Int J Theor Phys (2015) 54:1304–1315
DOI 10.1007/s10773-014-2328-x

Exact Solutions of Bianchi Type *V* Spacetime in *f(R, T)* Gravity

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Received: 1 April 2014 / Accepted: 3 September 2014 / Published online: 17 September 2014

Abstract The main purpose of this paper is to explore the exact solutions of Bianchi type *V* spacetime in *f(R, T)* theory of gravity (Harko et al. 2011). In this context, two exact solutions are investigated using assumptions of the variation law of Hubble parameter and constant deceleration parameter. The first solution corresponds to a singular model while the second solution gives a non-singular model of the universe. The physical quantities for these models are calculated. Moreover, the energy density and pressure of the universe is discussed in each case.

Keywords *f(R, T)* gravity · Bianchi type *V* · Deceleration parameter

1 Introduction

Recent observations from different sources such as cosmic microwave background [2, 3], Supernovae Ia (SNIa) experiments [4–9], X-ray experiments [10] and large scale structure [11, 12] suggest that our present universe is in expansion mode. The phenomenon of dark energy and dark matter is another interseting topic of discussion [13–22]. Einstein first introduced the concept of dark energy by including a small positive cosmological constant in the field equations. But after sometime, he rejected this idea. However, it is now thought that the cosmological constant is a suitable candidate for dark energy. Another justification of dark matter and expansion of universe comes from alternative theories of gravity. Recently developed *f(T)* gravity is an alternative theory which is a generalization of teleparallel gravity. This theory seems interesting as it may explain the cosmic acceleration without involving the dark energy. A considerable amount of work has been done in this theory so far [23–34].

f(R) theory of gravity is another example which involves a general function of Ricci scalar in the Lagrangian. Many authors have investigated *f(R)* gravity in different con-

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texts [36–55]. Some review articles [56–59] may be helpful to better understand the theory. Hendi and Momeni [60] explored black hole solutions in *f(R)* gravity with conformal anomaly. Noether symmetries have been used by Jamil et al. to investigate *f(R)* tachyon model [61]. Spherically symmetry has been used extensively to investigate *f(R)* theory in different context. Multamӓki and Vilja [62, 63] explored vacuum and perfect fluid solutions of spherically symmetric spacetime in metric *f(R)* gravity for constant scalar curvature. They found that the solutions corresponded to the already existing solutions in general relativity (GR). Capozziello et al. [64] studied spherically symmetric solutions in *f(R)* gravity using Noether symmetries. Conserved quantities in metric *f(R)* gravity via Noether symmetry approach have been recently calculated [65]. Vacuum and non-vacuum cylindrically symmetric solutions have also been explored in this theory [66–68]. Sharif and Shamir [69] gave plane symmetric constant curvature solutions.

Recently, Harko et al. [1] proposed a new theory named as *f(R,T)* gravity. In this theory, an arbitrary function of the scalar curvature R and the trace of the energy-momentum tensor T is used in gravitational Lagrangian. Jamil et al. [70] reconstructed some cosmological models in *f(R,T)* gravity and it was concluded that the dust fluid reproduced ΛCDM, phantom-non-phantom era and the phantom cosmology. Gödel type universe is studied in the framework of *f(R,T)* modified theories of gravity by Santos [71]. Sharif and Zubair [72] discussed the reconstruction and stability of *f(R,T)* gravity with Ricci and modified Ricci dark energy. The same authors [73] analyzed the laws of thermodynamics in this theory. However, it has been proved that the first law of black bole thermodynamics is violated for *f(R,T)* gravity [74]. Houndjo [75] reconstructed *f(R,T)* gravity by taking $f(R,T) = f_1(R) + f_2(T)$ where it was shown that *f(R,T)* gravity allowed transition of matter from dominated phase to an acceleration phase.

The investigation of Bianchi Type models in modified theories is another interesting topic of discussion. Jamil et al. [76] studied Bianchi Type I cosmology in generalized Saez-Ballester theory using Noether gauge symmetry approach. Adhav [77] reported the exact solutions of *f(R,T)* field equations for locally rotationally symmetric Bianchi type I spacetime. Study of Bianchi I anisotropic model in *f(R,T)* gravity has been done by Sharif and Zubair [78]. In a recent paper [79], we have investigated the exact solutions of Bianchi type I cosmological model in *f(R,T)* gravity.

This paper is devoted to explore the exact solutions of Bianchi type V spacetime in the context of *f(R,T)* gravity. The paper is planned as follows: In section 2, we briefly introduce *f(R,T)* gravity. Section 3 is used to investigate the exact solutions for Bianchi type V spacetime. Concluding remarks are given in the last section.

2 *f(R,T)* Gravity and Field Equations

The *f(R,T)* theory of gravity is the generalization or modification of GR. The action for this theory is [1]

$$S = \int \sqrt{-g}\left(\frac{1}{16\pi G} f(R,T) + L_m\right) d^4x. \tag{1}$$

Here *f(R,T)* is an arbitrary function of the Ricci scalar R and T is the trace of energy momentum tensor $T_{\mu\nu}$ while L_m is the matter Lagrangian. It would be worthwhile to mention here that we get the action for *f(R)* gravity if we replace *f(R,T)* with *f(R)*. Further, the replacement of *f(R,T)* with R leads to the action of GR. The energy momentum tensor $T_{\mu\nu}$ is given by [80]

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{\mu\nu}}. \tag{2}$$

We assume that the matter Lagrangian merely depends on the metric tensor $g_{\mu\nu}$ rather than its derivatives. This case yields

$$T_{\mu\nu} = L_m g_{\mu\nu} - 2\frac{\delta L_m}{\delta g^{\mu\nu}}. \tag{3}$$

By varying the action S in Eq.(1) with respect to the metric tensor $g_{\mu\nu}$, we get the $f(R,T)$ gravity field equations

$$f_R(R,T)R_{\mu\nu}-\frac{1}{2}f(R,T)g_{\mu\nu}-(\nabla_\mu\nabla_\nu-g_{\mu\nu}\Box)f_R(R,T) = \kappa T_{\mu\nu}-f_T(R,T)(T_{\mu\nu}+\Theta_{\mu\nu}), \tag{4}$$

where ∇_μ denotes the covariant derivative and

$$\Box \equiv \nabla^\mu\nabla_\mu, \quad f_R(R,T) = \frac{\partial f_R(R,T)}{\partial R}, \quad f_T(R,T) = \frac{\partial f_R(R,T)}{\partial T}, \quad \Theta_{\mu\nu} = g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{\mu\nu}}.$$

Contraction of Eq. (4) provides

$$f_R(R,T)R + 3\Box f_R(R,T) - 2f(R,T) = \kappa T - f_T(R,T)(T+\Theta), \tag{5}$$

where $\Theta = \Theta_\mu{}^\mu$. This is an important equation because it gives a relationship between R and T. The standard matter energy-momentum tensor is

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}, \tag{6}$$

where $u_\mu = \sqrt{g_{00}}(1,0,0,0)$ is the four-velocity in co-moving coordinates and ρ and p are energy density and pressure of the fluid respectively. In modified theories, the problems involving energy density and pressure are not any easy task to deal with. Moreover, there does not exist any unique definition for matter Lagrangian. Thus we may assume the matter Lagrangian as $L_m = -p$ which gives

$$\Theta_{\mu\nu} = -pg_{\mu\nu} - 2T_{\mu\nu}, \tag{7}$$

and consequently the field equations (4) take the form

$$f_R(R,T)R_{\mu\nu}-\frac{1}{2}f(R,T)g_{\mu\nu}-(\nabla_\mu\nabla_\nu-g_{\mu\nu}\Box)f_R(R,T) = \kappa T_{\mu\nu}+f_T(R,T)(T_{\mu\nu}+pg_{\mu\nu}), \tag{8}$$

It is mentioned here that these field equations depend on the physical nature of matter field. Many theoretical models corresponding to different matter contributions for *f(R,T)* gravity are possible. However, Harko et al. [1] gave three classes of these models

$$f(R,T) = \begin{cases} R + 2f(T), \\ f_1(R) + f_2(T), \\ f_1(R) + f_2(R)f_3(T). \end{cases}$$

We consider the first class in this paper, i.e. *f(R,T)=R+2f(T)*. The field equations for this model take the form

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa T_{\mu\nu} + 2f'(T)T_{\mu\nu} + \Big[f(T) + 2pf'(T)\Big]g_{\mu\nu}, \tag{9}$$

where prime represents derivative with respect to T.

3 Exact Solutions of Bianchi Type *V* Model

Here we find the exact solutions of Bianchi *V* spacetime in the framework of *f(R,T)* gravity. For the sake of simplicity, we consider natural system of units ($G = c = 1$) and $f(T) = \lambda T$, where λ is an arbitrary constant. For Bianchi type *V* spacetime, the line element is

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{2mx}[B^2(t)dy^2 + C^2(t)dz^2]. \tag{10}$$

Here *A, B* and *C* are cosmic scale factors and *m* is an arbitrary constant. The Ricci scalar for this spacetime turns out to be

$$R = -2\left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - \frac{3m^2}{A^2} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA}\right]. \tag{11}$$

Using Eq. (9), we get

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{3m^2}{A^2} = (8\pi + 3\lambda)\rho - \lambda p, \tag{12}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} = \lambda\rho - (8\pi + 3\lambda)p, \tag{13}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{AC} - \frac{m^2}{A^2} = \lambda\rho - (8\pi + 3\lambda)p, \tag{14}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} = \lambda\rho - (8\pi + 3\lambda)p \tag{15}$$

and the 01-component turn out to be

$$2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0. \tag{16}$$

Here p denotes the average pressure along the directional axes and ρ is the energy density of the perfect fluid. These are five non-linear differential equations with five unknowns namely *A, B, C*, ρ and p. Following the approach of Saha and Rikhvitsky [81], we get respectively

$$\frac{B}{A} = d_1 \exp\left[c_1 \int \frac{dt}{a^3}\right], \tag{17}$$

$$\frac{C}{B} = d_2 \exp\left[c_2 \int \frac{dt}{a^3}\right], \tag{18}$$

$$\frac{A}{C} = d_3 \exp\left[c_3 \int \frac{dt}{a^3}\right], \tag{19}$$

where $c_1,\ c_2,\ c_3$ and $d_1,\ d_2,\ d_3$ are integration constants which satisfy the following relation

$$c_1 + c_2 + c_3 = 0, \quad d_1 d_2 d_3 = 1. \tag{20}$$

Using Eqs. (17)-(19), we can write the unknown metric functions in an explicit way

$$A = ap_1 \exp\left[q_1 \int \frac{dt}{a^3}\right], \tag{21}$$

$$B = ap_2 \exp\left[q_2 \int \frac{dt}{a^3}\right], \tag{22}$$

$$C = ap_3 \exp\left[q_3 \int \frac{dt}{a^3}\right], \tag{23}$$

where

$$p_1 = ({d_1}^{-2}{d_2}^{-1})^{\frac{1}{3}}, \quad p_2 = (d_1{d_2}^{-1})^{\frac{1}{3}}, \quad p_3 = (d_1{d_2}^{2})^{\frac{1}{3}} \tag{24}$$

and

$$q_1 = -\frac{2c_1 + c_2}{3}, \quad q_2 = \frac{c_1 - c_2}{3}, \quad q_3 = \frac{c_1 + 2c_2}{3}. \tag{25}$$

It is mentioned here that $p_1,\ p_2,\ p_3$ and $q_1,\ q_2,\ q_3$ also satisfy the relation

$$p_1p_2p_3 = 1, \quad q_1 + q_2 + q_3 = 0. \tag{26}$$

By making use of Eq. (16), we get the constraint equations as follows

$$p_1 = 1, \quad p_2 = {p_3}^{-1} = P, \quad q_1 = 0, \quad q_2 = -q_3 = Q. \tag{27}$$

Thus, the metric coefficients become

$$A = a, \quad B = aP \exp\left[Q \int \frac{dt}{a^3}\right], \quad C = aP^{-1} \exp\left[-Q \int \frac{dt}{a^3}\right]. \tag{28}$$

3.1 Some Important Physical Parameters

Now we give definitions of some important physical parameters. The average scale factor a and volume scale factor V are defined as

$$a = \sqrt[3]{ABC}, \quad V = a^3 = ABC. \tag{29}$$

The generalized mean Hubble parameter H is given by

$$H = \frac{1}{3}(H_1 + H_2 + H_3), \tag{30}$$

where

$H_1 = \frac{\dot{A}}{A},\ H_2 = \frac{\dot{B}}{B},\ H_3 = \frac{\dot{C}}{C}$ are defined as the directional Hubble parameters in the directions of *x, y* and *z* axis respectively. The mean anisotropy parameter A is

$$A = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2. \tag{31}$$

The expansion scalar θ and shear scalar σ^2 are defined as follows

$$\theta = u^{\mu}_{;\mu} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \tag{32}$$

$$\sigma^2 = \frac{1}{2}\sigma_{\mu\nu}\sigma^{\mu\nu} = \frac{1}{3}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2 - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{C}\dot{A}}{CA}\right], \tag{33}$$

where

$$\sigma_{\mu\nu} = \frac{1}{2}(u_{\mu;\alpha}h^{\alpha}_{\nu} + u_{\nu;\alpha}h^{\alpha}_{\mu}) - \frac{1}{3}\theta h_{\mu\nu}, \tag{34}$$

$h_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$ is the projection tensor. The deceleration parameter q is a measure of the cosmic accelerated expansion of the universe. It is defined as follows

$$q = -\frac{\ddot{a}a}{\dot{a}^2}. \tag{35}$$

The behavior of models is determined by the sign of q. The positive value of deceleration parameter indicates a decelerating model while the negative value gives inflation. Here we need an additional constraint as there are four equations and five unknowns. Thus we use a well-known relation [82–84] between the average scale factor a and average Hubble parameter H to solve the equations,

$$H = la^{-n}, \tag{36}$$

where l and n are positive constants. Using Eqs. (30) and (36), we get

$$\dot{a} = la^{1-n} \tag{37}$$

and the deceleration parameter becomes

$$q = n - 1. \tag{38}$$

Integrating Eq. (37), it follows that

$$a = (nlt + k_1)^{\frac{1}{n}}, \quad n \neq 0 \tag{39}$$

and

$$a = k_2 \exp(lt), \quad n = 0, \tag{40}$$

where k_1 and k_2 are constants of integration. Thus we get two different models of the universe corresponding to these values of the average scale factor.

3.2 Singular Model of the Universe

For the model of the universe when $n \neq 0$, the metric functions A, B and C become

$$A = (nlt + k_1)^{\frac{1}{n}}, \tag{41}$$

$$B = P(nlt + k_1)^{\frac{1}{n}} \exp\left[\frac{Q(nlt + k_1)^{\frac{n-3}{n}}}{l(n-3)}\right], \quad n \neq 3 \tag{42}$$

$$C = P^{-1}(nlt + k_1)^{\frac{1}{n}} \exp\left[\frac{-Q(nlt + k_1)^{\frac{n-3}{n}}}{l(n-3)}\right], \quad n \neq 3. \tag{43}$$

The directional Hubble parameters H_1, H_2 and H_3 take the form

$$H_1 = \frac{l}{nlt + k_1}, \tag{44}$$

$$H_2 = \frac{l}{nlt + k_1} + \frac{Q}{(nlt + k_1)^{\frac{3}{n}}}, \tag{45}$$

$$H_2 = \frac{l}{nlt + k_1} - \frac{Q}{(nlt + k_1)^{\frac{3}{n}}}. \tag{46}$$

The mean generalized Hubble parameter and volume scale factor are

$$H = \frac{l}{nlt + k_1}, \quad V = (nlt + k_1)^{\frac{3}{n}}. \tag{47}$$

The mean anisotropy parameter becomes

$$A = \frac{2Q^2}{3(nlt + k_1)^{(6-2n)/n}}. \tag{48}$$

The expansion scalar and shear scalar for this model are given by

$$\theta = \frac{3l}{nlt + k_1}, \quad \sigma^2 = \frac{Q^2}{(nlt + k_1)^{6/n}}. \tag{49}$$

The energy density and pressure of Bianchi V universe for this model turns out to be

$$\rho = \frac{1}{12(\lambda + 2\pi)(\lambda + 4\pi)} \left[4(\lambda + 3\pi) \left\{ \frac{3l^2}{(nlt + k_1)^2} - \frac{Q^2}{(nlt + k_1)^{\frac{6}{n}}} \right\} - \lambda \left\{ \frac{3l^2(1-n)}{(nlt + k_1)^2} + \frac{2Q^2}{(nlt + k_1)^{\frac{6}{n}}} \right\} \right], \tag{50}$$

$$p = \frac{-1}{12(\lambda + 2\pi)(\lambda + 4\pi)} \left[4\pi \left\{ \frac{3l^2}{(nlt + k_1)^2} - \frac{Q^2}{(nlt + k_1)^{\frac{6}{n}}} \right\} + (3\lambda + 8\pi) \left\{ \frac{3l^2(1-n)}{(nlt + k_1)^2} + \frac{2Q^2}{(nlt + k_1)^{\frac{6}{n}}} \right\} \right]. \tag{51}$$

The plots of energy density ρ, pressure p and equation of state parameter $w = p/\rho$ against time coordinate t are shown in Figs. 1 and 2 respectively. It is can be seen from Fig. 2 that $w \to \frac{1}{3}$ as $t \to \infty$ which indicates that the model corresponds to a radiation dominated universe as the time grows.

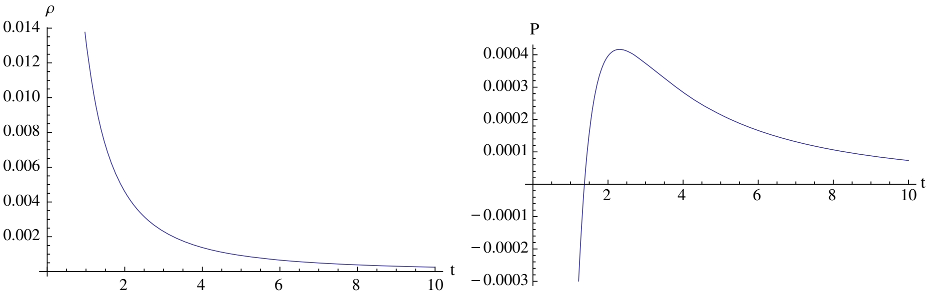

Fig. 1 Behavior of energy density and pressure versus time for $t > 0$ with $n = 2,\ \lambda = 1,\ l = 1,\ k_1 = 0$ and $Q = 1$

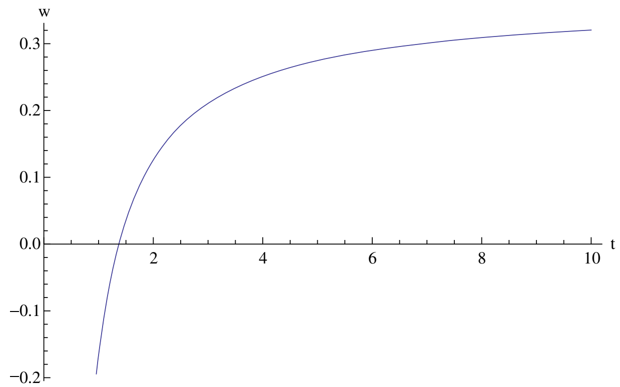

Fig. 2 Behavior of w versus time for $t > 0$ with $n = 2,\ \lambda = 1,\ l = 1,\ k_1 = 0$ and $Q = 1$

3.3 Non-singular Model of the Universe

For the model when $n = 0$, the metric coefficients A, B and C turn out to be

$$A = k_2 \exp(lt), \tag{52}$$

$$B = P k_2 \exp(lt) \exp\left[-\frac{Q\exp(-3lt)}{3lk_2{}^3}\right], \tag{53}$$

$$C = P^{-1} k_2 \exp(lt) \exp\left[\frac{Q\exp(-3lt)}{3lk_2{}^3}\right]. \tag{54}$$

The directional Hubble parameters $H_1,\ H_2$ and H_3 are

$$H_1 = l, \quad H_2 = l + \frac{Q\exp(-3lt)}{k_2{}^3}, \quad H_3 = l - \frac{Q\exp(-3lt)}{k_2{}^3}. \tag{55}$$

The mean anisotropy parameter and shear scalar for this model become

$$A = \frac{2Q^2}{3l^2 k_2{}^6 \exp(6lt)}, \qquad \sigma^2 = \frac{Q^2}{k_2{}^6 \exp(6lt)}. \tag{56}$$

The mean generalized Hubble parameter, expansion scalar and volume scale factor turn out to be

$$H = l, \quad \theta = 3l, \quad V = k_2{}^3 \exp(3lt). \tag{57}$$

The energy density and pressure of universe here become

$$\rho = \frac{1}{12(\lambda + 2\pi)(\lambda + 4\pi)}\left[4(\lambda + 3\pi)\left\{3l^2 - \frac{Q^2}{k_2{}^6 \exp(6lt)}\right\} - \lambda\left\{3l^2 + \frac{2Q^2}{k_2{}^6 \exp(6lt)}\right\}\right], \tag{58}$$

$$p = \frac{-1}{12(\lambda + 2\pi)(\lambda + 4\pi)}\left[4\pi\left\{3l^2 - \frac{Q^2}{k_2{}^6 \exp(6lt)}\right\} + (3\lambda + 8\pi)\left\{3l^2 + \frac{2Q^2}{k_2{}^6 \exp(6lt)}\right\}\right]. \tag{59}$$

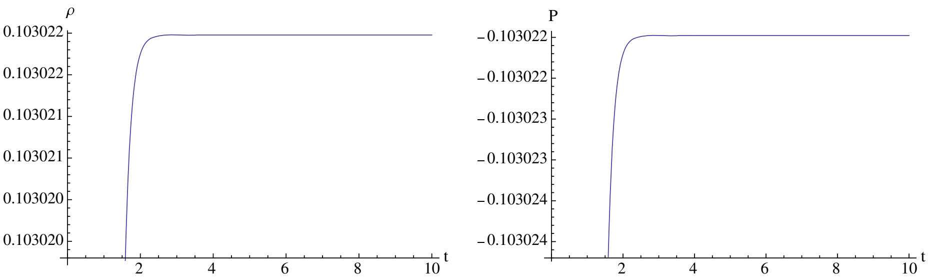

Fig. 3 Behavior of energy density and pressure versus time for $t > 0$ with $n = 2,\ \lambda = 1,\ l = 1,\ k_2 = 1$ and $Q = 1$

The plots of ρ, P and w against time coordinate t are shown in Figs. 3 and 4 respectively. It is evident from Fig. 4 that $w \to -1$ as $t \to \infty$ which indicates that the non-singular model corresponds to a vacuum fluid dominated universe.

4 Concluding Remarks

In this paper, we discuss the phenomenon of current accelerated expansion of universe in the context of recently proposed *f(R,T)* theory of gravity. For this purpose, we take $f(R, T) = R + 2\lambda T$ and investigate the exact solutions of Bianchi type *V* cosmological model. We use the assumption of constant value of deceleration parameter and the law of variation of Hubble parameter to find the solutions of field equations. We obtain two solutions that correspond to two different models of universe. The first solution gives a singular model with power law expansion while the second solution provides a non-singular model with exponential growth of universe. The physical parameters for these models are discussed below.

The singular model of the universe corresponding to $n \neq 0$ possesses a point singularity when $t \equiv t_s = -\frac{k_1}{nl}$. The volume scale factor and the metric coefficients $A,\ B$ and C vanish at this singularity point. The cosmological parameters $H_1,\ H_2,\ H_3,\ H,\ \theta$, and σ^2 are all

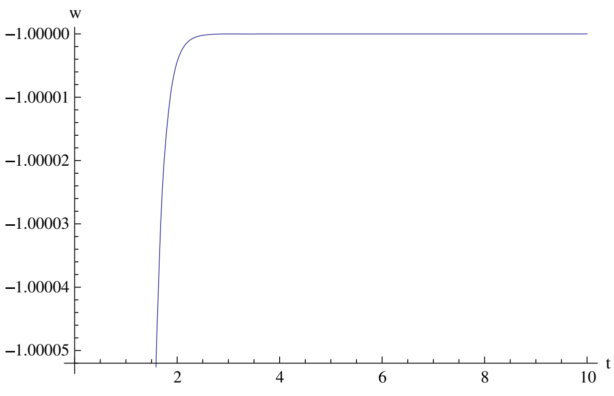

Fig. 4 Behavior of w versus time for $t > 0$ with $n = 2,\ \lambda = 1,\ l = 1,\ k_2 = 1$ and $Q = 1$

infinite at this point of singularity. The mean anisotropy parameter A also becomes infinite at this point for $0 < n < 3$ and vanishes for $n > 3$. Moreover, the isotropy condition, i.e., $\frac{\sigma^2}{\theta} \to 0$ as $t \to \infty$, is verified for this model. When we choose $k_1 = 0$, Fig. 1 suggests that energy density of the universe goes to zero as the time grows. The pressure approaches negative infinity as $t \to 0$. This strong negative pressure indicates the presence of dark energy in our universe. For this model, equation of state parameter $w \to \frac{1}{3}$ as $t \to \infty$ which shows that universe is radiation dominated. All these observations suggest that the universe starts its expansion with zero volume, strong negative pressure from $t = t_s$ and it will continue to expand for $0 < n < 3$.

The non-singular model of the universe corresponds to $n = 0$ with average scale factor $a = k_2 \exp(lt)$. The expansion scalar θ and mean generalized Hubble parameter H turn out to be constant in this case. The physical parameters $H_1,\ H_2,\ H_3,\ \sigma^2$ and A are all finite for finite values of t. Moreover, the metric functions are defined for finite time while the isotropy condition is satisfied. There is an exponential increase in the volume as the time grows. However, energy density is approximately zero initially and becomes constant after some time. Pressure of the universe is also in negative zone for this model which may be an indication of dark energy in the universe. It is evident from Fig. 4 that $w \to -1$ as $t \to \infty$. which shows that the exponential model corresponds to a vacuum fluid dominated universe. According to recent observations [85], the expansion of the universe is accelerating when $w \approx -1$. Therefore, the solution supports the phenomenon of expansion of universe and it is expected that the problematic issues such as dark energy and accelerated expansion of universe may be addressed using modified theories of gravity especially *f(R,T)* gravity.

Acknowledgments The author is thankful to National University of Computer and Emerging Sciences (NUCES) Lahore Campus for funding the PhD programme.

References

1. Harko, T., Lobo, F.S.N., Nojiri, S., Odintsov, S.D.: Phys. Rev. **D84**, 024020 (2011)
2. Seprgel, D.N. et al.: Astrophys. J. Suppl. Ser **148**, 175 (2003)
3. Bennett, C.L. et al.: Astrophys. J **148**, 1 (2003)
4. Perlmutter, S. et al.: Astrophys. J **517**, 565 (1999)
5. Riess, A.G. et al.: Astron. J **116**, 1009 (1998)
6. Riess, A.G. et al.: Astrophys. J **607**, 665 (2004)
7. Astier, P. et al. Astron. Astrophys. **447**, 31 (2006)
8. Spergel, D.N. et al.: Astrophys. J. Suppl. **170**, 377 (2007)
9. Astier, P. et al.: Astron. Astrophys. **447**, 31 (2006)
10. Allen, S.W. et al.: Mon. Not. R. Astron. Soc. **353**, 457 (2004)
11. Tegmark, M. et al.: Phys. Rev. **D69**, 103501 (2004)
12. Abazajian, K. et al.: Astron. J. **129**, 1755 (2005)
13. Nojiri, S., Odintsov, S.D.: Int. J. Geom. Meth. Mod. Phys. **4**, 115 (2007)
14. Turner, M.S., Huterer, D.: J. Phys. Soc. Jap. **76**, 111015 (2007)
15. Frieman, J., Turner, M., Huterer, D.: Ann. Rev. Astron. Astrophys. **46**, 385 (2008)
16. Li, M., Li, X.D., Wang, S., Wang, Y.: Commun. Theor. Phys **56**, 525 (2011)
17. Copeland, E.J., Sami, M., Tsujikawa, S.: Int. J. Mod. Phys. **D15**, 1753 (2006)
18. Sahni, V., Starobinsky, A.: Int. J. Mod. Phys. **D9**, 373 (2000)
19. Sahni, V.: Lect. Notes. Phys. **653**, 141 (2004)
20. Carroll, S.M.: Living Rev. Rel. **4**, 1 (2001)
21. Weinberg, D.H.: New. Astron. Rev. **49**, 337 (2005)
22. Straumann, N.: Mod. Phys. Lett. **A21**, 1083 (2006)

23. Jamil, M., Momeni, D., Myrzakulov, R.: Eur. Phys. J. **C72**, 1959 (2012). ibid. C72, 2075 (2012) ibid. C72, 2122 (2012) ibid. C72, 2137 (2012) ibid. C73, 2267 (2013)
24. Jamil, M., Yesmakhanova, K., Momeni, D., Myrzakulov, R.: Cent. Eur. J. Phys. **10**, 1065 (2012)
25. Houndjo, M.J.S., Momeni, D., Myrzakulov, R.: Int. J. Mod. Phys. **D21**, 1250093 (2012)
26. Rodrigues, M.E., Houndjo, M.J.S., Momeni, D., Myrzakulov, R.: Int. J. Mod. Phys. **D22**, 1350043 (2013)
27. Rodrigues, M.E., Houndjo, M.J.S., Tossa, J., Momeni, D., Myrzakulov, R.: JCAP **11**, 24 (2013)
28. Jamil, M., Momeni, D., Myrzakulov, R.: Gen. Rel.Grav. **45**, 263 (2013)
29. Jamil, M., Momeni, D., Myrzakulov, R., Rudra, P.: J. Phys. Soc. Jpn. **81**, 114004 (2012)
30. Yang, R.J.: Europhys. Lett. **93**, 60001 (2011)
31. Wei, H., Ma, X.P., Qi, H.Y.: Phys. Lett. **B703**, 74 (2011)
32. Wu, P.X., Yu, H.W.: Eur. Phys. J. **C71**, 1552 (2011)
33. Wu, P.X., Yu, H.W.: Phys. Lett. **B703**, 223 (2011)
34. Bamba, K., Geng, C.Q., Lee, C.C., Luo, L.W.: JCAP **1101**, 021 (2011)
35. Li, B., Sotiriou, T.P., Barrow, J.D.: Phys. Rev. **D83**, 104017 (2011)
36. Sharif, M., Zubair, M.: Adv. High Energy Phys. **2013**, 790967 (2013)
37. Sharif, M., Kausar, H.R.: JCAP **07**, 022 (2011)
38. Sharif, M., Kausar, H.R.: Int. J. Mod. Phys. **D20**, 2239 (2011)
39. Sharif, M., Kausar, H.R.: Astrophys. Space Sci. **331**, 281 (2011)
40. Sharif, M., Kausar, H.R.: Mod. Phys. Lett. **A25**, 3299 (2010)
41. Farooq, M.U., Jamil, M., Momeni, D., Myrzakulov, R.: Can. J. Phys. **91**, 703 (2013)
42. Bamba, K., Nojiri, S., Odintsov, S.D., Saez-Gomez, D.: Phys. Lett. **B730**, 136 (2014)
43. Bamba, K., Makarenko, A.N., Myagky, A.N., Nojiri, S., Odintsov, S.D.: JCAP **01**, 008 (2014)
44. Bamba, K., Nojiri, S., Odintsov, S.D.: Phys. Lett. **B698**, 451 (2011)
45. Capozziello, S., Vignolo, S.: Int. J. Geom. Meth. Mod. Phys. **8**, 167 (2011)
46. Capozziello, S., Darabi, F., Vernieri, D.: Mod. Phys. Lett. **A26**, 65 (2011)
47. Capozziello, S., Laurentis, M.D., Odintsov, S.D., Stabile, A.: Phys. Rev. **D83**, 064004 (2011)
48. Elizalde, E., Nojiri, S., Odintsov, S.D., Saez-Gomez, D.: Eur. Phys. J. **C70**, 351 (2010)
49. Bamba, K., Geng, C., Nojiri, S., Odintsov, S.D.: Mod. Phys. Lett. **A25**, 900 (2010)
50. Capozziello, S., Laurentis, M.D., Nojiri, S., Odintsov, S.D.: Gen. Rel. Grav. **41**, 2313 (2009)
51. Hollenstein, L., Lobo, F.S.N.: Phys. Rev. **D78**, 124007 (2008)
52. Sharif, M., Kausar, H.R.: J. Phys. Soc. Jpn. **80**, 044004 (2011)
53. Shojai, A., Shojai, F.: Gen. Relativ. Gravit. **44**, 211 (2011)
54. Sharif, M., Shamir, M.F.: Class. Quantum Grav. **26**, 235020 (2009)
55. Sharif, M., Shamir, M.F.: Gen. Relativ. Gravit **42**, 2643 (2010)
56. Felice, A.D., Tsujikawa, S.: Living Rev. Rel. **13**, 3 (2010)
57. Sotiriou, T.P., Faraoni, V.: Rev. Mod. Phys. **82**, 451 (2010)
58. Clifton, T., Ferreira, P.G., Padilla, A., Skordis, C.: Phys. Rep. **513**, 1 (2012)
59. Nojiri, S., Odintsov, S.D.: Phys. Rept. **505**, 59 (2011)
60. Hendi, S.H., Momeni, D.: Eur. Phys. J. **C71**, 1823 (2011)
61. Jamil, M., Mahomed, F.M., Momeni, D.: Phys. Lett. **B702**, 315 (2011)
62. Multamäki, T., Vilja, I.: Phys. Rev. **D74**, 064022 (2006)
63. Multamäki, T., Vilja, I.: Phys. Rev. **D76**, 064021 (2007)
64. Capozziello, S., Stabile, A., Troisi, A.: Class. Quantum Grav. **24**, 2153 (2007)
65. Shamir, M.F., Jhangeer, A., Bhatti, A.A.: Chin. Phys. Lett. **29**(8), 080402 (2012)
66. Azadi, A., Momeni, D., Nouri-Zonoz, M.: Phys. Lett. **B670**, 210 (2008)
67. Momeni, D., Gholizade, H.: Int. J. Mod. Phys. **D18**, 1719 (2009)
68. Sharif, M., Arif, S.: Mod. Phys. Lett. **A27**, 1250138 (2012)
69. Sharif, M., Shamir, M.F.: Mod. Phys. Lett. **A25**, 1281 (2010)
70. Jamil, M., Momeni, D., Raza, M., Myrzakulov, R.: Eur. Phys. J. **C72**, 1999 (2012)
71. Santos, A.F.: Mod. Phys. Lett. **A28**, 1350141 (2013)
72. Sharif, M., Zubair, M.: Astrophys. Space Sci. **349**, 529 (2014)
73. Sharif, M., Zubair, M.: JCAP **03**, 028 (2012)
74. Jamil, M., Momeni, D., Myrzakulov, R.: Chin. Phys. Lett. **29**, 109801 (2012)
75. Houndjo, M.J.S.: Int. J. Mod. Phys. **D21**, 1250003 (2012)
76. Jamil, M., Ali, S., Momeni, D., Myrzakulov, R.: Eur. Phys. J. **C72**, 1998 (2012)
77. Adhav, K.S.: Astrophys. Space Sci. **339**, 365 (2012)
78. Sharif, M., Zubair, M. Astrophys. Space Sci. **349**, 457 (2014)
79. Shamir, M.F.: Bianchi Type I Cosmology in $f(R,T)$ Gravity. J. Exp. Theor. Phy. **119**(2), 242–250 (2014)

80. Landau, L.D., Lifshitz E.M.: The Classical Theory of Fileds. Butterworth-Heinemann, Oxford (2002)
81. Saha, B., Rikhvitsky, V.: Physica **D219**, 168 (2006)
82. Shamir, M.F., Bhatti, A.A.: Can. J. Phys. **90**, 193 (2012)
83. Singh, C.P., Kumar, S.: Int. J. Theor. Phys. **47**, 3171–3179 (2008)
84. Singh, C.P., Zeyauddin, M., Ram, S.: Int. J. Theor. Phys. **47**, 3162–3170 (2008)
85. Hogan, J.: Nature **448**, 240 (2007)