

Geometric Phase of Time-Dependent Superconducting Qubit

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Abstract Geometric phases are studied in terms of invariant operator for a time-dependent superconducting qubit. The results show that the geometric phase depends on the dipole interaction strength between the qubit and a microwave field of frequency and phase, which provides a clue to realize the geometric quantum computation in the experiments.

Keywords Geometric phase · Solid-state qubit · Invariant operator

1 Introduction

For a closed quantum system, the wave function describing by Schrödinger equation can be separated into two parts with amplitude and phase [1]. In the representation of probability wave, such a phase is not correlated to the probability amplitude. Fortunately, a quantum system retains memory of its evolution in terms of a geometric (Berry) phase [2–4] when it undergoes a closed evolution. This Berry phase can be interpreted as a holonomy of the Hermitian fibre bundle over the parameter space [5, 6]. The Berry phase is proportional to the area spanned in parameter space and independent of the path traversed by the system during its evolution, which means that the geometric phase has an observable consequence.

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Such a potential value makes it importantly observe and further apply the geometric phase in different quantum systems [7–15].

In quantum information science, the phase of a wave function plays an important role in encoding information. Although some traditional experiments depend on dynamic effects to manipulate this information, an alternative approach is to use a geometric phase that is called as geometric quantum computation [16, 17]. The geometric quantum computation is a potentially approach to obtain an intrinsic fault tolerant scheme and therefore resilient to certain types of computational errors [18–21]. Such a holonomy can be generated when a quantum system is driven in a cyclic evolution through adiabatic or nonadiabatic change in the control parameters in the Hamiltonian.

The studies of the geometric phases under more realistic situations have been promoted by the fact that a physical system interacts irreversibly with its surrounding environment [22–27]. In a really closed system, a useful way to remove the adiabatic constraint in quantum computation is the theory of the dynamical invariant to treat time-dependent Hamiltonian. Indeed, the dynamically invariant theory was recently used in a proposal of an interferometric experiment to measure the nonadiabatic geometric phase in cavity quantum electrodynamics [28].

In the other hand, a superconducting nanostructure [29, 30] with its potential scalability leads to a promising solid-state platform for quantum information processing [31–34]. The coherent control of macroscopic quantum states in superconducting circuits [35–38], especially for the two-level system, makes it possibly observe the Berry phase.

In this work, we first find a corresponding invariant operator for a solid-state qubit interacting with a microwave field under the case of dipole coupling. Then an exact solution of Schrödinger equation is found. Further, we obtain the geometric phases of the solid-state qubit. By analyzing the geometric phases, we show that the geometric phases are relative to the microwave field of frequency and phase as well as the dipole coupling strength.

2 Invariant Operator and Solid-State Qubit

Let us consider a superconducting qubit in a time-dependent magnetic field. In the experiment, fast and accurate control of the magnetic field for this qubit is achieved through phase and amplitude modulation of microwave radiation coupled to the qubit through the input port of the resonator [38]. The qubit Hamiltonian in the presence of such radiation is

$$\mathcal{H}(t) = \frac{\hbar}{2}\omega_a\sigma_z + \hbar\Omega_R \cos(\omega_b t + \psi)\sigma_x, \quad (1)$$

where σ_z and σ_x are the Pauli operators, \hbar is Planck's constant divided by 2π , $\hbar\Omega_R$ is the dipole interaction strength between the qubit and a microwave field of frequency ω_b and phase ψ , and t is time.

The above Hamiltonian may be transformed to a dynamic invariant operator satisfying

$$\frac{\partial \mathcal{I}(t)}{\partial t} = \frac{i}{\hbar}[\mathcal{I}(t), \mathcal{H}(t)], \quad (2)$$

where $\mathcal{I}(t)$ is a Hermitian invariant operator with a member of a complete set of commuting observables.

For a two-level system, the invariant operator is a 2×2 matrix. Thus the unit matrix and three Pauli matrices (σ_x , σ_y , and σ_z) construct a complete basis of our density matrix. Therefore, we can expand the density matrix as

$$\mathcal{I}(t) = \alpha(t)\sigma_z + \beta(t)\sigma_y + \gamma(t)\sigma_x + c(t), \tag{3}$$

where $\alpha(t)$, $\beta(t)$, $\gamma(t)$ and $c(t)$ are determined by the invariant (2). Inserting (3) into (2), we find

$$\begin{aligned} \frac{i}{\hbar}[\mathcal{I}(t), \mathcal{H}(t)] &= i[\alpha(t)\sigma_z + \beta(t)\sigma_y + \gamma(t)\sigma_x + c(t), \frac{1}{2}\omega_a\sigma_z + \Omega_R \cos(\omega_b t + \psi)\sigma_x] \\ &= -2\Omega_R \cos(\omega_b t + \psi)\alpha(t)\sigma_y - \omega_a\beta(t)\sigma_x + 2\Omega_R \cos(\omega_b t + \psi)\beta(t)\sigma_z + \omega_a\gamma(t)\sigma_y, \end{aligned} \tag{4}$$

and

$$\frac{\partial \mathcal{I}(t)}{\partial t} = \frac{\partial \alpha(t)}{\partial t}\sigma_z + \frac{\partial \beta(t)}{\partial t}\sigma_y + \frac{\partial \gamma(t)}{\partial t}\sigma_x. \tag{5}$$

Comparing (4) with (5), one has

$$\frac{\partial \alpha(t)}{\partial t} = 2\Omega_R \cos(\omega_b t + \psi)\beta(t), \tag{6}$$

$$\frac{\partial \beta(t)}{\partial t} = -2\Omega_R \cos(\omega_b t + \psi)\alpha(t) + \omega_a\gamma(t), \tag{7}$$

$$\frac{\partial \gamma(t)}{\partial t} = -\omega_a\beta(t), \tag{8}$$

$$\frac{\partial c(t)}{\partial t} = 0. \tag{9}$$

Equations (6)–(8) can be rescaled as a matrix form, i.e.,

$$\frac{\partial}{\partial t} \begin{pmatrix} \alpha(t) \\ \beta(t) \\ \gamma(t) \end{pmatrix} = \begin{pmatrix} 0 & 2\Omega_R \cos(\omega_b t + \psi) & 0 \\ -2\Omega_R \cos(\omega_b t + \psi) & 0 & \omega_a \\ 0 & -\omega_a & 0 \end{pmatrix} \begin{pmatrix} \alpha(t) \\ \beta(t) \\ \gamma(t) \end{pmatrix}, \tag{10}$$

where $c(t) = 0$ are taken in terms of (10) and (4).

In order to seek for a solution of (10), we first consider an eigenequation, such as

$$\begin{pmatrix} 0 & 2\Omega_R \cos(\omega_b t + \psi) & 0 \\ -2\Omega_R \cos(\omega_b t + \psi) & 0 & \omega_a \\ 0 & -\omega_a & 0 \end{pmatrix} \begin{pmatrix} \alpha(t) \\ \beta(t) \\ \gamma(t) \end{pmatrix} = \lambda \begin{pmatrix} \alpha(t) \\ \beta(t) \\ \gamma(t) \end{pmatrix}. \tag{11}$$

We find that the solution of eigenequation (11) can be expressed as

$$| \vec{\lambda}_1(t) \rangle = \frac{1}{\lambda} \begin{pmatrix} \omega_a \\ 0 \\ 2\Omega_R \cos(\omega_b t + \psi) \end{pmatrix}, \tag{12}$$

$$| \vec{\lambda}_2(t) \rangle = \frac{1}{\sqrt{2}\lambda} \begin{pmatrix} -2\Omega_R \cos(\omega_b t + \psi) \\ i\lambda \\ \omega_a \end{pmatrix}, \tag{13}$$

$$| \vec{\lambda}_3(t) \rangle = \frac{1}{\sqrt{2}\lambda} \begin{pmatrix} -2\Omega_R \cos(\omega_b t + \psi) \\ -i\lambda \\ \omega_a \end{pmatrix}, \tag{14}$$

with the corresponding three eigenvalues, i.e., $\lambda_1 = 0$, $\lambda_2 = -i\lambda$ and $\lambda_3 = i\lambda$ with $\lambda = \sqrt{\omega_a^2 + 4\Omega_R^2 \cos^2(\omega_b t + \psi)}$, respectively.

Combining (10) with (11), (10) is rescaled as

$$\frac{\partial}{\partial t} \begin{pmatrix} \alpha(t) \\ \beta(t) \\ \gamma(t) \end{pmatrix} = \lambda(t) \begin{pmatrix} \alpha(t) \\ \beta(t) \\ \gamma(t) \end{pmatrix}, \tag{15}$$

which has a form solution under the adiabatic approximation,

$$\begin{pmatrix} \alpha(t) \\ \beta(t) \\ \gamma(t) \end{pmatrix} = \exp\left(\int_0^t \lambda(t) dt\right) \begin{pmatrix} \alpha(t=0) \\ \beta(t=0) \\ \gamma(t=0) \end{pmatrix}. \tag{16}$$

where $\lambda(t)$ is eigenvalues with a 3×3 diagonal matrix.

Under the adiabatic approximation, thus, the solution of (10) can be written as

$$\begin{pmatrix} \alpha(t) \\ \beta(t) \\ \gamma(t) \end{pmatrix} = c_1 | \vec{\lambda}_1(t=0) \rangle + c_2 e^{-ig(t)} | \vec{\lambda}_2(t=0) \rangle + c_3 e^{ig(t)} | \vec{\lambda}_3(t=0) \rangle, \tag{17}$$

where $g(t) = \int_0^t \lambda dt$, while c_1, c_2 and c_3 are determined by the initial conditions. Equation (14) can be further expressed as

$$\alpha(t) = \frac{1}{\lambda_0} \left(c_1 \omega_a + \sqrt{2} \Omega_R \cos(\psi) (-c_2 \cos(g(t)) + c_3 \sin(g(t))) \right), \tag{18}$$

$$\beta(t) = \frac{1}{\sqrt{2}} (c_2 \sin(g(t)) + c_3 \cos(g(t))), \tag{19}$$

$$\gamma(t) = \frac{1}{\lambda_0} \left(2c_1 \Omega_R \cos(\psi) + \frac{\omega_a}{\sqrt{2}} (c_2 \cos(g(t)) - c_3 \sin(g(t))) \right). \tag{20}$$

In terms of Pauli matrices, the invariant operators (3) can be rewritten as a matrix form, i.e.,

$$I = \begin{pmatrix} \alpha(t) & \gamma(t) - i\beta(t) \\ \gamma(t) + i\beta(t) & -\alpha(t) \end{pmatrix}, \tag{21}$$

with two normalized eigenstates,

$$| \vec{\lambda}_{m1} \rangle_N = \frac{1}{r_1(t)} \begin{pmatrix} \alpha(t) + \lambda_m \\ \gamma(t) + i\beta(t) \end{pmatrix}, \tag{22}$$

and

$$| \vec{\lambda}_{m2} \rangle_N = \frac{1}{r_2(t)} \begin{pmatrix} \alpha(t) - \lambda_m \\ \gamma(t) + i\beta(t) \end{pmatrix}, \tag{23}$$

where $\lambda_m = \sqrt{\alpha^2(t) + \beta^2(t) + \gamma^2(t)}$, $\lambda_{m1} = \lambda_m$, $\lambda_{m2} = -\lambda_m$, $r_1(t) = \sqrt{(\alpha(t) + \lambda_m)^2 + \beta^2(t) + \gamma^2(t)}$ and $r_2(t) = \sqrt{(\alpha(t) - \lambda_m)^2 + \beta^2(t) + \gamma^2(t)}$.

In order to simplify our equation, next, we define two parameters as

$$\theta(t) = \cos^{-1} \left(\frac{\alpha(t) + \lambda_m}{r_2(t)} \right), \phi(t) = \tan^{-1} \left(\frac{\beta(t)}{\gamma(t)} \right), \tag{24}$$

which is similar to the two azimuthal angles [39–43]. Thus the eigenstates (19) and (20) can be rewritten as

$$| \vec{\lambda}_{m1} \rangle_N = \begin{pmatrix} \cos(\theta(t)) \\ e^{i\phi(t)} \sin(\theta(t)) \end{pmatrix}, \tag{25}$$

and

$$|\vec{\lambda}_{m2}\rangle_N = \begin{pmatrix} \sin(\theta(t)) \\ -e^{i\phi(t)} \cos(\theta(t)) \end{pmatrix}. \tag{26}$$

Suppose that the initial state is given by $\theta(t = 0) = \frac{\pi}{4}$, $\phi(t = 0) = 0$ and $\beta(t = 0) = 0$, we find

$$c_1 = \frac{2\Omega_R \cos(\psi)}{\sqrt{\omega_a^2 + 4\Omega_R^2 \cos^2(\psi)}}, \tag{27}$$

$$c_2 = \frac{\sqrt{2}\omega_a}{\sqrt{\omega_a^2 + 4\Omega_R^2 \cos^2(\psi)}}, \tag{28}$$

$$c_3 = 0, \tag{29}$$

$$\alpha(t) = \frac{2\omega_a \Omega_R \cos(\psi)(1 - \cos(g(t)))}{\omega_a^2 + 4\Omega_R^2 \cos^2(\psi)}, \tag{30}$$

$$\beta(t) = \frac{\omega_a \sin(g(t))}{\sqrt{\omega_a^2 + 4\Omega_R^2 \cos^2(\psi)}}, \tag{31}$$

$$\gamma(t) = \frac{4\Omega_R^2 \cos^2(\psi) + \omega_a^2 \cos(g(t))}{\omega_a^2 + 4\Omega_R^2 \cos^2(\psi)}. \tag{32}$$

From (24)–(29), we can determine the invariant operator and its eigenstates.

3 Invariant Operator and Geometric Phase

It is known that $I(t)$ is one of a complete set of commuting observables. Thus there exists a complete set of eigenstates of $I(t)$. The nondegenerate eigenvalue equation of the time-dependent invariant operator is given by

$$I(t)|\lambda_n, t\rangle = \lambda_n |\lambda_n, t\rangle, \tag{33}$$

which is used to construct the solution of the Schrödinger equation because the eigenstate, $|\lambda_n, t\rangle$, is also an eigenstate of the Hamiltonians $H(t)$, i.e.,

$$|\Psi(t)\rangle = \sum_n c_n e^{i\chi_n} |\lambda_n, t\rangle, \tag{34}$$

where the c_n is independent of involving time and is chosen by the initial condition. The phases χ_n , called Lewis phases, are determined by the schrödinger equation, i.e.,

$$\dot{\chi}_n = \langle \lambda_n, t | i \frac{\partial}{\partial t} - H(t) | \lambda_n, t \rangle. \tag{35}$$

Inserting (22) and (23) into (32), we can exactly get a total phase of the solid-state qubit. Then substituting (32) into (31), we obtain the wave function satisfying the Schrödinger equation, which is spanned by the instantaneous eigenstates of the invariant operator $I(t)$.

In (32), the second terms is called as a dynamic phase because of relating to the Hamiltonian $H(t)$. The first term is a geometric phase and is written as

$$\gamma_n^B = i \int_0^\tau \langle \lambda_n, t | \frac{\partial}{\partial t} | \lambda_n, t \rangle dt, \tag{36}$$

which elucidates an intimate connection between the Aharonov and Anandan approach and the invariant approach to the geometric phase.

From (33), we see that the geometric phase is independent of the Hamiltonian $H(t)$. Therefore, the geometric phase exists even for Hamiltonians that do not have an explicit time-dependence. Especially, the geometric phase can be computed without the adiabatic hypothesis.

According to (33), the geometric phases for the solid-state qubit can be written as

$$\begin{aligned}
 \gamma_{1g}^B &= -Im \oint_0^T dt \left({}_N \langle \vec{\lambda}_{m1} | \frac{\partial}{\partial t} | \vec{\lambda}_{m1} \rangle_N \right) \\
 &= -Im \oint_0^T dt \left(\cos(\theta(t)) e^{-i\phi(t)} \sin(\theta(t)) \right) \frac{\partial}{\partial t} \left(\begin{matrix} \cos(\theta(t)) \\ e^{i\phi(t)} \sin(\theta(t)) \end{matrix} \right) \\
 &= -\oint_0^T \sin^2(\theta(t)) d\phi(t),
 \end{aligned} \tag{37}$$

and

$$\begin{aligned}
 \gamma_{2g}^B &= -Im \oint_0^T dt \left({}_N \langle \vec{\lambda}_{m2} | \frac{\partial}{\partial t} | \vec{\lambda}_{m2} \rangle_N \right) \\
 &= -Im \oint_0^T dt \left(\sin(\theta(t)) -e^{-i\phi(t)} \cos(\theta(t)) \right) \frac{\partial}{\partial t} \left(\begin{matrix} \sin(\theta(t)) \\ -e^{i\phi(t)} \cos(\theta(t)) \end{matrix} \right) \\
 &= -\oint_0^T \cos^2(\theta(t)) d\phi(t).
 \end{aligned} \tag{38}$$

4 Results and Discussion

The geometric phases γ_{1g}^B and γ_{2g}^B as a function of Ω_R with $\omega_a - \omega_b = 100\pi$ MHz for different initial angle ψ are shown in Figs. 1 and 2, respectively. We find that the geometric phases γ_{1g}^B and γ_{2g}^B are separated into two groups in terms of the initial angles. From Fig. 1, we see that the geometric phases γ_{1g}^B is a linear increasing function of Ω_R in the region of $\psi \in [0, \pi/2]$. In the region of $\psi \in [\pi/2, \pi]$, however, the geometric phases γ_{1g}^B is an linear decreasing function of Ω_R . In contrast with the geometric phases γ_{2g}^B as shown in Fig. 2, the geometric phases γ_{2g}^B is a linear decreasing function of Ω_R in the region of $\psi \in [0, \pi/2]$ and a linear increasing function. Obviously, the geometric phases γ_{1g}^B and γ_{2g}^B are symmetry relatively to the $\psi = \pi/2$.

The geometric phases γ_{1g}^B and γ_{2g}^B as functions of ψ with $\Omega_R = 30$ MHz for different initial $\omega_a - \omega_b$ are plotted in Figs. 3 and 4. The results show that there exists a maximum value for the geometric phase γ_{1g}^B and minimum one for the geometric phase γ_{2g}^B at point of $\psi = 0$ for all $\omega_a - \omega_b$. For the region of $\psi > 0$, the geometric phase γ_{1g}^B increases smoothly and crosses the maximum point and then decreases smoothly. Differently from the γ_{1g}^B , the geometric phase γ_{2g}^B decreases smoothly and crosses the maximum point and then increases smoothly. For both the γ_{1g}^B and γ_{2g}^B , we find that there exist symmetries between the $\omega_a - \omega_b > 0$ and $\omega_a - \omega_b < 0$.

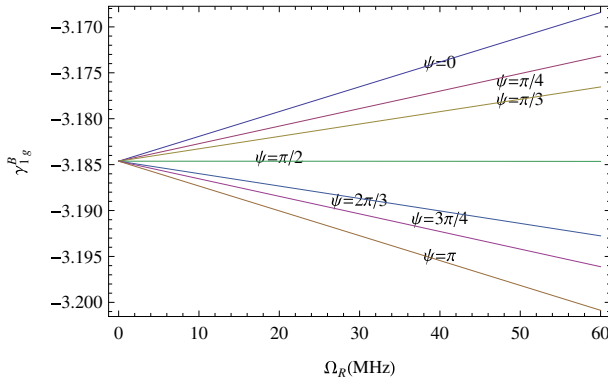


Fig. 1 (Color online) Geometric phase γ_{1g}^B as a function of Ω_R with $\omega_a - \omega_b = 100\pi$ MHz for different initial angles ψ

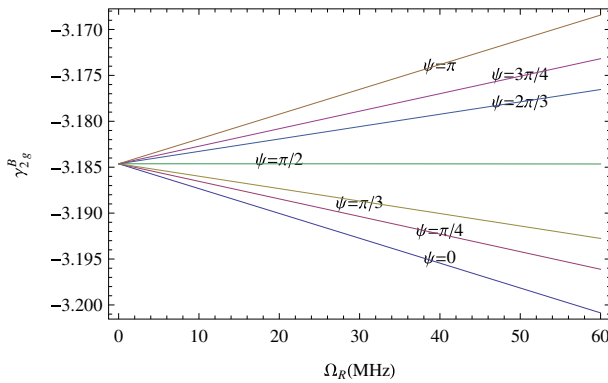


Fig. 2 (Color online) Geometric phase γ_{2g}^B as a function of Ω_R with $\omega_a - \omega_b = 100\pi$ MHz for different initial angles ψ

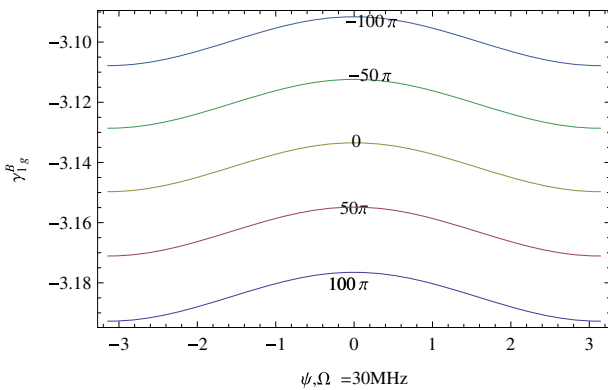


Fig. 3 (Color online) Geometric phase γ_{1g}^B as a function of ψ with $\Omega_R = 30$ MHz for different $\omega_a - \omega_b$

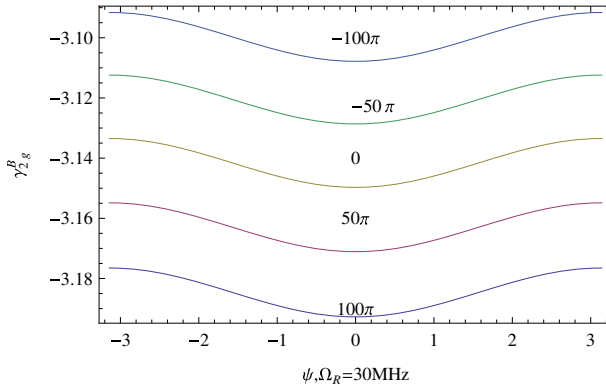


Fig. 4 (Color online) Geometric phase γ_{2g}^B as a function of ψ with $\Omega_R = 30$ MHz for different $\omega_a - \omega_b$

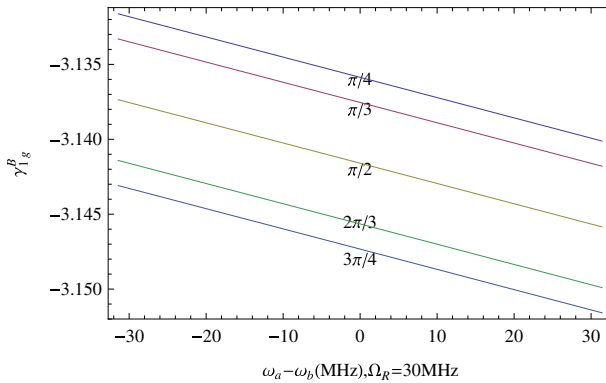


Fig. 5 (Color online) Geometric phase γ_{1g}^B as a function of $\omega_a - \omega_b$ with $\Omega_R = 30$ MHz for different initial angles ψ

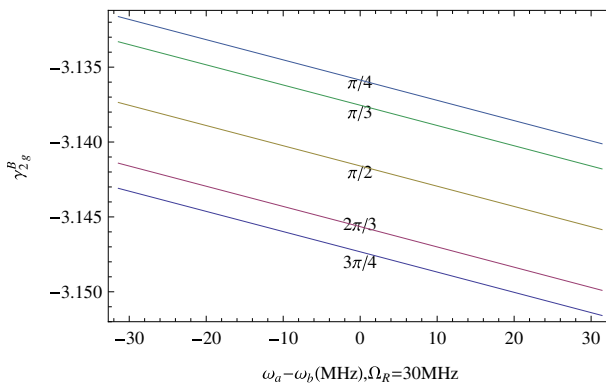


Fig. 6 (Color online) Geometric phase γ_{2g}^B as a function of $\omega_a - \omega_b$ with $\Omega_R = 30$ MHz for different initial angles ψ

For the different initial angles, the geometric phases γ_{1g}^B and γ_{2g}^B as a function of $\omega_a - \omega_b$ with $\Omega_R = 30$ MHz are shown in Figs. 5 and 6. We find that both the geometric phases γ_{1g}^B and γ_{2g}^B are linear decreasing functions of $\omega_a - \omega_b$.

5 Conclusions

In summary, we obtain an exact solution of the qubit Hamiltonian in the presence of such radiation in terms of the theory of the dynamical invariant to treat time-dependent Hamiltonian. We analyze all factors to affect the geometric phases in controlling physical quantities. We find that the geometric phases depend on the initial angle ψ and $\omega_a - \omega_b$, which provide a useful clue to control and implement a geometric quantum computation.

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