Int J Theor Phys (2015) 54:1515–1518
DOI 10.1007/s10773-014-2348-6

Splitting an Arbitrary Two-ubit State Via a Seven-qubit Maximally Entangled State

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Received: 3 July 2014 / Accepted: 15 September 2014 / Published online: 24 September 2014

Abstract We investigate the usefulness of a recently introduced seven-qubit maximally entangled state by Zha et al. (J. Phys. A: Math. Theor. 45:255–302 2012) for quantum information splitting. It is shown that such a seven-qubit entangled state can be utilized for quantum information splitting of an arbitrary two-qubit state by performing only the Bell-state measurements.

Keywords Quantum information splitting · Arbitrary two-qubit state · Seven-qubit maximally entangled state

1 Introduction

Quantum entanglement has been recognized as one of the most striking results of quantum information theory. It has played a key role in the development of quantum communication and quantum computation. In particular, it also has a significant impact on the quantum information splitting [1]. It is known that quantum information splitting (QIS) not only provides absolute security, but also likely plays an important role in protecting secret quantum information. Owing to its numerous practical application in quantum communication [2], many QIS schemes have been investigated [3–9].

Recently Zha et al. [10] have proposed a seven-qubit maximally entangled state. This state has many interesting entanglement properties and wide applications in quantum information processing and fundamental tests of quantum physics. In this work, we demonstrate that such a seven-qubit maximally entangled state can be used as the quantum channel to realize the deterministic QIS of an arbitrary two-qubit state by performing only the Bell-state measurements (BSMs).

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2 QIS of an Arbitrary Two-Qubit State

Our scheme can be described as follows. Suppose there are three legitimate parties, say, Alice, Charlie and Bob. Alice is the sender of quantum information. Charlie and Bob are two agents. Suppose the sender Alice has an arbitrary two-qubit state, which is given by

$$|\psi\rangle_{AB} = \alpha\,|00\rangle_{AB} + \beta\,|10\rangle_{AB} + \gamma\,|01\rangle_{AB} + \theta\,|11\rangle_{AB}\,, \tag{1}$$

where $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\theta|^2 = 1$. Now Alice wants to send the state to Bob who is assigned to reconstruct the original state with the help of Charlie. Hence, she prepares a seven-qubit maximally entangled state, which is in the form of [10]

$$\begin{aligned}|\psi\rangle_{1234567} = \frac{1}{4\sqrt{2}} &\big\{\big(|000\rangle + |111\rangle\big)_{127}\left[|0\rangle_3\,(|000\rangle + |111\rangle)_{456} + |1\rangle_3\,(|110\rangle - |001\rangle)_{456}\right] \\ &+ (|010\rangle - |101\rangle)_{127}\left[|0\rangle_3\,(|100\rangle + |011\rangle)_{456} + |1\rangle_3\,(|010\rangle - |101\rangle)_{456}\right] \\ &+ (|100\rangle - |011\rangle)_{127}\left[|0\rangle_3\,(|101\rangle - |010\rangle)_{456} - |1\rangle_3\,(|011\rangle + |100\rangle)_{456}\right] \\ &+ (|001\rangle + |110\rangle)_{127}\left[|0\rangle_3\,(|001\rangle + |110\rangle)_{456} + |1\rangle_3\,(|000\rangle - |111\rangle)_{456}\right]\big\}\,,\end{aligned} \tag{2}$$

in which the qubits 1, 2, 7 sent to Charlie, and 5 and 6 to Bob. The combined state of the nine-qubit system is given by,

$$\begin{aligned}|\Psi\rangle_{AB1234567} &= |\psi\rangle_{AB} \otimes |\psi\rangle_{1234567} \\ &= \frac{1}{4}\big[|\Phi^+\rangle_{A3}|\Phi^+\rangle_{B4}\,(\alpha|\xi_1\rangle + \beta|\xi_3\rangle + \gamma|\xi_2\rangle + \theta|\xi_4\rangle)_{12567} \\ &\quad + |\Phi^+\rangle_{A3}|\Phi^-\rangle_{B4}\,(\alpha|\xi_1\rangle + \beta|\xi_3\rangle - \gamma|\xi_2\rangle - \theta|\xi_4\rangle)_{12567} \\ &\quad + |\Phi^-\rangle_{A3}|\Phi^+\rangle_{B4}\,(\alpha|\xi_1\rangle - \beta|\xi_3\rangle + \gamma|\xi_2\rangle - \theta|\xi_4\rangle)_{12567} \\ &\quad + |\Phi^-\rangle_{A3}|\Phi^-\rangle_{B4}\,(\alpha|\xi_1\rangle - \beta|\xi_3\rangle - \gamma|\xi_2\rangle + \theta|\xi_4\rangle)_{12567} \\ &\quad + |\Phi^+\rangle_{A3}|\Phi^+\rangle_{B4}\,(\alpha|\xi_1\rangle + \beta|\xi_3\rangle + \gamma|\xi_2\rangle + \theta|\xi_4\rangle)_{12567} \\ &\quad + |\Phi^+\rangle_{A3}|\Phi^-\rangle_{B4}\,(\alpha|\xi_1\rangle + \beta|\xi_3\rangle - \gamma|\xi_2\rangle - \theta|\xi_4\rangle)_{12567} \\ &\quad + |\Phi^-\rangle_{A3}|\Phi^+\rangle_{B4}\,(\alpha|\xi_1\rangle - \beta|\xi_3\rangle + \gamma|\xi_2\rangle - \theta|\xi_4\rangle)_{12567} \\ &\quad + |\Phi^-\rangle_{A3}|\Phi^-\rangle_{B4}\,(\alpha|\xi_1\rangle - \beta|\xi_3\rangle - \gamma|\xi_2\rangle + \theta|\xi_4\rangle)_{12567} \\ &\quad + |\Psi^+\rangle_{A3}|\Phi^+\rangle_{B4}\,(\alpha|\xi_1\rangle + \beta|\xi_3\rangle + \gamma|\xi_2\rangle + \theta|\xi_4\rangle)_{12567} \\ &\quad + |\Psi^+\rangle_{A3}|\Phi^-\rangle_{B4}\,(\alpha|\xi_1\rangle + \beta|\xi_3\rangle - \gamma|\xi_2\rangle - \theta|\xi_4\rangle)_{12567} \\ &\quad + |\Psi^-\rangle_{A3}|\Phi^+\rangle_{B4}\,(\alpha|\xi_1\rangle - \beta|\xi_3\rangle + \gamma|\xi_2\rangle - \theta|\xi_4\rangle)_{12567} \\ &\quad + |\Psi^-\rangle_{A3}|\Phi^-\rangle_{B4}\,(\alpha|\xi_1\rangle - \beta|\xi_3\rangle - \gamma|\xi_2\rangle + \theta|\xi_4\rangle)_{12567} \\ &\quad + |\Psi^+\rangle_{A3}|\Phi^+\rangle_{B4}\,(\alpha|\xi_1\rangle + \beta|\xi_3\rangle + \gamma|\xi_2\rangle + \theta|\xi_4\rangle)_{12567} \\ &\quad + |\Psi^+\rangle_{A3}|\Phi^-\rangle_{B4}\,(\alpha|\xi_1\rangle + \beta|\xi_3\rangle - \gamma|\xi_2\rangle - \theta|\xi_4\rangle)_{12567} \\ &\quad + |\Psi^-\rangle_{A3}|\Phi^+\rangle_{B4}\,(\alpha|\xi_1\rangle - \beta|\xi_3\rangle + \gamma|\xi_2\rangle - \theta|\xi_4\rangle)_{12567} \\ &\quad + |\Psi^-\rangle_{A3}|\Phi^-\rangle_{B4}\,(\alpha|\xi_1\rangle - \beta|\xi_3\rangle - \gamma|\xi_2\rangle + \theta|\xi_4\rangle)_{12567}\big]\,,\end{aligned} \tag{3}$$

where

$$
\begin{aligned}
|\xi_1\rangle_{12567} &= \frac{1}{2\sqrt{2}}\left[(|000\rangle+|111\rangle)_{127}|00\rangle_{56}+(|010\rangle-|101\rangle)_{127}|11\rangle_{56}\right.\\
&\quad\left.+(|100\rangle-|011\rangle)_{127}|10\rangle_{56}+(|001\rangle+|110\rangle)_{127}|01\rangle_{56}\right\},\\
|\xi_2\rangle_{12567} &= \frac{1}{2\sqrt{2}}\left[(|000\rangle+|111\rangle)_{127}|11\rangle_{56}+(|010\rangle-|\rangle)_{127}|101\rangle_{56}\right.\\
&\quad\left.+(|100\rangle-|011\rangle)_{127}|01\rangle_{56}+(|001\rangle+|110\rangle)_{127}|10\rangle_{56}\right\},\\
|\xi_3\rangle_{12567} &= \frac{1}{2\sqrt{2}}\left[(|000\rangle+|111\rangle)_{127}|01\rangle_{56}+(|010\rangle-|101\rangle)_{127}|10\rangle_{56}\right.\\
&\quad\left.+(|011\rangle-|100\rangle)_{127}|11\rangle_{56}+(|001\rangle+|110\rangle)_{127}|00\rangle_{56}\right\},\\
|\xi_4\rangle_{12567} &= \frac{1}{2\sqrt{2}}\left[(|000\rangle+|111\rangle)_{127}|10\rangle_{56}+(|101\rangle-|010\rangle)_{127}|01\rangle_{56}\right.\\
&\quad\left.+(|011\rangle-|100\rangle)_{127}|00\rangle_{56}-(|001\rangle+|110\rangle)_{127}|11\rangle_{56}\right\},
\end{aligned}
$$

with $\left|\Phi^{\pm}\right\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$ and $\left|\Psi^{\pm}\right\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$.

To achieve the purpose of QIS, Alice first performs BSMs on her qubit pairs (A, 3) and (B, 4), respectively. It is known that Alice may obtain one of the 16 kinds of possible measured results with equal probability, and the remaining qubits may collapse into one of the 16 states after the measurement. Then Alice tells the BSMs result to Bob and Charlie. If Charlie allows Bob to reconstruct the initial state, he needs to carry out a single qubit measurement on his qubits 1, 2, 7 under the basis of $\{|0\rangle, |1\rangle\}$. By combining information from Alice and Charlie, Bob can reconstruct the original state $|\psi\rangle_{AB}$ with an appropriate unitary transformation on the qubits at hand.

Now, let us take an example to demonstrate the principle of this QIS protocol. Suppose Alice's BSMs outcome is $\left|\Phi^{+}\right\rangle_{A3}\left|\Phi^{+}\right\rangle_{B4}$, then the state of the remaining qubits collapse into the state

$$
\begin{aligned}
|\varphi\rangle_{12567} = \frac{\sqrt{2}}{4}\Big[&|000\rangle_{127}(\alpha|00\rangle+\beta|01\rangle+\gamma|11\rangle+\theta|10\rangle)_{56}\\
&+|111\rangle_{127}(\alpha|00\rangle+\beta|01\rangle+\gamma|11\rangle+\theta|10\rangle)_{56}\\
&+|001\rangle_{127}(\alpha|01\rangle+\beta|00\rangle+\gamma|10\rangle-\theta|11\rangle)_{56}\\
&+|110\rangle_{127}(\alpha|01\rangle+\beta|00\rangle+\gamma|10\rangle-\theta|11\rangle)_{56}\\
&+|010\rangle_{127}(\alpha|11\rangle+\beta|10\rangle+\gamma|00\rangle-\theta|01\rangle)_{56}\\
&-|101\rangle_{127}(\alpha|11\rangle+\beta|10\rangle+\gamma|00\rangle-\theta|01\rangle)_{56}\\
&+|100\rangle_{127}(\alpha|10\rangle-\beta|11\rangle+\gamma|01\rangle-\theta|00\rangle)_{56}\\
&-|011\rangle_{127}(\alpha|10\rangle-\beta|11\rangle+\gamma|01\rangle-\theta|00\rangle)_{56}\Big]. \qquad (4)
\end{aligned}
$$

Charlie can now make a single qubit measurement on qubits 1, 2, 7 under the basis of $\{|0\rangle, |1\rangle\}$, and then he sends the result of his measurement to Bob. If the measured result is $|000\rangle_{127}, |111\rangle_{127}, |001\rangle_{127}, |110\rangle_{127}, |010\rangle_{127}, |101\rangle_{127}, |100\rangle_{127}$ or $|011\rangle_{127}$, Bob needs to apply the local unitary operation $U_1, U_2, U_3, U_4, U_5, U_6, U_7$ or U_8 on his own qubits 5 and 6. And these unitary transformations are given by

$$
\begin{aligned}
U_1 &= |00\rangle\langle 00| + |10\rangle\langle 01| + |01\rangle\langle 11| + |11\rangle\langle 10|, \\
U_2 &= |00\rangle\langle 00| + |10\rangle\langle 01| + |01\rangle\langle 11| + |11\rangle\langle 10|, \\
U_3 &= |00\rangle\langle 01| + |10\rangle\langle 00| + |01\rangle\langle 10| - |11\rangle\langle 11|, \\
U_4 &= |00\rangle\langle 01| + |10\rangle\langle 00| + |01\rangle\langle 10| - |11\rangle\langle 11|, \\
U_5 &= |00\rangle\langle 11| + |10\rangle\langle 10| + |01\rangle\langle 00| - |11\rangle\langle 01|, \\
U_6 &= |00\rangle\langle 11| + |10\rangle\langle 10| + |01\rangle\langle 00| - |11\rangle\langle 01|, \\
U_7 &= |00\rangle\langle 10| - |10\rangle\langle 11| + |01\rangle\langle 01| - |11\rangle\langle 00|, \\
U_8 &= |00\rangle\langle 10| - |10\rangle\langle 11| + |01\rangle\langle 01| - |11\rangle\langle 00|.
\end{aligned}
$$

After doing those operations, Bob can successfully reconstruct the original unknown two-qubit state $|\psi\rangle_{AB}$.

3 Conclusions

In summary, we have proposed a scheme for QIS of an arbitrary two-qubit state by using a seven-particle maximally entangled state. In our scheme, Alice first performs two BSMs and announces her measurement outcome and assigns Bob to reconstruct the original unknown state. If the controller Charlie agrees to help Bob obtain the original state, he should perform single-qubit measurements on his qubits. With the sender's message and controller's help, the state receiver can reconstruct the original state via an appropriate local unitary operation.

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