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Faithful Remote Information Concentration Based on the Optimal Universal $1 \rightarrow 2$ Telecloning of Arbitrary Two-Qubit States

Jia-Yin Peng · Hong-Xuan Lei · Zhi-Wen Mo

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Abstract The previous protocols of remote quantum information concentration were focused on the reverse process of quantum telecloning of single-qubit states. We here investigate the reverse process of optimal universal $1 \rightarrow 2$ telecloning of arbitrary two-qubit states. The aim of this telecloning is to distribute respectively the quantum information to two groups of spatially separated receivers from a group of two senders situated at two different locations. Our scheme shows that the distributed quantum information can be remotely concentrated back to a group of two different receivers with 1 of probability by utilizing maximally four-particle cluster state and four-particle GHZ state as quantum channel.

Keywords Optimal universal $1 \rightarrow 2$ cloning · Telecloning of arbitrary two-qubit states · Remote information concentration · Cluster state · GHZ state

1 Introduction

Cloning is a process that shows the essential differences between the classical and quantum information processing [1, 2]. In contrast to the classical case, when one can generate as many copies of a system as one wishes, Wootters and Zurek [1] have shown that no such a machine exists, which can produce two perfect copies of an arbitrary quantum pure state. This statement is called the no-cloning theorem. There was an extension of this theorem for mixed states [3], where Barnum et al. have shown that one cannot broadcast two noncommuting mixed states. Because perfect coping is not possible, it is then natural to ask how one can copy quantum states with the highest fidelity. This problem was firstly addressed by Bužek and Hillery [4], and this scheme was proved to be optimal in [5]. The Bužek-Hillery theory actually exhibited a universal symmetric $1 \rightarrow 2$ quantum cloning machine,

J.-Y. Peng (✉) · H.-X. Lei
School of Mathematics and Information Science, Neijiang Normal University, Neijiang 641100, China
e-mail: pengjiayin62226@yahoo.com.cn

J.-Y. Peng · Z.-W. Mo
College of Mathematics and Software Science, Sichuan Normal University, Chengdu 610066, China

which exported two identical clones closest to the input pure qubit state with a constant fidelity. Correspondingly, Cerf has introduced the asymmetric $1 \to 2$ cloner that produced two states emerging from two different Heisenberg channels [6, 7]. Since then, Quantum cloning has been attracting considerable attention [8–20], due to its potential applications in the quantum-information sciences [21–24]. The essentiality of quantum cloning is to broadcast information to certain distributed objects, so it is regarded as a useful quantum information transmission. It is well-known that quantum teleportation [25] is the most effective technique for remotely broadcasting information. Murao et al. [10] have advanced the $1 \to M$ quantum telecloning which combined the tricks of both quantum teleportation and cloning. Although the fidelities of clones relating to the original state are less than one, the quantum information of the input system is not degraded but only distributed into a larger quantum system.

As one of the most intriguing features of quantum mechanics, quantum entanglement plays a crucial role in most applications of the quantum information processing [25–40], and maximally entangled states are resources preferred in many quantum tasks. Obviously, many quantum tasks involving telecloning cannot be performed without entanglement. As the reverse process of telecloning, remote information concentration (RIC) was introduced by Murao and Vedral [41]. They showed that the quantum information, which was originally distributed into three spatially separated qubits from a single qubit, can be remotely concentrated back to a single qubit via a four-qubit unlockable bound entangled state without performing any global operations. Since the seminal work of Murao and Vedral [41], some quantum entangled states have been employed as quantum channels for implementing the reversal of the optimal universal $1 \to 2$ telecloning, such as GHZ state [42], W state [43], four-particle cluster state [44], generalized Smolin state [45], and so on. Recently, RIC has been generalized to many-particle and high-dimensional systems [45–47]. In addition, Wang et al. [48, 49] proposed a RIC scheme for implementing the reversal of ancilla-free phase-covariant telecloning (AFPCT), and they demonstrated that the quantum information, which was originally distributed into two spatially separated qubits from a single qubit via the optimal AFPCT procedure, can be remotely concentrated back to a single qubit with a certain probability by using an asymmetric W state as the quantum channel. Particularly, they presented a general scheme for RIC in d-level systems, in which the quantum information initially distributed in many spatially separated qudits could be remotely and deterministically concentrated to a single qudit via an entangled channel without performing any global operations. And they showed that their many-to-one RIC protocol could be slightly modified to perform some types of many-to-many RIC tasks.

All the previous RIC protocols were focused on the reverse process of telecloning of single-qubit states. Differently to the optimal universal telecloning of the single-qubit states, Chen et al. [50] proposed a scheme of optimal universal asymmetric $1 \to 2$ telecloning for the entangled inputs $|\phi\rangle = \alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle$, which is an arbitrary two-qubit state. Up to now, there are no reports for implementing the reversal of optimal universal asymmetric $1 \to 2$ telecloning of two-qubit states. Furthermore, there is no uniform approach to achieve RIC task by using diverse quantum entangled states as quantum channels. Lately, Briegel and Raussendorf introduced a new kind of entangled state named as a cluster state, which can be created efficiently in any system with an Ising-type interaction [51]. The cluster states share the properties of both the GHZ states and W class entangled states. However, they still also have some special properties. For example, they have a large persistency of entanglement, that is, they are harder to be destroyed than GHZ class states by local operations. Based on these reasons, we investigate the reverse process of optimal universal asymmetric $1 \to 2$ telecloning for the entangled inputs $|\phi\rangle = \alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle$

[50] in this paper. In detail, we choose a four-particle cluster state and a four-particle GHZ state as quantum channel to achieve information concentration task. The result indicates that our RIC task can be successfully realized with one of probability by utilizing maximally cluster state and GHZ state as quantum channel.

The structure of this letter is as following. In Sect. 2, we address the RIC via a four-particle cluster state and a four-particle GHZ state as quantum channel. And we give a brief conclusion involving a further issue in Sect. 3.

2 RIC Via Four-Particle Entangled States

Before describing our RIC protocol, we briefly summarize the telecloning process: the optimal universal asymmetric $1 \rightarrow 2$ telecloning of arbitrary two-qubit states in 2-level systems [50]. This scheme aims at simultaneously distributing two optimal clones of an arbitrary two-qubit state

$$|\phi\rangle_{AA'} = \alpha_0|00\rangle_{AA'} + \alpha_1|01\rangle_{AA'} + \alpha_2|10\rangle_{AA'} + \alpha_3|11\rangle_{AA'} \tag{1}$$

from two spatially separated observers, A and A', to two groups of observers located at different places B, B', and C, C', respectively. Here α_i $(i = 0, 1, 2, 3)$ are arbitrary four complex numbers and satisfy $|\alpha_0|^2 + |\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 = 1$. The entangled channel is a maximally entangled state [50] shared between the senders and receivers:

$$\begin{aligned} |\psi\rangle = \frac{1}{2}\{&|00\rangle_{\dot{A}\dot{A}'}|\eta_0\rangle_{BB'CC'aa'} + |01\rangle_{\dot{A}\dot{A}'}|\eta_1\rangle_{BB'CC'aa'} \\ &+ |10\rangle_{\dot{A}\dot{A}'}|\eta_2\rangle_{BB'CC'aa'} + |11\rangle_{\dot{A}\dot{A}'}|\eta_3\rangle_{BB'CC'aa'}\}, \end{aligned} \tag{2}$$

where the particles denoted by $\dot{A}$, $\dot{A}'$ belong to the senders A, A', while a, a' are the two observers who hold the ancillas. The ancilla particles are necessary for the Heisenberg QCM, otherwise it cannot reach the optimal fidelity [6, 7].

The states $|\eta_0\rangle_{BB'CC'aa'}$ and $|\eta_1\rangle_{BB'CC'aa'}$ are defined as

$$\begin{aligned} |\eta_0\rangle_{BB'CC'aa'} = {}& \frac{1}{\sqrt{1+3(p^2+q^2)}}(|000000\rangle + p|000101\rangle + p|001010\rangle \\ &+ p|001111\rangle + q|010001\rangle + q|100010\rangle + q|110011\rangle)_{BB'CC'aa'}, \end{aligned} \tag{3}$$

$$\begin{aligned} |\eta_1\rangle_{BB'CC'aa'} = {}& \frac{1}{\sqrt{1+3(p^2+q^2)}}(|010101\rangle + p|010000\rangle + p|011010\rangle \\ &+ p|011111\rangle + q|000100\rangle + q|100110\rangle + q|110111\rangle)_{BB'CC'aa'}, \end{aligned} \tag{4}$$

$$\begin{aligned} |\eta_2\rangle_{BB'CC'aa'} = {}& \frac{1}{\sqrt{1+3(p^2+q^2)}}(|101010\rangle + p|100000\rangle + p|100101\rangle \\ &+ p|101111\rangle + q|001000\rangle + q|011001\rangle + q|111011\rangle)_{BB'CC'aa'}, \end{aligned} \tag{5}$$

and

$$\begin{aligned} |\eta_3\rangle_{BB'CC'aa'} = {}& \frac{1}{\sqrt{1+3(p^2+q^2)}}(|111111\rangle + p|110000\rangle + p|110101\rangle \\ &+ p|111010\rangle + q|001100\rangle + q|011101\rangle + q|101110\rangle)_{BB'CC'aa'}, \end{aligned} \tag{6}$$

where p and q are the real constants satisfying $p + q = 1$ and their concrete meaning is to generate a universal OCM and keep the optimality of it. The telecloning can be accomplished by the following procedure: (1) Each sender performs a Bell-state measurement on his particles. (2) The senders communicate the measurement outcomes to the receivers. (3) The receivers apply local unitary operations depending on the outcomes of the senders' measurement. The collective output state is the six-particle state:

$$|\chi\rangle = \sum_{j=0}^{3} \alpha_j |\eta_j\rangle_{BB'CC'aa'}, \tag{7}$$

which contains the optimal two clones of system BB' and CC' as well as one ancilla of system aa' due to the universal Heisenberg QCM.

Now we describe our RIC protocol for implementing the reversal of the aforementioned telecloning. And our goal is to concentrate on the information $|\chi\rangle$ back to a two-qubit state $|\phi\rangle_{RR'}$ with form of Eq. (1), i.e., $|\chi\rangle \to |\phi\rangle_{RR'}$.

We consider employing the following maximally entangled cluster state $|C\rangle$

$$|C\rangle = \frac{1}{2}(|0000\rangle + |0011\rangle + |1100\rangle + |1111\rangle)_{123R}, \tag{8}$$

and the GHZ-type state

$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle)_{456R'}, \tag{9}$$

as the quantum channel, where particles $1, 2, 3, 4, 5, 6$ belong to the observer B, C, a, B', C', a', respectively, while particles R and R' are at two separated receivers. The state of the whole system is

$$|T\rangle = |\chi\rangle \otimes |C\rangle \otimes |\mathrm{GHZ}\rangle. \tag{10}$$

Since each observer owns his two particles being in the state $|T\rangle$, they can individually perform a joint measurement on their respective two-qubit system, and inform the receivers of the results of the measurements. Likewise, each of them can obtain one of the possible measurement outcomes: $\{\Phi^+, \Phi^-, \Psi^+, \Psi^-\}$, where $\Phi^\pm = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$, $\Psi^\pm = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$.

After straightforward calculation, we classify the RIC process into the following sixteen cases according to the form of the collapse states of particles R and R' to obtain the original state.

Case 1. If the combining result of the Bell state measurement is one of the following forms

$$\begin{array}{ll}
\Phi^\pm_{B1}\Phi^\pm_{B'4}\Phi^\pm_{C2}\Phi^\pm_{C'5}\Phi^\pm_{a3}\Phi^\pm_{a'6}, & \Psi^\pm_{B1}\Phi^\pm_{B'4}\Psi^\pm_{C2}\Phi^\pm_{C'5}\Phi^\pm_{a3}\Phi^\pm_{a'6}, \\
\Phi^+_{B1}\Phi^\pm_{B'4}\Psi^+_{C2}\Phi^\pm_{C'5}\Psi^\pm_{a3}\Phi^\pm_{a'6}, & \Phi^-_{B1}\Phi^\pm_{B'4}\Psi^-_{C2}\Phi^\pm_{C'5}\Psi^\pm_{a3}\Phi^\pm_{a'6}, \\
\Psi^+_{B1}\Phi^\pm_{B'4}\Phi^+_{C2}\Phi^\pm_{C'5}\Psi^\pm_{a3}\Phi^\pm_{a'6}, & \Psi^-_{B1}\Phi^\pm_{B'4}\Phi^-_{C2}\Phi^\pm_{C'5}\Psi^\pm_{a3}\Phi^\pm_{a'6},
\end{array}$$

the states of particles R and R' will become

$$\pm\frac{1}{16\sqrt{2}\sqrt{1+3(p^2+q^2)}}\{\alpha_0|00\rangle \pm \alpha_1|01\rangle \pm \alpha_2|10\rangle \pm \alpha_3|11\rangle\}.$$

It is worth emphasizing that the positive term must be even in polynomial $\alpha_0|00\rangle \pm \alpha_1|01\rangle \pm \alpha_2|10\rangle \pm \alpha_3|11\rangle$. It is the same specification to the following Cases 2–16. The two receivers can successfully recover the original two-particle state by applying one of unitary operations $\{I \otimes I, I \otimes \sigma_z, \sigma_z \otimes I, \sigma_z \otimes \sigma_z\}$. For instance, when measurement outcome is $\Phi^+_{B1}\Phi^+_{B'4}\Phi^+_{C2}\Phi^+_{C'5}\Phi^+_{a3}\Phi^+_{a'6}$, the corresponding collapse state of particles R and R' is

$$\frac{1}{16\sqrt{2}\sqrt{1+3(p^2+q^2)}}\{\alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle\}.$$

Then, the receivers perform a unitary operation $I \otimes I$ on particles R and R' to have obtained the original state with the probability $[\frac{1}{16\sqrt{2}\sqrt{1+3(p^2+q^2)}}]^2 = \frac{1}{512[1+3(p^2+q^2)]}$.

Since the above measurement results add up to 192, the total probability of successful concentration is $\frac{1}{512[1+3(p^2+q^2)]} \times 192 = \frac{3}{8[1+3(p^2+q^2)]}$.

Case 2. If the combined Bell state measurement outcome is one of the following forms

$$\begin{aligned}
&\Phi^\pm_{B1}\Psi^\pm_{B'4}\Phi^\pm_{C2}\Psi^\pm_{C'5}\Phi^\pm_{a3}\Psi^\pm_{a'6}, \quad \Psi^\pm_{B1}\Psi^\pm_{B'4}\Psi^\pm_{C2}\Psi^\pm_{C'5}\Phi^\pm_{a3}\Psi^\pm_{a'6},\\
&\Phi^+_{B1}\Psi^\pm_{B'4}\Psi^+_{C2}\Psi^\pm_{C'5}\Psi^\pm_{a3}\Psi^\pm_{a'6}, \quad \Phi^-_{B1}\Psi^\pm_{B'4}\Psi^-_{C2}\Psi^\pm_{C'5}\Psi^\pm_{a3}\Psi^\pm_{a'6},\\
&\Psi^+_{B1}\Psi^\pm_{B'4}\Phi^+_{C2}\Psi^\pm_{C'5}\Psi^\pm_{a3}\Psi^\pm_{a'6}, \quad \Psi^-_{B1}\Psi^\pm_{B'4}\Phi^-_{C2}\Psi^\pm_{C'5}\Psi^\pm_{a3}\Psi^\pm_{a'6},
\end{aligned}$$

the states of particles R and R' will become

$$\pm\frac{1}{16\sqrt{2}\sqrt{1+3(p^2+q^2)}}\{\alpha_0|01\rangle \pm \alpha_1|00\rangle \pm \alpha_2|11\rangle \pm \alpha_3|10\rangle\}.$$

Performing unitary operator $I \otimes \sigma_x$ or $I \otimes i\sigma_y$ or $\sigma_z \otimes i\sigma_y$ or $\sigma_z \otimes \sigma_z$ on R and R' by the receivers, the original state is successfully reconstructed with the probability $\frac{3}{8[1+3(p^2+q^2)]}$.

Case 3. If the joint Bell state measurement result is one of the following states

$$\begin{aligned}
&\Phi^\pm_{B1}\Phi^\pm_{B'4}\Phi^\pm_{C2}\Phi^\pm_{C'5}\Psi^\pm_{a3}\Phi^\pm_{a'6}, \quad \Psi^\pm_{B1}\Phi^\pm_{B'4}\Psi^\pm_{C2}\Phi^\pm_{C'5}\Psi^\pm_{a3}\Phi^\pm_{a'6},\\
&\Phi^+_{B1}\Phi^\pm_{B'4}\Psi^+_{C2}\Phi^\pm_{C'5}\Phi^\pm_{a3}\Phi^\pm_{a'6}, \quad \Phi^-_{B1}\Phi^\pm_{B'4}\Psi^-_{C2}\Phi^\pm_{C'5}\Phi^\pm_{a3}\Phi^\pm_{a'6},\\
&\Psi^+_{B1}\Phi^\pm_{B'4}\Phi^+_{C2}\Phi^\pm_{C'5}\Phi^\pm_{a3}\Phi^\pm_{a'6}, \quad \Psi^-_{B1}\Phi^\pm_{B'4}\Phi^-_{C2}\Phi^\pm_{C'5}\Phi^\pm_{a3}\Phi^\pm_{a'6},
\end{aligned}$$

the state of particles R and R' is one of the following forms

$$\pm\frac{1}{16\sqrt{2}\sqrt{1+3(p^2+q^2)}}\{\alpha_0|10\rangle \pm \alpha_1|11\rangle \pm \alpha_2|00\rangle \pm \alpha_3|01\rangle\}.$$

Performing unitary operator $\sigma_x \times I$ or $\sigma_x \otimes \sigma_z$ or $i\sigma_y \otimes \sigma_z$ or $i\sigma_y \otimes I$ on R and R', the receivers can restore the original state with the probability $\frac{3}{8[1+3(p^2+q^2)]}$.

Case 4. If the joint Bell state measurement result is one of the following states

$$\begin{aligned}
&\Phi^\pm_{B1}\Psi^\pm_{B'4}\Phi^\pm_{C2}\Psi^\pm_{C'5}\Psi^\pm_{a3}\Psi^\pm_{a'6}, \quad \Psi^\pm_{B1}\Psi^\pm_{B'4}\Psi^\pm_{C2}\Psi^\pm_{C'5}\Psi^\pm_{a3}\Psi^\pm_{a'6},\\
&\Phi^+_{B1}\Psi^\pm_{B'4}\Psi^+_{C2}\Psi^\pm_{C'5}\Phi^\pm_{a3}\Psi^\pm_{a'6}, \quad \Phi^-_{B1}\Psi^\pm_{B'4}\Psi^-_{C2}\Psi^\pm_{C'5}\Phi^\pm_{a3}\Psi^\pm_{a'6},\\
&\Psi^+_{B1}\Psi^\pm_{B'4}\Phi^+_{C2}\Psi^\pm_{C'5}\Phi^\pm_{a3}\Psi^\pm_{a'6}, \quad \Psi^-_{B1}\Psi^\pm_{B'4}\Phi^-_{C2}\Psi^\pm_{C'5}\Phi^\pm_{a3}\Psi^\pm_{a'6},
\end{aligned}$$

the state of the qubits R and R' is one of the following forms

$$\pm\frac{1}{16\sqrt{2}\sqrt{1+3(p^2+q^2)}}\{\alpha_0|11\rangle\pm\alpha_1|10\rangle\pm\alpha_2|01\rangle\pm\alpha_3|00\rangle\}.$$

In order to recover the original two-particle state, the receivers should perform one of unitary operations $\{\sigma_x\otimes\sigma_x, \sigma_x\otimes i\sigma_y, i\sigma_y\otimes i\sigma_y, i\sigma_y\otimes\sigma_x\}$. For these measurement results, the probability of information concentration is $\frac{3}{8[1+3(p^2+q^2)]}$.

In the discussion of Cases 5–8 in the following, we assume that $p\neq\frac{1}{2}$, because the inner products of these measurements and the state $|T\rangle$ are zero if $p=\frac{1}{2}$. This results that Cases 5–8 do not happen for information concentration.

Case 5. If the combining result of the Bell state measurements is arbitrary member of the set

$$\begin{array}{ll}\Phi^+_{B1}\Phi^\pm_{B'4}\Psi^-_{C2}\Phi^\pm_{C'5}\Psi^\pm_{a3}\Phi^\pm_{a'6}, & \Phi^-_{B1}\Phi^\pm_{B'4}\Psi^+_{C2}\Phi^\pm_{C'5}\Psi^\pm_{a3}\Phi^\pm_{a'6},\\ \Psi^+_{B1}\Phi^\pm_{B'4}\Phi^-_{C2}\Phi^\pm_{C'5}\Psi^\pm_{a3}\Phi^\pm_{a'6}, & \Psi^-_{B1}\Phi^\pm_{B'4}\Phi^+_{C2}\Phi^\pm_{C'5}\Psi^\pm_{a3}\Phi^\pm_{a'6},\end{array}$$

the state of particles R and R' will become

$$\pm\frac{p-q}{16\sqrt{2}\sqrt{1+3(p^2+q^2)}}\{\alpha_0|00\rangle\pm\alpha_1|01\rangle\pm\alpha_2|10\rangle\pm\alpha_3|11\rangle\}.$$

Preforming the same unitary operation as Case 1, the receivers can retrieve the original state with the probability $\frac{(p-q)^2}{8[1+3(p^2+q^2)]}$.

Case 6. If the joint Bell state measurement result is one of the following states

$$\begin{array}{ll}\Phi^+_{B1}\Psi^\pm_{B'4}\Psi^-_{C2}\Psi^\pm_{C'5}\Psi^\pm_{a3}\Psi^\pm_{a'6}, & \Phi^-_{B1}\Psi^\pm_{B'4}\Psi^+_{C2}\Psi^\pm_{C'5}\Psi^\pm_{a3}\Psi^\pm_{a'6},\\ \Psi^+_{B1}\Psi^\pm_{B'4}\Phi^-_{C2}\Psi^\pm_{C'5}\Psi^\pm_{a3}\Psi^\pm_{a'6}, & \Psi^-_{B1}\Psi^\pm_{B'4}\Phi^+_{C2}\Psi^\pm_{C'5}\Psi^\pm_{a3}\Psi^\pm_{a'6},\end{array}$$

the state of the qubit R and R' is one of the following forms

$$\pm\frac{p-q}{16\sqrt{2}\sqrt{1+3(p^2+q^2)}}\{\alpha_0|01\rangle\pm\alpha_1|00\rangle\pm\alpha_2|11\rangle\pm\alpha_3|10\rangle\}.$$

Similar to Case 2, the original state can be reconstructed with the probability $\frac{(p-q)^2}{8[1+3(p^2+q^2)]}$.

Case 7. If the united Bell state measurement outcome is one of the following states

$$\begin{array}{ll}\Phi^+_{B1}\Phi^\pm_{B'4}\Psi^-_{C2}\Phi^\pm_{C'5}\Phi^\pm_{a3}\Phi^\pm_{a'6}, & \Phi^-_{B1}\Phi^\pm_{B'4}\Psi^+_{C2}\Phi^\pm_{C'5}\Phi^\pm_{a3}\Phi^\pm_{a'6},\\ \Psi^+_{B1}\Phi^\pm_{B'4}\Phi^-_{C2}\Phi^\pm_{C'5}\Phi^\pm_{a3}\Phi^\pm_{a'6}, & \Psi^-_{B1}\Phi^\pm_{B'4}\Phi^+_{C2}\Phi^\pm_{C'5}\Phi^\pm_{a3}\Phi^\pm_{a'6},\end{array}$$

the state of particles R and R' has one of the following forms

$$\pm\frac{p-q}{16\sqrt{2}\sqrt{1+3(p^2+q^2)}}\{\alpha_0|10\rangle\pm\alpha_1|11\rangle\pm\alpha_2|00\rangle\pm\alpha_3|01\rangle\}.$$

Applying an appropriate unitary operation in Case 3, the original state can be recovered with the probability $\frac{(p-q)^2}{8[1+3(p^2+q^2)]}$.

Case 8. If the associated Bell state measurement is one of the following states

$$\Phi^{+}_{B1}\Psi^{\pm}_{B'4}\Psi^{-}_{C2}\Psi^{\pm}_{C'5}\Phi^{\pm}_{a3}\Psi^{\pm}_{a'6},\quad \Phi^{-}_{B1}\Psi^{\pm}_{B'4}\Psi^{+}_{C2}\Psi^{\pm}_{C'5}\Phi^{\pm}_{a3}\Psi^{\pm}_{a'6},$$
$$\Psi^{+}_{B1}\Psi^{\pm}_{B'4}\Phi^{-}_{C2}\Psi^{\pm}_{C'5}\Phi^{\pm}_{a3}\Psi^{\pm}_{a'6},\quad \Psi^{-}_{B1}\Psi^{\pm}_{B'4}\Phi^{+}_{C2}\Psi^{\pm}_{C'5}\Phi^{\pm}_{a3}\Psi^{\pm}_{a'6},$$

the state of particles R and R' will be one of the following forms

$$\pm\frac{p-q}{16\sqrt{2}\sqrt{1+3(p^2+q^2)}}\{\alpha_0|11\rangle\pm\alpha_1|10\rangle\pm\alpha_2|01\rangle\pm\alpha_3|00\rangle\}.$$

In order to reconstruct the original two-particle state, the receivers can do the similar operation on the particles R and R' as Case 4 shows with the probability $\frac{(p-q)^2}{8[1+3(p^2+q^2)]}$.

Case 9. If the combined Bell state measurement outcome is one of the following states

$$\Phi^{\pm}_{B1}\Phi^{\pm}_{B'4}\Phi^{\pm}_{C2}\Psi^{\pm}_{C'5}\Phi^{\pm}_{a3}\Psi^{\pm}_{a'6},\quad \Psi^{\pm}_{B1}\Phi^{\pm}_{B'4}\Psi^{\pm}_{C2}\Psi^{\pm}_{C'5}\Phi^{\pm}_{a3}\Psi^{\pm}_{a'6},$$
$$\Phi^{\pm}_{B1}\Phi^{\pm}_{B'4}\Psi^{\pm}_{C2}\Psi^{\pm}_{C'5}\Psi^{\pm}_{a3}\Psi^{\pm}_{a'6},\quad \Psi^{\pm}_{B1}\Phi^{\pm}_{B'4}\Phi^{\pm}_{C2}\Psi^{\pm}_{C'5}\Psi^{\pm}_{a3}\Psi^{\pm}_{a'6},$$

the state of particles R and R' will be one of the following states

$$\pm\frac{p}{16\sqrt{2}\sqrt{1+3(p^2+q^2)}}\{\alpha_0|00\rangle\pm\alpha_1|01\rangle\pm\alpha_2|10\rangle\pm\alpha_3|11\rangle\}.$$

These states of particles R and R' can be restored to the original state with the probability $\frac{p^2}{2[1+3(p^2+q^2)]}$ by the similar method as in Case 1.

Case 10. The projected states

$$\pm\frac{p}{16\sqrt{2}\sqrt{1+3(p^2+q^2)}}\{\alpha_0|01\rangle\pm\alpha_1|00\rangle\pm\alpha_2|11\rangle\pm\alpha_3|10\rangle\}$$

of particles R and R' correspond to the combined measurement results

$$\Phi^{\pm}_{B1}\Psi^{\pm}_{B'4}\Phi^{\pm}_{C2}\Phi^{\pm}_{C'5}\Phi^{\pm}_{a3}\Phi^{\pm}_{a'6},\quad \Psi^{\pm}_{B1}\Psi^{\pm}_{B'4}\Psi^{\pm}_{C2}\Phi^{\pm}_{C'5}\Phi^{\pm}_{a3}\Phi^{\pm}_{a'6},$$
$$\Phi^{\pm}_{B1}\Psi^{\pm}_{B'4}\Psi^{\pm}_{C2}\Phi^{\pm}_{C'5}\Psi^{\pm}_{a3}\Phi^{\pm}_{a'6},\quad \Psi^{\pm}_{B1}\Psi^{\pm}_{B'4}\Phi^{\pm}_{C2}\Phi^{\pm}_{C'5}\Psi^{\pm}_{a3}\Phi^{\pm}_{a'6}.$$

Adopting the similar method as does in Case 2, the receivers can restore the original state with the probability $\frac{p^2}{2[1+3(p^2+q^2)]}$.

Case 11. If the combined Bell state measurement outcome is one of the following states

$$\Phi^{\pm}_{B1}\Phi^{\pm}_{B'4}\Phi^{\pm}_{C2}\Psi^{\pm}_{C'5}\Psi^{\pm}_{a3}\Psi^{\pm}_{a'6},\quad \Psi^{\pm}_{B1}\Phi^{\pm}_{B'4}\Psi^{\pm}_{C2}\Psi^{\pm}_{C'5}\Psi^{\pm}_{a3}\Psi^{\pm}_{a'6},$$
$$\Phi^{\pm}_{B1}\Phi^{\pm}_{B'4}\Psi^{\pm}_{C2}\Psi^{\pm}_{C'5}\Phi^{\pm}_{a3}\Psi^{\pm}_{a'6},\quad \Psi^{\pm}_{B1}\Phi^{\pm}_{B'4}\Phi^{\pm}_{C2}\Psi^{\pm}_{C'5}\Phi^{\pm}_{a3}\Psi^{\pm}_{a'6},$$

the state of particles R and R' has one of the following forms

$$\pm\frac{p}{16\sqrt{2}\sqrt{1+3(p^2+q^2)}}\{\alpha_0|10\rangle\pm\alpha_1|11\rangle\pm\alpha_2|00\rangle\pm\alpha_3|01\rangle\}.$$

In order to reconstruct the original two-particle state, the receivers do the similar operation as they do in Case 3, the original state can be concentrated back with the probability $\frac{p^2}{2[1+3(p^2+q^2)]}$.

Case 12. If the combined Bell state measurement outcome is one of the following states

$$\Phi^{\pm}_{B1}\Psi^{\pm}_{B'4}\Phi^{\pm}_{C2}\Phi^{\pm}_{C'5}\Psi^{\pm}_{a3}\Phi^{\pm}_{a'6},\quad \Psi^{\pm}_{B1}\Psi^{\pm}_{B'4}\Psi^{\pm}_{C2}\Phi^{\pm}_{C'5}\Psi^{\pm}_{a3}\Phi^{\pm}_{a'6},$$
$$\Phi^{\pm}_{B1}\Psi^{\pm}_{B'4}\Psi^{\pm}_{C2}\Phi^{\pm}_{C'5}\Phi^{\pm}_{a3}\Phi^{\pm}_{a'6},\quad \Psi^{\pm}_{B1}\Psi^{\pm}_{B'4}\Phi^{\pm}_{C2}\Phi^{\pm}_{C'5}\Phi^{\pm}_{a3}\Phi^{\pm}_{a'6},$$

the state of particles R and R' is one of the following states

$$\pm\frac{p}{16\sqrt{2}\sqrt{1+3(p^2+q^2)}}\{\alpha_0|11\rangle \pm \alpha_1|10\rangle \pm \alpha_2|01\rangle \pm \alpha_3|00\rangle\}.$$

Adopting the similar method as Case 4, the original state can be concentrated back with the probability $\frac{p^2}{2[1+3(p^2+q^2)]}$.

Case 13. If the combined Bell state measurement outcome is one of the following states

$$\Phi^{\pm}_{B1}\Psi^{\pm}_{B'4}\Phi^{\pm}_{C2}\Phi^{\pm}_{C'5}\Phi^{\pm}_{a3}\Psi^{\pm}_{a'6},\quad \Psi^{\pm}_{B1}\Psi^{\pm}_{B'4}\Psi^{\pm}_{C2}\Phi^{\pm}_{C'5}\Phi^{\pm}_{a3}\Psi^{\pm}_{a'6},$$
$$\Phi^{\pm}_{B1}\Psi^{\pm}_{B'4}\Psi^{\pm}_{C2}\Phi^{\pm}_{C'5}\Psi^{\pm}_{a3}\Psi^{\pm}_{a'6},\quad \Psi^{\pm}_{B1}\Psi^{\pm}_{B'4}\Phi^{\pm}_{C2}\Phi^{\pm}_{C'5}\Psi^{\pm}_{a3}\Psi^{\pm}_{a'6},$$

the state of particles R and R' is one of the following forms

$$\pm\frac{q}{16\sqrt{2}\sqrt{1+3(p^2+q^2)}}\{\alpha_0|00\rangle \pm \alpha_1|01\rangle \pm \alpha_2|10\rangle \pm \alpha_3|11\rangle\}.$$

Applying the similar method as Case 1 does, the original state can be reconstructed with the probability $\frac{q^2}{2[1+3(p^2+q^2)]}$.

Case 14. If the joint Bell state measurement outcome has one of the following

$$\Phi^{\pm}_{B1}\Phi^{\pm}_{B'4}\Phi^{\pm}_{C2}\Psi^{\pm}_{C'5}\Phi^{\pm}_{a3}\Phi^{\pm}_{a'6},\quad \Psi^{\pm}_{B1}\Phi^{\pm}_{B'4}\Psi^{\pm}_{C2}\Psi^{\pm}_{C'5}\Phi^{\pm}_{a3}\Phi^{\pm}_{a'6},$$
$$\Phi^{\pm}_{B1}\Phi^{\pm}_{B'4}\Psi^{\pm}_{C2}\Psi^{\pm}_{C'5}\Psi^{\pm}_{a3}\Phi^{\pm}_{a'6},\quad \Psi^{\pm}_{B1}\Phi^{\pm}_{B'4}\Phi^{\pm}_{C2}\Psi^{\pm}_{C'5}\Psi^{\pm}_{a3}\Phi^{\pm}_{a'6},$$

the state of particles R and R' will become one of the following

$$\pm\frac{q}{16\sqrt{2}\sqrt{1+3(p^2+q^2)}}\{\alpha_0|01\rangle \pm \alpha_1|00\rangle \pm \alpha_2|11\rangle \pm \alpha_3|10\rangle\}.$$

Doing the similar operation as Case 2, the receivers can recover the original state with the probability $\frac{q^2}{2[1+3(p^2+q^2)]}$.

Case 15. If the united Bell state measurement outcome is one of the following states

$$\Phi^{\pm}_{B1}\Psi^{\pm}_{B'4}\Phi^{\pm}_{C2}\Phi^{\pm}_{C'5}\Psi^{\pm}_{a3}\Psi^{\pm}_{a'6},\quad \Psi^{\pm}_{B1}\Psi^{\pm}_{B'4}\Psi^{\pm}_{C2}\Phi^{\pm}_{C'5}\Psi^{\pm}_{a3}\Psi^{\pm}_{a'6},$$
$$\Phi^{\pm}_{B1}\Psi^{\pm}_{B'4}\Psi^{\pm}_{C2}\Phi^{\pm}_{C'5}\Phi^{\pm}_{a3}\Psi^{\pm}_{a'6},\quad \Psi^{\pm}_{B1}\Psi^{\pm}_{B'4}\Phi^{\pm}_{C2}\Phi^{\pm}_{C'5}\Phi^{\pm}_{a3}\Psi^{\pm}_{a'6},$$

the state of particles R and R' will project to one of the following states

$$\pm\frac{q}{16\sqrt{2}\sqrt{1+3(p^2+q^2)}}\{\alpha_0|10\rangle \pm \alpha_1|11\rangle \pm \alpha_2|00\rangle \pm \alpha_3|01\rangle\}.$$

To achieve the information concentration task, the receivers should do the similar unitary operation as Case 3, the probability of recovering the information is $\frac{q^2}{2[1+3(p^2+q^2)]}$.

Case 16. If the associated Bell state measurement outcome has one of the following forms

$$\Phi^{\pm}_{B1}\Phi^{\pm}_{B'4}\Phi^{\pm}_{C2}\Psi^{\pm}_{C'5}\Psi^{\pm}_{a3}\Phi^{\pm}_{a'6}, \quad \Psi^{\pm}_{B1}\Phi^{\pm}_{B'4}\Psi^{\pm}_{C2}\Psi^{\pm}_{C'5}\Psi^{\pm}_{a3}\Phi^{\pm}_{a'6},$$
$$\Phi^{\pm}_{B1}\Phi^{\pm}_{B'4}\Psi^{\pm}_{C2}\Psi^{\pm}_{C'5}\Phi^{\pm}_{a3}\Phi^{\pm}_{a'6}, \quad \Psi^{\pm}_{B1}\Phi^{\pm}_{B'4}\Phi^{\pm}_{C2}\Psi^{\pm}_{C'5}\Phi^{\pm}_{a3}\Phi^{\pm}_{a'6},$$

the state of particles R and R' is one of the following states

$$\pm\frac{q}{16\sqrt{2}\sqrt{1+3(p^2+q^2)}}\{\alpha_0|11\rangle \pm \alpha_1|10\rangle \pm \alpha_2|01\rangle \pm \alpha_3|00\rangle\}.$$

Performing the similar unitary operation as Case 4 does, the quantum information of the original two-particle state can be concentrated back on the particles R and R' with the probability $\frac{q^2}{2[1+3(p^2+q^2)]}$.

To sum up, noting that $p+q=1$, if $p \neq \frac{1}{2}$, the total probability of successful concentration is $\frac{4(p^2+q^2)+2pq}{1+3(p^2+q^2)} = 1$. If $p = \frac{1}{2}$, Cases 5–8 do not occur, and the total probability of successful concentration is still one for Cases 1–4 and 9–16 by careful calculation. In a words, no matter how parameters p and q range in the interval $(0, 1)$, the successful probability of our scheme is always one. This is shown that our protocol is perfect and faithful.

3 Conclusion

Telecloning and its reverse process, namely, remote quantum information concentration (RIC), have been attracting considerable interest because of their potential applications in quantum information precessing. Since Murao and Vedral [41] introduced the concept of RIC, the vast majority of the RIC protocols were focused on the reverse process of quantum telecloning of single-qubit states. So far, there are no results for studying the reversal of the optimal universal asymmetric telecloning of arbitrary two-qubit states. In this letter, we have investigated the RIC based on Ref. [50], i.e., the reverse process of optimal universal asymmetric $1 \rightarrow 2$ telecloning of unknown two-qubit states $|\phi\rangle = \alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle$. Because cluster states have some properties of both the GHZ states and W class entangled states, and they also have some peculiar properties, they are important resources in quantum information processing. In this paper, we have proposed a RIC protocol using four-particle cluster and GHZ states as quantum channel. Our scheme can be remotely concentrated back to a two-qubit state with 1 of probability. Although our quantum channel is chosen as special states, we have strikingly found that common states such as W state and GHZ state used solely as quantum channel can not complete quantum information concentration task about telecloning of arbitrary two-particle states.

Our further work is to discuss the reverse process of optimal universal $1 \rightarrow 2$ telecloning of arbitrary two-particle state using non-maximally entangled four-particle cluster and GHZ states or others as quantum channel.

As to the physical realization of the above RIC protocol, we mainly need to consider two points: (a) the preparations of the four-qubit cluster state, and (b) the Bell-state measurement. Recently, Refs. [52, 53] proposed schemes for generating a four-atom entangled and n-atom cluster states via cavity quantum electrodynamics, respectively. Lu et al. [54] proposed a scheme for generating cluster states in linear optics system, and Wang et al. [55] presented two feasible schemes for preparing cluster states with ion-trap setup. The Bell-state measurement has been well realized for both atomic and photic qubits [56, 57]. All these achievements may contribute to our RIC scheme in physical realization.

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