion of *verifiability* in a quantum framework.

## References


1. Busch, P., Lahti, P.J., Mittelstaedt, P.: The Quantum Theory of Measurement. Springer, Berlin (1991, 1996)
2. Bell, J.S.: On the problem of hidden variables in quantum mechanics. Rev. Mod. Phys. **38**, 447–452 (1966)
3. Kochen, S., Specker, E.P.: The problem of hidden variables in quantum mechanics. J. Math. Mech. **17**, 59–87 (1967)
4. Bell, J.S.: On the Einstein-Podolski-Rosen paradox. Physics **1**, 195–200 (1964)
5. Zeilinger, A.: A foundational principle for quantum mechanics. Found. Phys. **29**, 631–643 (1999)
6. Clifton, R., Bub, J., Halvorson, H.: Characterizing quantum theory in terms of information theoretic constraints. Found. Phys. **33**, 1561–1591 (2003)
7. Aspect, A., Grangier, P., Roger, G.: Experimental realization of Einstein-Podolski-Rosen-Bohm gedankenexperiment: a new violation of Bell's inequalities. Phys. Rev. Lett. **49**, 91–94 (1982)
8. Aspect, A., Dalibard, J., Roger, G.: Experimental test of Bell's inequalities using time-varying analyzers. Phys. Rev. Lett. **49**, 1804–1807 (1982)
9. Genovese, M.: Research on hidden variables theories: a review on recent progresses. Phys. Rep. **413**, 319–396 (2005)
10. Garola, C., Sozzo, S.: The ESR model: a proposal for a noncontextual and local Hilbert space extensions of QM. Europhys. Lett. **86**, 20009–20015 (2009)
11. Garola, C., Sozzo, S.: Embedding quantum mechanics into a broader noncontextual theory: a conciliatory result. Int. J. Theor. Phys. **49**, 3101–3117 (2010)
12. Garola, C., Sozzo, S.: Generalized observables, Bell's inequalities and mixtures in the ESR model. Found. Phys. **41**, 424–449 (2011)
13. Garola, C., Sozzo, S.: The modified Bell inequality and its physical implications in the ESR model. Int. J. Theor. Phys. **50**, 3787–3799 (2011)
14. Garola, C., Sozzo, S.: Representation and interpretation of mixtures in the ESR model. Theor. Math. Phys. **168**, 912–923 (2011)
15. Garola, C., Sozzo, S.: Extended representations of observables and states for a noncontextual reinterpretation of QM. J. Phys. A: Math. Theor. **45**, 075303–075315 (2012)
16. Garola, C., Persano, M., Pykacz, J., Sozzo, S.: Finite local models for the GHZ experiment. Int. J. Theor. Phys. **53**, 622–644 (2014)
17. Garola, C., Sozzo, S.: Recovering quantum logic within an extended classical framework. Erkenn **78**, 399–419 (2013)
18. Garola, C., Persano, M.: Embedding quantum mechanics into a broader noncontextual theory. Found. Sci. **19**(3), 217–239 (2014)
19. Garola, C., Sozzo, S., Wu, J.: Outline of a generalization and reinterpretation of quantum mechanics recovering obectivity. ArXiv:1402.4394v2 [quant-ph] (2014)
20. Santos, E.: The failure to perform a loophole-free test of Bell's inequality supports local realism. Found. Phys. **34**, 1643–1673 (2004)
21. Santos, E.: Bell's theorem and the experiments: increasing empirical support for local realism? Stud. Hist. Philos. Mod. Phys. **36**, 544–565 (2005)
22. Garola, C.: The ESR model: reinterpreting quantum probabilities within a realistic and local framework. In: Adenier, G., Khrennikov, A., Lahti, P., Man'ko, V., Nieuwenhuizen, T. (eds.) Quantum Theory: Reconsideration of Foundations-4, pp. 247–252. American Institute of Physics, Ser. Conference Proceedings 962, Melville (2007)
23. Accardi, L.: Some loopholes to save quantum nonlocality. In: Adenier, G., Khrennikov, A. (eds.) Foundations of Probability and Physics-3, pp. 1–20. American Institute of Physics, Ser. Conference Proceedings 750, Melville (2005)
24. Khrennikov, A.: Interpretations of Probability. De Gruyter, Berlin (1998, 2009)
25. Khrennikov, A., Smolyanov, O.G., Truman, A.: Kolmogorov probability spaces describing Accardi models for quantum correlations. Open. Syst. Inf. Dyn. **12**(4), 371–384 (2005)
26. Hess, K., Philipp, W.: Exclusion of time in Mermin's proof of Bell-type inequalities. In: Khrennikov, A. (ed.) Quantum Theory: Reconsideration of Foundations-2. Ser. Math. Model. 10, pp. 243–254. Växjö University Press, Växjö (2003)
27. Hess, K., Philipp, W.: Bell's theorem: critique of proofs with and without inequalities. In: Adenier, G., Khrennikov, A. (eds.) Foundations of Probability and Physics-3, pp. 150–155. American Institute of Physics, Ser. Conference Proceedings 750, Melville (2005)

28. Khrennikov, A.: Quantum probabilities and violation of CHSH-inequality from classical random signals and threshold type detection scheme. Prog. Theor. Phys. **128**, 31–58 (2012)
29. Khrennikov, A.: Born's rule from measurements of classical signals by threshold detectors which are properly calibrated. J. Mod. Opt. **59**, 667–678 (2012)
30. Greenberger, D.M., Horne, M.A., Shimony, A., Zeilinger, A.: Bell's theorem without inequalities. Am. J. Phys. **58**, 1131–1143 (1982)
31. Mermin, N.D.: Hidden variables and the two theorems of John Bell. Rev. Mod. Phys. **65**, 803–815 (1993)
32. Szabó, L.E., Fine, A.: A local hidden variable theory for the GHZ experiment. Phys. Lett. A **295**, 229–240 (2002)
33. Beltrametti, E.G., Cassinelli, G.: The Logic of Quantum Mechanics. Addison-Wesley, Reading (1981)
34. Aerts, D.: Foundations of quantum physics: a general realistic and operational approach. Int. J. Theor. Phys. **38**, 289–358 (1999)
35. d'Espagnat, B.: Conceptual Foundations of Quantum Mechanics. Benjamin, Reading (1976)
36. Timpson, C.G., Brown, H.R.: Proper and improper separability. Int. J. Quant. Inf. **3**, 679–690 (2005)