Int J Theor Phys (2014) 53:1322–1324
DOI 10.1007/s10773-013-1928-1

Quantum Teleportation of a Two Qubit State Using GHZ-*Like* State

Kaushik Nandi · Chandan Mazumdar

Received: 2 July 2013 / Accepted: 14 November 2013 / Published online: 27 November 2013

Abstract Recently Yang et al. (Int. J. Theor. Phys. 48:516, 2009) had shown that using a particular type of GHZ- Like state as quantum channel, it is possible to teleport an arbitrary unknown qubit. We investigate this channel for the teleportation of a particular type of two qubit state.

Keywords Quantum teleportation · GHZ-*Like* state · Two qubit state

1 Introduction

Quantum Teleportation is a process of transferring an arbitrary unknown qubit to a distant receiver with the help of shared entanglement and some classical information without any transfer of mass or energy. In their initial proposal, Bennett et al. [1] proposed quantum teleportation using maximally entangled two qubit states. Thereafter, this was realized in tripartite GHZ state [2], four partite GHZ state [3], SLOCC equivalent W-class state [4] and the cluster state [5], etc. The perfect teleportation of an arbitrary two-qubit state was proposed using quantum channels formed by the tensor product of two Bell states [6], tensor product of two orthogonal states [7], genuinely entangled five qubit state [8], five qubit cluster state [9] and six qubit genuinely entangled states [10], etc.

Yang et al. [11] had shown that perfect teleportation of a single qubit is possible using a GHZ-*Like* state. Tsai et al. [12] had shown that this state can also be used successfully to teleport a pure EPR state $\alpha|01\rangle + \beta|10\rangle$. However, we show that a more general two qubit state can be teleported with this GHZ-*Like* state.

Apart from this GHZ-*Like* state, deterministic quantum teleportation of a single arbitrary qubit state through a tripartite state as quantum channel is possible through the GHZ state [2]

K. Nandi (✉)
Physics Department, Gurudas College, Narkeldanga, Kolkata 700 054, India
e-mail: mail.knandi@gmail.com

C. Mazumdar
Department of Computer Science and Engineering, Jadavpur University, Kolkata 700 032, India

and a SLOCC equivalent W-class states [4]. Of the three, GHZ-*Like* state is more robust from the application point of view. For a pure state $|\Psi\rangle_{AB}$, the entanglement between the two subsystems A and B are given by,

$$E_{A|B}\big(|\Psi\rangle_{AB}\big) = S(\rho_A) \tag{1}$$

where $\rho_A = \mathrm{tr}_B(|\Psi\rangle\langle\Psi|)$ and $S(\rho_A)$ is the von Neumann entropy. For the tripartite GHZ state and the GHZ-*Like* states, $E_{1|23} = E_{2|13} = E_{3|12} = 1$ while for the other state $E_{3|12} = 1$ and $E_{1|23} = E_{2|13} < 1$. As a result, quantum teleportation of a single qubit is only possible through any bipartite cut of the first two states while it is possible through the 12|3 cut of the third system. Now if we consider these states for the teleportation of a two qubit state, it can be seen that only a pure EPR state of the form $\alpha|00\rangle + \beta|11\rangle$ can be teleported by the GHZ state while it is not possible through the asymmetric W state in a deterministic manner. The GHZ-*Like* state on the other hand can be used for perfect teleportation of a more general two-qubit state.

2 Quantum Teleportation of a Two Qubit Entangled State via the GHZ-*Like* State

The most general form of a two qubit state is given by,

$$|\chi\rangle_{12} = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle \tag{2}$$

with α, β, γ and δ being complex and $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$. Let us consider that, Alice had an unknown two qubit state, which is a special form of Eq. (2) with $\alpha = \delta$ and $\beta = \gamma$,

$$|\tau\rangle_{12} = \alpha\big(|00\rangle + |11\rangle\big) + \beta\big(|01\rangle + |10\rangle\big) \tag{3}$$

where $|\alpha|^2 + |\beta|^2 = \frac{1}{2}$. She wants to send this state to a distant receiver Bob. The quantum channel linking Alice and Bob is a GHZ-*Like* state given by,

$$|\phi_G\rangle_{345} = \frac{1}{2}\big[|001\rangle + |010\rangle + |100\rangle + |111\rangle\big] \tag{4}$$

with Alice owning qubit 3 and qubits 4 and 5 belong to Bob. The combined state of the five qubit system is given by,

$$\begin{aligned}
|\Gamma\rangle_{12345} &= |\tau\rangle_{12} \otimes |\phi_G\rangle_{345} \\
&= \frac{1}{2}\alpha\big(|00001\rangle + |00010\rangle + |00100\rangle + |00111\rangle + |11001\rangle + |11010\rangle \\
&\quad + |11100\rangle + |11111\rangle\big) + \frac{1}{2}\beta\big(|01001\rangle + |01010\rangle + |01100\rangle + |01111\rangle \\
&\quad + |10001\rangle + |10010\rangle + |10100\rangle + |10111\rangle\big)_{12345}
\end{aligned} \tag{5}$$

Thereafter, Alice performs a three-qubit von-Neumann measurement on her qubits 1, 2 and 3 in the following basis,

$$\begin{aligned}
\big|\xi^{\pm}\big\rangle &= \frac{1}{2}\big[|001\rangle + |111\rangle \pm \big(|010\rangle + |100\rangle\big)\big] \\
\big|\eta^{\pm}\big\rangle &= \frac{1}{2}\big[|000\rangle + |110\rangle \pm \big(|011\rangle + |101\rangle\big)\big]
\end{aligned} \tag{6}$$

Alice then sends her measurement results via two bits of classical information to Bob and Bob applies suitable recovery operations to reconstruct the two qubit unknown state. Alice's measurement results and Bob's recovery operations are listed in Table 1.

Table 1 Strategy for recovering the two qubit state at Bob's end

Alice's measurement result	Classical information from Alice	Bob's state	Bob's operation
$\|\xi^+\rangle$	00	$\alpha(\|00\rangle+\|11\rangle)+\beta(\|01\rangle+\|10\rangle)$	$I\otimes I$
$\|\xi^-\rangle$	01	$\alpha(\|00\rangle+\|11\rangle)-\beta(\|01\rangle+\|10\rangle)$	$\sigma_z\otimes\sigma_z$
$\|\eta^+\rangle$	10	$\alpha(\|01\rangle+\|10\rangle)+\beta(\|00\rangle+\|11\rangle)$	$I\otimes\sigma_x$
$\|\eta^-\rangle$	11	$\alpha(\|01\rangle+\|10\rangle)-\beta(\|00\rangle+\|11\rangle)$	$\sigma_z\otimes\sigma_z\sigma_x$

In Tsai et al.'s [12] proposal, after measurement in the $\{+,-\}$ basis and Bell basis, Bob's state becomes a product state and Bob has to use a specific entangled unitary operation U_s to recover the EPR state. In contrast, in our case, Alice had to perform a single three qubit measurement and thereafter, Bob had to perform simple Pauli rotations to recover the state. Again, in [12], the classical communication required is 3 cbits while in our case it is 2 cbits.

3 Conclusion

Here we have shown that a more general form of two qubit state than that shown by Tsai et al. [12] can be teleported by using a GHZ-*Like* state. In [12], the measurement cost consist of a measurement in the diagonal basis followed by a measurement in the Bell basis. In our case, one has to perform a single three qubit measurement. The classical communication cost is also less in our case.

Acknowledgements The authors would like to thank the University Grants Commission of India, New Delhi for partially supporting this research under Faculty Development Programme [Grant No. F. TF.WC2-005-04/08-09 (ERO)].

References

1. Bennett, C.H., Brassard, G., Crépeau, C., Jozsa, R., Peres, A., Wootters, W.K.: Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. Phys. Rev. Lett. **70**, 1895 (1993)
2. Karlsson, A., Bourennane, M.: Quantum teleportation using three-particle entanglement. Phys. Rev. A **58**, 4394 (1998)
3. Pati, A.: Assisted cloning and orthogonal complementing of an unknown state. Phys. Rev. A **61**, 022308 (2000)
4. Agrawal, P., Pati, A.: Perfect teleportation and superdense coding with W states. Phys. Rev. A **74**, 062320 (2006)
5. Briegel, H.J., Raussendorf, R.: Persistent entanglement in arrays of interacting particles. Phys. Rev. Lett. **86**, 910 (2001)
6. Rigolin, G.: Quantum teleportation of an arbitrary two-qubit state and its relation to multipartite entanglement. Phys. Rev. A **71**, 032303 (2005)
7. Yeo, Y., Chua, W.K.: Teleportation and dense coding with genuine multipartite entanglement. Phys. Rev. Lett. **96**, 060502 (2006)
8. Muralidharan, S., Panigrahi, P.K.: Perfect teleportation, quantum-state sharing, and superdense coding through a genuinely entangled five-qubit state. Phys. Rev. A **77**, 032321 (2008)
9. Liu, J.-C., Li, Y.-H., Nie, Y.-Y.: Controlled teleportation of an arbitrary two-particle pure or mixed state by using a five-qubit cluster state. Int. J. Theor. Phys. **49**, 1976 (2010)
10. Li, Y.-H., Liu, J.-C., Nie, Y.-Y.: Quantum teleportation and quantum information splitting by using a genuinely entangled six-qubit state. Int. J. Theor. Phys. **49**, 2592 (2010)
11. Yang, K., Huang, L., Yang, W., Song, F.: Quantum teleportation via GHZ-like state. Int. J. Theor. Phys. **48**, 516 (2009)
12. Tsai, C.-W., Hwang, T.: Teleportation of a pure EPR state via GHZ-like state. Int. J. Theor. Phys. **49**, 1969 (2010)