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Bidirectional Quantum Controlled Teleportation by Using a Genuine Six-qubit Entangled State

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Abstract A bidirectional quantum controlled teleportation scheme by using a genuine six-qubit entangled state is proposed. In our scheme, such a six-qubit entangled state is employed as the quantum channel linking three legitimate participants. And Alice may transmit an arbitrary single qubit state of qubit A to Bob and Bob may transmit an arbitrary single qubit state of qubit B to Alice via the control of the supervisor Charlie. This bidirectional quantum controlled teleportation is deterministic.

Keywords Quantum information · Bidirectional quantum controlled teleportation · Six-qubit entangled state

1 Introduction

Quantum teleportation [1], one of the most surprising features of quantum information theory, has been studied by many authors theoretically [2–16] and experimentally [17–19] since the original quantum teleportation protocol was proposed. The first controlled teleportation protocol was introduced by Karlsson and Bourennane, and it is known that controlled teleportation equals to quantum information splitting and quantum state sharing [20–24]. After Zha et al. [25] demonstrated that the five-qubit cluster state can be used for bidirectional quantum controlled teleportation (BQCT), various BQCT protocols have been devised with the help of multi-particle entangled states, such as five-qubit entangled state [26–29], six-qubit cluster state [30] and seven-qubit entangled state [31].

Recently Chen et al. [32] introduced a genuine six-qubit entangled state. In this work, we present a protocol for implementing BQCT by using such a genuine six-qubit entangled state. In this scheme, the six-qubit entangled state is initially shared by the senders (receivers) Alice, Bob and supervisor Charlie. Using teleportation of an arbitrary single-qubit state, Alice and Bob can exchange their quantum state simultaneously under the

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control of the supervisor Charlie. This means that Alice has qubit A in an unknown state, she wants to transmit an arbitrary single qubit state of qubit A to Bob; at the same time, Bob has a qubit B in an unknown state, he wants to transmit the state of qubit B to Alice. The BQCT task is completed following the typical procedure that, the receiver applies an appropriate unitary transformation to his qubit, after receiving the measurement results of both the sender Alice (Bob) and the controller Charlie on their separate qubits. In our scheme, only two Bell-state measurements and single-qubit measurements are necessary.

2 Bidirectional Quantum Controlled Teleportation

Our scheme can be described as follows. Suppose Alice has an arbitrary single qubit state $|\psi\rangle_A = a_0|0\rangle + a_1|1\rangle$ and that Bob has qubit B in an unknown state $|\psi\rangle_B = b_0|0\rangle + b_1|1\rangle$. Now Alice wants to transmit the state $|\psi\rangle_A$ of qubit A to Bob and at the same time, Bob wants to transmit the state $|\psi\rangle_B$ of qubit B to Alice. Assume that Alice, Bob and Charlie share a genuine six-qubit entangled state, which has the form [32]

$$\begin{aligned}|\psi\rangle_{123456} = \frac{\sqrt{2}}{4}&(|000000\rangle + |011100\rangle + |111000\rangle + |100100\rangle \\ +\ &|001111\rangle + |010011\rangle + |110111\rangle + |101011\rangle)_{123456}\,. \end{aligned} \tag{1}$$

where the qubits 1 and 5 belong to Alice, qubits 3 and 6 belong to Charlie and qubits 2 and 4 belong to Bob, respectively. The initial state of the total system can be expressed as

$$|\Psi\rangle_{123456AB} = |\psi\rangle_{123456} \otimes |\psi\rangle_A \otimes |\psi\rangle_B\,. \tag{2}$$

To achieve the purpose of BQCT, then Alice and Bob performs a Bell-state measurement on own qubit pairs (A, 1) and (B, 2), respectively. It is known that one may obtain one of the 16 kinds of possible measured results with equal probability, and the remaining qubits may collapse into one of the 16 states after the measurement.

$$\begin{aligned}{}_{B2}\langle\Phi^{\pm}|_{A1}\langle\Phi^{\pm}|\,|\Psi\rangle_{123456AB} &= a\frac{1}{4}[a_0b_0(|0000\rangle + |1111\rangle) \pm +a_1b_0(|0100\rangle + |1011\rangle) \\ &+\ \pm a_0b_1(|1100\rangle + |0011\rangle) \pm \pm a_1b_1(|1000\rangle + |0111\rangle)]_{3456}\,, \quad (3)\\ {}_{B2}\langle\Psi^{\pm}|_{A1}\langle\Phi^{\pm}|\,|\Psi\rangle_{123456AB} &= \frac{1}{4}[a_0b_0(|1100\rangle + |0011\rangle) \pm +a_1b_0(|1000\rangle + |0111\rangle) \\ &+\ \pm a_0b_1(|0000\rangle + |1111\rangle) \pm \pm a_1b_1(|0100\rangle + |1011\rangle)]_{3456}\,, \quad (4)\\ {}_{B2}\langle\Phi^{\pm}|_{A1}\langle\Psi^{\pm}|\,|\Psi\rangle_{123456AB} &= \frac{1}{4}[a_0b_0(|0100\rangle + |1011\rangle) \pm +a_1b_0(|0000\rangle + |1111\rangle) \\ &+\ \pm a_0b_1(|1000\rangle + |0111\rangle) \pm \pm a_1b_1(|1100\rangle + |0011\rangle)]_{3456}\,, \quad (5)\\ {}_{B2}\langle\Psi^{\pm}|_{A1}\langle\Psi^{\pm}|\,|\Psi\rangle_{123456AB} &= \frac{1}{4}[a_0b_0(|1000\rangle + |0111\rangle) \pm +a_1b_0(|1100\rangle + |0011\rangle) \\ &+\ \pm a_0b_1(|0100\rangle + |1011\rangle) \pm \pm a_1b_1(|0000\rangle + |1111\rangle)]_{3456}\,, \quad (6)\end{aligned}$$

where $|\Phi^{\pm}\rangle = \frac{\sqrt{2}}{2}(|00\rangle \pm |11\rangle)$ and $|\Psi^{\pm}\rangle = \frac{\sqrt{2}}{2}(|01\rangle \pm |10\rangle)$ are Bell states. In (3)–(6), the notes "±" or "+" from right to left correspond to the Bell-state measurements of qubits 'A1' and 'B2', respectively, and they mean multiplication of ± signs.

Then Alice (Bob) tells the result to Bob (Alice) and Charlie. If Charlie allows Bob and Alice to reconstruct the initial unknown state, he needs to carry out a single-qubit measurement in the basis of $\{|0\rangle, |1\rangle\}$ on qubit 3 and makes a single-qubit projective measurement on qubit 6 under the basis $|\pm\rangle = (|0\rangle \pm |1\rangle) \pm \sqrt{2}$, respectively, and then tells the receivers his result. By combining information from Alice, Bob and Charlie, Alice and Bob can obtain the secret quantum information with an appropriate unitary transformation on the qubit at hand, the BQCT is easily realized.

Now, let us take an example to demonstrate the principle of this BQCT protocol. Suppose Alice's measurement outcome is $\left|\Phi^+\right\rangle_{A1}$, at the same time, Bob's measurement outcome is $\left|\Phi^+\right\rangle_{B2}$, then the state of the remaining qubits collapse into the state

$$\begin{aligned} |\phi\rangle_{3456} &= \frac{1}{2}\big[|0\rangle_3 |+\rangle_6 (a_0 |0\rangle + a_1 |1\rangle)_4 (b_0 |0\rangle + b_1 |1\rangle)_5 \\ &+ |0\rangle_3 |-\rangle_6 (a_0 |0\rangle + a_1 |1\rangle)_4 (b_0 |0\rangle - b_1 |1\rangle)_5 \\ &+ |1\rangle_3 |+\rangle_6 (a_0 |1\rangle + a_1 |0\rangle)_4 (b_0 |1\rangle + b_1 |0\rangle)_5 \\ &+ |1\rangle_3 |-\rangle_6 (a_0 |1\rangle + a_1 |0\rangle)_4 (-b_0 |1\rangle + b_1 |0\rangle)_5\big], \end{aligned} \tag{7}$$

where $|\pm\rangle_6 = \frac{\sqrt{2}}{2}(|0\rangle \pm |1\rangle)_6$.

Charlie can now make the single-qubit measurements on qubits 3 and 6, respectively, and then he sends the result of his measurement to Bob and Alice. If the result of the single-qubit measurement is $|0\rangle_3 |+\rangle_6$, $|0\rangle_3 |-\rangle_6$, $|1\rangle_3 |+\rangle_6$ or $|1\rangle_3 |-\rangle_6$, Bob and Alice need to apply the local unitary operation $I_4 \otimes I_5$, $I_4 \otimes \sigma_5^z$, $\sigma_4^x \otimes \sigma_5^x$ or $\sigma_4^x \otimes -i\sigma_5^y$. After doing those operations, Bob and Alice can successfully exchange their quantum state. Thus the BQCT is successfully realized.

3 Conclusion

In summary, we have demonstrated that such a genuine six-qubit entangled state can be used as the quantum channel to realize the deterministic BQCT. In our scheme, Alice may transmit an arbitrary single qubit state of qubit A to Bob and at same time Bob may transmit an arbitrary single qubit state of qubit B to Alice via the control of the supervisor Charlie. In the scheme only two Bell-state measurements and single qubit measurements, and the appropriate single qubit unitary operation are necessary. We hope that such a BQCT scheme can be realized experimentally in the future.

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