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The Logic of Quantum Measurements

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Abstract We apply our previously developed formalism of contexts of histories, suitable to deal with quantum properties at different times, to the measurement process. We explore the logical implications which are allowed by the quantum theory, about the realization of properties of the microscopic measured system, before and after the measurement process with a given pointer value.

Keywords Quantum measurements · Quantum histories · Quantum interpretations

1 Introduction

Under the influence of the views of Bohr, during much time quantum mechanics was conceived as a theory for the description of the microscopic world, whereas the macroscopic phenomena were considered the realm of classical theories. This limitation in the application domain of quantum mechanics was designed to save the theory from the contradictions with common sense, constructed by the human beings from the experience on the macroscopic world and organized according to the classical theories. This approach faced the problem of where to place the boundary between the quantum and the classical worlds. A further problem derived from that position was to understand why it was not possible to apply quantum mechanics to a macroscopic world constituted by small particles obeying the quantum laws.

It was recently proved that the properties represented in classical mechanics by domains of the phase space with regular boundaries and volumes much greater than the Planck con-

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stant can also be represented in the quantum theory by projection operators [1]. Another important result is that, in most cases, the quantum description of the interaction between the relevant macroscopic variables and the huge number of microscopic variables of a macroscopic physical system leads to the approximated coincidence between the quantum and the classical statistics for the macroscopic relevant variables [2, 3]. These theoretical results point toward the understanding of the cinematic, dynamic and logic of the macroscopic world as a special application of quantum mechanics. From this point of view, the measurement process is an interaction between a microscopic system and a special macroscopic device, which should be fully described by quantum theory [4–6].

The quantum description of the measurement process should provide the logical relations between the pointer readings of the measuring instrument when the measurement ends and certain properties of the microscopic system before the measurement [5, 7, 8]. Therefore, the conjunction of properties at different times, for instance, the value of an observable of a microscopic system before the measurement *and* a pointer position of the measuring instrument after the measurement, should be part of the universe of the discourse about the composite system consisting of the microscopic system to be measured and the macroscopic instrument. For a given state of the composite system, if not the truth value, at least the probability of these conjunctions of properties at different times should be obtained from the theory.

Starting from the notion of time translation of quantum properties, we developed a formalism that, by extending the usual notion of *context* in quantum mechanics, is capable of dealing with descriptions and reasonings involving properties at different times [9, 10]. The probabilities of the properties involving different times are obtained by the Born rule. The formalism was developed in reference [10], where it was also compared with the theory of consistent histories [11–13].

In this work, we will apply our formalism to the quantum measurement process. With the notion of *context of histories*, we will study what can be said about the microscopic system when the measuring instrument shows a given position of its pointer observable. In Sect. 2 we introduce a brief summary of our formalism of generalized contexts. In Sect. 3 we analyze a non-ideal measurement, and obtain the implications of a given pointer value on the properties of the microscopic system before the measurement. The conclusions are summarized in Sect. 4.

2 Contexts of Histories

For the sake of completeness we present in this section a brief summary of our formalism of generalized contexts [9, 10].

Let us represent a quantum property p at time t by the pair $(p;t)$, or by $(\Pi_p;t)$, where Π_p is the projector representing the property p in the Hilbert space $\mathcal{H}$ of the system. The time translation of the property p at time t to time t' is defined by the pair $(p';t')$, or by $(\Pi_{p'};t')$, where p' is the quantum property represented by $\Pi_{p'} \equiv U(t',t)\Pi_p U^{-1}(t',t)$. The unitary operator $U(t',t) = \exp(-iH(t'-t)/\hbar)$ is the time evolution operator generated by the Hamiltonian operator H of the system. The relation between time translated pairs is transitive, reflexive and symmetric and, therefore, it is an equivalence relation. We use $[(p;t)]$ (or $[(\Pi_p;t)]$) to name the class of pairs equivalent to $(p;t)$ (or to $(\Pi_p;t)$). It is interesting to note that the Born rule assigns the same probability to all the pairs of the same equivalence class in a given state, i.e.

$$\Pr(p;t) = \mathrm{Tr}(\rho_t \Pi_p) = \mathrm{Tr}(\rho_{t'} \Pi_{p'}) = \Pr\bigl(p';t'\bigr) = \Pr\bigl[(p;t)\bigr].$$

By definition, the equivalence class $[(\Pi^{(1)}; t_1)]$ *implies* the equivalence class $[(\Pi^{(2)}; t_2)]$ if the representative elements of the classes at a common time t_0 verify the implication of the usual formalism of quantum mechanics, i.e.

$$\Pi^{(1,0)}\mathcal{H} \subset \Pi^{(2,0)}\mathcal{H},$$

where

$$\begin{aligned} \Pi^{(1,0)} &\equiv U(t_0, t_1)\Pi^{(1)}U^{-1}(t_0, t_1), \\ \Pi^{(2,0)} &\equiv U(t_0, t_2)\Pi^{(2)}U^{-1}(t_0, t_2). \end{aligned} \tag{1}$$

The two properties represented by projectors $\Pi^{(1)}$ and $\Pi^{(2)}$ at the times t_1 and t_2 transform into two new properties represented by the projectors $\Pi^{(1,0)}$ and $\Pi^{(2,0)}$ at a common time t_0. The meaning of the previous equations is that the property represented by $\Pi^{(1,0)}$ imply the property represented by $\Pi^{(2,0)}$ in the usual sense of the inclusion of the corresponding Hilbert subspaces.

The *conjunction* (*disjunction*) of two classes $[(\Pi; t)]$ and $[(\Pi'; t')]$ can be obtained as the greatest lower (least upper) bound, i.e.

$$\begin{aligned} \big[(\Pi; t)\big] \wedge \big[\big(\Pi'; t'\big)\big] &= \mathrm{Inf}\big\{\big[(\Pi; t)\big], \big[\big(\Pi'; t'\big)\big]\big\} \\ \big[(\Pi; t)\big] \vee \big[\big(\Pi'; t'\big)\big] &= \mathrm{Sup}\big\{\big[(\Pi; t)\big], \big[\big(\Pi'; t'\big)\big]\big\}. \end{aligned} \tag{2}$$

The *negation* of an equivalence class $[(\Pi; t)]$ is defined by

$$\overline{\big[(\Pi; t)\big]} = \big[(\overline{\Pi}; t)\big] = \big[\big((I - \Pi); t\big)\big].$$

With the implication, disjunction, conjunction and negation previously obtained, the set of equivalence classes has the structure of an orthocomplemented nondistributive lattice.

The usual concept of context is a subset of all possible simultaneous properties which can be organized as a meaningful description of a quantum system at a given time, and can be endowed with a boolean logic with well-defined probabilities. Our formalism supplies a prescription to obtain, from the nondistributive lattice of equivalence classes of pairs, the valid descriptions involving properties at different times, which we called *generalized contexts* or *contexts of histories*.

Let us consider a context of properties at time t_1, generated by atomic properties $p_j^{(1)}$ represented by projectors $\Pi_j^{(1)}$ verifying

$$\Pi_i^{(1)}\Pi_j^{(1)} = \delta_{ij}\Pi_i^{(1)}, \qquad \sum_{j\in\sigma^{(1)}} \Pi_j^{(1)} = I, \quad i, j \in \sigma^{(1)}.$$

Let us also consider a context of properties at time t_2, generated by atomic properties $p_\mu^{(2)}$ represented by projectors $\Pi_\mu^{(2)}$ verifying

$$\Pi_\mu^{(2)}\Pi_\nu^{(2)} = \delta_{\mu\nu}\Pi_\mu^{(2)}, \qquad \sum_{\mu\in\sigma^{(2)}} \Pi_\mu^{(2)} = I, \quad \mu, \nu \in \sigma^{(2)}.$$

We wish to represent with our formalism a universe of discourse able to incorporate expressions like "the property $p_j^{(1)}$ at time t_1 *and* the property $p_\mu^{(2)}$ at time t_2". The conjunction of the classes with representative elements $\Pi_i^{(1)}$ at t_1 and $\Pi_\mu^{(2)}$ at t_2 is also the conjunction of the classes with representative elements $\Pi_i^{(1,0)} \equiv U(t_0,t_1)\Pi_i^{(1)}U^{-1}(t_0,t_1)$ and $\Pi_\mu^{(2,0)} \equiv U(t_0,t_2)\Pi_\mu^{(2)}U^{-1}(t_0,t_2)$ at the common time t_0.

In usual quantum theory the conjunction of simultaneous properties represented by non-commuting operators has no meaning. So, it seems natural to consider quantum descriptions of a system, involving the properties generated by the projectors $\Pi_i^{(1)}$ at time t_1 and $\Pi_\mu^{(2)}$ at time t_2, *only for the cases* in which the projectors $\Pi_i^{(1)}$ and $\Pi_\mu^{(2)}$ commute when translated to a common time t_0, i.e.

$$\Pi_i^{(1,0)}\Pi_\mu^{(2,0)} - \Pi_\mu^{(2,0)}\Pi_i^{(1,0)} = 0.$$

If this is the case, for the equivalence class of composite properties representing "the property $p_j^{(1)}$ at time t_1 *and* the property $p_\mu^{(2)}$ at time t_2" we obtain

$$h_{i\mu} = \big[\big(\Pi_i^{(1)};t_1\big)\big] \wedge \big[\big(\Pi_\mu^{(2)};t_2\big)\big] = \Big[\Big(\lim_{n\to\infty}\big(\Pi_i^{(1,0)}\Pi_\mu^{(2,0)}\big)^n;t_0\Big)\Big] = \big[\big(\Pi_i^{(1,0)}\Pi_\mu^{(2,0)};t_0\big)\big]. \quad (3)$$

As we can see, the conjunction of properties at different times t_1 and t_2 is equivalent to a single property, represented by the projector $\Pi_{i\mu}^{(0)} \equiv \Pi_i^{(1,0)}\Pi_\mu^{(2,0)}$ at a single time t_0.

If the different contexts at times t_1 and t_2 produce commuting projectors $\Pi_i^{(1,0)}$ and $\Pi_\mu^{(2,0)}$ at the common time t_0, it is easy to prove that

$$\Pi_{i\mu}^{(0)}\Pi_{j\nu}^{(0)} = \delta_{ij}\delta_{\mu\nu}\Pi_{i\mu}^{(0)}, \qquad \sum_{i\mu}\Pi_{i\mu}^{(0)} = I.$$

Therefore, we realize that the composite properties $h_{i\mu}$, represented at time t_0 by the complete and exclusive set of projectors $\Pi_{i\mu}^{(0)}$, can be interpreted as the atomic properties generating a usual context in the sense described above. More general properties are obtained from the atomic ones by means of the disjunction operation. For instance, we can represent the property "$p_j^{(1)}$ at time t_1 *and* $p_\mu^{(2)}$ at time t_2, with j and μ having any value in the subsets $\Delta^{(1)} \subset \sigma^{(1)}$ and $\Delta^{(2)} \subset \sigma^{(2)}$", as

$$h_{\Delta^{(1)},\Delta^{(2)}} = \Big[\Big(\sum_{i\in\Delta^{(1)}}\sum_{\mu\in\Delta^{(2)}}\Pi_{i\mu}^{(0)};t_0\Big)\Big].$$

The set of properties obtained in this way is an orthocomplemented and distributive lattice.

If the state of the system at time t_0 is represented by ρ_{t_0}, the Born rule gives the following expression for the probability of the class of properties $h_{\Delta^{(1)},\Delta^{(2)}}$,

$$\Pr(h_{\Delta^{(1)},\Delta^{(2)}}) = \sum_{i\in\Delta^{(1)}}\sum_{\mu\in\Delta^{(2)}}\mathrm{Tr}\big(\rho_{t_0}\Pi_{i\mu}^{(0)}\big).$$

As a natural extension of the notion of context, we *postulate* that a description of a physical system involving properties at two different times t_1 and t_2 is valid if these properties are represented by commuting projectors when they are translated to a single time t_0. We will call each one of those valid descriptions *context of histories*. On each context of histories, the probabilities given by the Born rule are well-defined (i.e. they are positive, normalized and additive) and, therefore, they may be meaningful in terms of frequencies.

In summary, our formalism is based on the notion of time-translation, allowing to transform the properties at a sequence of different times into properties at a single common time. A usual context of properties is first considered for each time of the sequence. If the projectors representing the atomic properties of each context commute when they are translated to a common time, the contexts at different times can be organized in a context of histories. A context of histories is a distributive and orthocomplemented lattice, a boolean logic with well-defined implication, negation, conjunction and disjunction. This logic can be used to speak and make inferences about the selected properties of the system at different times. Well-defined probabilities on the elements of the lattice of properties are obtained by means of the well-known Born rule.

In the usual formalism of quantum mechanics, the order relation $p_1 \leq p_2$ on two quantum properties is represented by the inclusion of the corresponding Hilbert subspaces ($\mathcal{H}_{p_1} \subset \mathcal{H}_{p_2}$, $\mathcal{H}_{p_1} = \Pi_{p_1}\mathcal{H}$, $\mathcal{H}_{p_2} = \Pi_{p_2}\mathcal{H}$). If the two properties belong to the same context, and they are considered at the same time t, the implication corresponds to the conditional probability

$$\Pr(p_2 \mid p_1) = \frac{\Pr(p_2 \wedge p_1)}{\Pr(p_1)} = \frac{\mathrm{Tr}(\rho_t \Pi_{p_2} \Pi_{p_1})}{\mathrm{Tr}(\rho_t \Pi_{p_1})} = \frac{\mathrm{Tr}(\rho_t \Pi_{p_1})}{\mathrm{Tr}(\rho_t \Pi_{p_1})} = 1,$$

where ρ_t is the state of the system at time t. We can give a physical interpretation to the last equation by saying that *if property p_1 is realized at time t, then property p_2 is also realized at the same time*.

Quantum histories give a meaning to the implication between properties at different times. If $[(\Pi^{(1)}; t_1)] \leq [(\Pi^{(2)}; t_2)]$, and both classes belong to the same context of histories, we have

$$\begin{aligned}
\Pr\big(\big[\big(\Pi^{(2)}; t_2\big)\big] \,\big|\, \big[\big(\Pi^{(1)}; t_1\big)\big]\big) &\equiv \frac{\Pr([(\Pi^{(2)}; t_2)] \wedge [(\Pi^{(1)}; t_1)])}{\Pr([(\Pi^{(1)}; t_1)])} \\
&= \frac{\Pr([(\Pi^{(2,0)}; t_0)] \wedge [(\Pi^{(1,0)}; t_0)])}{\Pr([(\Pi^{(1,0)}; t_0)])} \\
&= \frac{\Pr([(\Pi^{(2,0)}\Pi^{(1,0)}; t_0)])}{\Pr([(\Pi^{(1,0)}; t_0)])} = \frac{\Pr([(\Pi^{(1,0)}; t_0)])}{\Pr([(\Pi^{(1,0)}; t_0)])} = 1,
\end{aligned}$$

where $\Pi^{(1,0)}$ and $\Pi^{(2,0)}$ have been defined in (1). The physical interpretation is that *if the property represented by $\Pi^{(1)}$ is realized at the time t_1, then the property represented by $\Pi^{(2)}$ is realized at the time t_2.*

3 Context of Histories for a Measurement Process

In this section we will consider a measurement process where a microscopic system S interacts with a measuring instrument M. The possible states of the composite system $S + M$ are represented in a Hilbert space $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_M$. We assume that the interaction between the systems S and M begins at time t_1 and ends at time t_2. We also assume that the microscopic system S has an observable Q represented by the self-adjoint operator Q, with eigenvalues q_j ($j \in \sigma^{(S)}$), and that the measuring instrument has a macroscopic observable represented by an operator A with eigenvalues a_i ($i \in \sigma^{(A)}$, $\sigma^{(S)} \subset \sigma^{(A)}$).

Any vector of the space $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_M$ can be written as a linear combination of the orthonormal vectors $|q_j; a_i, r\rangle = |q_j\rangle \otimes |a_i, r\rangle$, where the indexes r in the vector $|a_i, r\rangle \in$

$\mathcal{H}_M$ correspond to the huge number of microscopic variables of the measuring instrument, different from the pointer observable a_i.

We consider the case of a non-ideal (or type II) measurement of the Q observable, where the system-apparatus interaction is represented by a unitary transformation $U(t_2, t_1)$ satisfying

$$|q_j\rangle|a_0, r\rangle \to U(t_2, t_1)|q_j\rangle|a_0, r\rangle = \sum_{j'r'} C_{rr'jj'}|q_{j'}\rangle|a_j, r'\rangle, \tag{4}$$

where a_0 represents the reference value of the pointer observable before the measurement process [15]. This last equation expresses the requirements that must satisfy the evolution operator $U(t_2, t_1)$ for the calibration of measurement [14]. There is a correlation between value q_j of the microscopic observable Q at the time t_1 and the value a_j of the pointer observable at a later time t_2. The sums over j' and r' indicate that the observable Q and the variable r do not have well-defined values after the measurement process starting from the pure state $|q_j; a_0, r\rangle$.

In the general case, the state $|\varphi\rangle$ of the microscopic system S previous to the measurement is not an eigenstate of the observable Q, and we have

$$|\varphi\rangle|a_0, r\rangle = \sum_j c_j|q_j\rangle|a_0, r\rangle \to U(t_2, t_1)\sum_j c_j|q_j\rangle|a_0, r\rangle = \sum_j c_j \sum_{j'r'} C_{rr'jj'}|q_{j'}\rangle|a_j, r'\rangle. \tag{5}$$

In what follows we are going to define a possible universe of discourse, i.e. a context of histories involving relevant properties before and after the measurement process. As we showed in Sect. 2, a context of histories can be obtained from two ordinary contexts at the times t_1 and t_2, provided the atomic properties of this two context are compatible, i.e. they are represented by commuting projectors when translated to a common time.

A relevant context at the time t_1 should include the possible values q_j of the microscopic observable. We are going to choose an ordinary context at t_1 generated by the atomic properties q_j, represented by the following projectors on the total Hilbert space $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_M$

$$\Pi_{q_j} \equiv |q_j\rangle\langle q_j| \otimes \sum_{jr} |a_j, r\rangle\langle a_j, r|, \quad j \in \sigma^{(S)}. \tag{6}$$

These properties are exclusive ($\Pi_{q_j}\Pi_{q_i} = 0$ if $q_j \neq q_i$) and exhaustive ($\sum_{q_j} \Pi_{q_j} = I_S \otimes I_M$). They are *atomic properties* (i.e. directly connected with the zero element in the Hasse diagram). The set of all possible disjunctions obtained from these atomic properties is a distributive lattice of properties, a context for the time t_1. We will say that the atomic properties *generate* the context.

For the time t_2 our interest is about the possible final values a_i of the pointer observable A. Therefore, we are going to consider an ordinary context generated by all the possible values of the pointer positions, represented by the projectors

$$\Pi_{a_i} \equiv I_S \otimes \sum_r |a_i, r\rangle\langle a_i, r|, \quad i \in \sigma^{(A)}. \tag{7}$$

The measurement process is always started with the pointer indicating the reference value a_0. Therefore, to apply our formalism of section II, we only need to verify that the generators of the two ordinary contexts commute when they are time translated to the common time t_1, and acting on vectors of the form $|\varphi\rangle|a_0, r\rangle$.

Using (4), (6) and (7), it is easy to prove that

$$\Pi_{q_j}\big(U^{-1}\Pi_{a_i}U\big)|\varphi\rangle|a_0,r\rangle = \big(U^{-1}\Pi_{a_i}U\big)\Pi_{q_j}|\varphi\rangle|a_0,r\rangle = \delta_{ij}\Pi_{q_j}|\varphi\rangle|a_0,r\rangle, \quad U\equiv U(t_2,t_1). \tag{8}$$

The projectors $(U^{-1}\Pi_{a_i}U)$, representing the translation to time t_1 of the relevant properties at time t_2, commute with the projectors Π_{q_j}, representing the relevant properties at time t_1. Therefore, as we showed in (3), it is possible to have a context of histories generated by the atomic properties q_j *at time* t_1, *together with* a_i *at the time* t_2. These properties will be denoted by $[(q_j;t_1)]\wedge[(a_i;t_2)]$, and they are represented by the following equivalence classes of pairs projector-time,

$$h_{q_j;a_i}\equiv\big[(\Pi_{q_j};t_1)\big]\wedge\big[(\Pi_{a_i};t_2)\big]=\big[\big(\Pi_{q_j}U^{-1}\Pi_{a_i}U;t_1\big)\big], \quad j\in\sigma^{(S)},\ i\in\sigma^{(A)}.$$

By disjunction of these atomic histories, we obtain a distributive lattice of properties involving both times t_1 and t_2. This two-times *context of histories* is a possible universe of discourse about the measurement process. Now we have a logical structure for the properties we can legitimately talk about, according with quantum mechanics.

If we sum over the index j in (8) we obtain $(U^{-1}\Pi_{a_i}U)|\varphi\rangle|a_0,r\rangle=\Pi_{q_i}|\varphi\rangle|a_0,r\rangle$, and

$$\big[(a_i,t_2)\big]=\big[(q_i,t_1)\big]. \tag{9}$$

Therefore

$$\Pr\big\{\big[(a_i,t_2)\big]\big\}=\Pr\big\{\big[(q_i,t_1)\big]\big\}. \tag{10}$$

This equation gives us information about the values of the observable Q before the measurement process in terms of the readings of the pointer observable A. By repeating the measurement procedure several times, always with the same preparation of the microscopic system S, and always with the pointer indicating a_0 at the beginning of the measurement process, it is possible to obtain the relative frequency of the value a_i of the pointer observable at time t_2. If this relative frequency is identified with the probability $\Pr\{[(a_i,t_2)]\}$ on the left hand side of (10), the right hand side $\Pr\{[(q_i,t_1)]\}$ of this equation turns out to be the relative frequency for the value q_i of the observable Q in the microscopic system S, at the time t_1. Let us emphasize that this microscopic relative frequency is indirectly obtained through (10) from the empirical information about the macroscopic frequency of the pointer value a_i.

From (9) we obtain the following conditional probability

$$\Pr\big\{\big[(a_i,t_2)\big]\,\big|\,\big[(q_i,t_1)\big]\big\}=\frac{\Pr\{[(a_i,t_2)]\wedge[(q_i,t_1)]\}}{\Pr\{[(q_i,t_1)]\}}=\frac{\Pr\{[(q_i,t_1)]\}}{\Pr\{[(q_i,t_1)]\}}=1. \tag{11}$$

As we discussed at the end of the previous section, we can give a direct physical interpretation to this result: *if the value* q_i *can be assigned to the observable* Q *of the microscopic system* S *at the time* t_1, *then the value* a_i *can be assigned to the pointer observable of the measurement instrument at the later time* t_2. Equation (11) allows the *prediction* of the pointer position after the measurement process, and it gives validity to the calibration process as described in [14]. Although it is not a new result, it is now recovered within the logical framework of a context of histories, involving properties at different times which are relevant to the measurement process.

We stress that the conjunction $\wedge$ in (11) would not be correct if the properties (a_i, t_2) and (q_i, t_1) do not belong to the same context of histories.

From (9) we also obtain

$$\Pr\{[(q_j, t_1)] \mid [(a_j; t_2)]\} \equiv \frac{\Pr\{[(q_j, t_1)] \wedge [(a_j; t_2)]\}}{\Pr\{[(a_j; t_2)]\}} = \frac{\Pr\{[(a_j; t_2)]\}}{\Pr\{[(a_j; t_2)]\}} = 1. \tag{12}$$

This equation provides the possibility of *retrodiction* of previous values of properties of the microscopic system from the later measurement results. The physical interpretation of (12) is that *if the value a_j can be assigned to the pointer observable at the time t_2, then the value q_j can be assigned to the observable Q of the microscopic system at the time t_1 before the measurement process started.*

The interpretation given to the conditional probabilities of (11) and (12) must be analyzed carefully because it call attention to the conceptual difficulties that are characteristic of quantum mechanics. The problem that arises here is how to consider the assignment of a value for the observable Q at the time t_1 previous to the measurement, if at this time the system was in the state $|\varphi\rangle|a_0, r\rangle = \sum_j c_j|q_j\rangle|a_0, r\rangle$, a superposition of eigenstates of Q. Moreover, the interpretation of (11) and (12) involve the assignment of a value for the pointer observable at the time t_2, but the unitary time evolution gives at t_2 a state with a superposition of different pointer values, as shown in (5).

To circumvent this problem we must accept that the attribution of value of the variable Q can not be understood as an attribution completely objective, but as an "effective" assignment that is not independent of the given experimental conditions.

This is just a manifestation of the entanglement between system and measuring instrument. In the context of histories formalism, which led us to (12), the experimental setup is implicit in the fact that the conditional is defined under the particular time evolution determined by the measurement. The properties q_j at t_1 and a_j at t_2 are not independent. They are linked to each other by the measurement.

Through the formalism of contextual histories, and as a result of the entanglement between system and the instrument, we conclude that the assignment of the value q_j at the time t_1 is valid provided it is performed by recording the value of a_j in the pointer at the later time t_2. That is, *the assignment of value of variable Q makes sense only through the result obtained in the measurement of the variable Q.*

The relations we have obtained are understood in terms of properties, which can be translated into propositions only through some certainties obtained from probability calculations, as in (11) and (12). In this sense, the context of histories formalism fits naturally into the set of modal interpretations [16, 17], because it gives properties as potentialities, independently of their realization.

When the histories are constructed with properties of the system and the instrument, before and after the measurement, the context of histories formalism allows to establish a link between the possible and actual: possible values in the microscopic system with actualized values in the instrument. As the context of histories formalism is faithful to quantum mechanics, this link is established through probability calculations.

4 Conclusions

While ideal measurements are widely used to discuss quantum measurements, it is rarely the case in a real situation. In most of the cases the measured state is modified by the measurement. for a non ideal measurement, the correlation is stablished between a property of the system before the measurement process and the pointer position after the measurement.

Choosing properties of the system before measurement, and properties of the instrument after the measurement, the contexts of histories formalism has allowed us to establish the logical structure of properties behind the measurement process, which defines a logical structure for the possibilities, independent of the realizations. This approach places the contexts of histories according to a modal interpretation of the quantum mechanic.

Using the certainties obtained from the calculation of some conditional probabilities, the possible becomes related with the actual. As contextual histories enables us to deal with properties at different times, this link is possible even when the process corresponds to a non-ideal measurement, i.e. when no information is preserved on the state of the system. Since the formalism includes properties at different times, it is not necessary to maintain information until a later time by an ideal measurement.

The obtained results are based on the biconditional $[(a_j, t_2)] \Leftrightarrow [(q_j, t_1)]$. If we decompose the biconditional, we have the relationship given by $[(q_j, t_1)] \Rightarrow [(a_j, t_2)]$. This relationship allows the prediction, and it is the more immediate because it defines the calibration of the measurement process. The reciprocal relation $[(a_j, t_2)] \Rightarrow [(q_j, t_1)]$ which allows retrodiction is more delicate: it assigns a value for the measured variable even when the system is in a superposition of states with different values of the variable. We understand that it is an *effective* assignment, because it makes sense only if it is realized through of the corresponding pointer value after measurement. It is an expression of the entanglement determined by the measurement process: the relations between $[(a_j, t_2)]$ and $[(q_j, t_1)]$, are determined by the same temporal evolution of the measurement process defining these properties.

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