Int J Theor Phys (2014) 53:1346–1360
DOI 10.1007/s10773-013-1931-6

An Application of the Theory of Open Quantum Systems to Model the Dynamics of Party Governance in the US Political System

Polina Khrennikova · Emmanuel Haven · Andrei Khrennikov

Received: 29 July 2013 / Accepted: 19 November 2013 / Published online: 30 November 2013

Abstract The Gorini-Kossakowski-Sudarshan-Lindblad equation allows us to model the process of decision making in US elections. The crucial point we attempt to make is that the voter's mental state can be represented as a superposition of two possible choices for either republicans or democrats. However, reality dictates a more complicated situation: typically a voter participates in two elections, i.e. the congress and the presidential elections. In both elections the voter has to decide between two choices. This very feature of the US election system requires that the mental state is represented by a 2-qubit state corresponding to the superposition of 4 different choices. The main issue is to describe the dynamics of the voters' mental states taking into account the mental and political environment. What is novel in this paper is that we apply the theory of open quantum systems to social science. The quantum master equation describes the resolution of uncertainty (represented in the form of superposition) to a definite choice.

Keywords Quantum master equation · Decoherence · Superposition · Voter's state · US election system

1 Introduction

For the last ten years, the mathematical formalism of quantum theory has been actively applied outside of the domain of quantum physics. We have seen applications in cognitive science and decision making (both in cognitive science and economics) and finance, see for instance [1–22].

More recently the *quantum-like* (QL) approach started to be explored in political science. Some of the QL features of the behavior of voters in the US political system were discussed

P. Khrennikova · E. Haven
School of Management and Institute of Finance, University of Leicester, Leicester, UK

A. Khrennikov (✉)
International Center for Mathematical Modeling, Linnaeus University, Växjö, Sweden
e-mail: andrei.khrennikov@lnu.se

in Zorn and Smith [17]. The authors start with a comparison of the notions of state separability in conventional models of party governance and in quantum information theory (see Zorn and Smith [17]) and they then show that the QL model might provide a more adequate description of the voters' state space—'mental space'. The authors present a strong motivation of the usage of the complex Hilbert space as the voters' 'mental space.'

In this paper we present a QL-model describing the dynamics of the voters' state (as represented in the complex Hilbert space). First, we consider what we could call 'a free QL-dynamics', when a voter is not under the pressure of mass media and the social environment. Following the tradition of quantum information theory, we call such a voter 'Alice'. In game theory 'Alice' is also often used. By applying the quantum approach we describe the dynamics of her state by using an analogue of the Schrödinger equation. A simple mathematical analysis implies that Alice's preferences encoded in her state-vector ('mental wave function') fluctuate *without* stabilization to the definite state. Hence, such a dynamics can describe the unstable part of the electorate: those voters who have no firm preferences. In quantum physics, stabilization and damping of fluctuations is a typical consequence of interaction with the environment (such environment in physics is also known as 'bath'). We apply this approach to the problem of the stabilization of fluctuations of voters' preferences, see Asano et al. [13, 18].

An essential part of the paper is devoted to the analysis of the applicability of quantum dynamics to a social system (e.g. a voter) which is coupled to the social environment. The main problem is that the exact quantum dynamics of a system coupled to the physical environment is extremely complicated. Therefore, to simplify matters, typically a quantum Markov approximation is applied. This approximation is applicable under a number of nontrivial conditions (see Ingarden et al. [23]). Our aim is to translate these conditions into the language of social science and to analyze their applicability to the dynamics of voters' preferences. In this connection the quantum Markovian dynamics, especially via the quantum master equation, can model (approximately) voters' preference dynamics. Our approach is based on the quantum master equation which describes the interaction of a social system with a 'social bath'. We use a very general framework which can be applied to a variety of problems in politics, social science, economics, and finance. The main problem of any concrete application is to analyze the conditions of applicability of the quantum master equation (the quantum Markov approximation) to the corresponding problem in decision making.

We remark that the work of Fiorina [24] played an important role in the motivation of the quantum model based on the use of entangled quantum states (see Zorn and Smith [17] for the two institutional choices in U.S. politics—the congress and the presidency). Zorn and Smith [17] also present a detailed analysis of the inter-relation between classical and quantum models. Such an analysis is very important to attract the interest of mainstream researchers in decision making to quantum models. For such researchers, the applications of the quantum formalism to social science may on prima facie be considered as quite exotic. Therefore, in this paper, we begin with an extended section in which we compare classical and quantum probabilistic approaches to decision making. Our aim is not only to stress the differences, but also to find the commonalities. Our findings argue for an important degree of similarity between quantum and subadditive probabilistic descriptions of decision making. We also emphasize the vital role of contextuality.

2 Description of Election Campaign by the Theory of Open Quantum Systems

With the help of the above mentioned features of QL models we now attempt to describe the dynamics of the process of decision making within the problem setting of party governance in the US-type two party system. This system allows voters to cast partisan ballots in two contests: executive and legislative. By so doing they can thus choose for instance ‘Republican’ in one institutional choice setting and ‘Democratic’ in the other (see Zorn and Smith [17]).

It is well known from physics that the quantum state dynamics are described by Schrödinger’s equation. This type of dynamics is unitary. Roughly speaking it is combined of a family of rotations and in principle, this family can be infinite. This equation describes the dynamics of an isolated system, i.e., a system which does not interact with the environment. However, a voter in the context of the election campaign definitely cannot be considered as an isolated social system. She, say Alice, is in permanent contact with mass media (whether TV or INTERNET). Such an influence of the environment induces random fluctuations of opinions and choices in Alice’s mind.

For the purposes of our research, we are interested in the ‘unstable’ part of the electorate which is composed of citizens who have no concrete opinions and who will make their electoral choice very close to the actual day of the elections (see Zaller and Feldman [25]).

If Alice could be considered as an isolated social system, then the only possibility to describe a transition from the mental state of superposition of choices to the state corresponding to the concrete choice was to use the projection postulate of quantum mechanics (the so called ‘von Neumann postulate’). This state reduction process, from superposition to one of its components, is called *the state collapse*. The state collapse is considered as one of the main mysteries of quantum physics. This notion is still a subject of intensive debate. Such collapse is imagined as an instantaneous (the jump-type) transition from one state to another. The state collapse might be used to describe the situation in which Alice makes her choice precisely at the moment of completing the voting bulletin. This type of behavior cannot be completely excluded from consideration, but such a case is probably not statistically significant. Moreover, mainstream quantum mechanical thought will tell us that the state collapse occurs when an isolated system driven by Schrödinger’s equation interacts practically instantaneously with a measurement device. Thus when Alice is totally isolated from the election campaign, she is suddenly asked to make her choice. It is evident that the process of decision making for the majority of the ‘unstable population’ in the electorate differs in essential ways from this collapse-type behavior.

Therefore, let us take more seriously the role which the social environment plays in the process of decision making. We apply to social science the theory of *open quantum systems*, i.e., systems which interact with a large thermostat (‘bath’). Since a bath is a huge physical system with millions of variables (the complexity of the “social bath” around an American citizen who will cast his/her vote in the election campaign is huge), it is in general impossible to provide a reasonable mathematical description of the dynamics of a quantum system interacting with such a bath. Physicists proceed under a few assumptions which allow then for the possibility to describe those dynamics in an approximate way. In quantum physics the interaction of a quantum system with a bath is described by a quantum version of the master equation for Markovian dynamics. The quantum Markovian dynamics are given by the *Gorini-Kossakowski-Sudarshan-Lindblad* (GKSL) equation. See Ingarden et al. [23] for details. This GSKL equation is the most popular approximation of quantum dynamics in the presence of interaction with a bath.

We briefly remind the origins of the GKSL-dynamics. The starting point is that the state of a composite system, a quantum system s combined with a bath, is a pure quantum state, complex vector Ψ. The evolution of Ψ is described by Schrödinger’s equation. This is an evolution in a Hilbert space of a huge dimension, since a bath has so many degrees of freedom. The existence of the Schrödinger dynamics in the huge Hilbert space has a merely theoretical value. Observers are interested in the dynamics of the state ϕ_s of the quantum system s. The next fundamental assumption in the derivation of the GKSL-equation is the Markovian character of the evolution, i.e. the absence of long term memory effects. It is assumed that interaction with the bath destroys such effects. Thus, the GKSL-evolution is a Markovian evolution. Finally, we point to the condition of the 'factorizability' of the initial state of a composite system (a quantum system coupled with a bath), $\Psi = \phi_s \otimes \phi_{\text{bath}}$, where $\otimes$ is the sign of the tensor product. Physically factorization is equivalent to the absence of correlations. One of the distinguishing features of the evolution under the mentioned assumptions is the existence of one or few *equilibrium points*. The state of the quantum system s stabilizes to one of such points in the process of evolution: a pure initial state, a complex vector ψ_s, is transformed into a mixed state, a density matrix $\rho_s(t)$ (classical state without superposition effects).

In contrast to the GKSL-evolution, the Schrödinger evolution does *not* induce stabilization. Any solution different from an eigenvector of the Hamiltonian will oscillate for ever. Another property of the Schrödinger dynamics is that it *always* transfers a pure state into a pure state, i.e., a vector into a vector: quantumness if it was originally present in a state (in the form of superposition) cannot disappear in the process of a continuous dynamical evolution. The transition from quantum indeterminism to classical determinism can happen only as the result of the collapse of the quantum state.

On the one hand, in our model of the decision making for party governance we would like to avoid the usage of the state collapse. On the other hand, to make a decision, Alice has to make a transition from a quantum to a classical representation of her preferences. We note that in quantum physics all experimentally obtained information is classical as well. The GKSL-evolution provides for such a possibility (and without 'quantum jumps'). Alice's mental state evolves in a smooth way (fluctuations exist but they are damped) to the final classical decision state.

3 Matching of Assumptions of Applicability of the Quantum Master Equation with Conditions of the Modern Election Campaign

We now list the social conditions corresponding to the above mentioned physical conditions. This will allow us for a possibility to apply the GKSL-equation:

- (**COMPL**) *complexity*: the social environment (election bath) influencing a voter has huge complexity
- (**FREE**) *freedom*: the mental state of a society under consideration is a pure QL state, i.e., a superposition of various opinions and expectations;
- (**DEM**) *democracy*: the feedback reaction of a voter to the election bath is negligibly small, it cannot essentially change the mental state of the bath;
- (**SEP**) *separability*: before the start of the election campaign a voter was independent of the election bath;
- (**MARK**) *Markovness*: a voter does not use a long range memory on interaction with the election bath to update her state.

We surely need to make some comments on those assumptions.

1. The assumption (**COMPL**), *complexity*, is definitely justified. Nowadays an election campaign has huge information complexity: the richness of media sources accounts for such complexity. We can even speculate that the proposed QL model is more adequate than say 50 years ago: the phenomenal increase of information complexity makes the usage of the (quantum, quantum-like) open systems approach more reasonable.

2. The (**FREE**), *freedom*, can be interpreted as guaranteeing the freedom of political opinions. The opposite to the (**FREE**)-society, is a totalitarian society where its mental state is a classical state in which all superpositions have been resolved (collapsed).
3. The (**DEM**), *democracy*, encodes the democratic system: one voter cannot change the mental state of society in a crucial way.
4. The (**SEP**), *separability*, describes a sample of voters who are not that interested in politics: they will determine their positions through an interaction with the election bath during the election campaign. This part of the electorate is the most interesting from the point of view of political technologies.
5. The (**MARK**)-assumption, *Markovness*, also reflects the fact that voters under study are not that interested in politics. They do not spend a lot of time analyzing the dynamics of the election campaign. However, they are not isolated from the election bath; they watch TV, read newspapers and use the Internet. From a pragmatic point of view, they unconsciously update their mental states each day by taking into account recent news.

Remark 1 (Markovness) We remark that the Markovness of the dynamics may induce the impression that voter's preferences would fluctuate forever. However, this is not the case. The mathematical formalism of quantum mechanics implies that quantum Markovean fluctuations stabilize to steady solutions. In physics, this theoretical prediction was confirmed by numerous experiments. Although the social counterparts of physical assumptions seem to be natural and this motivates the applicability of our theoretical model, the final justification can come only from the testing of our hypothesis by experimental data. This is a very complex problem.

Remark 2 (Decoherence) In quantum physics the process of transformation of a pure (superposition-type) state into a classical state (given by a diagonal density matrix) is called decoherence. A proper interpretation of this process is still one of the hardest problems *in the* foundations of quantum mechanics. Some authors present the viewpoint that superposition is in some way conserved: the disappearance of superposition in a subsystem increases it in the total system. In our model this would mean that the determination of states of voters in the process of interaction with the election bath will *transfer political uncertainty into an increase of political uncertainty in society in general, after elections*. At the moment it is not clear whether this interpretation is meaningful in social sciences.

4 Schrödinger's Dynamics

The state space of a voter (Alice) can be represented as the tensor product of two Hilbert spaces (each of them is two dimensional). One Hilbert space describes the election to the congress, and we denote it by the symbol $\mathcal{H}_{\text{congress}}$, and another describes the presidential election, denote it by the symbol $H_{\text{president}}$. In each of them we can select the basis corresponding to the definite strategies $e_1 = |d\rangle$, $e_2 = |r\rangle$. If Alice was thinking only about the election to congress, her mental state would be represented as the superposition of these two basis vectors:

$$\psi_{\text{congress}} = \alpha_c|d\rangle + \beta_c|r\rangle; \tag{1}$$

where α_c, β_c are complex numbers and they are normalized by the condition: $|\alpha_c|^2 + |\beta_c|^2 = 1$. By knowing the representation of (1) one can find the probabilities of intentions to vote for democrats and republicans in the election to the congress:

$$p_{\text{congress}}(d) = |\alpha_c|^2, \qquad p_{\text{congress}}(r) = |\beta_c|^2. \tag{2}$$

However, the quantum dynamics of the state, $\psi_{\text{congress}}(t)$, in the absence of interactions with the political bath (environment), see (9) below—'social Schrödinger equation', is such that the probabilities $p_{\text{congress}}(d;t)$, $p_{\text{congress}}(r;t)$ fluctuate. Therefore, even if Alice wanted to vote for republicans at $t = t_0$, in the process of mental evolution she will change her mind many times.

In the same way, if Alice was thinking only about the election of the president, her mental state would be represented as a superposition of the two basis vectors

$$\psi_{\text{president}} = \alpha_p |d\rangle + \beta_p |r\rangle; \tag{3}$$

where $|\alpha_p|^2 + |\beta_p|^2 = 1$. The corresponding probabilities are given by

$$p_{\text{president}}(d) = |\alpha_p|^2, \qquad p_{\text{president}}(r) = |\beta_p|^2. \tag{4}$$

For a moment, let us forget about the quantum model and turn to classical probability theory. Suppose that the classical probabilities $p_{\text{congress}}(d)$, $p_{\text{congress}}(r)$, $p_{\text{president}}(d)$, $p_{\text{president}}(r)$ are given. Furthermore, suppose that voters do not have any kind of correlations between two elections: their choice in the election to the congress does not depend on their choice of the president and vice versa. To avoid the problem of time fluctuations of probabilities, we may assume that both elections are performed at the same time. In this case independence implies factorization of the joint probability distribution:

$$p_{\text{congress,president}}(dd) = p_{\text{congress}}(d)\, p_{\text{president}}(d), \ldots \tag{5}$$

However, in the case of non-trivial correlations between the congress- and president-elections, the factorization condition is violated. In the quantum formalism, the models described by the two Hilbert spaces are unified in the model described by the tensor product of these two spaces. In our case we use the space $\mathcal{H} = \mathcal{H}_{\text{congress}} \otimes H_{\text{president}}$. Its elements are of the form:

$$\psi = \psi_{\text{congress}} \otimes \psi_{\text{president}} \tag{6}$$

which describe the states corresponding to uncorrelated choices in two elections. In quantum information such states are called *separable*. In general a state $\psi \in \mathcal{H}$ cannot be factorized. Nonseparable states describe correlations between choices in the two elections:

$$\psi = c_{dd}|dd\rangle + c_{dp}|dp\rangle + c_{pd}|pd\rangle + c_{pp}|pp\rangle; \tag{7}$$

where $|c_{dd}|^2 + |c_{dp}|^2 + |c_{pd}|^2 + |c_{pp}|^2 = 1$ and $|dd\rangle = |d\rangle \otimes |d\rangle, \ldots$. The main point of usage of the quantum formalism is that quantum correlations are not reduced to classical correlations (as described in the framework of the Kolmogorov model). Roughly speaking the quantum correlations can be stronger than the classical correlations. This is the essence of Bell's theorem. We also state again that the question of inter-relation between quantum and classical separability in the election framework was studied in Zorn and Smith [17]. However, the authors did not appeal directly to Bell's theorem, but to the more delicate condition of quantum (non-) separability.

The Schrödinger quantum dynamical equation has the form:

$$ih\frac{\partial \psi}{\partial t}(t) = H\psi(t); \tag{8}$$

where H is the operator of energy, the Hamiltonian, and h is the Planck constant. The mental interpretation of an analog of the Planck constant h is a complicated problem. We shall interpret it as the time scale parameter. In quantum physics the Planck constant has the dimension of action: time $\times$ energy. In this paper we do not want to speculate on such a controversial topic as "mental energy". Therefore, we proceed formally by considering the evolution generator H as a dimensionless quantity. Since the usage of the symbol h may be a source of misunderstanding (especially for physical science educated readers), we shall use a new scaling parameter, say τ having the dimension of time. It determines the time scale of updating of the mental state of Alice during the election campaign. We rewrite the dynamical equation as:

$$i\tau \frac{\partial \psi}{\partial t}(t) = H\psi(t). \tag{9}$$

And we call the operator H, the *decision Hamiltonian.*

The most general Hamiltonian H in the space of mental states in the two-party systems (wherein voters can cast partisan ballots in two contests, executive and legislative) has the form

$$H = H_{\text{stab}} + H_{\text{flip}}, \tag{10}$$

where H_{stab} is the part of the Hamiltonian responsible for the stability of the distribution of opinions about various possible selections of decisions. It is given by

$$H_{\text{stab}} = \lambda_{dd,dd}|dd\rangle\langle dd| + \lambda_{rr,rr}|rr\rangle\langle rr| + \lambda_{dr,dr}|dr\rangle\langle dr| + \lambda_{rd,rd}|rd\rangle\langle rd|. \tag{11}$$

And H_{flip} is the part of Hamiltonian responsible for flipping from one selection of the pair of strategies (for executive and legislative branches) to another. It is given by

$$H_{\text{flip}} = \lambda_{dd,rr}|dd\rangle\langle rr| + \lambda_{rr,dd}|rr\rangle\langle dd| + \lambda_{rd,dr}|rd\rangle\langle dr| + \lambda_{rd,dr}|rd\rangle\langle dr|. \tag{12}$$

To induce a unitary evolution, the Hamiltonian has to be Hermitian. This induces the following restrictions to its coefficients: $\lambda_{dd,dd}, \ldots, \lambda_{rd,rd}$ are real and $\overline{\lambda}_{dd,rr} = \lambda_{rr,dd}, \ldots, \overline{\lambda}_{rd,dr} = \lambda_{dr,rd}$.

In the absence of the H_{flip}-component, the probabilistic structure of superposition is preserved. Only phases of choices evolve in the rotation-like way, e.g., $|dd\rangle$ evolves as

$$\psi_{dd}(t) = e^{-\frac{it}{\tau}\lambda_{dd,dd}}|dd\rangle$$

which corresponds to a "rotation" of the strategy dd for the "angle" $\Delta\theta = t\lambda_{dd,dd}$. A larger λ induces quicker rotation. The meaning of such rotations of mental states has to be clarified in the process of the model's development. We can speculate that the coefficient $\lambda_{dd,dd}$ correspond to the speed of self-analysis (by Alice) of the choice dd.

In the presence of the flipping component H_{flip} the distribution of probabilities of choices of various strategies changes in the process of evolution. Such flipping from one strategy to another makes the state dynamics really quantum. In fact, for political technologies per sé, the most important component is the flipping part of the Hamiltonian. Of course, at the moment we proceed at a very abstract theoretical level. However, one may hope to develop the present QL model to the level of real applications.

Example 1 Suppose that Alice has neither a firm association with democrats nor with republicans, i.e., the diagonal elements of the decision Hamiltonian are equal to zero. Suppose also that the flipping part of the Hamiltonian contains only the transition:

$$|dr\rangle \to |rd\rangle, \tag{13}$$

which expresses the combination (democrats, republicans) into the combination (republicans, democrats), and vice versa. Let $\lambda_{dr,rd} = \lambda_{rd,dr} = \lambda > 0$. The Schrödinger equation has the form of a system of linear ordinary differential equations. The dynamics of coincidence of choices is trivial: $i\tau\frac{dx_{dd}(t)}{dt}=0$, $i\tau\frac{dx_{rr}(t)}{dt}=0$. Hence, $x_{dd}(t)=x_{dd}(0)$, $x_{rr}(t)=x_{rr}(0)$. However, the presence of a nontrivial transition channel, (13), induces fluctuations of Alice's preferences for choices dr and rd. Here we have the system of two equations:

$$i\tau\frac{dx_{dr}(t)}{dt}=\lambda x_{rd}(t), \qquad i\tau\frac{dx_{rd}(t)}{dt}=\lambda x_{dr}(t). \tag{14}$$

Its solutions have the form:

$$x_{dr}(t)=x_{dr}(0)\cos\frac{\lambda t}{\tau}-ix_{rd}(0)\sin\frac{\lambda t}{\tau}, \tag{15}$$

$$x_{rd}(t)=-ix_{dr}(0)\sin\frac{\lambda t}{\tau}+x_{rd}(0)\cos\frac{\lambda t}{\tau}. \tag{16}$$

5 Dynamics in the Election Bath

In physics the dynamics of a system in a bath is described by the quantum analog of the master equation, the GKSL-equation, see Sect. 3. We write this equation by using the time scaling constant τ, instead of the Planck constant:

$$i\tau\frac{\partial\rho}{\partial t}(t)=\big[H,\rho(t)\big]+L\big(\rho(t)\big); \tag{17}$$

where L is a linear operator acting in the space of operators on the complex Hilbert space. In the dynamics described by (17), density operators are transformed into density operators. The general form of L was found by Gorini, Kossakowski, Sudarshan, and Lindblad (see Ingarden et al. [23]). For now, we are not interested in (the rather complex) structure of L. For our applications, it is sufficient to know that it can be expressed through matrix multiplication for a family of matrices. The simplest dynamics of interaction of Alice with the two party election campaign is determined by two matrices V_d and V_r corresponding to advertising of democrats and republicans, respectively. Under natural selection of the matrices H, V_d, V_r any solution of this equation stabilizes to a diagonal density matrix

$$\rho_{\mathrm{decision}}=\mathrm{diag}(\mathrm{p_{dd}},\mathrm{p_{dr}},\mathrm{p_{rd}},\mathrm{p_{rr}}). \tag{18}$$

This matrix describes the distribution of firmly established decisions for voting strategies xy, where $x,y=d,r$.

The density matrix ρ_{decision} describes a population of voters who finally determine their choices. Denote the number of people in this population by N. There are then (approximately) $n_{dd}=p_{dd}N$ people in the mental state $|dd\rangle,\ldots$ and $n_{rr}=p_{rr}N$ people in the mental state $|rr\rangle$. For example, people in the mental state $|dd\rangle$ have firmly selected to vote for democrats both in the executive and legislative branches. Their decision is stable. From a pragmatic point of view there is no possibility to manipulate opinions of people in this population.

5.1 Comparing Mental States Given by Vectors and Density Matrices

Consider two populations, say $\mathcal{E}_1$ and $\mathcal{E}_2$. Suppose that in our QL model the first one is described by a pure state

$$\psi=c_{dd}|dd\rangle+c_{dr}|dr\rangle+c_{rd}|rd\rangle+c_{rr}|dd\rangle; \tag{19}$$

and the second one by the density matrix given by (18). Moreover, suppose that the complex amplitudes given by the coefficients in the expansion (19) produce the same probabilities as the density matrix, i.e.:

$$p_{xy} = |c_{xy}|^2. \tag{20}$$

One may ask: "What is the difference?" At first sight there is no difference at all[1] since we obtain the same probability distribution of preferences. However, the distributions of mental states in ensembles $\mathcal{E}_1$ and $\mathcal{E}_2$ are totally different. All people in $\mathcal{E}_1$ are in the same state of indeterminacy (superposition) ψ. They are in doubt. They are ready to change their opinion (to create a new superposition of opinions). The $\mathcal{E}_1$ is a proper population for political manipulations. To the opposite of population $\mathcal{E}_1$, the population $\mathcal{E}_2$ consists of people who have already resolved their doubts. Their mental states have already been reduced to states of the form xy, i.e. definite choices. Please see the discussion under (21) below.

5.2 Independence from the Initial State

The general theory of quantum master equations implies that for some important open system dynamics, the limiting probability distribution *does not depend on the initial state*! This mathematical fact has important consequences for our QL model of elections. It tells us that in principle it is possible to create such a quantum open system dynamics (voters interacting with some election bath) such that the desired state ρ_{decision} would be obtained *independently* of the initial mental state of Alice. This theoretical result may play an important role in QL election technologies. Even if a quantum master equation does not have the unique limiting state, there are typically just a few of them. In this case, we can split the set of all pure states (the unit sphere in the complex Hilbert state space) into clusters of voters. For each cluster, we can predict the final distribution of decisions.

We shall illustrate the above discussion by numerical simulation (using Mathematica software) of the dynamics of preferences of voters interacting with the "election environment".

Example 2 We consider only the two dimensional submodel of the general four dimensional model corresponding to a part of the electorate which have "double preferences"—democrats in one of the elections and republicans in another election. So, we reduce the modeling to the subspace with the basis $|dr\rangle$, $|rd\rangle$. It is assumed that at the beginning (i.e., before interaction with the 'election environment') voters are in a superposition of the basic states:

$$|\psi\rangle = c_1|dr\rangle + c_2|rd\rangle, \qquad |c_1|^2 + |c_2|^2 = 1. \tag{21}$$

We also assume that in the absence of interaction with the 'election campaign' the state of preferences fluctuations are driven by the Schrödinger dynamics considered in Example 1 above. In the matrix form the corresponding Hamiltonian can be written as

$$\mathcal{H} = \begin{pmatrix} 0 & \lambda \\ \lambda & 0 \end{pmatrix}; \tag{22}$$

where $\lambda > 0$ is the parameter describing the intensity of flipping from dr to rd and vice versa. The simplest perturbation of this Schrödinger equation is given by the Lindblad term of the form given by Ingarden et al. [23]:

$$C\rho C^* - \big(C^*C\rho + \rho C^*C\big)/2 = C\rho C^* - \frac{1}{2}\big\{C^*C, \rho\big\};$$

[1] See A. Plotnitsky [26, 27] for an analysis of the inter-relation between classical and quantum probability.

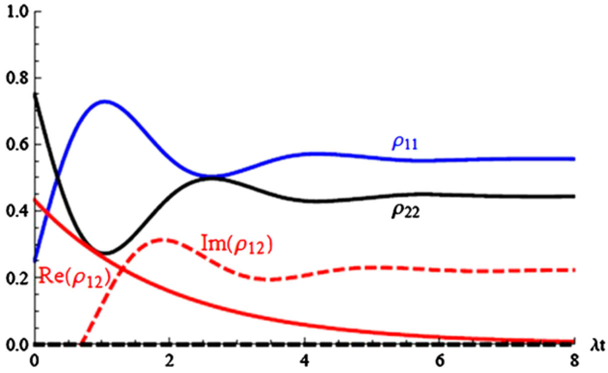

Fig. 1 Stabilization of the matrix elements of the density operator; the initial state is a symmetric superposition of state $|dr\rangle$ and $|rd\rangle$

where C^* denotes the operator which is the Hermitian adjoint to the operator C. As always in quantum formalism,

$$\{U, V\} = UV + VU,$$

which denotes the anticommutator of two operators U, V. We select the operator C by using its matrix in the basis $|dr\rangle$, $|rd\rangle$:

$$C = \begin{pmatrix} 0 & \lambda \\ 0 & 0 \end{pmatrix};$$

hence,

$$C^* = \begin{pmatrix} 0 & 0 \\ \lambda & 0 \end{pmatrix};$$

where the parameter λ is responsible for interaction between the voter's state. For simplicity, the 'election campaign' is selected in the same way as in the Hamiltonian (22). Thus, we proceed with the quantum master equation:

$$\frac{d\rho}{dt}(t) = -i\big[\mathcal{H}, \rho(t)\big] + C\rho(t)C^* - \frac{1}{2}\big\{C^*C, \rho(t)\big\}. \tag{23}$$

We present the dynamics corresponding to symmetric superposition,

$$c_1 = c_2 = \frac{1}{\sqrt{2}}. \tag{24}$$

See Fig. 1.

$$c_1 = \sqrt{0,9}, \qquad c_2 = \sqrt{0.1}. \tag{25}$$

For a strongly asymmetric superposition. See Fig. 2.

The interaction with the 'election environment' plays a crucial role. Strong oscillations of the dynamics, given by (15), (16) in the absence of interaction with the 'election bath' are quickly damped and the matrix elements $\rho_{11} \equiv \rho_{dr,dr}$, $\rho_{22} \equiv \rho_{rd,rd}$, $\rho_{12} \equiv \rho_{dr,rd}$, and $\bar{\rho}_{12} = \rho_{21} \equiv \rho_{rd,dr}$ stabilize to the definite values. Thus the preferences of population of voters who were in fluctuating superposition of choices stabilize under the pressure of the 'election bath'. We selected such a form of interaction between a voter and the 'election bath' such that both initial states, the totally symmetric state, i.e., no preference to dr nor rd, and the state with very strong preference for the dr combination in votes to congress

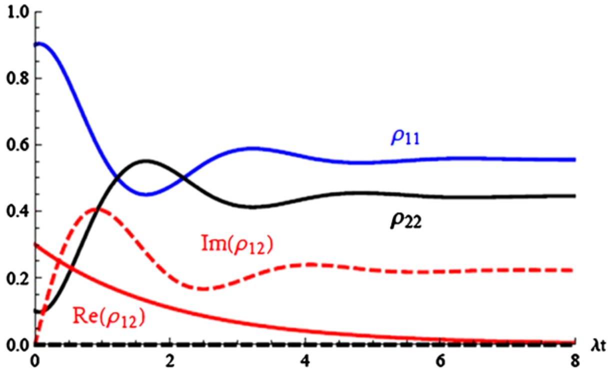

Fig. 2 Stabilization of the matrix elements of the density operator; the initial state is a strongly asymmetric superposition of states $|dr\rangle$ and $|rd\rangle$

and of president, $p(dr) = 0.9$, $p(rd) = 0.1$, induce dynamics with stabilization to the same density matrix ρ_{lim}. This example demonstrates the power of the social environment which, in fact, determines the choices of voters.

In the ρ_{lim} the elements $\rho_{dr,dr} \approx 0.6$, $\rho_{rd,rd} \approx 0.4$ determine corresponding probabilities $p(dr) \approx 0.6$, $p(rd) = 0,4$. Under the pressure of the social environment those who started with a superposition as indicated in (24) increase the dr-preference and those who started with the superposition in (25) decrease this preference, and the resulting distribution of choices is the same in both populations (with the initial state (24) and with the initial state (25)).

We stress that manipulation by the preferences described by the dynamics in (23) is sufficiently smooth. Those dynamics are an extension of the 'free thinking' dynamics given by the Schrödinger equation, the first term in the right-hand side of (23). Hence, in this model the social environment does not prohibit internal fluctuations of individuals, but instead damps them to obtain a 'peaceful' stabilization.

We emphasize that the degree of quantum uncertainty decreases in the process of evolution. One of the standard measures of uncertainty which is used in quantum information theory is given by so called *linear entropy* (see Ingarden et al. [23]) defined as:

$$S_L = 1 - \operatorname{Tr} \rho^2.$$

For a pure state (which has the highest degree of uncertainty), the linear entropy $S_{L,\min} = 0$. It increases with degeneration of purity in a quantum state and it approaches its maximal value $S_{L,\max} = 0,5$ for a maximally mixed state. Here we consider the two dimensional case; in the general case $S_{L,\max} = 1 - 1/d$, where d is the dimension of the state space. The dynamics of linear entropy corresponding to the initial states as per (24) and (25), respectively, are presented in Figs. 3 and 4.

We see that the entropy behaves in different ways, but finally it stabilizes to the same value $S_L \approx 0.4$. This value corresponds to a very large decreasing of purity—uncertainty of the superposition type. Numerical simulation demonstrated that, for other choices of pure initial states, the density matrix and the linear entropy stabilize to the same values. Our conjecture is that it may be possible to prove theoretically that this is really the case. However, at the moment we have only results of numerical simulation supporting this conjecture.

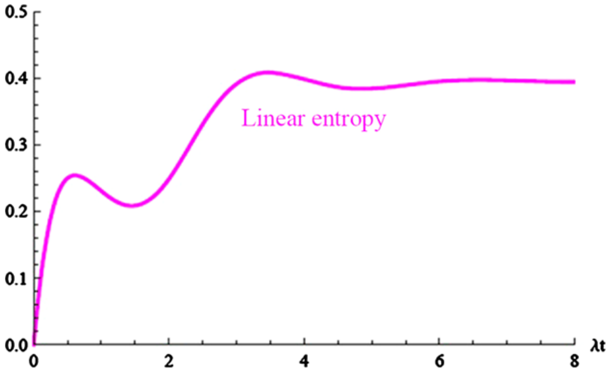

Fig. 3 Stabilization of linear entropy: the initial state is a symmetric superposition of states $|dr\rangle$ and $|rd\rangle$

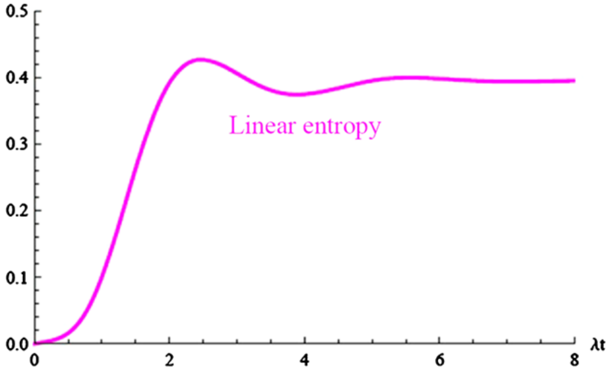

Fig. 4 Stabilization of linear entropy: the initial state is a strongly asymmetric superposition of states $|dr\rangle$ and $|rd\rangle$

6 Quantum Foundational Matters

6.1 Application of the Quantum Formalism to Microscopic Systems

In this paper the methods of quantum theory are applied to macroscopic systems (such as the cognitive systems of voters and the socio political environment) without attributing a definite quantum type of processes to the voters' brains themselves. As we mentioned in the introduction, the quantum-like paradigm aims to take advantage of the mathematical framework of quantum physics and to some extend of quantum logic. It has to be emphasized that the authors do not assume that the quantum informational structure of decisions performed by human beings emerges from quantum physical processes in the neural system. At the same time the mathematical apparatus of quantum theory can be applied not only in quantum physics, but in many different domains of science, where the *incompatibility* of events/observables prevents the usage of the so called 'classical' probabilistic apparatus. Here classicality is identified with commutativity and quantumness is associated with *non-commutativity of variables*. Incompatibility of various mental actions is an important feature of the human cognitive system where the decision making context has key implications for the decision outcome (Khrennikov [3], Busemeyer et al. [4], Khrennikov [16], Busemeyer and Bruza [21], Haven and Khrennikov [22], Khrennikova [28]. This feature of the human cognitive system follows quantum logic which gives a motivation to apply the calculus of quantum physics to the modeling of decision making problems in cognitive science, psychology, social science, economics, finance and, as we demonstrated in this paper, political science.

6.2 Entanglement of Mental States

Entanglement is one of the most fundamental concepts of quantum theory. As the result of the special structure of the US election system, entanglement provides an adequate representation of correlations between mental variables. We comment that the mainstream literature on voting behavior in US elections implies *voter's preference separability*: if a voter prefers one party to another he or she would do so for both types of elections; the presidential and the congressional. However, more recent studies on the US political system show that a so called phenomenon of '*ticket splitting takes place*'. In other words the voters cast partisan ballots for different parties in legislative and executive elections (Zorn and Smith [17], Fiorina [24], Smith et al. [29]). This type of behavior can be regarded as irrational from the traditional perspective, but what happens is that the choices of the voters are strongly correlated. In this regard Zorn and Smith [17] advise that a QL representation of voters' state of non-separability might provide a more adequate 'mental state' representation than the conventional models, by using the notion of entangled states. In our paper we developed this idea and modeled the evolution of entangled states with the help of the GKSL master equation.

In quantum physics entanglement is a complicated notion and its proper interpretation is still the subject of many hot debates. One of the possible interpretations is based on the idea of *quantum nonlocality.* In principle, the 'nonlocality interpretation' can also be used in social science. Its straightforward interpretation would imply the acceptance of the presence of an instantaneous action at a distance. However, in our model there is no need for such, so to say, "rigid nonlocality" based on *instantaneous action* at the distance. The time scale of social processes (in any of the events of the election campaigns) is slow enough to create nonlocal correlation without a violation of the laws of classical physics. The congress and the presidential elections are separated in time. At the same time those correlations, if regarded on the time scale of each particular decision maker, can strongly influence his or her decisions and therefore this can have implications on the future outcomes of the election.

6.3 Implications of Quantum-Like Socio Political Studies for the Foundations of Quantum Physics

It is important to mention that the applied model based on the theory of open quantum systems has important implications for the foundations of quantum physics. We demonstrated that an essential part of the quantum apparatus is not rigidly coupled with physics. This means that the mathematical calculus applied in the quantum framework could be successfully used to provide for an effective description of various phenomena, in particular in political science. We note that we did not explore the existence of the elementary quantum measure of action given by the *Planck constant* h. One can guess that only the latter represents the essence of quantum physics. The domain of interdisciplinary applications of the quantum framework is an emerging field and the meaning of quantum constants (such as the Planck constant) has to be delineated in more depth.

7 Conclusion

The theoretical application of the GKSL master equation to the social science context is a novel, yet promising domain that already demonstrates high predictive power of this model. In this paper we aimed to contribute to the field of decision making and cognition in political science by a numerical simulation with the help of the quantum master equation. The purpose was to model decision making dynamics of the US electorate in two step elections. The essential conclusion we can draw from the numerical simulations, is that the limiting probability distribution of the voters' preferences does not depend on their initial mental states. This result indicates that the 'environmental bath' (created by the context of election campaign) may play a decisive role in the formation of voters' final choices. In regards to the numerical simulation of the decision making dynamics studied in this paper, we can point out that the next step in the usage of our proposed theory will be to carefully analyze specific socio-political conditions and incorporate those into the presented model.

In summary, we believe that the future practical application of our model could be of vast interest to different political institutions. As mentioned above, those voters who did not make up their decisions yet (especially if they are situated in so called 'swing states') are mostly determining the success of which party will govern. Those types of voters can be to a high degree reached and influenced through media, newspapers, Internet and other mass-media sources. The spending on agitation campaigns is constantly growing. A table in the Economist [30], shows that the number of adds placed in the 2012 campaign increased with more than 50 % relative to the 2008 campaign. Websites are influenced by so called digital campaign spending. The Economist [31] reports that online campaign spending in 2012 has increased dramatically relative to the 2008 campaign. If the findings of a careful study of how the election campaign caninfluence voter behavior, were to be incorporated in our proposed model then this could provide for an accurate description and prediction of voters' decision making dynamics.

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