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Torsion and Particle Horizons

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Abstract In the present work we show that the existence of non-vanishing torsion field may solve, at least, one of the problems FRW-cosmology, the particle horizons problem. The field equations of general relativity (GR) are written in a space having non-vanishing torsion, the absolute parallelism (AP) space. An AP-Structure, satisfying the cosmological principle, is used to construct a world model. Energy density and pressure, purely induced by torsion, are defined from the building blocks of the AP-geometry using GR. When these quantities are used in the FRW-dynamical equations, we get a world model free from particle horizons.

Keywords Cosmology · Geeral relativity · Inflation

1 Introduction

Standard cosmology is a branch of science that deals with the structure and evaluation of the Universe as one system. Theoretically, it depends mainly on the theory of General Relativity (GR). Just after its birth in 1915, GR has succeeded in predicting important features of our Universe (e.g. The expansion of the Universe, and afterwards the Cosmic Microwave Background Radiation(CMBR), the abundance of light elements,...etc). Some of these predictions have been directly confirmed by observations afterwards [1–4]. However, standard cosmology suffers from some problems as singularity, particle horizons, flatness, and the accelerating expansions of the Universe [5, 6],...etc.

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Many authors have attempted to get rid of these problems by reinserting the cosmological term in GR (cf. [7]), writing alternative theories depending on the curvature scalar R, f(R) theories (cf. [8, 9]), or inventing some scenarios as inflation [10, 11], baryogenesis [12, 13]. All the above mentioned attempts have been done in the context theories written in Riemannian geometry. Another class of attempts has been done by writing other theories, alternative to GR, in the context of geometries with non-vanishing torsion (T). This class is known in the literature as f(T) theories (cf. [14–18]), and also dealing with f(R) and f(T) theories together in one theory called F(R,T) gravity[19].

The aim of the present article is to study the effect of torsion on the dynamics of the Friedmann-Robertson-Walker(FRW)-Cosmology. For this aim, the article is arranged as follows. In Section 2, a brief overview of the main feature of a geometry with non-vanishing torsion, the Absolute Parallelism (AP-)geometry together with some issues necessary for the present application. In Section 3, we investigate the effect of presence of a non-vanishing torsion on the FRW-dynamics. Discussion and some concluding remarks are given in Section 4.

2 A Geometry with Non-Vanishing Torsion

In the present section we use a simple type of geometry with non-vanishing torsion, the AP-geometry (cf.[20–22]). We are going to review briefly some of its properties and the geometric entities necessary for the present work. Also, we review the most general AP-structure, satisfying the cosmological principle, used usually for cosmological applications. At the end of this section we write GR field equations in the context of AP-geometry. The AP-geometry is used, frequently to solve physical problems in gravity theories (cf. [16, 32, 33]). It is worth of mention that calculations in the AP-geometry are easier compared to other types of geometries with non-vanishing torsion.

2.1 Brief review of the AP-space

In 4-dimensions, the structure of an AP-Space is defined completely in terms of a tetrad vector field $\underset{i}{\lambda}^{\mu}$(the building blocks of any AP-structure), where $i(= 0, 1, 2, 3)$ represents the vector number and $\mu(= 0, 1, 2, 3)$ denotes the coordinate component of the vector. In the present article we are going to use Latin (mesh) indices for the vector numbers and Greek (world) indices for coordinate components. Einstein summation convention is carried out over Greek indices in the usual manner, while for Latin indices, it is carried out over repeated indices wherever their positions. It is assumed that the tetrad vectors are totaly independent i.e. the determinant $\lambda \left(\overset{def.}{=} \| \underset{i}{\lambda}^{\mu} \| \right)$ is non-vanishing. This implies that, for any tetrad vector field there is a conjugate $\underset{i}{\lambda}_{\nu}$ such that:

$$\underset{i}{\lambda}^{\mu} \underset{i}{\lambda}_{\nu} = \delta^{\mu}_{\nu}, \tag{1}$$

$$\underset{i}{\lambda}^{\nu} \underset{j}{\lambda}_{\nu} = \delta_{ij}. \tag{2}$$

Using the tetrad vector field and its conjugate we can define the following second order symmetric tensors,

$$g^{\mu\nu} \overset{def.}{=} \underset{i}{\lambda}^{\mu} \underset{i}{\lambda}^{\nu}, \tag{3}$$

$$g_{\alpha\beta} \overset{def.}{=} \underset{i}{\lambda}_{\alpha} \underset{i}{\lambda}_{\beta}. \tag{4}$$

It is easy to show that,

$$g^{\mu\alpha} g_{\alpha\nu} = \delta^{\mu}_{\nu}, \tag{5}$$

and

$$g \overset{def.}{=} \| g_{\mu\nu} \| \neq 0. \tag{6}$$

The properties of $g_{\mu\nu}$ and its conjugate given by (3) - (6), show that this tensor can be used as a metric defining a Riemannian structure, associated with the AP-structure.

The AP-condition,

$$\underset{i}{\lambda}{}^{\mu}{}_{\overset{}{+}|\nu} = 0, \tag{7}$$

implies the existence of a linear connection,

$$\Gamma^{\alpha}_{.\mu\nu} \overset{def.}{=} \underset{i}{\lambda}{}^{\alpha} \underset{i}{\lambda}_{\mu,\nu}, \tag{8}$$

which is the direct solution of (7). Here we use the stroke (|) and the (+) sign to characterize tensor derivatives, using the connection (8), and the comma (,) is used for ordinary partial differentiation.

It is clear that the linear connection (8) is non-symmetric. Consequently, it has a torsion defined by,

$$\Lambda^{\alpha}_{.\mu\nu} \overset{def.}{=} \Gamma^{\alpha}_{.\mu\nu} - \Gamma^{\alpha}_{.\nu\mu}. \tag{9}$$

Now, since (8) is non-symmetric, so its dual $\widetilde{\Gamma}^{\alpha}_{.\mu\nu} \left(\overset{def.}{=} \Gamma^{\alpha}_{\nu\mu} \right)$ and its symmetric part $\Gamma^{\alpha}_{(\mu\nu)} \left(\overset{def.}{=} \frac{1}{2} \left(\Gamma^{\alpha}_{\mu\nu} + \Gamma^{\alpha}_{\nu\mu} \right) \right)$ are also linear connections. Imposing a metricity condition on (4), we can define another symmetric linear connection, Christoffel symbol of the second kind $\{^{\alpha}_{\mu\nu}\}$, as usually defined. Now, we can define the 3^{rd} order tensor,

$$\gamma^{\alpha}_{.\mu\nu} = \underset{i}{\lambda}{}^{\alpha} \underset{i}{\lambda}_{\mu;\nu} \tag{10}$$

where (;) is used to characterized covariant differentiation using Christoffel symbol. The tensor defined by (10) is the contortion of the space. It is easy to derive the following relations (cf.[20]),

$$\Gamma^{\alpha}_{.\mu\nu} = \{^{\alpha}_{\mu\nu}\} + \gamma^{\alpha}_{.\mu\nu}. \tag{11}$$

$$\Lambda^{\alpha}_{.\mu\nu} = \gamma^{\alpha}_{.\mu\nu} - \gamma^{\alpha}_{.\nu\mu}, \tag{12}$$

$$C_{\mu} \overset{def.}{=} \Lambda^{\alpha}_{.\mu\alpha} = \gamma^{\alpha}_{.\mu\alpha}. \tag{13}$$

The vector given by (13) is known as the basic vector of the space (cf. [23]). Using (9), (10), (13), a set of second order tensors has been defined [23], which are, usually used for physical applications (cf. [24–27]).

2.2 AP-Structure for Cosmological Applications

Robertson [28] has derived the most general building blocks of two AP-structures with homogeneity and isotropy, i.e. satisfy the cosmological principle. The structures are usually used for cosmological applications. Further investigation of the two structures [29] show that they have the types [30] FOGIII and FOGI, respectively. In the present work we use the first one whose structure is given by the matrix (coordinate system used, $x^0 = t$, $x^1 = x$, $x^2 = y$, $x^3 = z$),

$$
\begin{array}{ll}
\underset{\underset{\downarrow}{j\,(0,1,2,3)}}{\mu \rightarrow} & (0,1,2,3) \\
\underset{j}{\lambda}{}^{\mu} = &
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \frac{i\left(1-\frac{1}{4}kr^2+\frac{k}{2}x^2\right)}{a} & \frac{i\left(\frac{k}{2}xy-k^{\frac{1}{2}}z\right)}{a} & \frac{i\left(\frac{k}{2}xz+k^{\frac{1}{2}}y\right)}{a} \\
0 & \frac{i\left(\frac{k}{2}yx+k^{\frac{1}{2}}z\right)}{a} & \frac{i\left(1-\frac{1}{4}kr^2+\frac{k}{2}y^2\right)}{a} & \frac{i\left(\frac{k}{2}yz-k^{\frac{1}{2}}x\right)}{a} \\
0 & \frac{i\left(\frac{k}{2}zx-k^{\frac{1}{2}}y\right)}{a} & \frac{i\left(\frac{k}{2}zy+k^{\frac{1}{2}}x\right)}{a} & \frac{i\left(1-\frac{1}{4}kr^2+\frac{k}{2}z^2\right)}{a}
\end{pmatrix},
\end{array}
\tag{14}
$$

where a is a function of time only, $i \stackrel{def.}{=} \sqrt{-1}$, $h \stackrel{def.}{=} \frac{1}{1+\frac{1}{4}kr^2}$, $r = \sqrt{x^2+y^2+z^2}$ and $k(= 0, +1, -1)$ is the sectional curvature.

The covariant components of this tetrad are given in the matrix:

$$
\begin{array}{ll}
\underset{\underset{\downarrow}{j\,(0,1,2,3)}}{\mu \rightarrow} & (0,1,2,3) \\
\underset{j}{\lambda}{}_{\mu} = &
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -i\left(1-\frac{1}{4}kr^2+\frac{k}{2}x^2\right)ah^2 & -i\left(\frac{k}{2}xy-k^{\frac{1}{2}}z\right)ah^2 & -i\left(\frac{k}{2}xz+k^{\frac{1}{2}}y\right)ah^2 \\
0 & -i\left(\frac{k}{2}yx+k^{\frac{1}{2}}z\right)ah^2 & -i\left(1-\frac{1}{4}kr^2+\frac{k}{2}y^2\right)ah^2 & -i\left(\frac{k}{2}yz-k^{\frac{1}{2}}x\right)ah^2 \\
0 & -i\left(\frac{k}{2}zx-k^{\frac{1}{2}}y\right)ah^2 & -i\left(\frac{k}{2}zy+k^{\frac{1}{2}}x\right)ah^2 & -i\left(1-\frac{1}{4}kr^2+\frac{k}{2}z^2\right)ah^2
\end{pmatrix}.
\end{array}
\tag{15}
$$

where $h \stackrel{def.}{=} \frac{1}{1+\frac{1}{4}kr^2}$.

Using definitions (3), (4) we get the following non-vanishing components of the metric tensor of the Riemannian structure associated with the AP-structure(14),

$$
\begin{aligned}
g^{00} &= 1,\ g^{11} = g^{22} = g^{33} = -h^{-2}a^{-2}, \\
g_{00} &= 1,\ g_{11} = g_{22} = g_{33} = -h^2a^2.
\end{aligned}
\tag{16}
$$

In this article, we use relativistic system of units $c = G = 1$, where c is the speed of light and G is Newton's gravitational constant. The metric (16) implies that $a(t)$ is the scale factor of the FRW-cosmology. Now, using the building blocks (14)and definition (8) for the linear connection, we can get the following non-vanishing components of the torsion tensor, using definition (9)[26],

$$
\begin{aligned}
\Lambda^{1}_{.10} &= \Lambda^{2}_{.20} = \Lambda^{3}_{.30} = -\Lambda^{1}_{.01} = -\Lambda^{2}_{.02} = -\Lambda^{3}_{.03} = \frac{\dot{a}}{a}, \\
\Lambda^{1}_{.32} &= \Lambda^{2}_{.13} = \Lambda^{3}_{.21} = -\Lambda^{1}_{.23} = -\Lambda^{2}_{.31} = -\Lambda^{3}_{.12} = 2\sqrt{k}h,
\end{aligned}
\tag{17}
$$

where the dot (.) represents differentiation w.r.t. time. Now the only non-vanishing components of the basic vector C_μ, using (13), is:

$$
C_0 = -3\frac{\dot{a}}{a},
\tag{18}
$$

from which we can get a scalar $\mathcal{T}$ defined by,

$$
\mathcal{T} \stackrel{def.}{=} \sqrt{g^{\mu\nu}C_\mu C_\nu},
\tag{19}
$$

which, in the present case using (16),(18), gives,

$$\mathcal{T} = 3\frac{\dot{a}}{a}. \tag{20}$$

This is the value of the torsion scalar characterizing the AP structure (14).

2.3 GR in The AP-Space

Many authors have attempted to solve problems of GR by constructing new theories. In most cases this procedure does not focus on the weaknesses in GR that cause the problems. In the present work we are going to deal with GR , written in the AP-geometry. This will enable us to focus on the weaknesses of the theory and find a solution, if any.

The field equation of GR can be written as:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}(R - 2\Lambda) = -kT_{\mu\nu} \tag{21}$$

where $R_{\mu\nu}$ is Ricci tensor, R is Ricci scalar, Λ is the cosmological constant and $T_{\mu\nu}$ is the material-energy tensor defined from outside the geometric structure in a usual phenomenological manner. Ricci tensor and scalar can be evaluated, in the context of the AP-geometry, by sing the definition,

$$R_{\mu\nu} \stackrel{def.}{=} \{{}^{\alpha}_{\mu\alpha}\}_{,\nu} - \{{}^{\alpha}_{\mu\nu}\}_{,\alpha} + \{{}^{\epsilon}_{\mu\alpha}\}\{{}^{\alpha}_{\epsilon\nu}\} - \{{}^{\epsilon}_{\mu\nu}\}\{{}^{\alpha}_{\epsilon\alpha}\} \tag{22}$$

where {} is Christoffel symbol of the 2^{nd} kind constructed as stated above, by imposing a metricity condition on (4). By using (22) we can write the L.H.S. of the field equations of GR (21), in the context of AP-geometry. The R.H.S. of equations (21) are written from outside the objects of Riemannian geometry, as mentioned. In the next section, we are going to show how torsion affects the R.H.S. of the field equations of GR (21)in the case of FRW-cosmology.

3 Effect of Torsion on The Dynamics of FRW-Model

In the present Section we investigate the effect of torsion on the cosmological solutions of Einstein field equation (21). For this purpose we use the AP-structure (14) to evaluate Ricci tensor (21) and Ricci scalar. The following set of differential equations corresponding to (21), with $T_{\mu\nu}$ is the material-energy tensor of a perfect fluid (as usually done in standard cosmology cf.[31]),

$$3\left(\frac{\dot{a}}{a}\right)^2 = 8\pi\rho_0 - \frac{3k}{a^2} + \Lambda, \tag{23}$$

$$3\left(\frac{\ddot{a}}{a}\right) = -4\pi(\rho_0 + 3P_0) + \Lambda, \tag{24}$$

together with the conservation equation,

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho_0 + P_0) = 0, \tag{25}$$

where ρ_0 and P_0 are the proper density and the proper pressure of a perfect fluid, respectively. Equation (23) and (24) are the same FRW-dynamical equations of standard cosmology. Since we investigate the torsion sole effect on the dynamics, we take $\Lambda = 0$ since, as it is well known, Λ can be used as an alternative to solve some of such problems.

Also, we take $k = 0$ in order to facilitate comparison with standard cosmology. In this case (23),(24) will reduce to,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi\rho_0, \tag{26}$$

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4}{3}\pi(\rho_0 + 3P_0), \tag{27}$$

which are the dynamical equations, of FRW standard cosmology, written in the context of the AP-geometry (a geometry with non-vanishing torsion). It is obvious that all solution of (26) and (27) have some problems e.g. singularity, particle horizons, ...etc.

Now, to study the effect of the torsion field (19) on the solutions of FRW-dynamical equations (26), (27), we insert this field into the R.H.S. of (21) using the following scheme. The AP-structure (14) implies a relation between the scalar torsion field $\mathcal{T}$ and the scale factor as given by (20),

$$\mathcal{T} = 3\frac{\dot{a}}{a}, \tag{28}$$

Also it can be related to Hubble's parameter H as

$$\mathcal{T} = 3H. \tag{29}$$

From (28) we get,

$$\dot{\mathcal{T}} = 3\dot{H} = 3\left[\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2\right]. \tag{30}$$

From which we get,

$$\frac{\ddot{a}}{a} = \frac{1}{9}\mathcal{T}^2 + \frac{1}{3}\dot{\mathcal{T}}. \tag{31}$$

Note that relations (28)–(31) are theory independent. So, to get the effect of the scalar torsion $\mathcal{T}$ on the solution of FRW dynamical equations (26), (27), let us assume that the dominant energetic contents of the universe, at its very early stages, are purely induced by the torsion field $\mathcal{T}$, then the FRW-dynamical equations (26) and (27) may be written in the form:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi\rho_{\mathcal{T}}, \tag{32}$$

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4}{3}\pi(\rho_{\mathcal{T}} + 3P_{\mathcal{T}}). \tag{33}$$

where $\rho_{\mathcal{T}}$, $P_{\mathcal{T}}$ are the energy density and pressure induced by the torsion field $\mathcal{T}$, respectively. Now, the values of these quantities, in terms of the field $\mathcal{T}$, can be obtained by comparting (28) and (31) with the dynamical equations (32) and (33) respectively, we obtain

$$\rho_{\mathcal{T}} = \frac{1}{24\pi}\mathcal{T}^2, \tag{34}$$

$$P_{\mathcal{T}} = -\frac{1}{4\pi}\left(\frac{1}{6}\mathcal{T}^2 + \frac{1}{3}\dot{\mathcal{T}}\right). \tag{35}$$

In order to get an equation of state for a perfect fluid induced by the torsion field $\mathcal{T}$, (35) may be written, using (34), in the form:

$$P_{\mathcal{T}} = -\rho_{\mathcal{T}}(1 + \epsilon), \tag{36}$$

where

$$\epsilon \overset{def.}{=} \frac{2\dot{\mathcal{T}}}{\mathcal{T}^2}. \tag{37}$$

Equation (36) has the form of an equation of state of a fluid. The fluid in the present case is induced by the torsion field $\mathcal{T}$. To facilitate comparison with FRW-standard cosmology, let us write the equation of state of FRW- standard cosmology in the form

$$P_0 = \omega \rho_0. \tag{38}$$

Comparison of (38) with (36) gives

$$\omega(t) = -(1+\epsilon). \tag{39}$$

Now, we discuss the following possibility: In the case of $\epsilon = 0$: This implies $P_{\mathcal{T}} = -\rho_{\mathcal{T}}$ (from (36)). Substituting this relation into FRW-dynamical equations (32) and (33) we obtain

$$\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = 0,$$

which has the solution

$$a = e^{\alpha t}, \tag{40}$$

where α is constant. This solution has no particle horizons and will be discussed in the next section.

4 Discussion and Concluding Remarks

In the present work we used the AP-geometry to write the field equation of GR. In general, as stated in the text, the structure of an AP-space in 4-dimensions, is defined completely by 16-independent building blocks (a tetrad). So, one needs 16-conditions (equations) to fix these function. GR field equations (21) represent 10 of such conditions. Then we need 6 more conditions to fix the functions. Fortunately, in the AP-structure with homogeneity and isotropy (14), all skew 2^{nd} order tensors, vanish identically. And since the 6-conditions needed are expected to be given by a tensor equation including some combinations of the skew tensors [23], then we can say that the required six conditions are satisfied identically for the AP-structure(14).

It is to be noted that Riemannian geometry has no sufficient structure to accommodate any physical entities but the gravitational field and its background space-time. So, on one hand, it is preferable to use a more wider geometry than the Riemannian one, in order to represent more physical quantities, e.g. matter and energy , and to write a pure geometric theory for gravity. On the other hand, a more wider geometry would have torsion in addition to curvature (cf.[20–22]) as the two important geometric object characterizing the geometry. In this case, what would be the impact of torsion on the solutions and predictions of any suggested field theory, including GR, written in the context of this geometry?

The present work, gives an answer to the above question. Some of the problems of standard cosmology are solved. The solution (40) has no singularity and no particle horizon problems. The scheme leading to this achievement can be summarized as follows. The field equations of GR (21) are written in the AP-geometry and applied to an AP-structure having homogeneity and isotropy (14). The resulting differential equations (23),(24) are the FRW-dynamical equations, characterizing standard cosmology. Till this step, nothing is changed concerning standard cosmology, its problems still exist. But when torsion is inserted into the R.H.S. of the field equation of GR (see, (32)and (33)) as discussed above, a resulting solution has neither particle horizons nor singularity.

GR in free space is a geometric theory for the gravitational interaction. It gained most of its success from applications of its field equations (in **free** space) together with the geodesic

Table 1 Comparison between conventional inflation and the present work

	Conventional inflation [10, 11]	Present work
Conservation	Noether theorem	Bianchi identity
Energy Density	$\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi)$	$\rho_{\mathcal{T}} = \frac{1}{24\pi}\mathcal{T}^2$
Pressure	$P_\phi = \frac{\dot{\phi}^2}{2} - V(\phi)$	$P_{\mathcal{T}} = -\frac{1}{4\pi}\left(\frac{1}{6}\mathcal{T}^2 + \frac{1}{3}\dot{\mathcal{T}}\right)$
Equation of State Parameter	$w_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}$	$\omega_{\mathcal{T}} = -(1+\epsilon)$
Exponential expansion condition	$\dot{\phi}^2 << V(\phi)$	$\epsilon = 0$

equations. Such successful applications are carried out in the cases of spherical symmetry (The Schwarzschild solution) and axial symmetry (Kerr solution). In this sense, GR, constructed in Riemannian geometry, can be considered as a **pure** geometric theory for gravity, since all entities used in the theory are constructed from the building blocks of the geometry (the metric tensor). In contrast, many problems appeared when using the field equation within a **material distribution** (21), for applications. The main difference between GR in free space and within a material distribution is the presence of the tensor $T_{\mu\nu}$, which has no representative in Riemannian Geometry. In this sense, GR within a material distribution, cannot be considered as a pure geometric theory for gravity.

In the present work, we show that if energy density and pressure are purely induced by torsion (as defined by (34) and (35)), then some of the problems of FRW-cosmology are removed. Two entities are used to get the model (40), the GR field equations and the AP-structure (14). It is to be considered that the solution (40) cannot be obtained ether from GR field equations alone (26), (27) or from the AP-structure (14) alone. The result (40) is obtained from a common factor, presented in both entities, the scale factor a. The compassion between the derivatives of the scale factor in both entities gives the solution (40), which is free from particle horizon. This may imply an avenue to get rid of problems of GR, by writing its material-energy contents using pure geometric objects. In other words, this may be achieved by converting GR to a pure geometric or constructing a pure geometric theory for gravity.

In the literature, it is well known that the most famous solution of the particle horizons problem is the scheme called inflation [10, 11]. This scheme implies the existence of a scalar field, from outside Riemannian geometry. The present treatment throws some light on the nature of this field. The torsion field, used in the present work, may represent a good candidate for solving such problems. It is a part of the geometry used, the AP-geometry. Table 1 gives a brief comparison between the main features of conventional inflation and the present work.

From Table 1, we can note that in both cases, the field should vary very slowly in order to get exponential expansion (model free from particle horizons). In our treatment the source of the field is the torsion scalar, which is a pure geometric quantity.

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