

Bidirectional Quantum Controlled Teleportation via a Maximally Seven-qubit Entangled State

Ya-Jun Duan · Xin-Wei Zha · Xin-Mei Sun · Jia-Fan Xia

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Abstract A bidirectional quantum controlled teleportation scheme using a seven-qubit maximally entangled state as quantum channel is proposed. This means that Alice can transmit an arbitrary single qubit state of qubit a to Bob and Bob can transmit an arbitrary single qubit state of qubit b to Alice via the control of the supervisor Charlie.

Keywords Bidirectional quantum controlled teleportation · Seven-qubit maximally entangled state · von Neumann measurement

1 Introduction

Quantum teleportation, which allows transportation of an unknown state from a sender Alice to a spatially distant receiver Bob with the aid of the previously shared entanglement and classical communication, is regarded as one of the most striking results of quantum information theory [1]. Since the original quantum teleportation protocol was proposed by Bennett [2], many theoretical protocols of quantum teleportation have been presented [3–23], experimental development of teleportation has also been reported [24, 25]. In 1998, Karlsson and Bourennane introduced the concept of Controlled teleportation, actually, controlled teleportation equals to quantum state sharing [26–45]. In particular, in 2013, Zha et al. [46] presented the first bidirectional quantum controlled teleportation (BQCT) protocol by employing five-qubit cluster state as the quantum channel, After that, BQCT has received great attention, some BQCT protocols have been devised based on different kinds of entangled states, such as five-qubit entangled state [47–49], maximally six-qubit entangled state [50], six-qubit cluster state [51].

Y.-J. Duan (✉) · X.-W. Zha · X.-M. Sun · J.-F. Xia
School of Science, Xi'an University of Posts and Telecommunications,
Xi'an, 710121, People's Republic of China
e-mail: duanyajun501@126.com

Recently Zha et al. [52] put forward a maximally seven-qubit entangled state. In this paper, we utilize this seven-qubit maximally entangled state to present a BQCT scheme. In the scheme, the seven-qubit maximally entangled state is initially shared by the senders (receivers) Alice, Bob and supervisor Charlie. Suppose that Alice has particle a in an unknown state, she wants to transmit the state of particle a to Bob; at the same time, Bob has particle b in an unknown state, he wants to transmit the state of particle b to Alice. To achieve the purpose of BQCT, besides local operation and classical communications, it is necessary that three parties perform proper measurement, that is, Alice and Bob take the Bell measurement respectively, Charlie carries out three-qubit von Neumann measurement.

2 Bidirectional Quantum Controlled Teleportation

Now let us present our BQCT scheme in detail. The schematic demonstration is illustrated in Fig. 1.

Our scheme can be described as follows. Suppose Alice has an arbitrary single qubit a in a unknown state, which is described by

$$|\varphi\rangle_a = a_0|0\rangle + a_1|1\rangle \tag{1}$$

and that Bob has qubit b in an unknown state

$$|\varphi\rangle_b = b_0|0\rangle + b_1|1\rangle \tag{2}$$

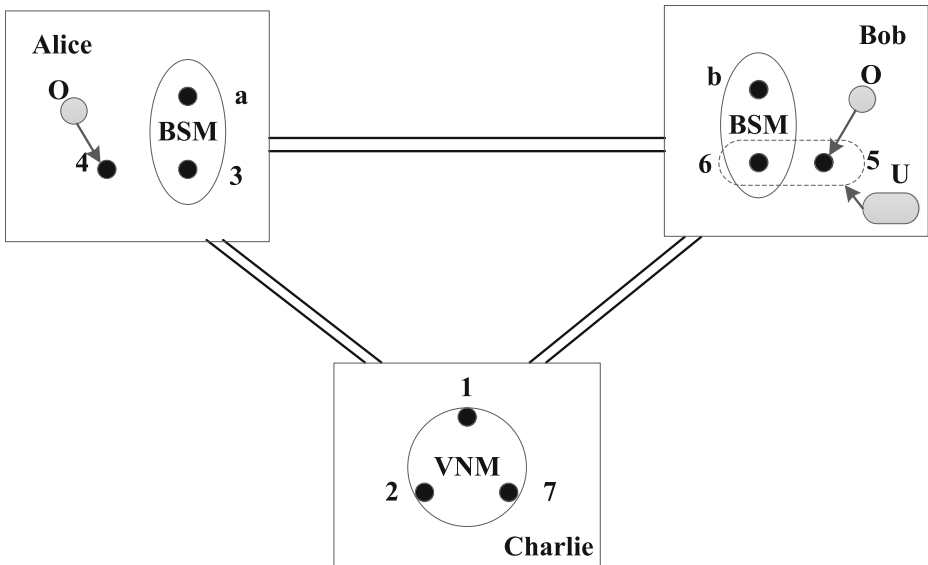


Fig. 1 Illustration of the BQCT scheme. The abbreviation *BSM* represents the Bell state measurement, *VNM* denotes the von Neumann measurement, *solid lines* among rectangles stand for classical channels, *solid dots* represent qubits, the *gray solid closed planes* label the single-qubit unitary operation *O* and two-qubit unitary operation *U*

Now Alice wants to transmit the state of qubit a to Bob and Bob wants to transmit the state of qubit b to Alice. The quantum channel linking Alice, Bob and Charlie is a maximally seven-qubit entangled state [52], which is in the form of

$$\begin{aligned}
 |\psi_M\rangle_{1234567} = & \frac{1}{4\sqrt{2}}[(|0000000\rangle + |0000011\rangle + |0001101\rangle + |0001110\rangle) \\
 & + (|0010001\rangle - |0010010\rangle + |0011100\rangle - |0011111\rangle) \\
 & + (-|0100101\rangle - |0100110\rangle + |0101000\rangle + |0101011\rangle) \\
 & + (|0110100\rangle - |0110111\rangle - |0111001\rangle + |0111010\rangle) \\
 & + (-|1000100\rangle - |1000111\rangle + |1001001\rangle + |1001010\rangle) \\
 & + (|1010101\rangle - |1010110\rangle - |1011000\rangle + |1011011\rangle) \\
 & + (|1100001\rangle + |1100010\rangle + |1101100\rangle + |1101111\rangle) \\
 & + (|1110000\rangle - |1110011\rangle + |1111101\rangle - |1111110\rangle)] \tag{3}
 \end{aligned}$$

Alice owns qubits 3 and 4, Bob holds qubits 5 and 6, qubits 1, 2 and 7 belong to Charlie. As a result, the entangled channel can also be rewritten as

$$\begin{aligned}
 |\psi_M\rangle_{C_1C_2C_3A_1A_2B_1B_2} = & \frac{1}{4\sqrt{2}}[(|000\rangle + |111\rangle)_{C_1C_2C_3} \otimes (|0000\rangle + |0111\rangle) \\
 & - |1001\rangle + |1110\rangle)_{A_1A_2B_1B_2} + (|010\rangle + |101\rangle)_{C_1C_2C_3} \\
 & \otimes (|0100\rangle - |0011\rangle + |1010\rangle + |1101\rangle)_{A_1A_2B_1B_2} \\
 & + (|100\rangle + |011\rangle)_{C_1C_2C_3} \otimes (-|0010\rangle + |0101\rangle - |1011\rangle) \\
 & - |1100\rangle)_{A_1A_2B_1B_2} + (|001\rangle + |110\rangle)_{C_1C_2C_3} \otimes (|0001\rangle \\
 & + |0110\rangle + |1000\rangle - |1111\rangle)_{A_1A_2B_1B_2}] \tag{4}
 \end{aligned}$$

So the system state of the total qubits can be expressed as

$$|\psi\rangle_t = |\psi\rangle_{C_1C_2C_3A_1A_2B_1B_2ab} = |\psi_M\rangle_{C_1C_2C_3A_1A_2B_1B_2} \otimes |\psi\rangle_a \otimes |\psi\rangle_b \tag{5}$$

In order to realize BQCT, Bob performs an unitary operator on qubits B_1 and B_2 . The unitary operator $U_{B_1B_2}$ is given by

$$U_{B_1B_2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{6}$$

Therefore, the state of the total system becomes

$$\begin{aligned}
 |\psi'\rangle_t = U_{B_1B_2}|\psi\rangle_t = & \frac{1}{4\sqrt{2}}|\varphi\rangle_a|\varphi\rangle_b[(|000\rangle + |111\rangle)_{C_1C_2C_3}(|0000\rangle + |0101\rangle) \\
 & + |1010\rangle + |1111\rangle)_{A_1A_2B_1B_2} + (|010\rangle + |101\rangle)_{C_1C_2C_3} \\
 & \times (-|0001\rangle + |0100\rangle + |1011\rangle - |1110\rangle)_{A_1A_2B_1B_2} \\
 & + (|100\rangle + |011\rangle)_{C_1C_2C_3}(-|0011\rangle - |0110\rangle - |1001\rangle \\
 & - |1100\rangle)_{A_1A_2B_1B_2} + (|001\rangle + |110\rangle)_{C_1C_2C_3}(-|0010\rangle \\
 & + |0111\rangle + |1000\rangle - |1101\rangle)_{A_1A_2B_1B_2}] \tag{7}
 \end{aligned}$$

Next, Alice performs a Bell-state measurement on particles (a, A_1) , at the same time, Bob also takes a Bell measurement on (b, B_2) , then they convey the measurement results to each other. In our paper, Bell states are given by

$$|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), |\varphi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \tag{8}$$

The outcomes of measurement performed by Alice and Bob, and the corresponding collapse state of qubits A_2, B_1, C_1, C_2 and C_3 are shown in Table 1.

In Table 1, the state $|\zeta^i\rangle (i = 0, \dots, 15)$ are given by

$$\begin{aligned} |\zeta^0\rangle_{A_2 B_1 C_1 C_2 C_3} &= (b_0|0\rangle + b_1|1\rangle)_{A_2} (a_0|0\rangle + a_1|1\rangle)_{B_1} (|000\rangle + |111\rangle)_{C_1 C_2 C_3} \\ &+ (b_0|1\rangle - b_1|0\rangle)_{A_2} (a_0|0\rangle - a_1|1\rangle)_{B_1} (|010\rangle + |101\rangle)_{C_1 C_2 C_3} \\ &- (b_0|1\rangle + b_1|0\rangle)_{A_2} (a_0|1\rangle + a_1|0\rangle)_{B_1} (|100\rangle + |011\rangle)_{C_1 C_2 C_3} \\ &- (b_0|0\rangle - b_1|1\rangle)_{A_2} (a_0|1\rangle - a_1|0\rangle)_{B_1} (|001\rangle + |110\rangle)_{C_1 C_2 C_3} \end{aligned}$$

$$\begin{aligned} |\zeta^1\rangle_{A_2 B_1 C_1 C_2 C_3} &= [(b_0|0\rangle - b_1|1\rangle)_{A_2} (a_0|0\rangle + a_1|1\rangle)_{B_1} (|000\rangle + |111\rangle)_{C_1 C_2 C_3} \\ &+ (b_0|1\rangle + b_1|0\rangle)_{A_2} (a_0|0\rangle - a_1|1\rangle)_{B_1} (|010\rangle + |101\rangle)_{C_1 C_2 C_3} \\ &- (b_0|1\rangle - b_1|0\rangle)_{A_2} (a_0|1\rangle + a_1|0\rangle)_{B_1} (|100\rangle + |011\rangle)_{C_1 C_2 C_3} \\ &- (b_0|0\rangle + b_1|1\rangle)_{A_2} (a_0|1\rangle - a_1|0\rangle)_{B_1} (|001\rangle + |110\rangle)_{C_1 C_2 C_3} \end{aligned}$$

$$\begin{aligned} |\zeta^2\rangle_{A_2 B_1 C_1 C_2 C_3} &= [(b_0|1\rangle + b_1|0\rangle)_{A_2} (a_0|0\rangle + a_1|1\rangle)_{B_1} (|000\rangle + |111\rangle)_{C_1 C_2 C_3} \\ &- (b_0|0\rangle - b_1|1\rangle)_{A_2} (a_0|0\rangle - a_1|1\rangle)_{B_1} (|010\rangle + |101\rangle)_{C_1 C_2 C_3} \\ &- (b_0|0\rangle + b_1|1\rangle)_{A_2} (a_0|1\rangle + a_1|0\rangle)_{B_1} (|100\rangle + |011\rangle)_{C_1 C_2 C_3} \\ &+ (b_0|1\rangle - b_1|0\rangle)_{A_2} (a_0|1\rangle - a_1|0\rangle)_{B_1} (|001\rangle + |110\rangle)_{C_1 C_2 C_3} \end{aligned}$$

Table 1 The outcomes of measurement performed by Alice and Bob, and the corresponding collapse state of qubits $(A_2, B_1, C_1, C_2, C_3)$

Alices result	Bob's result	The state of qubits $(A_2, B_1, C_1, C_2, C_3)$
$ \phi^+\rangle_{aA_1}$	$ \phi^+\rangle_{bB_2}$	$ \zeta^0\rangle_{A_2 B_1 C_1 C_2 C_3}$
$ \phi^+\rangle_{aA_1}$	$ \phi^-\rangle_{bB_2}$	$ \zeta^1\rangle_{A_2 B_1 C_1 C_2 C_3}$
$ \phi^+\rangle_{aA_1}$	$ \phi^+\rangle_{bB_2}$	$ \zeta^2\rangle_{A_2 B_1 C_1 C_2 C_3}$
$ \phi^+\rangle_{aA_1}$	$ \phi^-\rangle_{bB_2}$	$ \zeta^3\rangle_{A_2 B_1 C_1 C_2 C_3}$
$ \phi^-\rangle_{aA_1}$	$ \phi^+\rangle_{bB_2}$	$ \zeta^4\rangle_{A_2 B_1 C_1 C_2 C_3}$
$ \phi^-\rangle_{aA_1}$	$ \phi^-\rangle_{bB_2}$	$ \zeta^5\rangle_{A_2 B_1 C_1 C_2 C_3}$
$ \phi^-\rangle_{aA_1}$	$ \phi^+\rangle_{bB_2}$	$ \zeta^6\rangle_{A_2 B_1 C_1 C_2 C_3}$
$ \phi^-\rangle_{aA_1}$	$ \phi^-\rangle_{bB_2}$	$ \zeta^7\rangle_{A_2 B_1 C_1 C_2 C_3}$
$ \phi^+\rangle_{aA_1}$	$ \phi^+\rangle_{bB_2}$	$ \zeta^8\rangle_{A_2 B_1 C_1 C_2 C_3}$
$ \phi^+\rangle_{aA_1}$	$ \phi^-\rangle_{bB_2}$	$ \zeta^9\rangle_{A_2 B_1 C_1 C_2 C_3}$
$ \phi^+\rangle_{aA_1}$	$ \phi^+\rangle_{bB_2}$	$ \zeta^{10}\rangle_{A_2 B_1 C_1 C_2 C_3}$
$ \phi^+\rangle_{aA_1}$	$ \phi^-\rangle_{bB_2}$	$ \zeta^{11}\rangle_{A_2 B_1 C_1 C_2 C_3}$
$ \phi^-\rangle_{aA_1}$	$ \phi^+\rangle_{bB_2}$	$ \zeta^{12}\rangle_{A_2 B_1 C_1 C_2 C_3}$
$ \phi^-\rangle_{aA_1}$	$ \phi^-\rangle_{bB_2}$	$ \zeta^{13}\rangle_{A_2 B_1 C_1 C_2 C_3}$
$ \phi^-\rangle_{aA_1}$	$ \phi^+\rangle_{bB_2}$	$ \zeta^{14}\rangle_{A_2 B_1 C_1 C_2 C_3}$
$ \phi^-\rangle_{aA_1}$	$ \phi^-\rangle_{bB_2}$	$ \zeta^{15}\rangle_{A_2 B_1 C_1 C_2 C_3}$

$$\begin{aligned}
 |\zeta^{11}\rangle_{A_2B_1C_1C_2C_3} &= (b_0|1\rangle - b_1|0\rangle)_{A_2}(a_0|1\rangle + a_1|0\rangle)_{B_1}(|000\rangle + |111\rangle)_{C_1C_2C_3} \\
 &+ (b_0|0\rangle + b_1|1\rangle)_{A_2}(a_0|1\rangle - a_1|0\rangle)_{B_1}(|010\rangle + |101\rangle)_{C_1C_2C_3} \\
 &- (b_0|0\rangle - b_1|1\rangle)_{A_2}(a_0|0\rangle + a_1|1\rangle)_{B_1}(|100\rangle + |011\rangle)_{C_1C_2C_3} \\
 &- (b_0|1\rangle + b_1|0\rangle)_{A_2}(a_0|0\rangle - a_1|1\rangle)_{B_1}(|001\rangle + |110\rangle)_{C_1C_2C_3} \\
 \\
 |\zeta^{12}\rangle_{A_2B_1C_1C_2C_3} &= (b_0|0\rangle + b_1|1\rangle)_{A_2}(a_0|1\rangle - a_1|0\rangle)_{B_1}(|000\rangle + |111\rangle)_{C_1C_2C_3} \\
 &- (b_0|1\rangle - b_1|0\rangle)_{A_2}(a_0|1\rangle + a_1|0\rangle)_{B_1}(|010\rangle + |101\rangle)_{C_1C_2C_3} \\
 &- (b_0|1\rangle + b_1|0\rangle)_{A_2}(a_0|0\rangle - a_1|1\rangle)_{B_1}(|100\rangle + |011\rangle)_{C_1C_2C_3} \\
 &+ (b_0|0\rangle - b_1|1\rangle)_{A_2}(a_0|0\rangle + a_1|1\rangle)_{B_1}(|001\rangle + |110\rangle)_{C_1C_2C_3} \\
 \\
 |\zeta^{13}\rangle_{A_2B_1C_1C_2C_3} &= (b_0|0\rangle - b_1|1\rangle)_{A_2}(a_0|1\rangle - a_1|0\rangle)_{B_1}(|000\rangle + |111\rangle)_{C_1C_2C_3} \\
 &- (b_0|1\rangle + b_1|0\rangle)_{A_2}(a_0|1\rangle + a_1|0\rangle)_{B_1}(|010\rangle + |101\rangle)_{C_1C_2C_3} \\
 &- (b_0|1\rangle - b_1|0\rangle)_{A_2}(a_0|0\rangle - a_1|1\rangle)_{B_1}(|100\rangle + |011\rangle)_{C_1C_2C_3} \\
 &+ (b_0|0\rangle + b_1|1\rangle)_{A_2}(a_0|0\rangle + a_1|1\rangle)_{B_1}(|001\rangle + |110\rangle)_{C_1C_2C_3} \\
 \\
 |\zeta^{14}\rangle_{A_2B_1C_1C_2C_3} &= (b_0|1\rangle + b_1|0\rangle)_{A_2}(a_0|1\rangle - a_1|0\rangle)_{B_1}(|000\rangle + |111\rangle)_{C_1C_2C_3} \\
 &+ (b_0|0\rangle - b_1|1\rangle)_{A_2}(a_0|1\rangle + a_1|0\rangle)_{B_1}(|010\rangle + |101\rangle)_{C_1C_2C_3} \\
 &- (b_0|0\rangle + b_1|1\rangle)_{A_2}(a_0|0\rangle - a_1|1\rangle)_{B_1}(|100\rangle + |011\rangle)_{C_1C_2C_3} \\
 &- (b_0|1\rangle - b_1|0\rangle)_{A_2}(a_0|0\rangle + a_1|1\rangle)_{B_1}(|001\rangle + |110\rangle)_{C_1C_2C_3} \\
 \\
 |\zeta^{15}\rangle_{A_2B_1C_1C_2C_3} &= (b_0|1\rangle - b_1|0\rangle)_{A_2}(a_0|1\rangle - a_1|0\rangle)_{B_1}(|000\rangle + |111\rangle)_{C_1C_2C_3} \\
 &+ (b_0|0\rangle + b_1|1\rangle)_{A_2}(a_0|1\rangle + a_1|0\rangle)_{B_1}(|010\rangle + |101\rangle)_{C_1C_2C_3} \\
 &- (b_0|0\rangle - b_1|1\rangle)_{A_2}(a_0|0\rangle - a_1|1\rangle)_{B_1}(|100\rangle + |011\rangle)_{C_1C_2C_3} \\
 &- (b_0|1\rangle + b_1|0\rangle)_{A_2}(a_0|0\rangle + a_1|1\rangle)_{B_1}(|001\rangle + |110\rangle)_{C_1C_2C_3}
 \end{aligned} \tag{9}$$

For example, if Alice’s measurement outcome is $|\phi^+\rangle_{aA_1}$, Bob’s measurement outcome is $|\phi^+\rangle_{bB_2}$, the collapsed state of qubits A_2, B_1, C_1, C_2 and C_3 will be $|\zeta^0\rangle_{A_2B_1C_1C_2C_3}$.

Next, Charlie needs to perform a three-qubit von Neumann measurement on qubits (C_1, C_2, C_3) and then announces the measurement results to Alice and Bob. By combining information from the Charlie, Alice and Bob can perform appropriate unitary operations on particles A_2 and B_1 respectively to reconstruct the original unknown single qubit state. The outcomes of measurement performed by Alice, Bob and Charlie and the corresponding Alice and Bob’s operations are shown in Table 2. For example, if Alice’s measurement outcome is $|\phi^+\rangle_{aA_1}$, Bob’s measurement outcome is $|\phi^+\rangle_{bB_2}$, Charlie’s result is $|000\rangle_{C_1,C_2,C_3}$, consequently, the collapsed state of qubits A_2 and B_1 is

$$\begin{aligned}
 |\eta\rangle_{A_2} &= (b_0|0\rangle + b_1|1\rangle)_{A_2} \\
 |\eta\rangle_{B_1} &= (a_0|0\rangle + a_1|1\rangle)_{B_1}
 \end{aligned} \tag{10}$$

Thus the whole bidirectional quantum controlled teleportation has been realized.

Table 2 The outcomes of measurements performed by Alice, Bob and Charlie, and the corresponding Alice and Bob’s operations

Alice and Bob’s results	Charlie’s results	Alice and Bob’s operations
$ \phi^+\rangle_{aA_1} \phi^+\rangle_{bB_2}$	$ 000\rangle_{C_1 C_2 C_3}, 111\rangle_{C_1 C_2 C_3}$	$I_{A_2} \otimes I_{B_1}$
	$ 010\rangle_{C_1 C_2 C_3}, 101\rangle_{C_1 C_2 C_3}$	$i\sigma_{A_2y} \otimes \sigma_{B_1z}$
	$ 100\rangle_{C_1 C_2 C_3}, 011\rangle_{C_1 C_2 C_3}$	$\sigma_{A_2x} \otimes \sigma_{B_1x}$
	$ 001\rangle_{C_1 C_2 C_3}, 110\rangle_{C_1 C_2 C_3}$	$\sigma_{A_2z} \otimes i\sigma_{B_1y}$
$ \phi^+\rangle_{aA_1} \phi^-\rangle_{bB_2}$	$ 000\rangle_{C_1 C_2 C_3}, 111\rangle_{C_1 C_2 C_3}$	$\sigma_{A_2z} \otimes I_{B_1}$
	$ 010\rangle_{C_1 C_2 C_3}, 101\rangle_{C_1 C_2 C_3}$	$\sigma_{A_2x} \otimes \sigma_{B_1z}$
	$ 100\rangle_{C_1 C_2 C_3}, 011\rangle_{C_1 C_2 C_3}$	$i\sigma_{A_2y} \otimes \sigma_{B_1x}$
	$ 001\rangle_{C_1 C_2 C_3}, 110\rangle_{C_1 C_2 C_3}$	$I_{A_2z} \otimes i\sigma_{B_1y}$
$ \phi^+\rangle_{aA_1} \phi^+\rangle_{bB_2}$	$ 000\rangle_{C_1 C_2 C_3}, 111\rangle_{C_1 C_2 C_3}$	$\sigma_{A_2x} \otimes I_{B_1}$
	$ 010\rangle_{C_1 C_2 C_3}, 101\rangle_{C_1 C_2 C_3}$	$\sigma_{A_2z} \otimes \sigma_{B_1z}$
	$ 100\rangle_{C_1 C_2 C_3}, 011\rangle_{C_1 C_2 C_3}$	$I_{A_2z} \otimes \sigma_{B_1x}$
	$ 001\rangle_{C_1 C_2 C_3}, 110\rangle_{C_1 C_2 C_3}$	$i\sigma_{A_2y} \otimes i\sigma_{B_1y}$
$ \phi^+\rangle_{aA_1} \phi^-\rangle_{bB_2}$	$ 000\rangle_{C_1 C_2 C_3}, 111\rangle_{C_1 C_2 C_3}$	$i\sigma_{A_2y} \otimes I_{B_1}$
	$ 010\rangle_{C_1 C_2 C_3}, 101\rangle_{C_1 C_2 C_3}$	$I_{A_2} \otimes \sigma_{B_1z}$
	$ 100\rangle_{C_1 C_2 C_3}, 011\rangle_{C_1 C_2 C_3}$	$\sigma_{A_2z} \otimes \sigma_{B_1x}$
	$ 001\rangle_{C_1 C_2 C_3}, 110\rangle_{C_1 C_2 C_3}$	$\sigma_{A_2x} \otimes i\sigma_{B_1y}$
$ \phi^-\rangle_{aA_1} \phi^+\rangle_{bB_2}$	$ 000\rangle_{C_1 C_2 C_3}, 111\rangle_{C_1 C_2 C_3}$	$I_{A_2} \otimes \sigma_{B_1z}$
	$ 010\rangle_{C_1 C_2 C_3}, 101\rangle_{C_1 C_2 C_3}$	$i\sigma_{A_2y} \otimes I_{B_1}$
	$ 100\rangle_{C_1 C_2 C_3}, 011\rangle_{C_1 C_2 C_3}$	$\sigma_{A_2x} \otimes i\sigma_{B_1y}$
	$ 001\rangle_{C_1 C_2 C_3}, 110\rangle_{C_1 C_2 C_3}$	$\sigma_{A_2z} \otimes \sigma_{B_1x}$
$ \phi^-\rangle_{aA_1} \phi^-\rangle_{bB_2}$	$ 000\rangle_{C_1 C_2 C_3}, 111\rangle_{C_1 C_2 C_3}$	$\sigma_{A_2z} \otimes \sigma_{B_1z}$
	$ 010\rangle_{C_1 C_2 C_3}, 101\rangle_{C_1 C_2 C_3}$	$\sigma_{A_2x} \otimes I_{B_1}$
	$ 100\rangle_{C_1 C_2 C_3}, 011\rangle_{C_1 C_2 C_3}$	$i\sigma_{A_2y} \otimes i\sigma_{B_1y}$
	$ 001\rangle_{C_1 C_2 C_3}, 110\rangle_{C_1 C_2 C_3}$	$I_{A_2} \otimes \sigma_{B_1x}$
$ \phi^-\rangle_{aA_1} \phi^+\rangle_{bB_2}$	$ 000\rangle_{C_1 C_2 C_3}, 111\rangle_{C_1 C_2 C_3}$	$\sigma_{A_2x} \otimes \sigma_{B_1z}$
	$ 010\rangle_{C_1 C_2 C_3}, 101\rangle_{C_1 C_2 C_3}$	$\sigma_{A_2z} \otimes I_{B_1}$
	$ 100\rangle_{C_1 C_2 C_3}, 011\rangle_{C_1 C_2 C_3}$	$I_{A_2} \otimes i\sigma_{B_1y}$
	$ 001\rangle_{C_1 C_2 C_3}, 110\rangle_{C_1 C_2 C_3}$	$i\sigma_{A_2y} \otimes \sigma_{B_1x}$
$ \phi^-\rangle_{aA_1} \phi^-\rangle_{bB_2}$	$ 000\rangle_{C_1 C_2 C_3}, 111\rangle_{C_1 C_2 C_3}$	$i\sigma_{A_2y} \otimes \sigma_{B_1z}$
	$ 010\rangle_{C_1 C_2 C_3}, 101\rangle_{C_1 C_2 C_3}$	$I_{A_2} \otimes I_{B_1}$
	$ 100\rangle_{C_1 C_2 C_3}, 011\rangle_{C_1 C_2 C_3}$	$\sigma_{A_2z} \otimes i\sigma_{B_1y}$
	$ 001\rangle_{C_1 C_2 C_3}, 110\rangle_{C_1 C_2 C_3}$	$\sigma_{A_2x} \otimes \sigma_{B_1x}$
$ \phi^+\rangle_{aA_1} \phi^+\rangle_{bB_2}$	$ 000\rangle_{C_1 C_2 C_3}, 111\rangle_{C_1 C_2 C_3}$	$I_{A_2} \otimes \sigma_{B_1x}$
	$ 010\rangle_{C_1 C_2 C_3}, 101\rangle_{C_1 C_2 C_3}$	$i\sigma_{A_2y} \otimes i\sigma_{B_1y}$
	$ 100\rangle_{C_1 C_2 C_3}, 011\rangle_{C_1 C_2 C_3}$	$\sigma_{A_2x} \otimes I_{B_1}$
	$ 001\rangle_{C_1 C_2 C_3}, 110\rangle_{C_1 C_2 C_3}$	$\sigma_{A_2z} \otimes \sigma_{B_1z}$
$ \phi^+\rangle_{aA_1} \phi^-\rangle_{bB_2}$	$ 000\rangle_{C_1 C_2 C_3}, 111\rangle_{C_1 C_2 C_3}$	$\sigma_{A_2z} \otimes \sigma_{B_1x}$
	$ 010\rangle_{C_1 C_2 C_3}, 101\rangle_{C_1 C_2 C_3}$	$\sigma_{A_2x} \otimes i\sigma_{B_1y}$
	$ 100\rangle_{C_1 C_2 C_3}, 011\rangle_{C_1 C_2 C_3}$	$i\sigma_{A_2y} \otimes I_{B_1}$
	$ 001\rangle_{C_1 C_2 C_3}, 110\rangle_{C_1 C_2 C_3}$	$I_{A_2} \otimes \sigma_{B_1z}$

Table 2 (continued)

Alice and Bob's results	Charlie's results	Alice and Bob's operations
$ \varphi^+\rangle_{aA_1} \varphi^+\rangle_{bB_2}$	$ 000\rangle_{C_1C_2C_3}, 111\rangle_{C_1C_2C_3}$	$\sigma_{A_2x} \otimes \sigma_{B_1x}$
	$ 010\rangle_{C_1C_2C_3}, 101\rangle_{C_1C_2C_3}$	$\sigma_{A_2z} \otimes i\sigma_{B_1y}$
	$ 100\rangle_{C_1C_2C_3}, 011\rangle_{C_1C_2C_3}$	$I_{A_2} \otimes I_{B_1}$
	$ 001\rangle_{C_1C_2C_3}, 110\rangle_{C_1C_2C_3}$	$i\sigma_{A_2y} \otimes \sigma_{B_1z}$
$ \varphi^+\rangle_{aA_1} \varphi^-\rangle_{bB_2}$	$ 000\rangle_{C_1C_2C_3}, 111\rangle_{C_1C_2C_3}$	$i\sigma_{A_2y} \otimes \sigma_{B_1x}$
	$ 010\rangle_{C_1C_2C_3}, 101\rangle_{C_1C_2C_3}$	$I_{A_2} \otimes i\sigma_{B_1y}$
	$ 100\rangle_{C_1C_2C_3}, 011\rangle_{C_1C_2C_3}$	$\sigma_{A_2z} \otimes I_{B_1}$
	$ 001\rangle_{C_1C_2C_3}, 110\rangle_{C_1C_2C_3}$	$\sigma_{A_2x} \otimes \sigma_{B_1z}$
$ \varphi^-\rangle_{aA_1} \varphi^+\rangle_{bB_2}$	$ 000\rangle_{C_1C_2C_3}, 111\rangle_{C_1C_2C_3}$	$I_{A_2} \otimes i\sigma_{B_1y}$
	$ 010\rangle_{C_1C_2C_3}, 101\rangle_{C_1C_2C_3}$	$i\sigma_{A_2y} \otimes \sigma_{B_1x}$
	$ 100\rangle_{C_1C_2C_3}, 011\rangle_{C_1C_2C_3}$	$\sigma_{A_2x} \otimes \sigma_{B_1z}$
	$ 001\rangle_{C_1C_2C_3}, 110\rangle_{C_1C_2C_3}$	$\sigma_{A_2z} \otimes I_{B_1}$
$ \varphi^-\rangle_{aA_1} \varphi^-\rangle_{bB_2}$	$ 000\rangle_{C_1C_2C_3}, 111\rangle_{C_1C_2C_3}$	$\sigma_{A_2z} \otimes i\sigma_{B_1y}$
	$ 010\rangle_{C_1C_2C_3}, 101\rangle_{C_1C_2C_3}$	$\sigma_{A_2x} \otimes \sigma_{B_1x}$
	$ 100\rangle_{C_1C_2C_3}, 011\rangle_{C_1C_2C_3}$	$i\sigma_{A_2y} \otimes \sigma_{B_1z}$
	$ 001\rangle_{C_1C_2C_3}, 110\rangle_{C_1C_2C_3}$	$I_{A_2} \otimes I_{B_1}$
$ \varphi^-\rangle_{aA_1} \varphi^+\rangle_{bB_2}$	$ 000\rangle_{C_1C_2C_3}, 111\rangle_{C_1C_2C_3}$	$\sigma_{A_2x} \otimes i\sigma_{B_1y}$
	$ 010\rangle_{C_1C_2C_3}, 101\rangle_{C_1C_2C_3}$	$\sigma_{A_2z} \otimes \sigma_{B_1x}$
	$ 100\rangle_{C_1C_2C_3}, 011\rangle_{C_1C_2C_3}$	$I_{A_2} \otimes \sigma_{B_1z}$
	$ 001\rangle_{C_1C_2C_3}, 110\rangle_{C_1C_2C_3}$	$i\sigma_{A_2y} \otimes I_{B_1}$
$ \varphi^-\rangle_{aA_1} \varphi^-\rangle_{bB_2}$	$ 000\rangle_{C_1C_2C_3}, 111\rangle_{C_1C_2C_3}$	$i\sigma_{A_2y} \otimes i\sigma_{B_1y}$
	$ 010\rangle_{C_1C_2C_3}, 101\rangle_{C_1C_2C_3}$	$I_{A_2} \otimes \sigma_{B_1x}$
	$ 100\rangle_{C_1C_2C_3}, 011\rangle_{C_1C_2C_3}$	$\sigma_{A_2z} \otimes \sigma_{B_1z}$
	$ 001\rangle_{C_1C_2C_3}, 110\rangle_{C_1C_2C_3}$	$\sigma_{A_2x} \otimes I_{B_1}$

3 Discussion and Conclusion

Let's discuss amply the advantage of our scheme, our scheme is a bidirectional quantum controlled teleportation protocol, that is to say, if the controller Charlie takes right measurement, Alice and Bob's particles will simultaneously collapse to the corresponding state, we can see that from eq. (9), this characteristic can be employed to experimentally investigate the problem "spooky action at a distance" in quantum entanglement.

In fact, bidirectional quantum controlled teleportation protocols have already been reported by five-qubit and six-qubit entangled state [46–51], compared with those reports, our protocol that exploits the seven-qubit entangled state as quantum channel can improve greatly the security of the scheme, because controller Charlie needs to perform three-qubit von Neumann measurement on his particles, what's more, we consider a situation in which there are three controllers (i.e., Charlie1, Charlie2, Charlie3), in this circumstance, the BQOS can be completed successfully if and only if every controller carries out proper single-qubit von Neumann measurement on corresponding particle respectively.

Now Let's us briefly consider the feasibility of this scheme in experiment. It is found that the necessary local unitary operation in the protocol contains two-qubit unitary operation and single-qubit operation, the employed measurement includes Bell state measurement

and three-qubit von Neumann measurement. It is well known that Bell-state measurements can be decomposed into an ordering combination of a single-qubit Hadmard operation and a two-qubit CNOT operation as well as two single-qubit measurements. Up to now, the progress of Bell-state measurement and the single-qubit unitary operation in experiment in various quantum systems [24, 53–55] has been reported. In addition, the seven-qubit entangled state in our scheme has not been reported in experiment, but when combined with the advances in quantum information technology, we hope our scheme will be implemented in the future.

In summary, we have proposed a theoretical scheme for bidirectional quantum controlled teleportation via seven-qubit maximally entangled state. In the scheme, Alice and Bob are not only senders but also receivers. Based on local operations and classical communications (LOCC), Firstly, Bob needs to carries out an unitary operation on his particles and then Alice and Bob make the Bell-state measurement on the corresponding particles respectively, next Charlie performs a Von-Neumann measurement on his particles, after that Alice and Bob can perform appropriate unitary transformations on target particles to achieve the bidirectional state teleportation. However, if one agent does not cooperate, the receiver can not fully recover the original state of each qubit. Additionally, the advantage of the scheme is discussed. Finally, we depict concisely the experimental feasibility of our protocol.

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References

1. Nielsen, M.A., Chuang, I.L.: Quantum Computation and Quantum Information. Cambridge University Press, Cambridge (2002)
2. Bennett, C.H., Brassard, G., Crepeau, C., Jozsa, R., Peres, A., Wootters, W.K.: Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. *Phys. Rev. Lett.* **70**, 1895–1899 (1993)
3. Gorbachev, V.N., Trubilko, A.I., Rodichkina, A.A.: Can the states of the W-class be suitable for teleportation? *Phys. Lett. A* **314**, 267–271 (2003)
4. Zhang, Z.J., Zhong, X.M.: Many-agent controlled teleportation of multi-qubit quantum information. *Phys. Lett. A* **341**, 55–59 (2005)
5. Yuan, H.C., Qi, K.G.: Probabilistic and controlled teleportation of tripartite state in a general form and its quantum networks. *Chin. Phys. B* **14**, 1716–1723 (2005)
6. Deng, F.G., Li, C.Y., Li, Y.S.: Symmetric multiparty-controlled teleportation of an arbitrary two-particle entanglement. *Phys. Rev. A* **72**, 022338 (2005)
7. Cao, M., Zhu, S.: Probabilistic teleportation of n-particle state via n pairs of entangled particles. *Commun. Theor. Phys.* **43**, 69–72 (2005)
8. Yeo, Y., Chua, W.K.: Teleportation and dense coding with genuine multipartite entanglement. *Phys. Rev. Lett.* **96**, 060502 (2006)
9. Zhang, Z.J.: Controlled teleportation of an arbitrary n-qubit quantum information using quantum secret sharing of classical message. *Phys. Lett. A* **352**, 55–58 (2006)
10. Li, X.H., Deng, F.G., Zhou, H.Y.: Controlled Teleportation of an arbitrary multi-qudit state in a general form with d-Dimensional Greenberger-Horne-Zeilinger states. *Chin. Phys. Lett.* **24**, 1151–1153 (2007)
11. Zha, X.W., Song, H.Y.: Non-Bell-pair quantum channel for teleporting an arbitrary two-qubit state. *Phys. Lett. A* **369**, 377–379 (2007)
12. Zhou, P., Li, X.H., Deng, F.G., Zhou, H.Y.: Multiparty-controlled teleportation of an arbitrary m-qudit state with a pure entangled quantum channel. *J. Phys. A: Math. Theor.* **40**, 13121 (2007)
13. Jiang, W.X., Fang, J.X., Zhu, S.J., Sha, J.Q.: Probabilistic Controlled Teleportation of a Triplet W State. *Chin. Phys. Lett.* **24**, 1144–1146 (2007)

14. Zhan, X.G., Li, H.M., Ji, H., Zeng, H.S.: Controlled teleportation of multi-qudit quantum information. *Chin. Phys. B* **16**, 2880–2884 (2007)
15. Man, Z.X., Xia, J.J., An, N.B.: Economical and feasible controlled teleportation of an arbitrary unknown n -qubit entangled state. *J. Phys. B: At. Mol. Opt. Phys.* **40**, 1767–1774 (2007)
16. Jiang, W.X., Fang, J.X., Zhu, S.J., Sha, J.Q.: Controlled teleportation of an unknown n -qubit entangled GHZ state. *Commun. Theor. Phys.* **47**, 1045–1048 (2007)
17. Muralidharan, S., Panigrahi, P.K.: Quantum-information splitting using multipartite cluster states. *Phys. Rev. A* **78**, 062333 (2008)
18. Gao, T., Yan, F.L., Li, Y.C.: Optimal controlled teleportation. *Europhys. Lett.* **84**, 50001 (2008)
19. Dong, J., Teng, J.F.: Controlled teleportation of an arbitrary n -qudit state using nonmaximally entangled GHZ states. *Eur. Phys. J. D* **49**, 129–134 (2008)
20. Nie, Y.Y., Hong, Z.H., Huang, Y.B., Yi, X.J., Li, S.S.: Non-maximally entangled controlled teleportation using four particles cluster states. *Int. J. Theor. Phys.* **48**, 1485–1490 (2009)
21. Choudhury, S., Muralidharan, S., Panigrahi, P.K.J.: Quantum teleportation and state sharing using a genuinely entangled six-qubit state. *J. Phys. A: Math. Theor.* **42**, 115303 (2009)
22. Chen, X.B., Xu, G., Yang, Y.X., Wen, Q.Y.: Centrally controlled quantum teleportation. *Opt. Commun.* **283**, 4802–4809 (2010)
23. Hou, K., Liu, G.H., Zhang, X.Y., Sheng, S.Q.: An efficient scheme for five-party quantum state sharing of an arbitrary m -qubit state using multiqubit cluster states. *Quantum Inf. Process* **10**, 463–473 (2011)
24. Bouwmeester, D., Pan, J.W., Mattle, K., Eibl, M., Weinfurter, M., Zeilinger, A.: Experimental quantum teleportation. *Nature* **390**, 575–579 (1997)
25. Ursin, R., Jennewein, T., Aspelmeyer, M., Kaltenbaek, R., Lindenthal, M., Walther, P., Zeilinger, A.: Communications: Quantum teleportation across the danube. *Nature (London)* **430**, 849 (2004)
26. Cleve, R., Gottesman, D., Lo, H.K.: How to share a quantum secret. *Phys. Rev. Lett.* **83**, 648–651 (1999)
27. Lance, A.M., Symul, T., Bowen, W.P., Sanders, B.C., Lam, P.K.: Tripartite Quantum State Sharing. *Phys. Rev. Lett.* **92**, 177903 (2004)
28. Deng, F.G., Li, X.H., Li, C.Y., Zhou, P., Zhou, H.Y.: Multiparty quantum-state sharing of an arbitrary two-particle state with Einstein-Podolsky-Rosen pairs. *Phys. Rev. A* **72**, 044301 (2005)
29. Li, X.H., Zhou, P., Li, C.Y., Zhou, H.Y., Deng, F.G.: Efficient symmetric multiparty quantum state sharing of an arbitrary m -qubit state. *J. Phys. B: At. Mol. Opt. Phys.* **39**, 1975 (2006)
30. Deng, F.G., Li, X.H., Li, C.Y., Zhou, P., Zhou, H.Y.: Multiparty quantum secret splitting and quantum state sharing. *Phys. Lett. A* **354**, 190–195 (2006)
31. Deng, F.G., Li, X.H., Li, C.Y., Zhou, P., Zhou, H.Y.: Quantum state sharing of an arbitrary two-qubit state with two-photon entanglements and Bell-state measurements. *Eur. Phys. J. D* **39**, 459–464 (2006)
32. Man, Z.X., Xia, Y.J., An, N.B.: Quantum state sharing of an arbitrary multiqubit state using nonmaximally entangled GHZ states. *Eur. Phys. J. D* **42**, 333–340 (2007)
33. Wang, Z.Y., Yuan, H., Shi, S.H., Zhang, Z.J.: Three-party qutrit-state sharing. *Eur. Phys. J. D* **41**, 371–375 (2007)
34. Wang, Z.Y., Liu, Y.M., Wang, D., Zhang, Z.J.: Generalized quantum state sharing of arbitrary unknown two-qubit state. *Opt. Commun.* **276**, 322–326 (2007)
35. Dong, L., Xiu, X.M., Gao, Y.J.: Multiparty quantum state sharing of m -qubit state. *Int. J. Mod. Phys. C* **18**, 1699 (2007)
36. Tao, Y.J., Tian, D.P., Hu, M.L., Qin, M.: Quantum state sharing of an arbitrary qudit state by using nonmaximally generalized GHZ state. *Chin. Phys. B* **17**, 624–627 (2008)
37. Yuan, H., Liu, Y.M., Han, L.F., Zhang, Z.J.: Tripartite arbitrary two-qutrit quantum state sharing. *Commun. Theor. Phys.* **49**, 1191–1194 (2008)
38. Xia, Y., Song, J., Song, H.S.: Quantum state sharing using linear optical elements. *Opt. Commun.* **281**, 4946–4950 (2008)
39. Wang, T.J., Zhou, H.Y., Deng, F.G.: Quantum state sharing of an arbitrary m -qudit state with two-qudit entanglements and generalized Bell-state measurements. *Physica A* **387**, 4716–4722 (2008)
40. Sheng, Y.B., Deng, F.G., Zhou, H.Y.: Efficient and economic five-party quantum state sharing of an arbitrary m -qubit state. *Eur. Phys. J. D* **48**, 279–284 (2008)
41. Hou, K., Li, Y.B., Shi, S.H.: Quantum state sharing with a genuinely entangled five-qubit state and Bell-state measurements. *Opt. Commun.* **283**, 1961–1965 (2010)
42. Wang, T.J., Deng, F.G.: Remote three-party quantum state sharing based on three-atom entangled states assisted by cavity qED and flying qubits. *Commun. Theor. Phys.* **55**, 795–803 (2011)
43. Nie, Y.Y., Li, Y.H., Liu, J.C., Sang, M.H.: Quantum state sharing of an arbitrary three-qubit state by using four sets of W -class states. *Opt. Commun.* **284**, 1457–1460 (2011)
44. Shi, R., Huang, L., Yang, W., Zhong, H.: Asymmetric multi-party quantum state sharing of an arbitrary m -qubit state. *Quantum Inf. Process* **10**, 53–61 (2011)

45. Shi, R., Huang, L., Yang, W., Zhong, H.: Multi-party quantum state sharing of an arbitrary two-qubit state with Bell states. *Quantum Inf. Process* **10**, 231–239 (2011)
46. Zha, X.W., Zou, Z.C., Qi, J.X., Song, H.Y.: Bidirectional quantum controlled teleportation via five-qubit cluster state. *Int. J. Theor. Phys.* **52**, 1740–1744 (2013)
47. Li, Y.H., Nie, L.P.: Bidirectional controlled teleportation by using a five-qubit composite GHZ-Bell State. *Int. J. Theor. Phys.* **52**, 1630–1634 (2013)
48. Li, Y.H., Li, X.L., Sang, M.H., Nie, Y.Y., Wang, Z.S.: Bidirectional controlled quantum teleportation and secure direct communication using five-qubit entangled state. *Quantum Inf. Process* **12**, 3835–3844 (2013)
49. Chitra, S., Anindita, B., Anirban, P.: Bidirectional controlled teleportation by using 5-qubit states: a generalized view. *Int. J. Theor. Phys.* **52**, 3790–3796 (2013)
50. Sun, X.M., Zha, X.W.: A scheme of bidirectional quantum controlled teleportation via six-qubit maximally entangled state. *Acta. Photonica. Sinica* **48**, 1052–1056 (2013)
51. An, Y.: Bidirectional controlled teleportation via six-qubit cluster state. *Int. J. Theor. Phys.* **52**, 3870–3873 (2013)
52. Zha, X.W., Song, H.Y., Qi, J.X., Wang, D., Lan, Q.: A maximally entangled seven-qubit state. *J. Phys. A. Math. Theor.* **45**, 255302 (2012)
53. Boschi, D., Branca, S., Martini, F.D., Hardy, L., Popescu, S.: Experimental realization of teleporting an unknown pure quantum state via dual classical and Einstein-Podolsky-Rosen channels. *Phys. Rev. Lett.* **80**, 1121 (1998)
54. Riebe, M., Haffner, H., Roos, C.F., et al.: Deterministic quantum teleportation with atoms. *Nature* **429**, 734–737 (2004)
55. Barrett, M.D., Chiaverini, J., Schaetz, T., et al.: Deterministic quantum teleportation of atomic qubits. *Nature* **429**, 737–739 (2004)