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Biianchi Type II VIII and IX String Cosmological Models in $F(R)$ Gravity

S. D. Katore

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Abstract In this present paper, we investigate Bianchi type II VIII and IX string cosmological models in the context of $f(R)$ gravity. Exact solutions of the field equations are obtained. It is found that the scale factors (A, B) admit oscillatory nature in case of Bianchi type II and IX models. The strings do not exist in all three cosmological models for the $f(R)$ theory of gravitation.

Keywords Bianchi type II VIII and IX model · String · $f(R)$ gravity

1 Introduction

Large number of gravitational phenomena is explained by Einstein's general theory of relativity. It is well tested in laboratory. The results obtained in general relativity are agrees with the standard model of particles. The predictions of the theory are found analogous to solar system scales. However, some important issues like the accelerated expansion of the universe, cosmological constant or dark energy problem yet to be solved [1, 2].

To resolve these unsolved issues several models are suggested. $f(R)$ gravity is one of such effort [3–6], in which Einstein-Hilbert action is extended to obtain modification of gravity. In this action the Ricci scalar is replaced by function of Ricci scalar. This modified gravity explains the current accelerated expansion of the universe theoretically [7]. Modified gravity models also have been introduced as an alternative to the quintessence picture for dark energy [8, 9]. Recently, Sharif and Kausar [10] investigated non vacuum Bianchi type VI_0 cosmological models in $f(R)$ gravity. Larranaga [11] studied rotating charged black hole solutions in $f(R)$ gravity.

It is conjectured that universe might have experienced a number of phase transitions after big bang explosion. These phase transitions produces vacuum domain walls, strings and monopoles. Among these topological defects, strings play an important role in the formation

S. D. Katore (✉)
Department of Mathematics, S.G.B. Amravati University, Amravati 444602, India
e-mail: katore777@gmail.com

of galaxies. Observational data agrees with the existence of a large scale network of strings in the early universe. The cosmic strings have coupled stress energy to the gravitational field. Therefore, the study of gravitational effects of such strings will be interesting.

In recent years, cosmological string models have generated a lot of research interest. The gravitational effect of cosmic strings have been extensively discussed by Vilenkin [12], Gott [13], Latelier [14] and satchel [15] in general relativity. Reddy [16, 17], Pradhan et al. [18, 19], Tripathi et al. [20] have studied string cosmological models in different context. Bali et al. [21] investigated LRS Bianchi type II string dust cosmological model in general relativity. Amirhashchi and Mohamadian [22] have studied string cosmological models in LRS Bianchi type II dusty universe with time decaying vacuum energy density Λ.Rao et al. [23] has studied exact Bianchi type II, VIII and IX string cosmological models in General Relativity and Self creation theory of gravitation. This motivates us to investigate Bianchi type II, VIII and IX universe with effect of string in $f(R)$ gravity.

2 Basic Equation

Consider the action of modified gravity

$$S = \frac{1}{2k^2}\int d^4x\sqrt{-g}f(R) + S_m(g_{\mu\nu}, M), \tag{1}$$

where $k^2 = 8\pi G$, M refers collectively to all matter fields.

The field equations are

$$F(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu\nabla_\nu F(R) + g_{\mu\nu}\nabla^\mu\nabla_\nu F(R) = k^2 T_{\mu\nu}, \tag{2}$$

where $F = \frac{df}{dR}$, ∇ is usual covariant derivative. Very recently behavior of quark matter and strange quark matter which exist in the first seconds of the early universe in $f(R)$ gravity for Bianchi type I and V universe studied by Yilmaz et al. [24]. Vacuum and Non vacuum solution for Bianchi type I and V space-time in the framework of metric gravity studied by Sharif and Shamir [25, 26]. Bianchi type space times are significant to study the early stage of evolution of the universe. Bianchi type II, VIII and IX universe gives familiar solutions like FRW with positive curvature, the de Sitter universe, and the Taub-Nut solution. Katore et al. [27] obtained exact Bianchi type II, VIII and IX cosmological models in scale covariant theory of gravitation. We consider a spatially homogeneous Bianchi type – II, VIII and IX metric of the form

$$ds^2 = -dt^2 + A^2d\theta^2 + B^2d\psi^2 + \left(A^2 I^2(\theta) + B^2h^2(\theta)\right)d\varphi^2 + 2B^2h(\theta)d\psi d\theta, \tag{3}$$

where (θ, φ, ψ) are Eulerian angles, A and B are functions of cosmic time t only. It represents

Bianchi type II: if $I(\theta) = 1)$ and $h(\theta) = \theta$.

Bianchi type VIII: if $I(\theta) = \cosh\theta$ and $h(\theta) = \sinh\theta$.

Bianchi type IX: if $I(\theta) = \sin\theta$ and $h(\theta) = \cos\theta$

The Ricci scalar R is given by

$$R = 2\left[-\frac{I''}{IA^2} + \frac{2A_{44}}{A} + \frac{B_{44}}{B} + \frac{2A_4B_4}{AB} + \frac{A_4^2}{A^2} - \frac{B^2}{4A^4}\right],$$

where subscript '4' denote differentiation with respect to time t and superscript dash (') denote differentiation with respect to θ.

It is still challenging problem to know the exact physical situation at early stages of the formation of the universe. Concept of string theory is pondered to study the early stage of the universe. It is believed that strings are formed before the creation of the particle in the universe when the temperatures lower below some critical temperatures. Using these concepts, large number of researcher were investigated solutions for strings. The energy momentum tensor for string dust is taken as

$$T_i^j = \rho \nu_i \nu^j - \lambda x_i x^j, \tag{4}$$

With $\nu_i \nu^i = -x_i x^i = -1$ and $\nu_i x^i = 0$ where $\rho = \rho_p + \lambda$ is the rest energy density for a cloud of strings with particles attached to them. ρ_p, the density of particles, λ is the cloud strings tension density, ν^i the four velocity vector and x^i is the direction of the strings. In the commoving coordinate system, from (4), we have $T_1^1 = -\lambda$, $T_2^2 = T_3^3 = 0$, $T_4^4 = -\rho$ and $T_i^j = 0,\ for\ i \neq j$

The quantities $\rho$and $\lambda$are the functions of t only. Upadhaya and Dave [28] studied some Bianchi type III cosmological models with magnetic field for massive strings. Krori et al. [29] studied Bianchi type II, VI0, VIII and IX strings cosmological models. A Bianchi type I massive string cosmological model in the presence of a magnetic field is investigated by Saha and Visinescu [30].

3 Bianchi Type –II Model

The field equations corresponding to (2), (3) and (4) are as follows

$$F\left[-\frac{B_{44}}{B} - \frac{2A_{44}}{A}\right] + \frac{1}{2}f + \frac{B_4F_4}{B} + \frac{2A_4F_4}{A} = k^2\rho, \tag{5}$$

$$F\left[-\frac{B_{44}}{B} - \frac{2A_4B_4}{AB} - \frac{B^2}{2A^4}\right] + \frac{1}{2}f + \frac{2A_4F_4}{A} + F_{44} = 0, \tag{6}$$

$$\begin{aligned}\frac{F}{A^2+B^2\theta}&\left[A^2\left(\frac{A_4^2}{A^2} + \frac{A_{44}}{A} + \frac{A_4B_4}{AB} - \frac{B^2}{2A^4}\right) + B^2\theta\left(\frac{B_{44}}{B} + \frac{2A_4B_4}{AB} + \frac{B^2}{2A^4}\right)\right] \\ &+\frac{(AA_4 + BB_4\theta^2)}{A^2+B^2\theta^2}F_4 - \frac{1}{2}f - \frac{B_4F_4}{B} - \frac{2A_4F_4}{A} - F_{44} = 0,\end{aligned} \tag{7}$$

$$F\left[\frac{A_4^2}{A^2} + \frac{A_{44}}{A} + \frac{A_4B_4}{AB} - \frac{B^2}{2A^4}\right] - \frac{1}{2}f - \frac{B_4F_4}{B} - \frac{A_4F_4}{A} - F_{44} = k^2\lambda, \tag{8}$$

For simplicity we use the technique of Boutrous [31] and Ram [32] and assume adhoc relation.

$$\frac{B_{44}}{B} + \frac{A_4B_4}{AB} + \frac{B^2}{2A^4} = \frac{A_{44}}{A} + \frac{A_4B_4}{AB} + \frac{A_4^2}{A^2} - \frac{B^2}{2A^2}. \tag{9}$$

We also use the condition of shear scalar and expansion scalar with reference to Thorne [33]. Observations of the velocity redshift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropic today to within about 30 % range approximately [35, 36]. More precisely, redshift studies place the limit $\frac{\sigma}{H} \leq 0.30$ where σ is shear scalar and H the Hubble constant. This leads to the relation between scale factors.

$$B = A^n \tag{10}$$

Using (9) and (10), we have

$$2A_{44}+2\left(n+1\right)\frac{A_4^2}{A}=\frac{-2}{n-1}A^{2n-4}, n\neq 1. \tag{11}$$

Assume $A_4 = s$ so that $A_{44} = ss'$ then (9), leads to

$$s^2 = cA^{-2(n+1)}-\frac{A^{2n-2}}{2n(n-1)}. \tag{12}$$

Equation (12) further reduces to

$$\frac{dA}{dt}=\left[cA^{-2(n+1)}-\frac{A^{2n-2}}{2n(n-1)}\right]^{1/2}. \tag{13}$$

The line element (3) for this model becomes

$$\begin{array}{c} ds^2=-\left[cA^{-2(n+1)}-\frac{A^{2n-2}}{2n(n-1)}\right]^{1/2}dA^2 \\ +A^2d\theta^2+A^{2n}d\psi^2+\left(A^2+A^{2n}\theta\right)d\varphi^2+2A^{2n}\theta d\psi d\theta \end{array}. \tag{14}$$

In particular taking $n = 2$ (13) read as

$$\frac{A^3dA}{\left[4c^2-A^8\right]^{1/2}}=\frac{1}{2}dt. \tag{15}$$

Equation (15) on integrating gives us

$$A=\left[2c\sin 2t\right]^{1/4}. \tag{16}$$

Using (10) and (16), we obtain

$$B=\left[2c\sin 2t\right]^{1/2}. \tag{17}$$

From (16) and (17), it is clear that both the scale factors are functions of cosmic time. They admit oscillatory nature. This has physical meaning [36]. The oscillatory models are viable alternative to inflation [37].

These are the solution of the field equations subjected to the condition $F_4 = 0 \Rightarrow F = D, D = cons$. The value of the function of Ricci scalar becomes

$$f(R)=-3D. \tag{18}$$

From (8), we get $\lambda = 0$ which shows that in $f(R)$ theory strings do not exists. Then we get same result as obtained by Rao and Davuluri [38]. The volume of the universe found to be

$$V=BA^2=2c\sin 2t. \tag{19}$$

One can see that the volume of the universe is oscillatory.

Using (16), (17), (18), and (5), we have

$$\rho=\frac{5D}{2k^2}\cos ec^2(2t). \tag{20}$$

From the Fig. 1 it is apparent that $\rho \geq 0$ that means the model satisfies strong energy condition. We must note that the oscillation takes place in positive quadrant.

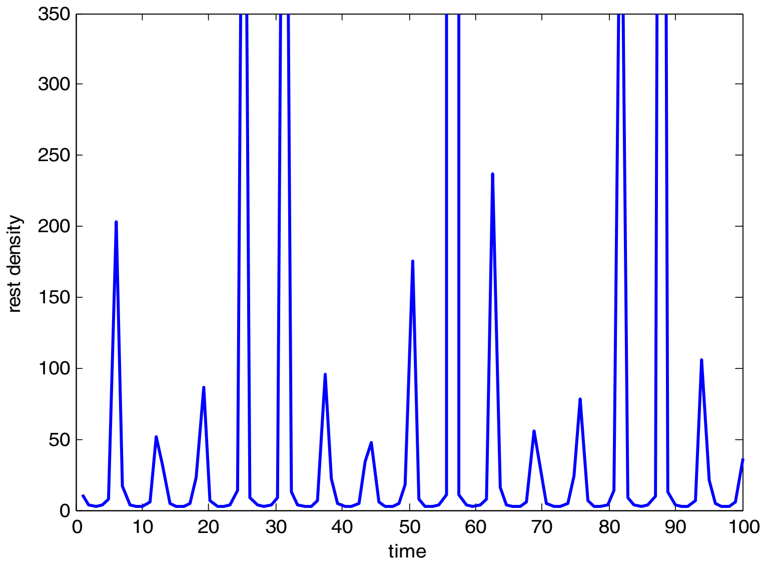

Fig. 1 Evolution of Rest density versus time

4 Kinematical Variables

The kinematical variables (Hubble, mean anisotropic parameter, shear scalar, expansion scalar and deceleration parameter) we wish to study in this model are

$$H = 2\cot(2t), \tag{21}$$

$$A_m = \frac{13}{12}, \tag{22}$$

$$\sigma = \pm\sqrt{\frac{13}{2}}\cot(2t), \tag{23}$$

$$\theta = 6\cot(2t), \tag{24}$$

$$q = \left(\sec^2(2t) - 1\right). \tag{25}$$

It is observed from (23) and (24) that the ratio of shear scalar to expansion scalar is non zero i.e. the universe is anisotropic. The deceleration parameter remains positive throughout the evolution that means the universe decelerates in standard way. The rate of expansion was large in the near past and slowing down at the present. The results are analogous to the result obtained by Beesham [39] and Rami [40] (Fig. 2).

5 Bianchi Type –VIII Model

The field equations for Bianchi type VIII model corresponding to (2), (3) and (4) are as follows

$$F\left[-\frac{B_{44}}{B} - \frac{2A_{44}}{A}\right] + \frac{1}{2}f + \frac{B_4F_4}{B} + \frac{2A_4F_4}{A} = k^2\rho, \tag{26}$$

$$F\left[-\frac{B_{44}}{B} - \frac{2A_4B_4}{AB} - \frac{B^2}{2A^4}\right] + \frac{1}{2}f + \frac{2A_4F_4}{A} + F_{44} = 0, \tag{27}$$

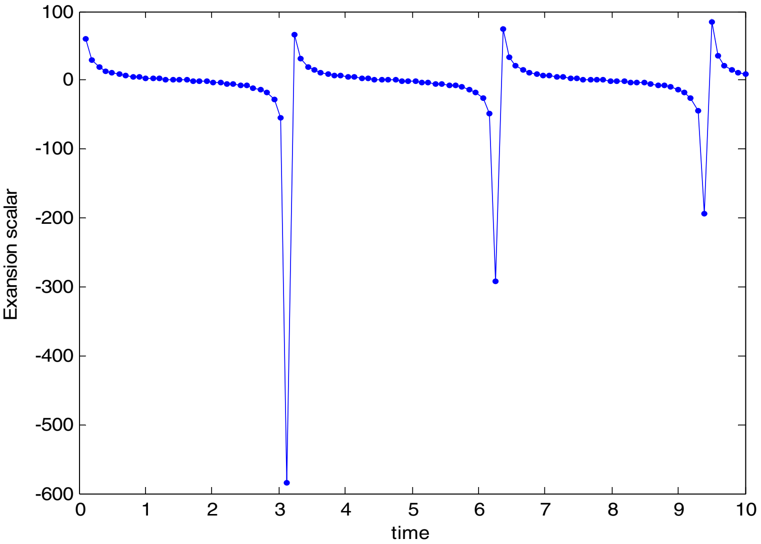

Fig. 2 Evolution of Expansion scalar versus time

$$\begin{aligned}&\frac{F}{A^2\cosh^2\theta+B^2\sinh^2\theta}\left[\begin{array}{l}A^2\cosh^2\theta\left(\frac{A_4^2}{A^2}+\frac{A_{44}}{A}+\frac{A_4B_4}{AB}-\frac{B^2}{2A^4}-\frac{1}{A^2}\right)+\\ B^2\sinh^2\theta\ (\ \frac{B_{44}}{B}+\frac{2A_4B_4}{AB}+\frac{B^2}{2A^4}\ )\end{array}\right]\\&+\frac{\left(AA_4\cosh^2\theta+BB_4\sinh^2\theta\right)F_4}{A^2\cosh^2\theta+B^2\sinh^2\theta}-\frac{1}{2}f-\frac{B_4F_4}{B}-\frac{2A_4F_4}{A}-F_{44}=0\end{aligned}, \tag{28}$$

$$F\left[\frac{A_4^2}{A^2}+\frac{A_{44}}{A}+\frac{A_4B_4}{AB}-\frac{B^2}{2A^4}-\frac{1}{A^2}\right]-\frac{1}{2}f-\frac{B_4F_4}{B}-\frac{A_4F_4}{A}-F_{44}=k^2\lambda. \tag{29}$$

For simplicity we assume adhoc relation [31, 32]

$$\frac{B_{44}}{B}+\frac{A_4B_4}{AB}+\frac{B^2}{2A^4}=\frac{A_{44}}{A}+\frac{A_4B_4}{AB}+\frac{A_4^2}{A^2}-\frac{B^2}{2A^2}-\frac{1}{A^2}\quad . \tag{30}$$

Using (10) and (30), we have

$$\frac{dA}{dt}=\left[\frac{1}{2n(1-n)}A^{2n-2}+c_1A^{-2n-2}+\frac{1}{1-n^2}\right]^{\frac{1}{2}}, n\neq\pm1, \tag{31}$$

where c_1 is constant of integration.

The line element (3) reduces to

$$\begin{aligned}ds^2=-&\left[\tfrac{1}{2n(1-n)}A^{2n-2}+\ c_1A^{-2n-2}+\tfrac{1}{1-n^2}\right]^{1/2}dA^2\\&+A^2d\theta^2+A^{2n}d\psi^2+\left(A^2\cosh^2\theta+A^{2n}\sinh^2\theta\right)d\varphi^2\cdot\\&+2A^{2n}\sinh\theta d\psi d\theta\end{aligned} \tag{32}$$

The field equations are satisfied if $F_4=0$.

In particular taking $n = {}^1/{}_2$ the Ricci scalar becomes

$$R(A)=2\left[A^{-3}-c_1A^{-5}+\frac{4}{3}A^{-2}-\frac{1}{4}\right]. \tag{33}$$

The function of Ricci scalar found to be

$$f(R) = \frac{2}{A^2} + \frac{3}{A^3}. \tag{34}$$

From (29) and (35), we get $\lambda = 0$. Thus in this model we again get that the strings do not exist, which corroborate the result obtained by Rao and Davuluri [38].

The volume of the universe is found to be

$$V = A^{5/2} \tag{35}$$

The volume of the universe is increase indefinitely with scale factor. This shows that the universe is expanding in nature (Fig. 3).

Using (26) and (31), we have

$$\rho = \frac{4}{3k^2A^2} + \frac{9}{2k^2A^3} + \frac{3c_1}{k^2A^5}. \tag{36}$$

Taking $A = T$ it is found that the density $\rho \geq 0$ that means the model satisfies strong the energy condition. The energy density is decreasing function. At singularity point it was high and as $T \to \infty, \quad \rho \to 0$ i.e. the universe is empty. This is similar to Katore and Rane [41].

6 Kinematical Variables

The kinematical variables (shear scalar, expansion scalar and deceleration parameter) in this model are

$$\sigma = \pm\frac{1}{\sqrt{12}}\left[2A^{-3} + c_1A^{-5} + \frac{4}{3}A^{-2}\right]^{1/2}, \tag{37}$$

$$\theta = \frac{5}{2}\left[2A^{-3} + c_1A^{-5} + \frac{4}{3}A^{-2}\right]^{1/2}, \tag{38}$$

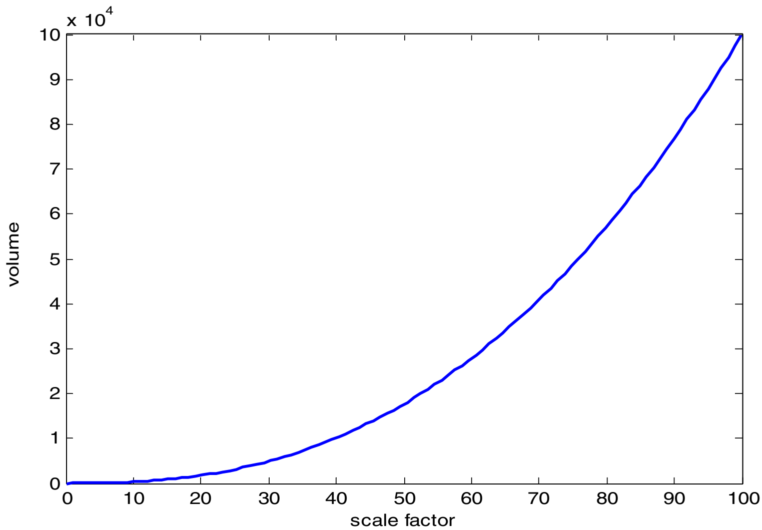

Fig. 3 Evolution of Volume versus time

$$q = \frac{3\left[6A^{-5} + 5c_1 A^{-7} + \frac{8}{3}A^{-4}\right]}{5\left[2A^{-1} + c_1 A^{-3} + \frac{4}{3}\right]} - 1. \tag{39}$$

If we set $A = T$ it is apparent that the shear scalar is decreasing function of time, at the early stage it was large and tends to zero for large time (Fig. 4). The deceleration parameter was positive at early epoch and tends to negative at later time i.e. the universe was decelerating in the past and accelerating at present which is consistent with the observational scenario. The expansion of the universe was very large initially and expansion rate is slowing down with time. The universe was anisotropic at the early stage of evolutions and tends to isotropy at late time. The rest density is decreasing function of time. In this model, the condition of homogeneity and isotropization, formulated by Collins and Hawking [42] is satisfied (Fig. 5).

7 Bianchi Type –IX Model

The field equations for Bianchi type IX model corresponding to (2), (3) and (4) are as follows

$$F\left[-\frac{B_{44}}{B} - \frac{2A_{44}}{A}\right] + \frac{1}{2}f + \frac{B_4 F_4}{B} + \frac{2A_4 F_4}{A} = k^2\rho, \tag{40}$$

$$F\left[-\frac{B_{44}}{B} - \frac{2A_4 B_4}{AB} - \frac{B^2}{2A^4}\right] + \frac{1}{2}f + \frac{2A_4 F_4}{A} + F_{44} = 0, \tag{41}$$

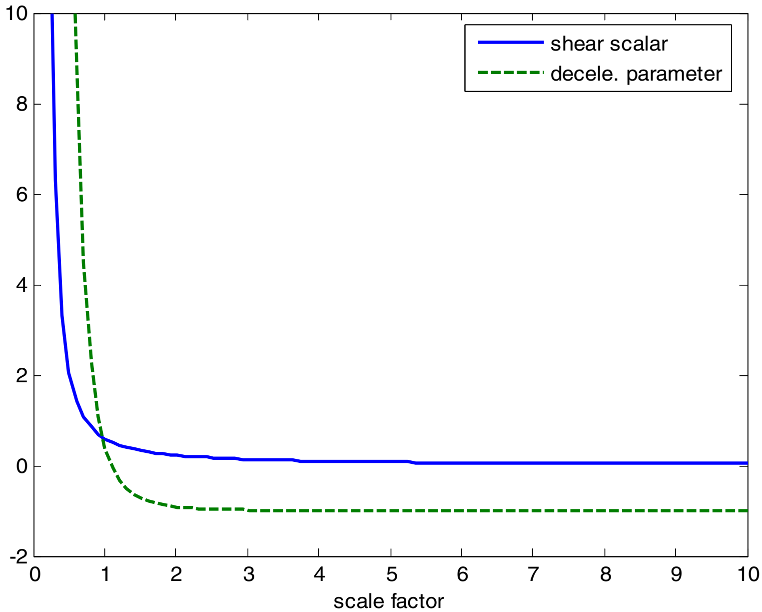

Fig. 4 Evolution of shear scalar and deceleration parameter versus scale factor (A)

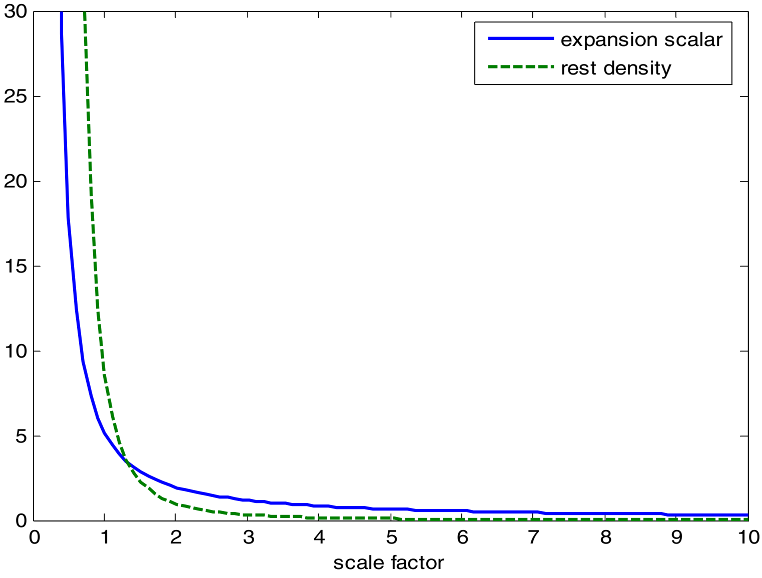

Fig. 5 Evolution of rest density and expansion scalar versus scalar factor (A)

$$\begin{array}{c}\frac{F}{A^2\sin^2\theta+B^2\cos^2\theta}\left[\begin{array}{l} A^2\sin^2\theta\left(\frac{A_4^2}{A^2}+\frac{A_{44}}{A}+\frac{A_4B_4}{AB}-\frac{B^2}{2A^4}+\frac{1}{A^2}\right)+\\ B^2\cos^2\theta\left(\frac{B_{44}}{B}+\frac{2A_4B_4}{AB}+\frac{B^2}{2A^4}\right)\end{array}\right] \\ +\frac{(AA_4\sin^2\theta+BB_4\cos^2\theta)}{A\sin^2\theta+B\cos^2\theta}F_4-\frac{1}{2}f-\frac{B_4F_4}{B}-\frac{2A_4F_4}{A}-F_{44}=0\end{array}, \tag{42}$$

$$F\left[\frac{A_4^2}{A^2}+\frac{A_{44}}{A}+\frac{A_4B_4}{AB}-\frac{B^2}{2A^4}+\frac{1}{A^2}\right]-\frac{1}{2}f-\frac{B_4F_4}{B}-\frac{A_4F_4}{A}-F_{44}=k^2\lambda. \tag{43}$$

For simplicity we assume adhoc relation [31, 32]

$$\frac{B_{44}}{B}+\frac{A_4B_4}{AB}+\frac{B^2}{2A^4}=\frac{A_{44}}{A}+\frac{A_4B_4}{AB}+\frac{A_4^2}{A^2}-\frac{B^2}{2A^2}+\frac{1}{A^2}. \tag{44}$$

Using (10) and (44), we get

$$\frac{dA}{dt}=\left[-\frac{1}{2n(1-n)}A^{2n-2}+c_2A^{-2n-2}+\frac{1}{1-n^2}\right]^{\frac{1}{2}}, n\neq\pm1. \tag{45}$$

Where c_2 is constant of integration.

The line element (3) reduces to

$$\begin{array}{l}ds^2=-\left[-\frac{1}{2n(1-n)}A^{2n-2}+c_2A^{-2n-2}+\frac{1}{1-n^2}\right]^{-1/2}, dA^2\\ +A^2d\theta^2+A^{2n}d\psi^2+\left(A^2\sin^2\theta+A^{2n}\cos^2\theta\right)d\varphi^2\\ +2A^{2n}\cos\theta d\psi d\theta\end{array}. \tag{46}$$

To find the solution for this model we take $c_2=0$ and $n=2$ so that the (45) reduces to

$$\frac{dA}{\left[\frac{4}{3}-A^2\right]^{-1/2}}=\frac{1}{2}dt. \tag{47}$$

Integrating (47), we obtain

$$A = \frac{2}{\sqrt{3}} \sin\left(\frac{1}{2}t\right), \tag{48}$$

$$B = \frac{4}{3} \sin^2\left(\frac{1}{2}t\right). \tag{49}$$

These are the solution of the field equations subjected to the condition $F_4 = 0$. From (48) and (49), it is observed that both the scale factors are functions of cosmic time. They admit oscillatory nature.

The volume of the universe becomes

$$V = \frac{16}{9} \sin^4\left(\frac{1}{2}t\right). \tag{50}$$

The Ricci scalar in this model found to be

$$R(t) = \frac{5}{\sin^2\left(\frac{1}{2}t\right)} - 6. \tag{51}$$

The function of Ricci scalar is obtained as

$$f(R) = -1 + 3\cot^2\left({}^{1}/{}_{2}t\right). \tag{52}$$

From (43) and (52), we get $\lambda = 0$. Thus, in the present model also strings do not survive, which corroborate to Rao and Davuluri [38].

Using (48), (49), (52) and (40), we yield

$$\rho = \frac{1}{k^2} \cos ec^2\left({}^{1}/{}_{2}t\right). \tag{53}$$

From Fig. 6, it is observed that the density $\rho \geq 0$ that means the model satisfies energy condition. The energy density oscillates in positive quadrant.

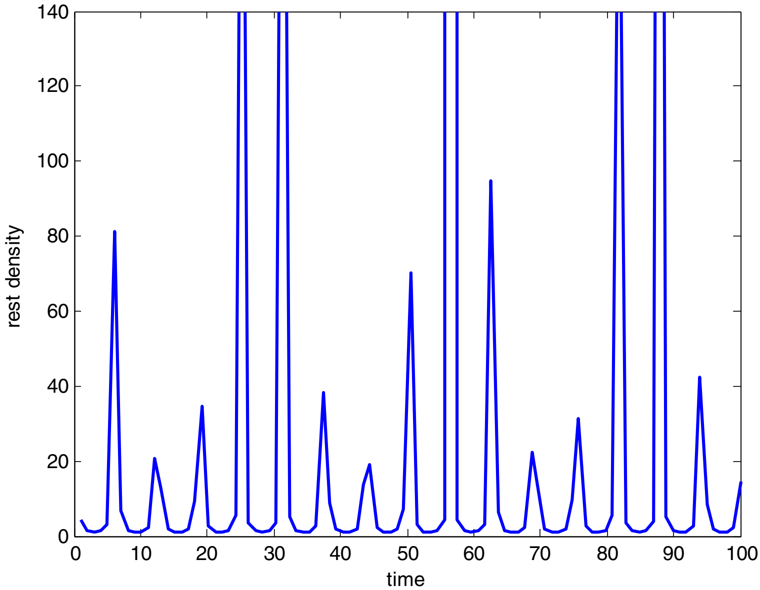

Fig. 6 Evolution of rest density versus time for $k = 1$

8 Kinematical Variables

The kinematical variables (shear scalar, expansion scalar and deceleration parameter) in this model are

$$\sigma = \pm\frac{1}{\sqrt{12}}\cot\left(\frac{1}{2}t\right), \tag{54}$$

$$\theta = 2\cot\left(\frac{1}{2}t\right), \tag{55}$$

$$q = \frac{3}{2}\sec^2\left(\frac{1}{2}t\right) - 1. \tag{56}$$

From equation (59), it is found that the deceleration parameter remain positive throughout the evolution i.e. the universe decelerate in standard way. The ratio of shear scalar to expansion scalar is obtained as $\frac{\sigma}{\theta} \cong 0.14$ which shows that the universe is anisotropic. The result resembles to result obtained by Katore and Rane [41].

9 Conclusion

In the present study we have developed Bianchi type II, VIII and IX space- time in the presence of string cloud in the $f(R)$ modified theory of gravitation. It is found that

1) Strings do not exist in the $f(R)$ theory of gravitation, which corroborate the investigation of Rao and Davuluri [38], Reddy and Naidu [43], Reddy et al. [44]. It should be note that even though strings are not survive we get similar type of solution for Bianchi type IX model in general relativity as obtained by Bali and Dave [45].
2) In case of Bianchi type II and IX cosmological models the energy density the scale factor admits oscillatory nature. The energy density oscillates in positive quadrant. The universe decelerates in standard way. The universe is anisotropic.
3) In case of Bianchi type VIII cosmological model the energy density is decreasing function of time. The volume of the universe is increasing indefinitely. The universe approaches isotropy at late time. The universe was decelerating in the past and accelerating at present.

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