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Geometric Phase of Two-Level Mixed State and Bloch Sphere Structure

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Abstract Geometric phase of two-level mixed state is investigated by comparing our approach with kinematic one. The results show that in the kinematic one, the Berry phases emerge discontinuous points because of losing a physical contribution from one of two eigenstates of density matrix. In contrast, our approach is a smooth curve of initial angle, evolving time and decay rate because of the interference between the two eigenstates with the probability ensemble.

Keywords Geometric phase · Mixed state · Bloch sphere

1 Introduction

The current quantum computation provides a wide range of challenges to quantum information [1–3], particularly for searching a quantum memory device in which allows temporal storage of quantum message in a long distance quantum information [4–6].

Geometric quantum computation is a potentially intrinsical fault tolerant scheme and therefore resilient to certain types of the experimental and fluctuation errors [7–10] because the geometric (Berry) phase is proportional to the area spanned in parameter space. The closed physical system retains a memory of its evolution in terms of the geometric phase [11–13], which makes that the geometric phase of pure state has an observable effect. Thus, it is interesting to investigate the geometric phase of mixed state.

Application of geometric phases in quantum computation has motivated one to include decoherent effect [14–26]. In a real situation, the physical system is inevitably affected by uncontrollable degrees of freedom in the environment. Differently from the pure state, the mixed state of the open system is always written in many different ways as a probabilistic mixture of distinct but not necessarily orthogonal pure states. Thus, the density matrix was introduced as an approach to describe the quantum open system and the state of the open

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system is not completely known. Up to now, therefore, the definition of the geometric phase for the open system is still a controversial issue. There have been many proposals tackling the problem from different generalizations of the parallel transport condition [12, 22]. In most of the cases, however, these definitions do not agree on account of different constraints imposed with different generalizations of the parallel transport condition. Thus, a comparison with these different approaches is useful for the study of the geometric phase of mixed state.

2 Representation of Mixed State

A mixed state density matrix $\rho(t)$ was introduced as a way of describing a quantum open system. Generally, the state for the open system can always be expressed in many different expansions as a classical probabilistic mixture of distinct but not necessarily orthogonal pure states. Such an expansion represents a distinct physical way in which is an ensemble of kinematically identical systems, characterized as a whole by the density matrix $\rho(t)$ with the synthesized properties,

$$\rho^+ = \rho, \quad \rho^2 < \rho, \quad \mathrm{Tr}\,\rho = 1. \tag{1}$$

In terms of a set of the normalized state vectors suitable for the description of pure states $|\psi_k(t)\rangle$ ($k = 1, 2, \ldots, N$, where N may be of finite or infinite dimensions) in a complex Hilbert subspace $\mathcal{H}_0 = \{|\psi_k(t)\rangle \in \mathcal{H}; \langle\psi_k(t)|\psi_k(t)\rangle = 1\}$, the density matrix can be expressed as

$$\rho(t) = \sum_{k=1}^{N} w_k^2(t)\big|\psi_k(t)\big\rangle\big\langle\psi_k(t)\big|, \tag{2}$$

where $0 \le w_k(t) \le 1$ are a set of classical probability amplitudes with the normalized condition,

$$\sum_{k=1}^{N} w_k^2(t) = 1. \tag{3}$$

Given a particular expansion (2) for the density matrix $\rho(t)$, an ensemble of a very large number of systems is consisted of a fraction $w_k^2(t)$ of which form a subensemble in the pure state $\rho_k(t) = |\psi_k(t)\rangle\langle\psi_k(t)|$. Thus the density matrix (2) can be rewritten as

$$\rho(t) = \sum_{k=1}^{N} w_k^2(t)\rho_k(t). \tag{4}$$

The average value of any hermitian operator $\mathcal{O}$ is given by

$$\langle\mathcal{O}\rangle = \sum_{k=1}^{N} w_k^2\langle\psi_k|\mathcal{O}|\psi_k\rangle = \sum_{k=1}^{N} w_k^2\,\mathrm{Tr}(\rho_k\mathcal{O}) = \mathrm{Tr}(\rho\mathcal{O}), \tag{5}$$

where only $\rho(t)$ appears, independently of a particular ensemble.

3 Bloch Sphere Structure

In our two-level system, the density matrix is a 2×2 matrix. Thus the unit matrix $\mathbf{1}_{2\times 2}$ and three Pauli matrices (σ_x, σ_y , and σ_z) construct a complete basis of our density matrix. Therefore, we can expand the density matrix as

$$\rho = a + \mathbf{b} \cdot \overrightarrow{\sigma}, \tag{6}$$

which is unique because of the following relations,

$$a = \frac{1}{2} \operatorname{Tr} \rho = \frac{1}{2}, \tag{7}$$

$$b_i = \frac{1}{2} \operatorname{Tr}(\rho \sigma_i), \quad i = x, y, z. \tag{8}$$

By inserting Eqs. (7) and (8) into Eq. (6), one finds

$$\rho = \frac{1}{2}(1 + \mathbf{n} \cdot \overrightarrow{\sigma}), \tag{9}$$

which defines a corresponding three-dimensional Bloch vector as

$$\mathbf{n} = \operatorname{Tr}(\rho \overrightarrow{\sigma}) = (n_x, n_y, n_z), \tag{10}$$

where

$$n_x = \operatorname{Tr}(\rho \sigma_x) = \rho_{12} + \rho_{21}, \tag{11}$$

which measures an overlap of real part between two-level open system,

$$n_y = \operatorname{Tr}(\rho \sigma_y) = i(\rho_{12} - \rho_{21}), \tag{12}$$

which measures an overlap of imaginary part, and

$$n_z = \operatorname{Tr}(\rho \sigma_z) = \rho_{11} - \rho_{22}, \tag{13}$$

which describes population inversion.

Inserting Eq. (9) into Eq. (1), one has

$$\mathbf{n}^* = \mathbf{n}, \qquad \mathbf{n} \cdot \mathbf{n} = r^2 \leq 1, \tag{14}$$

where r is called as a radius of the Bloch sphere with $r^2(t) = \mathbf{n} \cdot \mathbf{n} = n_x^2 + n_y^2 + n_z^2$.

From Eq. (14), we see that $\mathbf{n}$, its three components, and r are real. Therefore the Bloch vector along the tree pseudospin directions can be parameterized by two azimuthal angles, such as α and β. Next the two azimuthal angles are introduce as

$$\alpha = \cos^{-1} \frac{n_z}{r}, \qquad \beta = \tan^{-1} = \frac{n_y}{n_x}, \tag{15}$$

to parameterize the three-dimensional Bloch vector as

$$\mathbf{n} = (r \sin\alpha \cos\beta, r \sin\alpha \sin\beta, r \cos\alpha). \tag{16}$$

The set

$$\mathcal{S} = \left\{ \hat{\mathbf{n}} = \frac{\mathbf{n}}{r} \in \mathcal{R}^3 \middle| \hat{\mathbf{n}} \cdot \hat{\mathbf{n}} = 1, \hat{\mathbf{n}}^* = \hat{\mathbf{n}} \right\}, \tag{17}$$

leads to a unit Poincaré sphere construction. $\hat{\mathbf{n}} = \mathbf{n}/r$ corresponds to a diametrical point on the sphere surface $\mathcal{S}^2$, while $\mathbf{n} < \hat{\mathbf{n}}$ is the interior diametrical point on the corresponding Bloch sphere.

Using the two azimuthal angles α and β defined in Eq. (15), the eigenvectors and eigenvalues of $\hat{\mathbf{n}} \cdot \vec{\sigma} = \mathbf{n} \cdot \vec{\sigma}/r$ can be expressed as

$$\big|\chi_1(t)\big\rangle = e^{i\alpha_1(t)} \begin{pmatrix} \cos\frac{\alpha(t)}{2} \\ e^{i\beta(t)} \sin\frac{\alpha(t)}{2} \end{pmatrix}, \quad \chi_1 = 1, \tag{18}$$

$$\big|\chi_2(t)\big\rangle = e^{i\alpha_2(t)} \begin{pmatrix} \sin\frac{\alpha(t)}{2} \\ -e^{i\beta(t)} \cos\frac{\alpha(t)}{2} \end{pmatrix}, \quad \chi_2 = -1, \tag{19}$$

where $\alpha_1(t)$ and $\alpha_2(t)$ are the overall phase factors. It is interesting in noting that in the structure of Bloch sphere, the state vectors $|\chi_1(t)\rangle$ and $|\chi_2(t)\rangle$ are two orthogonal antipodal points in which lie on the azimuthal angles $\alpha(t)$ and $\beta(t)$.

According to these eigenvectors and eigenvalues in Eqs. (18) and (19), the density matrix of two-level open system is given by

$$\rho = \frac{1}{2}\big(1 + r(t)\big)\big|\chi_1(t)\big\rangle\big\langle\chi_1(t)\big| + \frac{1}{2}\big(1 - r(t)\big)\big|\chi_2(t)\big\rangle\big\langle\chi_2(t)\big|, \tag{20}$$

which is independent of the overall phase factors $\alpha_1(t)$ and $\alpha_2(t)$. From Eq. (20), the classical probabilities of the two-state mixture are given by

$$w_1^2(t) = \big(1 + r(t)\big)/2, \qquad w_2^2(t) = \big(1 - r(t)\big)/2, \tag{21}$$

with the normalized condition $w_1^2(t) + w_2^2(t) = 1$. We see that the classical probabilities are only a function of the Bloch radius $r(t)$, where $w_1^2(t)$ and $w_2^2(t)$ are just two eigenvalues of density matrix. Therefore, the Bloch radius geometrically describes the mixed degree of a quantum open two-level system. According to the probabilities $w_1^2(t)$ and $w_2^2(t)$, the surface points on the unit Poincaré sphere with $r(t) = 1$ identify with the pure states because of $w_1^2 = 1$ and $w_2^2 = 0$ so that the density matrix (20) becomes $\rho = |\chi_1(t)\rangle\langle\chi_1(t)|$, while the interior points in this sphere with $r(t) < 1$ are corresponding to the mixed states with $w_1^2 \neq 0$ and $w_2^2 \neq 0$ in which represent a classical mixture of two quantum states as shown in Eq. (20). The maximum mixed state is described by $r(t) = 0$ with $w_1^2 = 1/2$ and $w_2^2 = 1/2$.

4 Geometric Phase of Mixed State

From above analysis, we know that the pure and mixed states can be described by the structure of Bloch sphere in a unifying way, where the Bloch radius quantifies mixed degree of open system. Therefore, the geometric phase for the open system expressed in terms of geometric structures of Bloch sphere can avoid to find a proper generality of parallel transport condition.

According to a spinorial representation of mixed state in connecting the density matrix with mixed state vector as proposed by one of the authors [27], the Pancharatnam phase of mixed state is rewritten as

$$\begin{aligned}\gamma_g^P = {} & \frac{1}{N} \arg \sum_{k=1}^{N} \sqrt{\big(1 + br(t_0)\chi_k\big)\big(1 + br(t)\chi_k\big)}\big\langle\chi_k(t_0)\big|\chi_k(t)\big\rangle \\ & - \frac{1}{N} \Im \sum_{k=1}^{N} \int_{t_0}^{t} \big(1 + br(t)\chi_k\big)\big\langle\chi_k(t)\big|d\big|\chi_k(t)\big\rangle,\end{aligned} \tag{22}$$

where $|\chi_i(t)\rangle$ are parameterized by the Bloch azimuthal angles. Thus the geometric phase is fully expressed in terms of the geometric structure of the generalized Bloch sphere. In Eq. (22), the first term is a total phase and the second term is a dynamic phase. γ_g^P is a sum of contributions from all the eigenstates of density matrix in terms of the corresponding eigenvalues as the probabilities. Obviously, γ_g^P is a local $\bigotimes_{k=1}^N U(1)$ gauge invariant.

In the case of the two-level mixed state, $\chi_i(t)$ and $|\chi_i(t)\rangle$ are given by Eqs. (18) and (19). It is interesting to define a quasicyclitity T of mixed state in terms of $\phi(T)-\phi(t_0=0)=2\pi$. In this situation, the total phases from the first and second terms in Eq. (22) are constants 2π in which are not important and can be dropped off in quantum calculation. Thus the Berry phase of mixed state is given by

$$\gamma_g^B = -\frac{1}{2}\oint_0^T \big(1+r(t)\big)\sin^2\frac{\alpha(t)}{2}d\beta(t) - \frac{1}{2}\oint_0^T \big(1-r(t)\big)\cos^2\frac{\alpha(t)}{2}d\beta(t), \tag{23}$$

which is a symplectic area spanned by the evolving path in Bloch sphere with the time-dependent classical probabilities $w_1^2(t)=(1+r(t))/2$ and $w_2^2(t)=(1-r(t))/2$ from the two eigenvalues of the given density matrix, respectively.

In the kinematic approach to the Berry phase [28], the open system is purified as a pure state by combining the physical system with a ancilla. Thus the eigenfunctions of density matrix are directly taken as the state vectors of open system.

$$\gamma_g^K = \arg\left(\sum_k \sqrt{\lambda_k(0)\lambda_k(T)}\big\langle\lambda_k(0)\big|\lambda_k(T)\big\rangle \exp\left(\int_0^T dt\big\langle\lambda_k(t)\big|\frac{d}{dt}\big|\lambda_k(t)\big\rangle\right)\right), \tag{24}$$

where $\lambda_k(t)$ and $|\lambda_k(t)\rangle$ are eigenvalues and eigenfunctions of the reduced density matrix (2), respectively. In is noted that differently from our approach given by Eqs. (22) and (23), the probability ensemble $w_k=\sqrt{\lambda_k(0)\lambda_k(T)}$ are a product of the initial and final state eigenvalues so as not to depend on evolving time in the kinematic approach.

In terms of the representation (9) of Bloch sphere for the density matrix, the eigenvalues of density matrix may be expressed as

$$\lambda_1(t) = \frac{1}{2}\big(1-r(t)\big), \tag{25}$$

$$\lambda_2(t) = \frac{1}{2}\big(1+r(t)\big), \tag{26}$$

and the corresponding eigenvectors are given by

$$\big|\lambda_2(t)\big\rangle = \begin{pmatrix} n_z(t)+r(t) \\ n_x(t)+in_y(t) \end{pmatrix}, \tag{27}$$

$$\big|\lambda_1(t)\big\rangle = \begin{pmatrix} n_z(t)-r(t) \\ n_x(t)+in_y(t) \end{pmatrix}. \tag{28}$$

Inserting Eq. (16) into Eqs. (27) and (28), we find that the normalized eigenstates are exactly the same as Eqs. (18) and (19) besides an overall phase factor, i.e.,

$$\big|\lambda_i^R(t)\big\rangle = \frac{|\lambda_i(t)\rangle}{\langle\lambda_i(t)|\lambda_i(t)\rangle} = \big|\chi_i(t)\big\rangle, \quad i=1,2. \tag{29}$$

In the kinematic approach [28] to Berry phase of mixed state, however, an orthogonal pure initial state with the Bloch radius $r(0)=1$ was used so that $\lambda_1(0)=0$. Thus the Berry phase from the kinematic approach should be simplified as

$$\gamma_g^K = -\Im \int_0^T dt \langle \lambda_1^R(t) | \frac{d}{dt} | \lambda_1^R(t) \rangle$$
$$= -\frac{1}{2} \int_0^T \big(1 - \cos\alpha(t)\big) d\beta(t), \tag{30}$$

which includes only the contribution of eigenvector $|\lambda_1^R(t)\rangle$. Especially, the ensemble probabilities disappear in the kinematic approach to the Berry phase.

5 Mixed Sate in Fluctuating Magnetic Field

As an example, let us consider a two-level atom in the presence of an external magnetic field with a fluctuating component [29, 30]. The Hamiltonian of the system is

$$H(t) = H_0 + B(t)M, \tag{31}$$

where $H_0 = \frac{1}{2}\hbar\Omega\sigma_z$ with the atomic resonance frequency Ω, $B(t)$ is random field and $M = \frac{1}{2}\hbar\sigma_x$ is independent of time.

It is noted that the random field is a decoherence source in our system. After averaging on different trajectories induced by the noise, actually, the system at the end of the evolution is in a mixed state. Thus the Bloch vector does not return to its initial position since the Hamiltonian does not. To calculate the correct Bloch vector, we have to average the final positions [29, 30]. Thus, the master equation of reduced density matrix may be written as

$$\frac{d}{dt}\rho(t) = -i\big[H_0, \rho(t)\big] - \frac{\kappa}{2}\big[M(t), \big[M(t), \rho(t)\big]\big]. \tag{32}$$

The solution of Eq. (32) is direct. Under the initial condition $|\Psi(0)\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}|1\rangle$, the elements of the density matrix can be expressed by [29, 30]

$$\rho_{11} = \frac{1}{2}\big(1 + \cos\theta e^{-2\kappa t}\big), \tag{33}$$

$$\rho_{22} = \frac{1}{2}\big(1 - \cos\theta e^{-2\kappa t}\big), \tag{34}$$

$$\rho_{12} = \frac{1}{4}\sin\theta\left[-i\frac{\kappa}{\Omega_r} + \left(1 + \frac{\Omega}{\Omega_r}\right)\right]e^{-\kappa t - i\Omega_r t} + \frac{1}{4}\sin\theta\left[i\frac{\kappa}{\Omega_r} + \left(1 - \frac{\Omega}{\Omega_r}\right)\right]e^{-\kappa t + i\Omega_r t}, \tag{35}$$

$$\rho_{21} = \frac{1}{4}\sin\theta\left[-i\frac{\kappa}{\Omega_r} + \left(1 - \frac{\Omega}{\Omega_r}\right)\right]e^{-\kappa t - i\Omega_r t} + \frac{1}{4}\sin\theta\left[i\frac{\kappa}{\Omega_r} + \left(1 + \frac{\Omega}{\Omega_r}\right)\right]e^{-\kappa t + i\Omega_r t}, \tag{36}$$

where $\Omega_r = \sqrt{\Omega^2 - \kappa^2}$.

Inserting Eqs. (33)–(37) into Eqs. (14) and (15), the corresponding Bloch parameters are obtained by

$$r^2(t) = \sin^2\theta\left(\left(\cos\Omega_r t - \frac{\kappa}{\Omega_r}\sin\Omega_r t\right)^2 + \frac{\Omega^2}{\Omega_r^2}\sin^2\Omega_r t\right)e^{-2\kappa t} + \cos^2\theta e^{-4\kappa t}, \tag{37}$$

$$\cos\alpha = \frac{\cos\theta}{r}e^{-2\kappa t}, \tag{38}$$

$$\tan\beta = \frac{\Omega\sin\Omega_r t}{\Omega_r(\cos\Omega_r t - \frac{\kappa}{\Omega_r}\sin\Omega_r t)}. \tag{39}$$

Fig. 1 The kinematic approach to the Berry phase of mixed state as functions of the fluctuating rate κ/Ω and initial angle θ

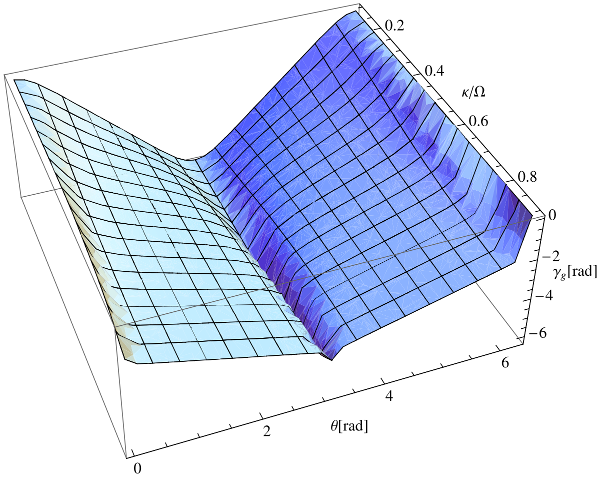
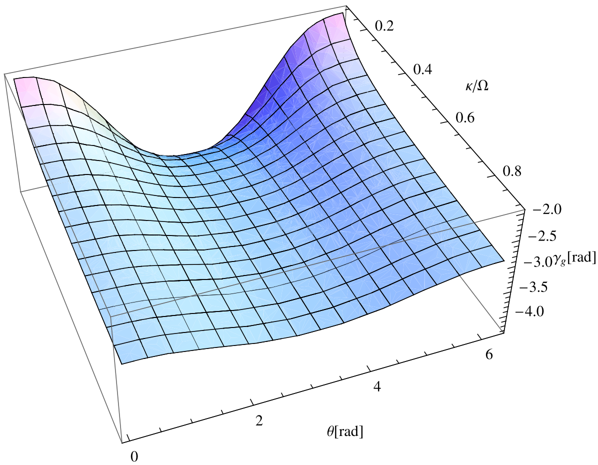


Fig. 2 Berry phase of mixed state as functions of the fluctuating rate κ/Ω and initial angle θ



From Eqs. (37)–(39), we see that the Bloch parameters, such as r, α and β, depend on the fluctuating rate and oscillate with the evolving time. Especially, β doesn't depend on the exponential decay. Thus, both the complex oscillations and exponential decay with the evolving time will be included in the nondiagonal elements of density matrix. Under the situation of the fluctuating parameter $\kappa = 0$, the oscillations become simple with the frequency Ω relating only to the magnetic field.

6 Discussions and Conclusions

The Berry phases of mixed state are shown in Fig. 1 for the kinematic approach and Fig. 2 for our approach. The results show that the Berry phases decrease with increasing of the fluctuating rate κ/Ω just like the radius $r(t)$ of Bloch sphere shown in Figs. 3 and 4.

It is noted that the behaviors of Berry phase as a function of initial angle are very different for our approach (see Fig. 2) and the kinematic approach (see Fig. 1). In our approach, the

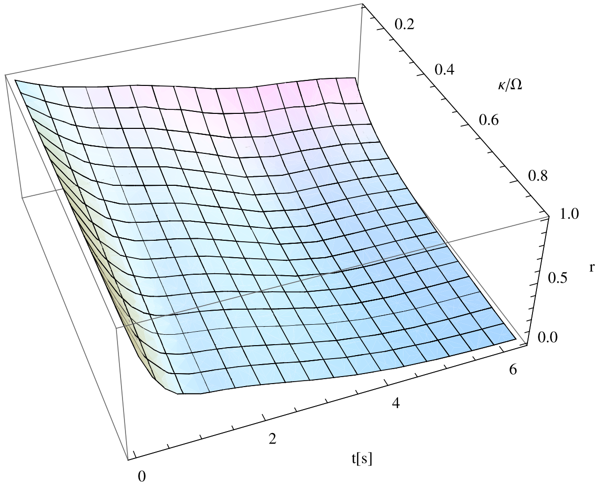


Fig. 3 The radius of Bloch sphere as functions of the evolving time t and the fluctuating rate κ/Ω in the case $\theta=\pi/4$

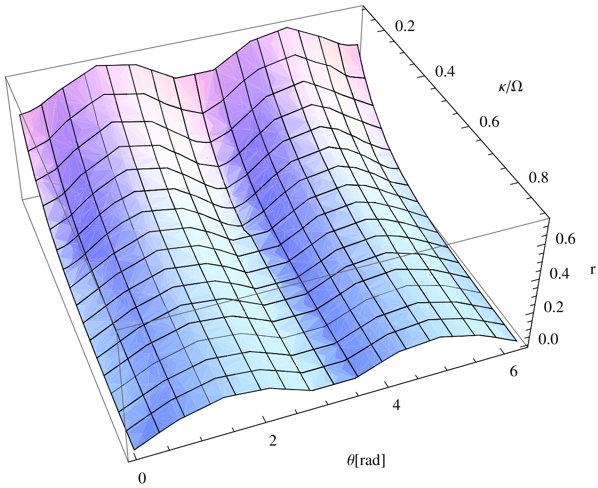


Fig. 4 The radius of Bloch sphere as functions of the initial angle θ and the fluctuating rate κ/Ω in the case $t=\pi[s]$

Berry phase decreases smoothly in the region $\theta \in [0,\pi]$ and then increases in the region $\theta \in [\pi, 2\pi]$, which is similar to the behavior of the radius $r(t)$ of Bloch sphere (see Fig. 4). In the kinematic approach, however, the discontinuous point of Berry phase is emerged at $\theta=\pi$. From Figs. 3 and 4, we see that the radius $r(t)$ of Bloch sphere is smooth and continuous functions of evolving time, initial angle and fluctuating rate. Therefore, the discontinuous property is difficult to be explained by a physical reason.

By comparing Eq. (30) from our approach with Eq. (23) from the kinematic approach, we find that the kinematic approach includes only the contribution of one eigenstate of density matrix similarly to the pure state, which may lose the physical content from another eigenstate. In our approach, the contributions of two eigenstates are exactly includes in the Berry phase of mixed state in terms of the ensemble probabilities. Because of the interference between the two eigenstates, the discontinuous point disappears in our approach to the Berry phase of mixed state.

Such a geometric phase factor can in principle be measured by interfering the atomic system which has undergone the above evolution with a coherent atomic system that did not evolve. An possible way to determine the correctness of definition for the geometric phase of mixed state can be provided by an analogous of the well-known single-photon Mach-Zehnder interferometer, which have already been reported in many Refs. [31, 32].

In summary, the geometric phase of mixed state is investigated in terms of Bloch sphere structure. We find that the kinematic approach does not include the contribution from one of two eigenstate of density matrix with the eigenvalue $(1-r(t))/2$. The results lead to the discontinuous geometric phase. In our approach, the geometric phase of mixed state is a smooth function of the initial angle.

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