Int J Theor Phys (2013) 52:1362–1369
DOI 10.1007/s10773-012-1454-6

Two Fluid Scenario for Dark Energy Model in a Scalar-Tensor Theory of Gravitation

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Received: 29 July 2012 / Accepted: 13 December 2012 / Published online: 30 December 2012

Abstract In this paper, we investigate the evolution of dark energy parameter in the spatially homogeneous and isotropic Friedmann-Robertson-Walker (FRW) model filled with barotropic fluid and dark energy in the framework of scalar-tensor theory of gravitation formulated by Saez and Ballester (Phys. Lett. A 113:467, 1986). To obtain a determinate solution special law of variation for Hubble's parameter proposed by Bermann (Nuovo Cimento B 74:183, 1983) is used. We consider the two cases of interacting and non-interacting fluid (barotropic and dark energy) scenario and obtained general results. The physical aspects of the results obtained are also discussed.

Keywords Dark energy · Scalar-tensor theory · Two fluid scenario

1 Introduction

In recent years there has been immense interest in cosmological models with dark energy in general relativity because of the fact that our observable universes is undergoing a phase of accelerated expansion which has been confirmed by several cosmological observations such as type 1a supernova [1–7]. Cosmic microwave background (CMB) anisotropy [8, 9] and large scale structure [10] strongly indicate that dark energy dominates the present universe, causing cosmic acceleration. Based on these observations, cosmologists have accepted the idea of dark energy, which is a fluid with negative pressure making up around 70 % of the present universe energy content to be responsible for this acceleration due to repulsive gravitation, cosmologists have proposed many candidates for dark energy to fit the current

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observations such as cosmological constant, tachyon, quintessence, phantom and so on. For instance, quintessence models involving scalar fields give rise to time dependent equation of state (EoS) parameter $\omega = p/\rho$ which is not necessarily constant where p is the fluid pressure and ρ is energy density [11]. several authors have investigated different aspects of dark energy models in general relativity with variable EoS parameter [12–21].

It is well known that there are two major approaches to address the problem of late time acceleration of the universe. One approach is by introducing a dark energy component in the universe and study its dynamics. Another alternative approach is modifying the general relativity itself [22–25]. This is known as 'modified gravity approach'. In spite of the fact both approaches have novel features with some deep theoretical problems we focus our attention on the modified gravity approach. Brans-Dicke gravity [26], which introduces, in addition to the metric tensor field, a dynamical scalar field to account for variable gravitational constant, was one of the earlier modifications of general relativity. This modification was introduced due to lack of compatibility of Einstein's theory with the Mach's Principle. Later Saez and Ballester [27] have formulated a scalar-tensor theory of gravity in which metric is coupled to a scalar field. This modification helped to solve the "missing mass problem". Several aspects of Saez-Ballester theory in relation to Bianchi cosmological models have been explored [28–31]. In particular, Bianchi type dark energy cosmological models have been investigated by several authors [32–35].

The cosmological evolution of a two field dilation model of dark energy was investigated by Liange et al. [36]. The viscous dark tachyon cosmology in interacting and non-interacting cases in non-flat FRW universe was studied by Setare et al. [37].Two fluid scenario for dark energy models was studied by Chimento et al. [38] and Chimento and Pavan (39). They have shown that such an interaction may help alleviate the coincidence problem. Two fluid dark energy models have been, recently, studied by many authors [40–43]. Very recently, Saha et al. [44] have revisited the two fluid scenario in FRW universe investigated by Amirhashchi et al. [39]. Motivated by the above investigations, in this paper, we study the evolution of dark energy parameter in FRW cosmological model filled with two fluids (barotropic fluid and dark energy) in the frame work of scalar-tensor theory of gravitation proposed by Saez and Ballester. The physical aspects of the two fluid scenario will be, also, discussed. We consider both interacting and non-interacting cases.

2 Metric and Field Equations

Assuming the universe to be homogeneous and isotropic, the FRW metric can be written as

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right] \tag{1}$$

where $a(t)$ is the scale factor and $k = -1, 0, +1$ respectively for open, flat and closed models of the universe. The field equations given by Saez and Ballester for the combined scalar and tensor fields (with $8\pi G = 1$ and $c = 1$) are

$$R_{ij} - \frac{1}{2}g_{ij}R - w\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -T_{ij} \tag{2}$$

and the scalar field satisfies the equation

$$2\phi^n\phi^{;i}_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{3}$$

Also, we have

$$T^{ij}_{;j} = 0 \tag{4}$$

which is a consequence of the field Eqs. (1) and (2). Here w and n are constants, T_{ij} is the two fluid energy momentum tensor consisting of dark energy and barotropic fluid and comma and semicolon denote partial and covariant differentiation respectively.

In a co-moving coordinate system Saez-Ballester field Eqs. (2)–(4) for the metric (1), in the two fluid scenario, lead to

$$2\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}-\frac{w}{2}\phi^n\dot{\phi}^2=-p_{tot} \tag{5}$$

$$3\left(\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\right)+\frac{w}{2}\phi^n\dot{\phi}^2=\rho_{tot} \tag{6}$$

$$\ddot{\phi}+3\frac{\dot{a}}{a}\dot{\phi}+\frac{n}{2}\frac{\dot{\phi}^2}{\phi}=0 \tag{7}$$

$$\dot{\rho}_{tot}+3(\rho_{tot}+p_{tot})\frac{\dot{a}}{a}=0 \tag{8}$$

where $p_{tot}=p_m+p_D$ and $\rho_{tot}=\rho_m+\rho_D$.

Here p_m and ρ_m are pressure and energy density of barotropic fluid and p_D and ρ_D are pressure and energy density of dark fluid respectively.

Also, The equations of state (EoS) parameters of the barotropic fluid and dark fluid are given by

$$\omega_m=\frac{p_m}{\rho_m} \tag{9}$$

and

$$\omega_D=\frac{p_D}{\rho_D} \tag{10}$$

respectively.

In the following section we consider two cases: non-interacting two fluid model and interacting fluid model. Solving the Saez-Ballester field equations in both the cases we determine $a(t)$, ρ_m, p_m, ρ_D, p_D, ω_m, ω_D and ϕ and then study their physical behavior.

3 Non-interacting Two-Fluid Model

First we consider that two fluid do not interact with each other. Hence the general form of conservation Eq. (8) leads us to write the conservation equations for the dark fluid and barotropic fluid separately as

$$\dot{\rho}_m+3\frac{\dot{a}}{a}(\rho_m+p_m)=0 \tag{11}$$

and

$$\dot{\rho}_D+3\frac{\dot{a}}{a}(\rho_D+p_D)=0 \tag{12}$$

We can observe that there is a structural difference between (11) and (12). Since EoS parameter of barotropic fluid ω_m is constant (Akarsu and Kilinc [13, 14]) while ω_D is allowed to be function of time, integration of (11) leads to

$$\rho_m=\rho_0 a^{-3(1+\omega_m)} \tag{13}$$

Using (13) in (5) and (6), we first obtain p_D and ρ_D, in terms of scale factor $a(t)$, as

$$\rho_D = 3\frac{\dot{a}^2}{a^2} + 3\frac{k}{a^2} + \frac{w}{2}\phi^n\dot{\phi}^2 - \rho_0 a^{-3(1+\omega_m)} \tag{14}$$

and

$$p_D = -\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) + \frac{w}{2}\phi^n\dot{\phi}^2 - \rho_0\omega_m a^{-3(1+\omega_m)} \tag{15}$$

To determine the scale factor $a(t)$, we consider the special law of variation for Hubble's parameter, proposed by Bermann [45] that yields constant deceleration parameter models of the universe. The constant deceleration parameter q is defined by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \text{constant} \tag{16}$$

which acts as an indicator of the existence of inflation of the model. If $q > 0$, the model decelerates in the standard way while $q < 0$ indicates inflation or accelerated expansion of the universe. Here we are dealing with accelerated expansion of the universe we take $q < 0$. Now, integration of (16) yields the solution

$$a(t) = (ct + d)^{1/1+q} \tag{17}$$

where $c \neq 0$ and d are constants of integration and $1 + q > 0$ for accelerated expansion of the universe, i.e. $-1 < q < 0$.

By the suitable choice of constants, we can write the metric (1), with the help of (17), as

$$ds^2 = -dt^2 + t^{\frac{2}{1+q}}\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right] \tag{18}$$

The model given by (18) represents non-interacting two fluid model in Saez-Ballester theory with the following physical properties.

Integrating (7) and using (18), the scalar field in the model is given by

$$\phi^{\frac{n+2}{2}} = \phi_0\left(\frac{n+2}{2}\right)\left(\frac{1+q}{q-2}\right)t^{\frac{q-2}{1+q}} \tag{19}$$

Using (18) and (19) in (14) and (15) we obtain p_D and ρ_D as

$$\rho_D = \frac{3}{(1+q)^2t^2} + \frac{3k}{t^{2/1+q}} + \frac{w}{2}\frac{\phi_0^2}{t^{6/1+q}} - \frac{\rho_0}{t^{\frac{3(1+\omega_m)}{1+q}}} \tag{20}$$

$$p_D = -\left[\frac{1-2q}{(1+q)^2}\frac{1}{t^2} + \frac{k}{t^{2/1+q}} - \frac{w\phi_0^2}{2t^{6/1+q}} + \frac{\rho_0\omega_m}{t^{\frac{3(1+\omega_m)}{1+q}}}\right] \tag{21}$$

Using (20) and (21) in (10) we obtain

$$\omega_D = -\left[\frac{\frac{1-2q}{(1+q)^2t^2} + \frac{k}{2t^{2/1+q}} - \frac{w\phi_0^2}{2t^{6/1+q}} + \frac{\rho_0\omega_m}{t^{\frac{3(1+\omega_m)}{1+q}}}}{\frac{3}{(1+q)^2t^2} + \frac{3k}{t^{2/1+q}} + \frac{w\phi_0^2}{2t^{6/1+q}} - \frac{\rho_0}{t^{\frac{3(1+\omega_m)}{1+q}}}}\right] \tag{22}$$

which is the equation of state (EoS) parameter of the dark fluid in terms of the cosmic time t.

It can be observed that for $t \to 0$, p_D and ρ_D diverge while for large t they vanish. Equation (22) gives the behavior of EoS in terms of cosmic time t. The behavior of ω_D in terms of cosmic time t is shown in Fig. 1. It is observed that ω_D is increasing function of cosmic time t. The rapidity of their growth at the early stage depends on the type of the

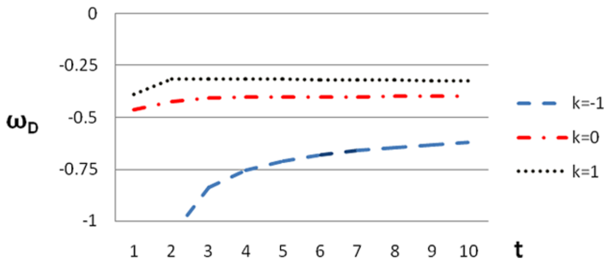

Fig. 1 The plot of EoS Parameter ω_D vs. t for non-interacting two-fluid scenario. Here $\rho_0 = 1, \omega_m = 0.5, \phi_0 = 1, w = 1, q = -0.1$

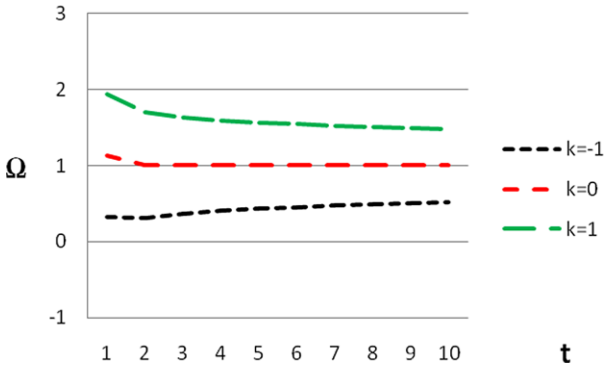

Fig. 2 The plot of average density parameter Ω vs. t for non-interacting two-fluid scenario. Here $\rho_0 = 1, \omega_m = 0.5, \phi_0 = 1, w = 1, q = -0.1$

universes, while later on they all tend to a constant value. From the Fig. 1 we observe that the EoS parameters of closed, open and flat universes are varying in quintessence ($\omega_D > -0.5$), phantom ($-1 < \omega_D < -0.5$) and super phantom ($\omega_D > -0.3$) regions respectively.

The expressions for the matter-energy density Ω_m and dark-energy density Ω_D are given by

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{\rho_0}{3}(1+q)^2 t^{2-\frac{3(1+\omega_m)}{(1+q)}} \tag{23}$$

$$\Omega_D = \frac{\rho_D}{3H^2} = 1 + (1+q)^2\left[kt^{\frac{2q}{1+q}} + \frac{w\phi_0^2}{6}t^{\frac{2(q-2)}{1+q}} - \frac{\rho_0}{3}t^{2-\frac{3(1+\omega_m)}{(1+q)}}\right] \tag{24}$$

respectively. Equations (23) and (24) give us the density parameter

$$\Omega = \Omega_m + \Omega_D = 1 + (1+q)^2\left[kt^{\frac{2q}{1+q}} + \frac{w}{6}\phi_0^2 t^{\frac{2(q-2)}{1+q}}\right] \tag{25}$$

From Eq. (25) it can be observed that in flat universe ($k = 0$), $\Omega \to 1$ and in open universe ($k = -1$), $0 < \Omega < 1$ and in closed universe ($k = 1$), $\Omega > 1$. But at late times, we observe, for all flat, open and closed universes $\Omega \to 1$. This result is compatible with the observational results. Since our model predicts a flat universe for large times and present day universe is very close to flat universe, the derived model is also in agreement with the observational results. The variation of density parameter with cosmic time is shown in Fig. 2.

4 Interacting Two Fluid Model

Here we consider the interaction between dark energy and barotropic fluid. For this purpose we can write the continuity equations for dark fluid and barotropic fluid as

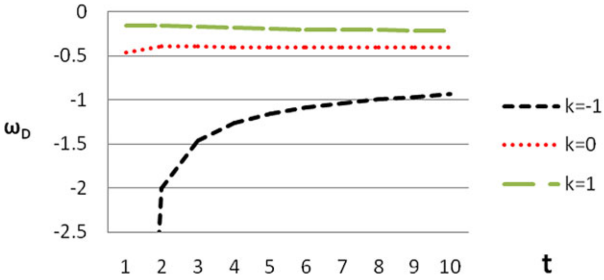

Fig. 3 The plot of EoS Parameter ω_D vs. t for interacting two-fluid scenario. Here $\rho_0 = 1, \omega_m = 0.5$, $\phi_0 = 1, w = 1, q = -0.1$ and $\sigma = 0.3$

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}(\rho_m + p_m) = Q \tag{26}$$

$$\dot{\rho}_D + 3\frac{\dot{a}}{a}(\rho_D + p_D) = -Q \tag{27}$$

where the quantity Q represents the interaction between dark energy components. Also $Q > 0$ ensure that the second law of thermodynamics is satisfied [46]. Following Amendola et al. [47] and Guo et al. [48], we consider

$$Q = 3H\sigma\rho_m \tag{28}$$

where σ is a coupling constant. Using (28) in (26) and integrating we obtain

$$\rho_m = \rho_0 a^{-3(1+a_m-\sigma)} \tag{29}$$

Now using (29) and (18) in (5) and (6) we get (by a straight forward calculation)

$$\rho_D = \frac{3}{(1+q)^2}\frac{1}{t^2} + \frac{3k}{t^{2/1+q}} + \frac{w}{2}\frac{\phi_0^2}{t^{6/1+q}} - \rho_0 t^{\frac{-3(1+\omega_m-\sigma)}{1+q}} \tag{30}$$

$$p_D = -\left[\frac{2}{1+q}\frac{1}{t^2} + \frac{3}{(1+q)^2}\frac{1}{t^2} - \frac{k}{t^{2/1+q}} + \frac{w}{2}\frac{\phi_0^2}{t^{6/1+q}} + \rho_0\omega_m t^{\frac{-3(1+\omega_m-\sigma)}{1+q}}\right] \tag{31}$$

$$\omega_D = -\left[\frac{\frac{1-2q}{(1+q)^2 t^2} - \frac{k}{2t^{2/1+q}} + \frac{w\phi_0^2}{2t^{6/1+q}} - \rho_0\omega_m t^{\frac{-3(1+\omega_m-\sigma)}{1+q}}}{\frac{3}{(1+q)^2}\frac{1}{t^2} + \frac{3k}{t^{2/1+q}} + \frac{w}{2}\frac{\phi_0^2}{t^{6/1+q}} - \rho_0 t^{\frac{-3(1+\omega_m-\sigma)}{1+q}}}\right] \tag{32}$$

The model (18) with ρ_D, p_D and ω_D given by (30)–(32) represent two fluid interacting dark energy model in Saez-Ballester scalar-tensor theory with the scalar field given by (19).

The behavior of ω_D in terms t is shown in Fig. 3. It is observed that for closed universe ω_D is decreasing function of time where as for open and flat universes it is an increasing function of time. From this figure we also observe that ω_D of closed open and flat universes are varying in quintessence ($\omega_{D>} > -1$), phantom ($-3 < \omega_D < -1$) and super phantom ($\omega_D < -0.3$) regions respectively, while later on they tend to same constant value -1 (that is cosmological constant). It is also observed that in interacting and non-interacting cases both open and flat universes can cross Phantom region. It can be seen that the closed universe corresponds to quintessence where as the flat and open universes correspond to phantom model of the universe. If we put $w = 0$ in the present paper, we obtain all the results of recent papers of Saha et al. Eq. (44). For large t, ρ_D and p_D vanish while the scalar field ϕ diverges. For $t \to 0$, ρ_D and p_D diverge and the scalar field vanishes.

The expressions for the matter-energy density Ω_m and dark-energy density Ω_D are given by

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{\rho_0}{3}(1+q)^2 t^{2-\frac{3(1+\omega_m-\sigma)}{(1+q)}} \tag{33}$$

$$\Omega_D = \frac{\rho_D}{3H^2} = 1 + (1+q)^2\left[kt^{\frac{2q}{1+q}} + \frac{w\phi_0^2}{6}t^{\frac{2(q-2)}{1+q}} - \frac{\rho_0}{3}t^{2-\frac{3(1+\omega_m-\sigma)}{(1+q)}}\right] \tag{34}$$

respectively. Equations (33) and (34) give us the density parameter

$$\Omega = \Omega_m + \Omega_D = 1 + (1+q)^2\left[kt^{\frac{2q}{1+q}} + \frac{w}{6}\phi_0^2 t^{\frac{2(q-2)}{1+q}}\right] \tag{35}$$

which is same as Eq. (25). Hence the behavior of the density parameter, in this case, is same as in the non-interacting case and is shown in Fig. 2.

5 Conclusions

Here, we have studied the two fluid scenario in the presence of Saez-Ballester scalar field in the spatially homogeneous and isotropic FRW space-time. It is observed that the EoS parameter turns out to be an increasing function of cosmic time for open, closed and flat FRW universes which explains the late time acceleration of the universe. It is also observed that in an interacting and non interacting cases studies of dynamics of scalar fields in connection with inflationary (accelerated) universe scenario are important because it can solve some of the outstanding problems of standard 'big bang' cosmology.

Acknowledgements The authors would like to thank the anonymous referee for his constructive comments, which have improved the quality of the paper considerably.

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