Int J Theor Phys (2014) 53:4275–4290
DOI 10.1007/s10773-014-2178-6

New Holographic Dark Energy in Chern-Simons Gravity and Cosmography

Ujjal Debnath

Received: 29 November 2013 / Accepted: 6 May 2014 / Published online: 30 May 2014
© Springer Science+Business Media New York 2014

Abstract We have considered a five-dimensional action, which is composed of a gravitational sector and a sector of matter, where the gravitational sector is given by a Chern-Simons gravity action instead of the Einstein-Hilbert action and where the matter sector is given by the so-called perfect fluid with barotropic EoS and new holographic dark energy. We will study the dynamic formulation of Chern-Simons gravity, where the coupling constant is promoted to a scalar field with potential. We have studied the implications of replacing the Einstein-Hilbert action by the Chern-Simons action on the cosmological evolution for a 5D FRW metric. The deceleration parameter shows that our considered model cannot cross the phantom divide. Also the natures of the cosmography parameters are examined in Chern-Simons gravity.

Keywords Cherns-Simons gravity · Dark energy · Cosmography

1 Introduction

Recent observations from type Ia Supernovae [1–4] and Cosmic Microwave Background (CMB) [5–7] radiation indicate that our present universe is accelerating which occurs due to some unknown matter called dark energy, which has the property that positive energy density and negative pressure. This accelerated expansion of the universe has also been strongly confirmed by some other independent experiments like Sloan Digital Sky Survey (SDSS) [8], Baryonic Acoustic Oscillation (BAO) [9], WMAP data analysis [10–12] etc. Also the observations indicate that the dominating component of the present universe is this dark energy. Dark energy occupies about 73% of the energy of our universe, while dark matter about 23% and the usual baryonic matter 4%. Although it is straightforward to explain the effect within the framework of Friedmann-Robertson-Walker cosmology by introducing a cosmological constant or a more general (dynamical) dark energy component,

U. Debnath (✉)
Department of Mathematics, Bengal Engineering and Science University, Shibpur, Howrah, 711 103, India
e-mail: ujjal@iucaa.ernet.in; ujjaldebnath@yahoo.com

all such explanations give rise to severe coincidence and fine-tuning problems. There are different candidates obey the property of dark energy to violate the strong energy condition $\rho + 3p > 0$ given by - quintessence [13, 14], K-essence [15], DBI-essence [16, 17], Hessence [18], Tachyon [19], Phantom [20], ghost condensate [21, 22], quintom [23, 24], interacting dark energy models [25–28], brane world models [29] and Chaplygin gas models [30]. Other types of dark energies are holographic dark energy [31–47], Ricci dark energy [48], agegraphic dark energy [49–52] and new holographic dark energy [53, 54].

Many theoretical models that describing the accelerated expansion of the universe and which appears to fit all recent available observations are affected by significant fine-tuning problems related to the vacuum energy scale and therefore it is important to investigate alternatives to this description of the Universe. There exist several other approaches to the theoretical description of the accelerated expansion of the universe. An alternative approach postulates that general relativity is only accurate on small scales and has to be modified on cosmological distances. One of these is a modified gravity theories. Modified gravity constitutes an interesting dynamical alternative to ΛCDM cosmology in that it is also able to describe the current acceleration in the expansion of our universe. One of the best-studied examples is the Dvali- Gabadadze-Porrati (DGP) brane-world model [55], in which the gravity leaks off the 4-dimensional Minkowski brane into the 5-dimensional bulk Minkowski space-time. On small scales the gravity is bound to the 4-dimensional brane and the Newtonian gravity is recovered to a good approximation. One of the simplest modified gravity is $f(R)$ gravity [56]. Other modified gravity includes $f(T)$ gravity [57], $f(G)$ gravity [58], Gauss-Bonnet gravity [59], Horava-Lifshitz gravity [60], Brans-Dicke gravity [61], Lovelock gravity [62], $f(R, T)$ gravity [63], $f(R, G)$ gravity [64], Galileon gravity [65] etc.

In GR, the space-time is a dynamical object which has independent degrees of freedom, and is governed by dynamical equations, eg, the Einstein field equations. Therefore, the construction of a gauge theory of gravity requires an action that does not consider a fixed space-time background. An action for gravity fulfilling these conditions, albeit only in odd-dimensional space-time, $d = 2n + 1$ was proposed by Chamseddine [66, 67]. One of the models of modified gravity that has stood out in recent years is the Chern-Simons modified gravity, which was initially developed in [68]. If Chern-Simons theories are the appropriate gauge theories in a framework for the gravitational interaction, then these theories must satisfy the correspondence principle i.e., they must be related to GR [69–72]. Five dimensional GR can be obtained from Chern-Simons gravity theory [71, 73] for a certain Lie algebra $\mathcal{B}$. The Chern-Simons Lagrangian is built from a $\mathcal{B}$-valued, one-form gauge connection A that depends on a scale parameter l which can be interpreted as a coupling constant that characterizes different regimes within this theory. There exist other Lie algebras that also allow for a similar identification and lead to a Chern-Simons Lagrangian that touches upon the Einstein-Hilbert term in a certain limit. Several works [74–81] on Chern-Simons gravity have been done on the context of GR and black hole.

We have considered a five-dimensional action for Lagrangian, which is composed of a gravitational sector and a sector of matter, where the gravitational sector is given by a Chern-Simons gravity action Lagrangian instead of the Einstein-Hilbert action Lagrangian and where the matter sector is given by the so-called perfect fluid and new holographic dark energy. We have studied the implications of replacing the Einstein-Hilbert action Lagrangian by the Chern-Simons action Lagrangian on the cosmological evolution for a FRW metric in five dimensions. We will study the dynamic formulation of Chern-Simons gravity, where the coupling constant is promoted to a scalar field with potential. The solutions of the field equations have been found for barotropic EoS and new holographic dark energy. We have also investigated the natures of scalar field and corresponding potential. We have analyzed the cosmography parameters for new holographic dark energy. Finally, we have given some concluding remarks.

2 Basic Equations of FRW universe for Chern-Simons Gravity

Chern-Simons (CS) models for gravity are interesting because they provide a truly gauge-invariant action principle in the fiber-bundle sense. So Chern-Simons gravity is a well defined gauge theory, but the presence of higher powers of the curvature makes its dynamics very remote from that for standard Einstein-Hilbert (EH) gravity. In order to write down the Chern-Simons Lagrangian for the $\mathcal{B}$ algebra, we start from the one form gauge connection (Lie group)

$$A = \frac{1}{2}\omega^{ab} J_{ab} + \frac{1}{l} e^a P_a + \frac{1}{2} k^{ab} Z_{ab} + \frac{1}{l} h^a Z_a \tag{1}$$

and the two-form curvature [71]

$$F = \frac{1}{2} R^{ab} J_{ab} + \frac{1}{l} T^a P_a + \frac{1}{2}\left(D_\omega k^{ab} + \frac{1}{l^2} e^a e^b\right) Z_{ab} + \frac{1}{l}\left(D_\omega h^a + k^a_b e^b\right) Z_a \tag{2}$$

where l is the coupling constant and D is the covariant derivative with respect to the Lorentz piece of the connection and $T^a = De^a$ is the spin tensor. The field content induced by $\mathcal{B}$ includes the gauge fields e^a and ω^{ab} as the veilbein and the spin connection respectively and two extra bosonic matter fields h^a and k^{ab}. Following the definitions of [82], consider the S expansion of the Lie algebra $SO(4,2)$ using as a semigroup $S_E^{(3)}$. After extracting a resonant subalgebra and performing its 0_S reduction, one finds a new Lie algebra, call it $\mathcal{B}$, whose generators $\{J_{ab}, P_a, Z_{ab}, Z_a\}$ satisfy the commutation relationships given in ref [73].

Chern-Simons (CS) gauge actions, defined in odd dimensions, arise as the boundary terms of the Chern classes $\int_{M_{2n}} (F)^n$. In fact, one has that $(F)^n = d\Omega_{2n-1}$, which defines the CS action to be [83]

$$S_{CS}(A) = \int_{M_{2n}} \Omega_{2n-1}$$

In field theory, the lowest dimensional case, corresponding to $n = 2$, has been profusely studied in the literature. Two of the most relevant examples are: (i) the explicit solution of $2 + 1$ dimensional gravity, rewritten as a CS theory [84] and (ii) the realization of fractional statistics via a three dimensional Abelian CS field [85]. In our work, we shall consider $n = 3$, i.e., 5D space-time.

Using the extended Cartan homotopy formula [86], the Chern-Simons Lagrangian in five dimensions for the $\mathcal{B}$ algebra can be written as as [73]

$$L_{CS} = \alpha_1 l^2 \epsilon_{abcde} R^{ab} R^{cd} e^e + \alpha_3 \epsilon_{abcde}\left(\frac{2}{3} R^{ab} e^c e^d e^e + 2l^2 k^{ab} R^{cd} T^e + l^2 R^{ab} R^{cd} h^e\right) \tag{3}$$

The Lagrangian is split into two independent pieces, one proportional to α_1 and the other to α_3. The piece proportional to α_1 corresponds to the Inonu-Wigner contraction and therefore it is the Chern-Simons Lagrangian for the Poincare-Lie algebra ISO(4,1). The field e^a with the vielbein, the piece proportional to α_3 contains the Einstein-Hilbert action plus non-linear couplings between the curvature and the bosonic matter fields given by h^a and k^{ab}, where the parameter l^2 can be interpreted as a kind of coupling constant.. We see that it is possible to recover the odd dimensional Einstein gravity theory [71] from a Chern-Simons gravity theory in the limit where the coupling constant l equals zero while keeping the effective Newtons constant fixed.

Now assume the Lagrangian $L = L_{CS} + L_m$ [73], where L_{CM} is the Lagrangian for Chern-Simons gravity and L_m is the corresponding matter Lagrangian. Here assume that $T^a = 0$ and $k^{ab} = 0$. We consider the homogeneous and isotropic Friedmann-Robertson-Walker (FRW) metric of the universe in five dimensions ($5D$) as

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left\{d\theta^2 + sin^2\theta\left(d\phi^2 + sin^2\phi d\psi^2\right)\right\}\right] \tag{4}$$

The energy-momentum tensor for matter is given by

$$T^m_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu} \tag{5}$$

Here, ρ and p are the energy density and pressure of matter respectively and $u_\mu = (1, 0, 0, 0, 0)$ is the velocity 5-vector. Also the energy momentum tensor for the field h^a is

$$T^{(h)}_{\mu\nu} = (\rho^{(h)} + p^{(h)})u_\mu u_\nu + p^{(h)}g_{\mu\nu} \tag{6}$$

Here, $\rho^{(h)}$ and $p^{(h)}$ are the energy density and pressure for field h^a respectively. The field equations are [73]

$$6\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) + \lambda l^2\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right)^2 = \kappa\rho\ , \tag{7}$$

$$3\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right)\right] + \lambda l^2\frac{\ddot{a}}{a}\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) = -\kappa p\ , \tag{8}$$

$$l^2\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right)^2 = \kappa_1\rho^{(h)}\ , \tag{9}$$

$$l^2\frac{\ddot{a}}{a}\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) = -\kappa_1 p^{(h)} \tag{10}$$

where κ is the gravitational constant, $\lambda = \frac{3\alpha_1}{\alpha_3}$ and κ_1 are constants. The above field equations are known as Einstein-Chern-Simons field equations for FRW universe. We see that when $l = 0$, the above equations reduce to usual Einstein's field equations in 5D in Einstein's gravity. We also see that the field equations (7) and (8) satisfy the conservation equation

$$\dot{\rho} + 4\frac{\dot{a}}{a}(\rho + p) = 0 \tag{11}$$

Also the field equations (9) and (10) satisfy the conservation equation

$$\dot{\rho}^{(h)} + 4\frac{\dot{a}}{a}\left(\frac{\dot{a}^2}{\dot{a}^2 + k}\right)\rho^{(h)} + 4\frac{\dot{a}}{a}p^{(h)} = 0 \tag{12}$$

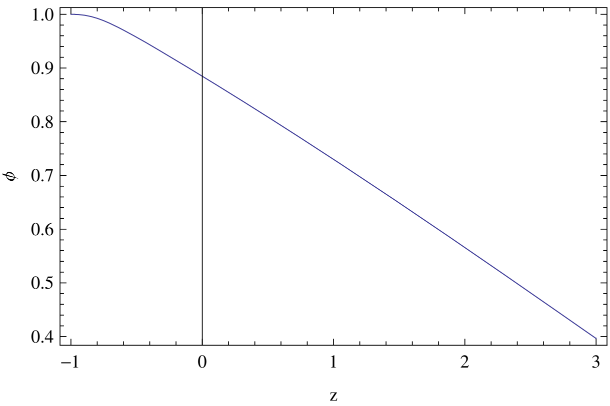

Fig. 1 Shows the variations of scalar field ϕ and potential V against redshift z for barotropic EoS

Since h^a is arbitrary vector field whose nature is not specified. Now assume h^a is the scalar field ϕ, whose action is given by

$$S = \int d^5x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] \tag{13}$$

where $V(\phi)$ is the corresponding potential for the scalar field ϕ. From the above action, we have the expressions of density and pressure as

$$\rho^{(h)} = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad and \quad p^{(h)} = \frac{1}{2}\dot{\phi}^2 - V(\phi) \tag{14}$$

Now it is assumed that the present universe is flat. So may choose $k = 0$. Now the conservation equation (12) yields the wave equation for ϕ as

$$\ddot{\phi} + 4\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0 \tag{15}$$

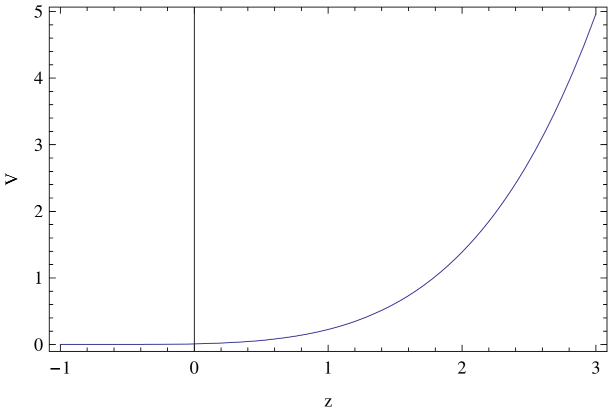

Fig. 2 Shows the variations of scalar field ϕ and potential V against redshift z for barotropic EoS

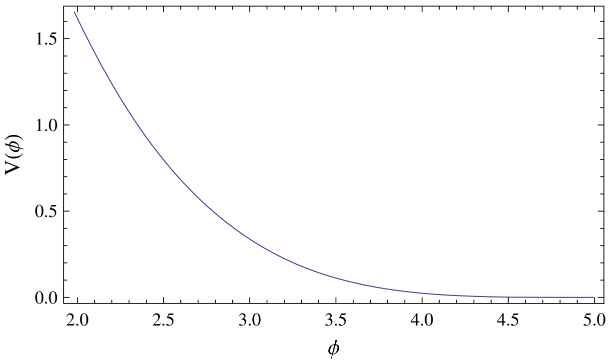

Fig. 3 Shows the variation of $V(\phi)$ against ϕ for barotropic EoS

So for flat model, the field equations (7)-(10) lead to

$$6\frac{\dot{a}^2}{a^2} + \lambda l^2 \frac{\dot{a}^4}{a^4} = \kappa\rho \ , \tag{16}$$

$$3\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) + \lambda l^2 \ \frac{\ddot{a}}{a}\ \frac{\dot{a}^2}{a^2} = -\kappa p \ , \tag{17}$$

$$l^2 \ \frac{\dot{a}^4}{a^4} = \kappa_1 \left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) \ , \tag{18}$$

$$l^2 \ \frac{\ddot{a}}{a}\ \frac{\dot{a}^2}{a^2} = -\kappa_1 \left(\frac{1}{2}\dot{\phi}^2 - V(\phi)\right) \tag{19}$$

Let, $H = \frac{\dot{a}}{a}$ is a Hubble parameter. Now solving equations (16)-(19), we obtain the solutions in terms of ρ and p as follows:

$$H^2 = \frac{-3 + \sqrt{9 + \lambda l^2 \kappa\rho}}{\lambda l^2} \ , \tag{20}$$

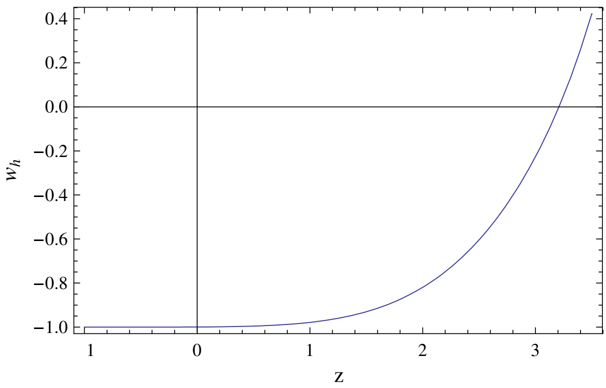

Fig. 4 Shows the variation of EoS parameter w for new holographic dark energy against redshift z

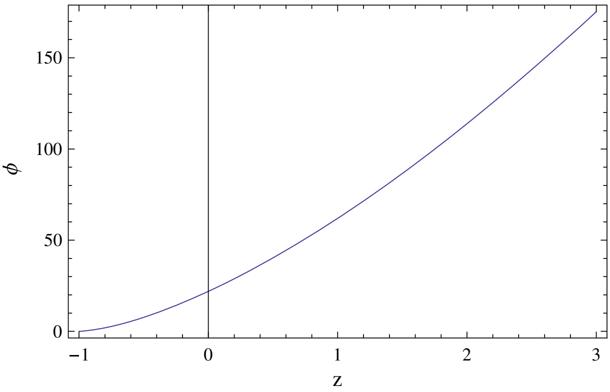

Fig. 5 Shows the variations of scalar field ϕ and potential V for new holographic dark energy against redshift z

$$\dot{H} = -\frac{\kappa(\rho + p)}{\sqrt{9 + \lambda l^2 \kappa\rho}} \ , \tag{21}$$

$$\dot{\phi}^2 = \frac{\kappa(\rho + p)(-3 + \sqrt{9 + \lambda l^2 \kappa\rho})}{\kappa_1 \lambda \sqrt{9 + \lambda l^2 \kappa\rho}} \ , \tag{22}$$

$$V = \frac{36 + \kappa\lambda l^2(\rho - p)}{2\kappa_1 \lambda^2 l^2} - \frac{108 + 3\kappa\lambda l^2(3\rho - p)}{2\kappa_1 \lambda^2 l^2 \sqrt{9 + \lambda l^2 \kappa\rho}} \tag{23}$$

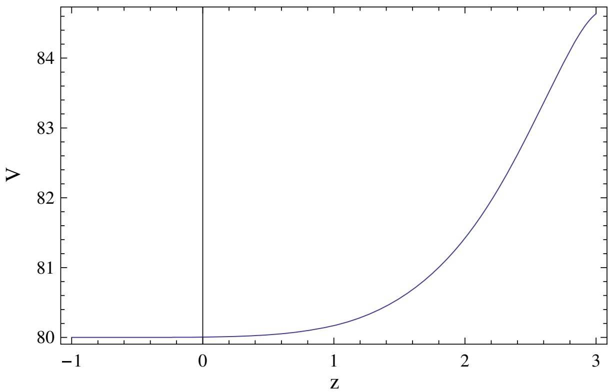

Fig. 6 Shows the variations of scalar field ϕ and potential V for new holographic dark energy against redshift z

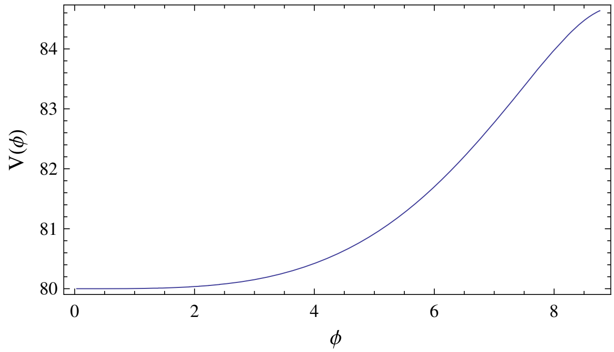

Fig. 7 Shows thevariation of $V(\phi)$ against ϕ for new holographic dark energy

3 Barotropic Equation of State

Now, assume that the matter fluid follows that barotropic equation of state (EoS) $p = w\rho$, where w is a constant. So the conservation equation (11) yields $\rho = \rho_0 a^{-4(1+w)}$, where ρ_0 is a constant. From equations (20) to (23) become

$$H^2 = \frac{-3 + \sqrt{9 + \lambda l^2 \kappa \rho_0 a^{-4(1+w)}}}{\lambda l^2} \quad , \tag{24}$$

$$\dot{H} = -\frac{\kappa(1+w)\rho_0 a^{-4(1+w)}}{\sqrt{9 + \lambda l^2 \kappa \rho_0 a^{-4(1+w)}}} \quad , \tag{25}$$

$$\dot{\phi}^2 = \frac{\kappa(1+w)\rho_0 a^{-4(1+w)}(-3 + \sqrt{9 + \lambda l^2 \kappa \rho_0 a^{-4(1+w)}})}{\kappa_1 \lambda \sqrt{9 + \lambda l^2 \kappa \rho_0 a^{-4(1+w)}}} \tag{26}$$

$$i.e., \quad \phi = \phi_0 - \sqrt{\frac{\kappa l^2 \rho_0}{24(1+w)}}\; a^{-2(1+w)}\; {}_2F_1[\frac{1}{2}, \frac{1}{4}, \frac{3}{2}, -\frac{1}{9}\, \kappa \lambda l^2 \rho_0 a^{-4(1+w)}], \tag{27}$$

$$V = \frac{36 + \kappa \lambda l^2 (1-w)\rho_0}{2\kappa_1 \lambda^2 l^2}\; a^{-4(1+w)} - \frac{108 + 3\kappa \lambda l^2 (3-w)\rho_0 a^{-4(1+w)}}{2\kappa_1 \lambda^2 l^2 \sqrt{9 + \lambda l^2 \kappa \rho_0 a^{-4(1+w)}}} \tag{28}$$

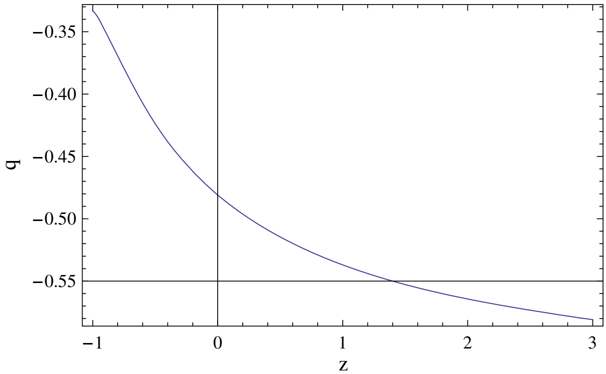

Fig. 8 Shows the variations of cosmography parameters q, J, S, L for *barotropic EoS* against redshift z

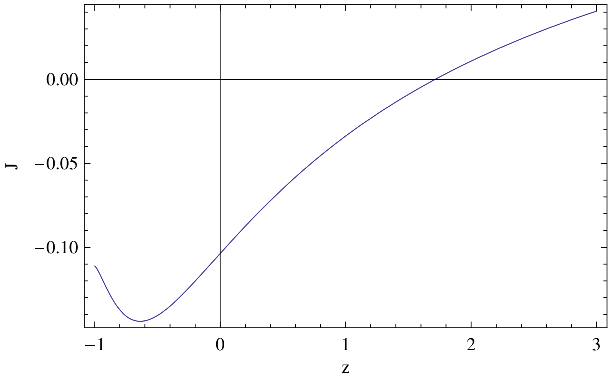

Fig. 9 Shows the variations of cosmography parameters q, J, S, L for *barotropic EoS* against redshift z

Here ${}_2F_1$ is hypergeometric function. Now we know that the expression of redshift z in term of scale factor a is governed by the relation $1 + z = \frac{1}{a}$. We have drawn the variations of ϕ and V with respect to redshift z in Figs. 1 and 2. From figures, we see that the scalar field increases and potential decreases due to the evolution of the universe. Also Fig. 3 shows the variation of V with respect to scalar field ϕ and the figure shows that potential decreases as scalar field increases.

4 New Holographic Dark Energy

In the work [87], it was suggested that in quantum field theory a short distance cut-off is related to a long distance cut-off due to the limit set by formation of a black hole, namely, if is the quantum zero-point energy density caused by a short distance cut-off, the total energy in a region of size L should not exceed the mass of a black hole of the same size, thus $L^3\rho_\Lambda \leq LM_p^2$. The largest L allowed is the one saturating this inequality, thus $\rho_\Lambda = 3c^2M_p^2L^{-2}$. In dark energy context, initially in the holographic principle it was assumed that $L = H^{-1}$, which implies $\rho_\Lambda = 3c^2H^2$ (assuming $M_p^2 = 1$). Since the origin of

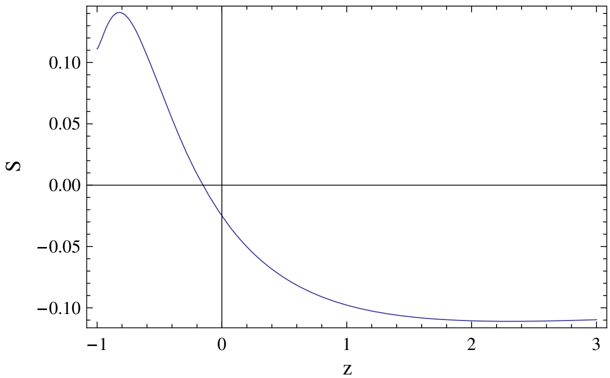

Fig. 10 Shows the variations of cosmography parameters q, J, S, L for *barotropic EoS* against redshift z

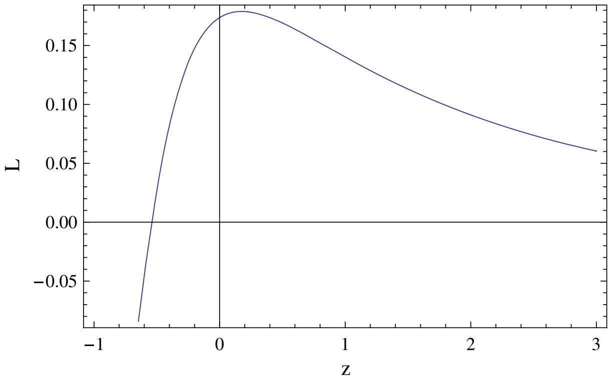

Fig. 11 Shows the variations of cosmography parameters q, J, S, L for *barotropic EoS* against redshift z

the holographic dark energy is still unknown and that the new term is contained in the expression for the Ricci scalar which scales as L^{-2} (a model with holographic dark energy proportional to the Ricci scalar was proposed in [88]). So the energy density of Ricci dark energy can be written as $\rho_\Lambda = 3\alpha\left(\dot{H} + 2H^2 + \frac{k}{a^2}\right)$. Granda et al [89] proposed, similar form of Ricci dark energy which is called *new holographic dark energy* (the holographic DE density with the new infrared cut-off). The density of new holographic dark energy is linear combination of square of Hubble parameter and it's derivative and is given by

$$\rho = \alpha H^2 + \beta \dot{H} \tag{29}$$

where, α and β are constants. Using equations (20)-(23) and (29), we have the solutions as

$$H^{-2} = \frac{\lambda l^2}{\kappa\alpha - 6} + C\, a^{\frac{2(\kappa\alpha-6)}{\kappa\beta}} \quad , \tag{30}$$

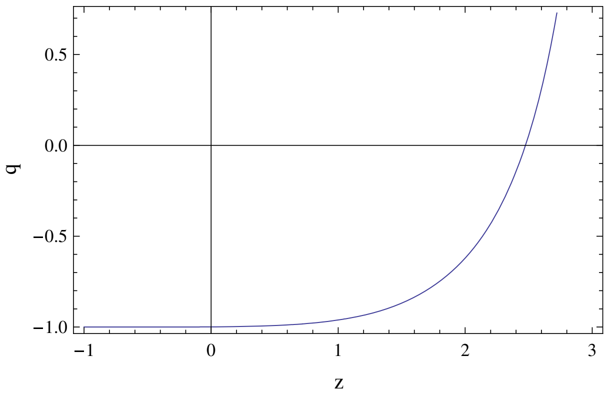

Fig. 12 Shows the variations of cosmography parameters q, J, S, L for *new holographic dark energy* against redshift z

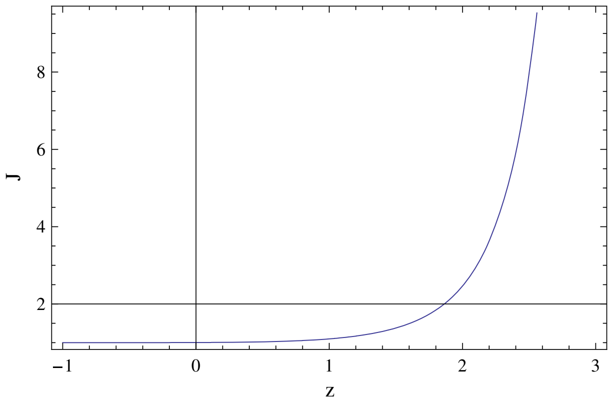

Fig. 13 Shows the variations of cosmography parameters q, J, S, L for *new holographic dark energy* against redshift z

$$\dot{H} = -\frac{C(\kappa\alpha - 6)^3 \, a^{\frac{2(\kappa\alpha+6)}{\kappa\beta}}}{\kappa\beta\left[\lambda l^2 a^{\frac{12}{\kappa\beta}} + C(\kappa\alpha - 6)a^{\frac{2\alpha}{\beta}}\right]^2} \, , \tag{31}$$

$$\dot{\phi}^2 = \frac{Cl^2(\kappa\alpha - 6)^4 \, a^{\frac{2(\kappa\alpha+12)}{\kappa\beta}}}{\kappa\kappa_1\beta\left[\lambda l^2 a^{\frac{12}{\kappa\beta}} + C(\kappa\alpha - 6)a^{\frac{2\alpha}{\beta}}\right]^2} \, , \tag{32}$$

$$i.e., \quad \phi = \frac{l(\kappa\alpha - 6)}{\alpha}\sqrt{\frac{\beta C}{\kappa\kappa_1}} \, a^{\frac{\alpha}{\beta}} \tag{33}$$

$$V = \frac{l^2(\kappa\alpha - 6)^2 \, a^{\frac{24}{\kappa\beta}}\left[2\kappa\beta\lambda l^2 a^{\frac{12}{\kappa\beta}} - C(\kappa\alpha - 6)\left\{\kappa(\alpha - 2\beta) - 6\right\} a^{\frac{2\alpha}{\beta}}\right]}{2\kappa\kappa_1\beta\left[\lambda l^2 a^{\frac{12}{\kappa\beta}} + C(\kappa\alpha - 6)a^{\frac{2\alpha}{\beta}}\right]^3} \, , \tag{34}$$

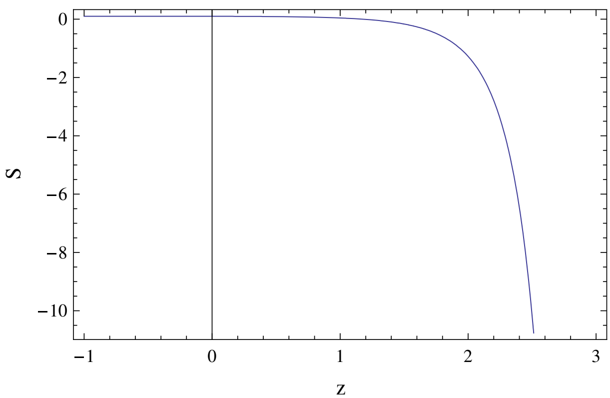

Fig. 14 Shows the variations of cosmography parameters q, J, S, L for *new holographic dark energy* against redshift z

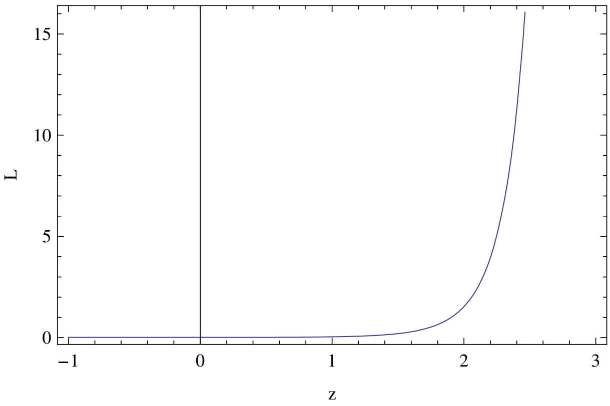

Fig. 15 Shows the variations of cosmography parameters q, J, S, L for *new holographic dark energy* against redshift z

$$i.e., \quad V(\phi)=\frac{\beta l^6(k\alpha-6)^4 k^{\frac{12}{k\alpha}} k_1^{-1+\frac{12}{k\alpha}}(\alpha\phi)^{\frac{24}{k\alpha}}\left[2\lambda\beta^2 l^4(k\alpha-6)(kk_1\alpha^2\phi^2)^{\frac{6}{k\alpha}}-k_1\alpha^2\phi^2(k(\alpha-2\beta)-6)(Cl^2\beta)^{\frac{6}{k\alpha}}(k\alpha-6)^{\frac{12}{k\alpha}}\right]}{2\left[\beta\lambda(k\alpha-6)l^4(kk_1\alpha^2\phi^2)^{\frac{6}{k\alpha}}+kk_1\alpha^2\phi^2(Cl^2\beta)^{\frac{6}{k\alpha}}(k\alpha-6)^{\frac{12}{k\alpha}}\right]^3} \tag{35}$$

$$\rho=\frac{\kappa\alpha\lambda l^2(\kappa\alpha-6)\,a^{\frac{24}{\kappa\beta}}+\beta C(\kappa\alpha-6)^2 a^{\frac{2(\kappa\alpha+6)}{\kappa\beta}}}{\kappa\left[\lambda l^2 a^{\frac{12}{\kappa\beta}}+C(\kappa\alpha-6)a^{\frac{2\alpha}{\beta}}\right]^2}\ , \tag{36}$$

$$p=\frac{(\kappa\alpha-\beta)a^{\frac{12}{\kappa\beta}}\left[3C^2(\kappa\alpha-6)^2\{\kappa(\alpha-2\beta)-6\}\,a^{\frac{4\alpha}{\beta}}-\alpha\beta\kappa^2\lambda^2 l^4 a^{\frac{24}{\kappa\beta}}+C\lambda l^2(\kappa\alpha-6)\{18+\kappa\alpha(\kappa\alpha-9)-\kappa\beta(\kappa\alpha+6)\}\,a^{\frac{2(\kappa\alpha+6)}{\kappa\beta}}\right]}{\kappa^2\beta\left[\lambda l^2 a^{\frac{12}{\kappa\beta}}+C(\kappa\alpha-6)a^{\frac{2\alpha}{\beta}}\right]^3} \tag{37}$$

Since we have found the expressions of density ρ and pressure p for new holographic dark energy model. So the equation of state parameter $w_h=p/\rho$ has been drawn in Fig. 4 against redshift z. We see that w_h decreases from positive value to negative value and ultimately tends to -1 in future. But w_h cannot be less than -1 in any case. So New holographic dark energy alongwith the Chern-Simons gravity cannot generate the phantom crossing. We have drawn the variations of ϕ and V with respect to redshift z in Figs. 5 and 6. From figures, we see that the scalar field potential decrease due to the evolution of the universe. Also Fig. 7 shows the variation of V with respect to scalar field ϕ and the figure shows that potential increases as scalar field increases.

5 Cosmography Parameters

To the property of acceleration of the universe, we need to consider after Hubble parameter (H). They are known as cosmographic parameters [90–93]. These parameters are related to the 2nd, 3rd, 4th, ... derivatives of scale factor a. The cosmographic parameters that are proportional to the coefficients of Taylor series expansion of the scale factor with respect to the cosmic time defined as

$$q = -\frac{1}{aH^2}\frac{d^2a}{dt^2} = -\frac{1}{H}\frac{d}{da}(aH) \tag{38}$$

$$J = \frac{1}{aH^3}\frac{d^3a}{dt^3} = -\frac{1}{H^2}\frac{d}{da}(aH^2q) \tag{39}$$

$$S = \frac{1}{aH^4}\frac{d^4a}{dt^4} = \frac{1}{H^3}\frac{d}{da}(aH^3J) \tag{40}$$

$$L = \frac{1}{aH^5}\frac{d^5a}{dt^5} = \frac{1}{H^4}\frac{d}{da}(aH^4S) \tag{41}$$

which are usually referred to as the deceleration (q), jerk (J), snap (S) and lerk (L) parameters, respectively. Analytically, we cannot examined the natures of the above parameters, because the expressions of the above parameters are very complicated form for barotropic EoS and new holographic dark energy models. We have drawn the deceleration (q), jerk (J), snap (S) and lerk (L) parameters in Figs. 8, 9, 10, 11 for barotropic EoS model. From Fig. 8, for barotropic EoS, the deceleration parameters keeps negative sign throughout the evolution of the universe. From fig. 9 we see that the jerk parameter (J) first decreases from positive state to negative stage and finally it slightly increases but keeps the negative sign. From Fig. 10, we see that the snap parameter (S) increases from negative to positive value and keeps the positive value in late stage. Figure 11 shows the lerk parameter (L) first increases with takes the positive value upto certain stage near $z = 0$ and then sharply decreases from the maximum positive value to negative value asymptotically. We have also drawn the deceleration (q), jerk (J), snap (S) and lerk (L) parameters in Figs. 12, 13, 14, 15 for new holographic dark energy model. From Fig. 12, we see that q decreases from positive value to negative value. From Fig. 13, we observe that the jerk parameter (J) decreases and keeps the positive sign. Figure 14 shows that the snap parameter (S) increases and keeps the negative value and tends to zero at final stage of the universe. Also Fig. 15 shows that the lerk parameter (L) decreases and keeps the positive value and finally approaches to zero at the late stage of the evolution of the universe.

6 Discussions and Conclusions

We have considered a five-dimensional action, which is composed of a gravitational sector and a sector of matter, where the gravitational sector is given by a Chern-Simons gravity action instead of the Einstein-Hilbert action and where the matter sector is given by the so-called perfect fluid with barotropic equation of state and new holographic dark energy. We have studied the implications of replacing the Einstein-Hilbert action Lagrangian by the Chern-Simons action Lagrangian on the cosmological evolution for a FRW metric in five dimensions. We have studied the dynamic formulation of Chern-Simons gravity, where the coupling constant is promoted to a scalar field ϕ with potential $V(\phi)$. The solutions of the field equations have been found for barotropic EoS and new holographic dark energy. We have also investigated the natures of scalar field and corresponding potential graphically. From Figs. 1 and 2, we see that the scalar field increases and potential decreases due to the evolution of the universe for barotropic EoS and Fig. 3 shows that potential decreases as scalar field increases. For new holographic dark energy model, we see that the EoS parameter w_h decreases from positive value to negative value (in Fig. 4) and ultimately tends to -1 in future. But w_h cannot be less than -1 in any case. So New holographic dark energy alongwith the Chern-Simons gravity cannot generate the phantom crossing. We have drawn the variations of ϕ and V with respect to redshift z in Figs. 5 and 6. From figures, we see that the scalar field potential decrease due to the evolution of the universe. Also Fig. 7 shows the variation of V with respect to scalar field ϕ and the figure shows that potential increases as scalar field increases. So we conclude that the scalar field always increases and potential always decreases for barotropic EoS whereas the scalar field and potential always decrease for new holographic dark energy.

We have analyzed the cosmography parameters for barotropic EoS and new holographic dark energy. Now we express the natures of the above parameters separately.

- Barotropic EoS:

We have drawn the deceleration (q), jerk (J), snap (S) and lerk (L) parameters in Figs. 8, 9, 10, 11 for barotropic EoS model. From Fig. 8, we observe that for barotropic EoS, the deceleration parameters keeps negative sign throughout the evolution of the universe in our considered Chern-Simons gravity model. So when the universe is filled with only dark matter with barotropic EoS in Chern-Simons gravity, the gravity generates the quintessence type dark energy. From Fig. 9 we see that the jerk parameter (J) first decreases from positive state to negative stage and finally it slightly increases but keeps the negative sign. From Fig. 10, we see that the snap parameter (S) increases from negative to positive value and keeps the positive value in late stage. Figure 11 shows the lerk parameter (L) first increases with takes the positive value upto certain stage near $z = 0$ and then sharply decreases from the maximum positive value to negative value asymptotically.

- New holographic dark energy:

We have drawn the deceleration (q), jerk (J), snap (S) and lerk (L) parameters in Figs. 12–15 for new holographic dark energy model. From Figs. 12, we see that q decreases from positive value to negative value and there is a sign flip of q and late stage of the evolution of the universe q approaches to -1 and q cannot less than -1 for new holographic dark energy. So we say that when the universe is filled with new holographic dark energy (without considering dark matter) in Chern-Simons gravity, the gravity with new holographic dark energy generate dark energy at present and late stage of the evolution of the universe. So barotropic fluid or new holographic dark energy always generate dark energy at present and future but phantom crossing cannot happened (because $q > -1$) in these models. From Fig. 13, we observe that the jerk parameter (J) decreases and keeps the positive sign. Figure 14 shows that the snap parameter (S) increases and keeps the negative value and tends to zero at final stage of the universe. Also Fig. 15 shows that the lerk parameter (L) decreases and keeps the positive value and finally approaches to zero at the late stage of the evolution of the universe which is filled with only the new holographic dark energy in our considered Chern-Simons gravity.

So we conclude that the deceleration parameter and EoS parameter the Chern-Simons gravity combined with barotropic EoS or new holographic dark energy stands another form of dark energy but they cannot cross the phantom divide. The jerk, snap and lerk parameters respectively decreases, increases and decreases for both barotropic EoS and new holographic dark energy models at present and future stages of the evolution of the universe.

Acknowledgments The author is thankful to Institute of Theoretical Physics, Chinese Academy of Science, Beijing, China for providing TWAS Associateship Programme under which part of the work was carried out. Also UD is thankful to CSIR, Govt. of India for providing research project grant (No. 03(1206)/12/EMR-II).

References

1. Perlmutter, S.J. et al.: Nat. **391**, 51 (1998)
2. Perlmutter, S. et al.: SNCP Collsboration. Astrophys. J. **517**, 565 (1999)
3. Riess, A.G. et al.: Supernova Search Team Collaboration. Astron. J., 116 (1009)
4. Riess, A.G. et al.: Astrophys. J. **607**, 665 (2004)
5. Miller, D. et al.: Astrophys. J. **524**, L1 (1999)
6. Bennet, C. et al.: Phys. Rev. Lett. **85**, 2236 (2000)
7. Spergel, D.N. et al.: Astrophys. J. Suppl. Ser. **170**, 377 (2007)
8. Adelman-McCarthy, J.K. et al.: Astrophys. J. Suppl. Ser. **175**, 297 (2008)
9. Eisenstein, D.J. et al.: SDSS Collaboration. Astrophys. J. **633**, 560 (2005)
10. Briddle, S. et al.: Sci **299**, 1532 (2003)
11. Spergel, D.N. et al.: Astrophys. J. Suppl. **148**, 175 (2003)
12. Spergel, D.N. et al.: Astrophys. J. Suppl. **170**, 377 (2007)
13. Peebles, P.J.E., Ratra, B.: Astrophys. J. **325**, L17 (1988)
14. Caldwell, R.R., Dave, R., Steinhardt, P.J.: Phys. Rev. Lett. **80**, 1582 (1998)
15. Armendariz - Picon, C., Mukhanov, V.F., Steinhardt, P.J.: Phys. Rev. Lett. **85**, 4438 (2000)
16. Gumjudpai, B., Ward, J.: Phys. Rev. D **80**, 023528 (2009)
17. Martin, J., Yamaguchi, M.: Phys. Rev. D **77**, 123508 (2008)
18. Wei, H., Cai, R.G., Zeng, D.F.: Class. Quantum Grav. **22**, 3189 (2005)
19. Sen, A.: JHEP **0207**, 065 (2002)
20. Caldwell, R.R.: Phys. Lett. B **545**, 23 (2002)
21. Arkani-Hamed, N., Cheng, H.C., Luty, M.A., Mukohyama, S.: JHEP **0405**, 074 (2004)
22. Piazza, F., Tsujikawa, S.: JCAP **0407**, 004 (2004)
23. Feng, B., Wang, X.L., Zhang, X.M.: Phys. Lett. B **607**, 35 (2005)
24. Guo, Z.K., Piao, Y.S., Zhang, X.M., Zhang, Y.Z.: Phys. Lett. B **608**, 177 (2005)
25. Amendola, L.: Phys. Rev. D **62**, 043511 (2000)
26. Zhang, X.: Mod. Phys. Lett. A **20**, 2575 (2005)
27. Jamil, M., Rashid, M.A.: Eur. Phys. J. **C60**, 141–147 (2009)
28. Jamil, M., Rashid, M.A.: Eur. Phys. J. **C56**, 429–434 (2008)
29. Sahni, V., Shtanov, Y.: JCAP **0311**, 014 (2003)
30. Kamenshchik, A.Y., Moschella, U., Pasquier, V.: Phys. Lett. B **511**, 265 (2001)
31. Fischler, W., Susskind, L. arXiv:hep-th/9806039
32. Tavakol, R., Ellis, G.: Phys. Lett. B **469**, 33 (1999)
33. Setare, M.R.: Phys. Lett. **B642**, 1–4 (2006)
34. Setare, M.R.: Phys. Lett. **B642**, 421–425 (2006)
35. Setare, M.R.: Phys. Lett. **B644**, 99–103 (2007)
36. Setare, M.R.: Eur. Phys. J. **C50**, 991–998 (2007)
37. Setare, M.R., Zhang, J., Zhang, X.: JCAP **0703**, 007 (2007)
38. Sadjadi, H.M., Jamil, M.: Gen. Rel. Grav. **43**, 1759–1775 (2011)
39. Setare, M.R., Jamil, M.: Europhys. Lett. **92**, 49003 (2010)
40. Farooq, M.U., Jamil, M., Momeni, D., Myrzakulov, R.: Can. J. Phys. **91**, 703–708 (2013)
41. Karami, K., Jamil, M., Ghaffari, S., Fahimi, K., Myrzakulov, R.: Can. J. Phys. **91**(10), 770–776 (2013)
42. Pasqua, A., Jamil, M., Myrzakulov, R., Majeed, B.: Phys. Scr. **86**, 045004 (2012)
43. Karami, K., Khaledian, M.S., Jamil, M.: Phys. Scr. **83**, 025901 (2011)
44. Sheykhi, A., Jamil, M.: Gen. Rel. Grav. **43**, 2661–2672 (2011)
45. Setare, M.R., Jamil, M.: Gen. Relativ. Gravit. **43**, 293–303 (2011)
46. Setare, M.R., Jamil, M.: Phys. Lett. **B690**, 1–4 (2010)
47. Jamil, M., Saridakis, E.N., Setare, M.R.: Phys. Lett. **B679**, 172–176 (2009)
48. Gao, C., Wu, F., Chen, X., Shen, Y.-G.: Phys. Rev. D **79**, 043511 (2009)
49. Cai, R.G.: Lett. Phys. B **657**, 228 (2007)
50. Wei, H., Cai, R.G.: Phys. Lett. B **660**, 113 (2008)
51. Jamil, M., Sheykhi, A.: Int. J. Theor. Phys. **50**, 625–636 (2011)

52. Jamil, M., Karami, K., Sheykhi, A.: Int. J. Theor. Phys. **50**, 3069–3077 (2011)
53. Granda, L.N., Oliveros, A.: Phys. Lett. B **669**, 275 (2008)
54. Granda, L.N., Oliveros, A.: Phys. Lett. B **671**, 199 (2009)
55. Dvali, G.R., Gabadadze, G., Porrati, M.: Phys. Lett. B **484**, 112 (2000)
56. De Felice, A., Tsujikawa, T. arXiv:1002.4928 [gr-qc]
57. Yerzhanov, K.K. et al. arXiv:1006.3879v1 [gr-qc] (2010)
58. Nojiri, S., Odintsov, S.D.: Phys. Lett. **B631**, 1 (2005)
59. Antoniadis, I., Rizos, J., Tamvakis, K.: Nucl. Phys. B **415**, 497 (1994)
60. Horava, P.: JHEP **0903**, 020 (2009)
61. Brans, C., Dicke, H.: Phys. Rev. **124**, 925 (1961)
62. Lovelock, D.: J. Math. Phys. **12**, 498 (1971)
63. Harko, T., Lobo, F.S.N., Nojiri, S., Odintsov, S.D.: Phys. Rev. **D84**, 024020 (2011)
64. Cognola, G., Elizalde, E., Nojiri, S., Odintsov, S.D., Zerbini, S. arXiv:hep-th/0601008
65. Chow, N., Khoury, J.: Phys. Rev. D **024037**, 80 (2009)
66. Chamseddine, A.H.: Phys. Lett. B **233**, 291 (1989)
67. Chamseddine, A.H.: Nucl. Phys. **B346**, 213 (1990)
68. Jackiw, R., Pi, S.Y.: Phys. Rev. D **68**, 104012 (2003)
69. Edelstein, J., Hassaine, M., Troncoso, R., Zanelli, J.: Phys. Lett. B **640**, 278 (2006)
70. Hassaine, M., Romo, M.: J. High Energy Phys. **06**, 018 (2008)
71. Izaurieta, F., Minning, P., Perez, A., Rodriguez, E., Salgado, P.: Phys. Lett. B **678**, 213 (2009)
72. de Azcarraga, J.A., Izquierdo, J.M., Picon, M., Varela, O.: Nucl. Phys. **B662**, 185 (2003)
73. Gomez, F., Minning, P., Salgado, P.: Phys. Rev. D **84**, 063506 (2011)
74. Silva, J.G., Santos, A.F.: Eur. Phys. J. C **73**, 2500 (2003)
75. Myung, Y.S. arXiv:1307.6265 [gr-qc]
76. DasGupta, P. arXiv:0905.1621 [gr-qc]
77. Urrutia, L.F. arXiv:gr-qc/0402001
78. Alexander, S., Yunes, N. arXiv:0907.2562v3 [hep-th]
79. Furtado, C., Mariz, T., Nascimento, J.R., Petrov, A.Yu., Santos, A.F. arXiv:0906.0554v3 [hep-th]
80. Yunes, N., Pretorius, F. arXiv:0902.4669v3 [gr-qc]
81. Grumiller, D., Yunes, N. arXiv:0711.1868v1 [gr-qc]
82. Izaurieta, F., Rodriguez, E., Salgado, P.: J. Math. Phys. (N.Y.) **47**, 123512 (2006)
83. Birmingham, D., Blau, M., Rakowsky, M., Thompson, G.: Phys. Rep. **209**, 129 (1991)
84. Witten, E.: Nucl. Phys. B **311**, 46 (1988)
85. Lerda, A.: Anyons, quantum mechanics of particles with fractional statistics: Lecture Notes in Physics, Vol. m14. Springer-Verlag (1992)
86. Izaurieta, F., Rodriguez, E., Salgado, P.: Lett. Math. Phys. **80**, 127 (2007)
87. Cohen, A., Kaplan, D., Nelson, A.: Phys. Rev. Lett. **82**, 4971 (1999)
88. Gao, C., Wu, F., Chen, X., Shen, Y.G.: Phys. Rev. D **79**, 043511 (2009)
89. Granda, L.N., Oliveros, A.: Phys. Lett. B **669**, 275 (2008)
90. Visser, M., Rel, Gen.: Grav. **37**, 1541 (2005)
91. Cattoen, C., Visser, M. arXiv:gr-qc/0703122
92. Capozziello, S., Cardone, V.F., Salzano, V.: Phys. Rev. D **063504**, 78 (2008)
93. Setare, M.R., Mohammadipour, N. arXiv:1206.0245 [physics.gen-ph]