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Bidirectional Controlled Quantum Teleportation by Using Five-Qubit Entangled State

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Abstract We propose a scheme for bidirectional controlled quantum teleportation by using a genuine five-qubit entangled state. In our scheme, Alice may transmit an arbitrary single qubit state of qubit A to Bob and at the same time, Bob may transmit an arbitrary single qubit state of qubit B to Alice via the control of the supervisor Charlie.

Keywords Quantum information · Bidirectional controlled quantum teleportation · Five-qubit entangled state

1 Introduction

Quantum teleportation (QT) is currently a vital area of research and discussion. Since the first scheme of QT was proposed [1], QT have been investigated extensively [2–10]. And QT was improved and modified as quantum information splitting [11], controlled teleportation [12], quantum operation sharing [13] and quantum secret sharing [14]. Recently, Zha et al. [15] and Li et al. [16] have reported tripartite schemes for bidirectional controlled QT by using five-qubit entangled states as the quantum channel.

In 2005, a genuine five-qubit entangled state through an extensive numerical optimization procedure was introduced by Brown et al. [17], which can not decompose into pairs of Bell states and therefore it exhibits genuine multipartite entanglement according to negative partial transpose measurement. Very importantly, such a state is more than the entanglement exhibited by the prototype-GHZ states, generalized W states and cluster states, which has been shown useful in perfect QT, quantum state sharing and superdense coding [18–20].

In this work, we present a protocol for implementing bidirectional controlled QT by using a genuine five-qubit entangled state. Using teleportation of an arbitrary single-qubit state, Alice and Bob can exchange their quantum state simultaneously under the control of the supervisor Charlie. This means that Alice has qubit A in an unknown state, she wants to transmit an arbitrary single qubit state of qubit A to Bob; at the same time, Bob has a qubit

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B in an unknown state, he wants to transmit the state of qubit B to Alice. The bidirectional controlled QT task is completed following the typical procedure that, the receiver applies an appropriate unitary transformation to his qubit, after receiving the measurement results of both the sender Alice (Bob) and the controller Charlie on their separate qubits. In our scheme, only two Bell-state measurements and a single-qubit measurement are necessary, which is more simple and feasible in experiment.

2 Bidirectional Controlled QT

Our scheme can be described as follows. Suppose Alice has an arbitrary single qubit state $|\psi\rangle_A = a_0|0\rangle + a_1|1\rangle$ and that Bob has qubit B in an unknown state $|\psi\rangle_B = b_0|0\rangle + b_1|1\rangle$. Now Alice wants to transmit the state $|\psi\rangle_A$ of qubit A to Bob and Bob wants to transmit the state $|\psi\rangle_B$ of qubit B to Alice. Assume that Alice, Bob and Charlie share a genuine five-qubit entangled state, which has the form

$$|\psi\rangle_{12345} = \frac{\sqrt{2}}{4}\big[|001\rangle_{123}\big(|01\rangle - |10\rangle\big)_{45} + |010\rangle_{123}\big(|00\rangle - |11\rangle\big)_{45} + |100\rangle_{123}\big(|01\rangle + |10\rangle\big)_{45} + |111\rangle_{123}\big(|00\rangle + |11\rangle\big)_{45}\big], \tag{1}$$

where the qubits 1 and 4 belong to Alice, qubit 5 belongs to Charlie and qubits 2 and 3 belong to Bob, respectively. The initial state of the total system can be expressed as

$$|\Psi\rangle_{12345AB} = |\psi\rangle_{12345} \otimes |\psi\rangle_A \otimes |\psi\rangle_B. \tag{2}$$

To achieve the purpose of bidirectional controlled QT, then Alice and Bob performs a Bell-state measurement on own qubit pairs (A, 1) and (B, 2), respectively. It is known that one may obtain one of the 16 kinds of possible measured results with equal probability, and the remaining qubits may collapse into one of the 16 states after the measurement.

$${}_{B2}\langle\Phi^{\pm}|_{A1}\langle\Phi^{\pm}||\Psi\rangle_{12345AB} = \frac{1}{4}\big[a_0b_0|1\rangle\big(|01\rangle - |10\rangle\big) \pm +a_1b_0|0\rangle\big(|01\rangle + |10\rangle\big) + \pm a_0b_1|0\rangle\big(|00\rangle - |11\rangle\big) \pm \pm a_1b_1|1\rangle\big(|00\rangle + |11\rangle\big)\big]_{345}, \tag{3}$$

$${}_{B2}\langle\Psi^{\pm}|_{A1}\langle\Phi^{\pm}||\Psi\rangle_{12345AB} = \frac{1}{4}\big[a_0b_0|0\rangle\big(|00\rangle - |11\rangle\big) \pm +a_1b_0|1\rangle\big(|00\rangle + |11\rangle\big) + \pm a_0b_1|1\rangle\big(|01\rangle - |10\rangle\big) \pm \pm a_1b_1|0\rangle\big(|01\rangle + |10\rangle\big)\big]_{345}, \tag{4}$$

$${}_{B2}\langle\Phi^{\pm}|_{A1}\langle\Psi^{\pm}||\Psi\rangle_{12345AB} = \frac{1}{4}\big[a_0b_0|0\rangle\big(|01\rangle + |10\rangle\big) \pm +a_1b_0|1\rangle\big(|01\rangle - |10\rangle\big) + \pm a_0b_1|1\rangle\big(|00\rangle + |11\rangle\big) \pm \pm a_1b_1|0\rangle\big(|00\rangle - |11\rangle\big)\big]_{345}, \tag{5}$$

$${}_{B2}\langle\Psi^{\pm}|_{A1}\langle\Phi^{\pm}||\Psi\rangle_{12345AB} = \frac{1}{4}\big[a_0b_0|1\rangle\big(|00\rangle + |11\rangle\big) \pm +a_1b_0|0\rangle\big(|00\rangle - |11\rangle\big) + \pm a_0b_1|0\rangle\big(|01\rangle + |10\rangle\big) \pm \pm a_1b_1|1\rangle\big(|01\rangle - |10\rangle\big)\big]_{345}, \tag{6}$$

where $|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$ and $|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$ are Bell states. In Eqs. (3)–(6), the notes "±" or "+" from right to left correspond to the Bell-state measurements of qubits 'A1' and 'B2', respectively, and they mean multiplication of ± signs.

Then Alice (Bob) tells the result to Bob (Alice) and Charlie. Whether it is possible for Bob and Alice to exchange their secret quantum information is dependent on the controller Charlie. If Charlie allows Bob and Alice to reconstruct the initial unknown state, he needs to carry out the single qubit measurement in the basis of $\{|0\rangle, |1\rangle\}$ on qubit 5 and tells the receivers his result. By combining information from Alice, Bob and Charlie, Alice and Bob can co-operate and make a quantum controlled phase gate (QCPG) operation and a controlled-NOT operation to obtain the secret quantum information with an appropriate unitary transformation on the qubit at hand, the bidirectional controlled QT is easily realized. The appropriate unitary transformation for Bob and Alice for different scenarios are presented in the Appendix.

Now, let us take an example to demonstrate the principle of this bidirectional controlled QT protocol. Suppose Alice's measurement outcome is $|\Phi^+\rangle_{A1}$, at the same time, Bob's measurement outcome is $|\Phi^+\rangle_{B2}$, then the state of the remaining qubits collapse into the state

$$|\varphi\rangle_{345} = \frac{1}{2}\big[|0\rangle_5\big(-a_0b_0|11\rangle + a_1b_0|01\rangle + a_0b_1|00\rangle + a_1b_1|10\rangle\big)_{34} + |1\rangle_5\big(a_0b_0|10\rangle + a_1b_0|00\rangle - a_0b_1|01\rangle + a_1b_1|11\rangle\big)_{34}\big]. \tag{7}$$

Then Alice and Bob can co-operate and make a QCPG operation on qubits 3 (set as a target qubit) and 4 (set as a control qubit), and the state $|\varphi\rangle_{345}$ will become the following state,

$$|\eta\rangle_{345} = \frac{1}{2}\big[|0\rangle_5\big(a_0b_0|11\rangle + a_1b_0|01\rangle + a_0b_1|00\rangle + a_1b_1|10\rangle\big)_{34} + |1\rangle_5\big(a_0b_0|10\rangle + a_1b_0|00\rangle - a_0b_1|01\rangle - a_1b_1|11\rangle\big)_{34}\big]. \tag{8}$$

Then Alice and Bob can co-operate and make a controlled-NOT operation on qubits 3 (set as a target qubit) and 4 (set as a control qubit), then the Eq. (8) becomes

$$|\xi\rangle_{345} = \frac{1}{2}\big[|0\rangle_5\big(a_0|0\rangle + a_1|1\rangle\big)_3\big(b_0|1\rangle + b_1|0\rangle\big)_4 + |1\rangle_5\big(a_0|1\rangle + a_1|0\rangle\big)_3\big(b_0|0\rangle - b_1|1\rangle\big)_4\big]. \tag{9}$$

Charlie can now make the single qubit measurement in the basis of $\{|0\rangle, |1\rangle\}$ on qubit 5, and then he sends the result of his measurement to Bob and Alice. If the result of the single qubit measurement is $|0\rangle_5$ or $|1\rangle_5$, Bob and Alice need to apply the local unitary operation $I_3 \otimes \sigma_4^x$ or $\sigma_3^x \otimes \sigma_4^z$. After doing those operations, Bob and Alice can successfully exchange their quantum state. Thus the bidirectional controlled QT is successfully realized.

3 Conclusion

In this paper, we have demonstrated that such a genuine five-qubit entangled state can be used as the quantum channel to realize the deterministic bidirectional controlled QT. In our scheme, Alice may transmit an arbitrary single qubit state of qubit A to Bob and at same time Bob may transmit an arbitrary single qubit state of qubit B to Alice via the control of the supervisor Charlie. In the scheme only two Bell-state measurements, a single qubit measurement, a QCPG operation, a controlled-NOT operation, and the appropriate single qubit unitary operation are necessary. We hope that such a bidirectional controlled QT scheme can be realized experimentally with photons in the future.

Appendix

Alice's possible measurement result, Bob's possible measurement result, Charlie's possible measurement result, final state with the receiver after the QCPG operation and CNOT operation, and the corresponding transformation performed by Bob and Alice on qubits 3 and 4, respectively, where σ^i, $i \in \{x, y, z\}$are Pauli matrices (Table 1).

Table 1 The outcome of measurements performed by Alice, Bob and Charlie, and the final state with the receiver after the QCPG operation and CNOT operation, and the corresponding unitary transformation performed by Bob and Alice

Alice's result	Bob's result	Charlie's result	Final state with the receiver after the QCPG operation and CNOT operation	Unitary transformation
$\|\Phi^+\rangle_{A1}$	$\|\Phi^+\rangle_{B2}$	$\|0\rangle_5$	$(a_0\|0\rangle + a_1\|1\rangle)_3 \otimes (b_0\|1\rangle + b_1\|0\rangle)_4$	$I_3 \otimes \sigma_4^x$
$\|\Phi^+\rangle_{A1}$	$\|\Phi^-\rangle_{B2}$	$\|0\rangle_5$	$(a_0\|0\rangle - a_1\|1\rangle)_3 \otimes (b_0\|1\rangle + b_1\|0\rangle)_4$	$\sigma_3^z \otimes \sigma_4^x$
$\|\Phi^-\rangle_{A1}$	$\|\Phi^+\rangle_{B2}$	$\|0\rangle_5$	$(a_0\|0\rangle + a_1\|1\rangle)_3 \otimes (b_0\|1\rangle - b_1\|0\rangle)_4$	$I_3 \otimes -i\sigma_4^y$
$\|\Phi^-\rangle_{A1}$	$\|\Phi^-\rangle_{B2}$	$\|0\rangle_5$	$(a_0\|0\rangle - a_1\|1\rangle)_3 \otimes (b_0\|1\rangle - b_1\|0\rangle)_4$	$\sigma_3^z \otimes -i\sigma_4^y$
$\|\Phi^+\rangle_{A1}$	$\|\Psi^+\rangle_{B2}$	$\|0\rangle_5$	$(a_0\|0\rangle + a_1\|1\rangle)_3 \otimes (b_0\|0\rangle + b_1\|1\rangle)_4$	$I_3 \otimes I_4$
$\|\Phi^+\rangle_{A1}$	$\|\Psi^-\rangle_{B2}$	$\|0\rangle_5$	$(a_0\|0\rangle - a_1\|1\rangle)_3 \otimes (b_0\|0\rangle + b_1\|1\rangle)_4$	$\sigma_3^z \otimes I_4$
$\|\Phi^-\rangle_{A1}$	$\|\Psi^+\rangle_{B2}$	$\|0\rangle_5$	$(a_0\|0\rangle + a_1\|1\rangle)_3 \otimes (b_0\|0\rangle - b_1\|1\rangle)_4$	$I_3 \otimes \sigma_4^z$
$\|\Phi^-\rangle_{A1}$	$\|\Psi^-\rangle_{B2}$	$\|0\rangle_5$	$(a_0\|0\rangle - a_1\|1\rangle)_3 \otimes (b_0\|0\rangle - b_1\|1\rangle)_4$	$\sigma_3^z \otimes \sigma_4^z$
$\|\Psi^+\rangle_{A1}$	$\|\Phi^+\rangle_{B2}$	$\|0\rangle_5$	$(a_0\|1\rangle + a_1\|0\rangle)_3 \otimes (b_0\|1\rangle + b_1\|0\rangle)_4$	$\sigma_3^x \otimes \sigma_4^x$
$\|\Psi^+\rangle_{A1}$	$\|\Phi^-\rangle_{B2}$	$\|0\rangle_5$	$(a_0\|1\rangle - a_1\|0\rangle)_3 \otimes (b_0\|1\rangle + b_1\|0\rangle)_4$	$-i\sigma_3^y \otimes \sigma_4^x$
$\|\Psi^-\rangle_{A1}$	$\|\Phi^+\rangle_{B2}$	$\|0\rangle_5$	$(a_0\|1\rangle + a_1\|0\rangle)_3 \otimes (b_0\|1\rangle - b_1\|0\rangle)_4$	$\sigma_3^x \otimes -i\sigma_4^y$
$\|\Psi^-\rangle_{A1}$	$\|\Phi^-\rangle_{B2}$	$\|0\rangle_5$	$(a_0\|1\rangle - a_1\|0\rangle)_3 \otimes (b_0\|1\rangle - b_1\|0\rangle)_4$	$-i\sigma_3^y \otimes -i\sigma_4^y$
$\|\Psi^+\rangle_{A1}$	$\|\Psi^+\rangle_{B2}$	$\|0\rangle_5$	$(a_0\|1\rangle + a_1\|0\rangle)_3 \otimes (b_0\|0\rangle + b_1\|1\rangle)_4$	$\sigma_3^x \otimes I_4$
$\|\Psi^+\rangle_{A1}$	$\|\Psi^-\rangle_{B2}$	$\|0\rangle_5$	$(a_0\|1\rangle - a_1\|0\rangle)_3 \otimes (b_0\|0\rangle + b_1\|1\rangle)_4$	$-i\sigma_3^y \otimes I_4$
$\|\Psi^-\rangle_{A1}$	$\|\Psi^+\rangle_{B2}$	$\|0\rangle_5$	$(a_0\|1\rangle + a_1\|0\rangle)_3 \otimes (b_0\|0\rangle - b_1\|1\rangle)_4$	$\sigma_3^x \otimes \sigma_3^z$
$\|\Psi^-\rangle_{A1}$	$\|\Psi^-\rangle_{B2}$	$\|0\rangle_5$	$(a_0\|1\rangle - a_1\|0\rangle)_3 \otimes (b_0\|0\rangle - b_1\|1\rangle)_4$	$-i\sigma_3^y \otimes \sigma_3^z$
$\|\Phi^+\rangle_{A1}$	$\|\Phi^+\rangle_{B2}$	$\|1\rangle_5$	$(a_0\|1\rangle + a_1\|0\rangle)_3 \otimes (b_0\|0\rangle - b_1\|1\rangle)_4$	$\sigma_3^x \otimes \sigma_4^z$
$\|\Phi^+\rangle_{A1}$	$\|\Phi^-\rangle_{B2}$	$\|1\rangle_5$	$(a_0\|1\rangle - a_1\|0\rangle)_3 \otimes (b_0\|0\rangle - b_1\|1\rangle)_4$	$-i\sigma_3^y \otimes \sigma_4^z$
$\|\Phi^-\rangle_{A1}$	$\|\Phi^+\rangle_{B2}$	$\|1\rangle_5$	$(a_0\|1\rangle + a_1\|0\rangle)_3 \otimes (b_0\|0\rangle + b_1\|1\rangle)_4$	$\sigma_3^x \otimes I_4$
$\|\Phi^-\rangle_{A1}$	$\|\Phi^-\rangle_{B2}$	$\|1\rangle_5$	$(a_0\|1\rangle - a_1\|0\rangle)_3 \otimes (b_0\|0\rangle + b_1\|1\rangle)_4$	$-i\sigma_3^y \otimes I_4$
$\|\Phi^+\rangle_{A1}$	$\|\Psi^+\rangle_{B2}$	$\|1\rangle_5$	$(a_0\|1\rangle + a_1\|0\rangle)_3 \otimes (-b_0\|1\rangle + b_1\|0\rangle)_4$	$\sigma_3^x \otimes \sigma_4^z\sigma_4^x$
$\|\Phi^+\rangle_{A1}$	$\|\Psi^-\rangle_{B2}$	$\|1\rangle_5$	$(a_0\|1\rangle - a_1\|0\rangle)_3 \otimes (-b_0\|1\rangle + b_1\|0\rangle)_4$	$-i\sigma_3^y \otimes \sigma_4^z\sigma_4^x$
$\|\Phi^-\rangle_{A1}$	$\|\Psi^+\rangle_{B2}$	$\|1\rangle_5$	$(a_0\|1\rangle + a_1\|0\rangle)_3 \otimes (-b_0\|1\rangle - b_1\|0\rangle)_4$	$\sigma_3^x \otimes -i\sigma_4^y\sigma_4^z$
$\|\Phi^-\rangle_{A1}$	$\|\Psi^-\rangle_{B2}$	$\|1\rangle_5$	$(a_0\|1\rangle - a_1\|0\rangle)_3 \otimes (-b_0\|1\rangle - b_1\|0\rangle)_4$	$-i\sigma_3^y \otimes -i\sigma_4^y\sigma_4^z$
$\|\Psi^+\rangle_{A1}$	$\|\Phi^+\rangle_{B2}$	$\|1\rangle_5$	$(a_0\|0\rangle + a_1\|1\rangle)_3 \otimes (b_0\|0\rangle - b_1\|1\rangle)_4$	$I_3 \otimes \sigma_4^z$
$\|\Psi^+\rangle_{A1}$	$\|\Phi^-\rangle_{B2}$	$\|1\rangle_5$	$(a_0\|0\rangle - a_1\|1\rangle)_3 \otimes (b_0\|0\rangle - b_1\|1\rangle)_4$	$\sigma_3^z \otimes \sigma_4^z$
$\|\Psi^-\rangle_{A1}$	$\|\Phi^+\rangle_{B2}$	$\|1\rangle_5$	$(a_0\|0\rangle + a_1\|1\rangle)_3 \otimes (b_0\|0\rangle + b_1\|1\rangle)_4$	$I_3 \otimes I_4$
$\|\Psi^-\rangle_{A1}$	$\|\Phi^-\rangle_{B2}$	$\|1\rangle_5$	$(a_0\|0\rangle - a_1\|1\rangle)_3 \otimes (b_0\|0\rangle + b_1\|1\rangle)_4$	$\sigma_3^z \otimes I_4$
$\|\Psi^+\rangle_{A1}$	$\|\Psi^+\rangle_{B2}$	$\|1\rangle_5$	$(a_0\|0\rangle + a_1\|1\rangle)_3 \otimes (-b_0\|1\rangle + b_1\|0\rangle)_4$	$I_3 \otimes \sigma_4^z\sigma_4^x$
$\|\Psi^+\rangle_{A1}$	$\|\Psi^-\rangle_{B2}$	$\|1\rangle_5$	$(a_0\|0\rangle - a_1\|1\rangle)_3 \otimes (-b_0\|1\rangle + b_1\|0\rangle)_4$	$\sigma_3^z \otimes \sigma_4^z\sigma_4^x$
$\|\Psi^-\rangle_{A1}$	$\|\Psi^+\rangle_{B2}$	$\|1\rangle_5$	$(a_0\|0\rangle + a_1\|1\rangle)_3 \otimes (-b_0\|1\rangle - b_1\|0\rangle)_4$	$I_3 \otimes -i\sigma_4^y\sigma_4^z$
$\|\Psi^-\rangle_{A1}$	$\|\Psi^-\rangle_{B2}$	$\|1\rangle_5$	$(a_0\|0\rangle - a_1\|1\rangle)_3 \otimes (-b_0\|1\rangle - b_1\|0\rangle)_4$	$\sigma_3^z \otimes -i\sigma_4^y\sigma_4^z$

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