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Higher Dimensional Cosmological Models Filled with Perfect Fluid in $f(R,T)$ Theory of Gravity

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Abstract The new class of higher dimensional cosmological model of the early universe filled with perfect fluid source in the framework of $f(R,T)$ theory of gravity (Harko et al. in Phys. Rev. D 84, 024020, 2011) is considered. A cosmological model with an appropriate choice of the function $f(T)$ has been constructed. The physical behavior of the model is studied. The well known astrophysical phenomena, namely the Hubble parameter $H(z)$, luminosity distance (d_L) and distance modulus $\mu(z)$ with redshift are discussed.

Keywords $f(R,T)$ gravity · Higher dimension · Perfect fluid · Cosmological parameters

1 Introduction

The discovery of the modern cosmology is that the current universe is not only expanding but also accelerating. This late time accelerated expansion of the universe has been confirmed by the high red-shift supernove experiments [2–4]. Also observations such as cosmic microwave background radiation [5, 6] and large scale structure [7] provide an indirect evidence for the late time accelerated expansion of the universe. The recent evidence coming from observational data [8–10] on the late time acceleration of the universe and the existence of dark matter have posed a fundamental theoretical challenge to gravitational theories. One possibility in explaining the observation is by assuming that at large scales the Einstein gravity model of general relativity breaks down, and a more general action describes the gravitational field. Theoretical models, in which the standard Einstein–Hilbert action is replaced by an arbitrary function of the Ricci scalar R [11] have been extensively investigated lately. The presence of a late time cosmic acceleration of the universe can indeed be explained by $f(R)$ theory of gravity [12]. The conditions of the existence of viable cosmological models

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have been found in [13] and serve weak field constraints obtained from the classical tests of general relativity for the solar system regime seem to rule out most of the models proposed so far [14, 15].

Now a days, there has been a lot of interest of cosmologists in alternative theories of gravitation [16–18]. A generalization of $f(R)$ modified theories of gravity was proposed in [19] by including in the theory an explicit coupling of an arbitrary function of the Ricci scalar R with the matter Lagrangian density L_m. As a result of the coupling the motion of the massive particles is non geodesic, and an extra-force orthogonal to the four velocity, arises. The connections with modified Newtonian dynamics and the pioneer anomaly were also explored. This model was extended to the case of the arbitrary coupling in both geometry and matter in [20]. The astrophysical and cosmological implications of the non minimal coupling matter-geometry coupling were extensively investigated in [21, 22] and the Palatini formulation of the non minimal geometry-coupling models was considered in [23]. In this context, a maximal extension of the Hilbert-Einstein action was proposed [24] by assuming that the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and of the matter Lagrangian L_m. The gravitational field equations have been obtained in the metric formalism, as well as the equations of motion for test particles, which follow from the covariant divergence of the stress-energy tensor.

Using a scalar field Lagrangian as matter, it is shown that the emerging FRW cosmology may lead either to an effective quintessence phase (cosmic speed-up) or to an effective phantom phase. Here the gravity matter coupling part is assumed to be $f(R)$ theory like Lagrangian, non-minimally coupled with a scalar field Lagrangian. Nojiri and Odintsov [25] developed the general scheme for modified $f(R)$ gravity reconstruction from any realistic FRW cosmology. They proved that the modified $f(R)$ gravity indeed represents the realistic alternative to general relativity, being more consistent in dark epoch. Further, Nojiri et al. [26] developed the general programme of the unification of matter-dominated era with acceleration epoch for scalar tensor theory or dark fluid. Nojiri and Odintsov [27] reviewed various modified gravities which have been considered as gravitational alternative for dark energy. Specifically, they have considered the versions of $f(R)$, $f(G)$ or $f(R,G)$ gravity, model with non linear gravitational coupling or sting inspired model with Gauss-Bonnet-dilation coupling in the late universe when they lead to cosmic speed-up.

Very recently, Harko et al. [1] proposed another extension of standard general relativity, $f(R,T)$ modified theories of gravity, where in the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and of the trace of the stress energy tensor T. It is to be noted that the dependence from T may be induced by exotic imperfect fluid or quantum effects. They have derived the field equations from a Hilbert-Einstein type variational principle and also obtained the covariant divergence of the stress energy tensor. The $f(R,T)$ gravity model depends on a source term, representing the variation of the matter stress energy tensor with respect to the metric. A general expression for this source term is obtained as a function of the matter Lagrangian L_m so that each choice of L_m would generate a specific set of field equations. Some particular models corresponding to specific choices of the function $f(R,T)$ are also represented, they have also demonstrated the possibility of reconstruction of arbitrary FRW cosmologies by an appropriate choice of a function $f(T)$. In the present model the covariant divergence of the stress energy tensor is non zero. Hence the motion of test particles is non geodesic and an extra acceleration due to the coupling between matter and geometry is always present. Recently, Adhav [28], Reddy et al. [29] and Shamir et al. [30] studied different cosmological models in $f(R,T)$ theory of gravity.

The purpose of our present paper is to study higher dimensional viable cosmological models in the newly established extension of the standard general relativity which is known as the $f(R,T)$ theory of gravity. In this paper, we study higher dimensional spherical symmetric cosmological model filled with perfect fluid source in $f(R,T)$ gravity proposed by [1]. In Sect. 1, a brief introduction of $f(R,T)$ gravity is given. In Sect. 2, the field equations in metric versions of $f(R,T)$ gravity is given. In Sect. 3, we obtain explicit form of field equations in $f(R,T)$ gravity by using the particular form of $f(T)=\lambda T$, which are used by [1]. The different cosmological solutions of the field equations are obtained and also discussed some physical properties of the model in Sect. 4. In Sect. 5, deals with some observational parameters $H(z)$ and $\mu(z)$ in physically viable cosmological model. In Sect. 6, some concluding remarks are given.

2 Gravitational Field Equations of $f(R,T)$ Gravity Theory

The $f(R,T)$ gravity is the modification of General Relativity (GR). The field equations of this theory are obtained from the Hilbert-Einstein theory variational principle. The action for this theory is given by [1]

$$S=\int\sqrt{-g}\left(\frac{1}{16\pi G}f(R,T)+L_m\right)d^4x \tag{1}$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar R and the trace T of energy momentum tensor $T_{\mu\gamma}$ while L_m is the usual matter Lagrangian. If we replace $f(R,T)$ with $f(R)$, we get the action for $f(R)$ gravity and replacement of $f(R,T)$ with R leads to the action of general relativity (GR). The energy momentum tensor $T_{\mu\gamma}$ of matter is defined as [31]

$$T_{\mu\gamma}=-\frac{2\delta(\sqrt{-g}L_m)}{\sqrt{-g}\delta g^{\mu\gamma}} \tag{2}$$

And its trace by $T=g^{\mu\gamma}T_{\mu\gamma}$ respectively. In this paper we assume that the dependence of matter Lagrangian is merely on the metric tensor $g_{\mu\gamma}$ rather than its derivatives.

$$T_{\mu\gamma}=g_{\mu\gamma}L_m-2\frac{\delta L_m}{\delta g^{\mu\gamma}} \tag{3}$$

Now by varying the action S of the gravitational field with respect to the metric tensor components $g^{\mu\gamma}$, we obtain the field equations of $f(R,T)$ gravity theory as

$$\begin{aligned} &f_R(R,T)R_{\mu\gamma}-\frac{1}{2}f(R,T)g_{\mu\gamma}+(g_{\mu\gamma}\Box-\nabla_\mu\nabla_\gamma)f_R(R,T)\\ &\quad=8\pi T_{\mu\gamma}-f_T(R,T)T_{\mu\gamma}-f_T(R,T)\Theta_{\mu\gamma} \end{aligned} \tag{4}$$

where

$$\Theta_{\mu\gamma}=-2T_{\mu\gamma}+g_{\mu\gamma}L_m-2g^{lk}\frac{\delta^2 L_m}{\delta g^{\mu\gamma}\delta g^{lk}} \tag{5}$$

Here $f_R\equiv\frac{\delta f(R,T)}{\delta R}$, $f_T\equiv\frac{\delta f(R,T)}{\delta T}$, $\Box\equiv\nabla^\mu\nabla_\mu$, ∇_μ is the covariant derivative and $T_{\mu\gamma}$ is the standard matter energy momentum tensor derived from the Lagrangian L_m. One should note that when $f(R,T)\equiv f(R)$ then Eq. (4) reduces to the field equations of $f(R)$ theory of gravity. Now, contraction of Eq. (4) gives

$$f_R(R,T)R+3\Box f_R(R,T)-2f(R,T)=8\pi T-f_T(R,T)(T+\Theta) \tag{6}$$

where $\Theta = \Theta^{\mu}_{\mu}$. Equation (6) gives a relationship between Ricci scalar (R) and trace T of energy momentum tensor. Using matter Lagrangian L_m, the stress energy tensor of the matter is given by

$$T_{\mu\gamma} = (p+\rho)U_\mu U_\gamma - pg_{\mu\gamma} \tag{7}$$

where $U^\mu = (1, 0, 0, 0, 0)$ is the five velocity in commoving coordinates which satisfies the conditions $U^\mu U_\mu = 1$ and $U^\mu \nabla_\gamma U_\mu = 0$. The problem of the perfect fluids described by energy density ρ, pressure p and matter Lagrangian can be taken as $L_m = -p$, since there is no unique definition of the matter Lagrangian. Then with the use of a perfect fluid, the expression

$$\Theta_{\mu\gamma} = -2T_{\mu\gamma} - pg_{\mu\gamma} \tag{8}$$

Generally, the field equations also depend through the tensor $\Theta_{\mu\gamma}$, on the physical nature of the matter field. Hence in the case of $f(R,T)$ theory of gravity depending on the nature of the matter source, we obtain several theoretical models corresponding to different matter contributions for $f(R,T)$ gravity are possible. However, Harko et al. [1] give three classes of these models

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T) \end{cases} \tag{9}$$

In this paper we are going to focus to the first class i.e. $f(R,T) = R + 2f(T)$, where $f(T)$ is an arbitrary function of the trace of stress energy tensor of matter, we get the gravitational field equations of $f(R,T)$ theory of gravity from Eq. (4) as

$$R_{\mu\gamma} - \frac{1}{2}Rg_{\mu\gamma} = 8\pi T_{\mu\gamma} - 2f'(T)T_{\mu\gamma} - 2f'(T)\Theta_{\mu\gamma} + f(T)g_{\mu\gamma} \tag{10}$$

where the prime denotes differentiation with respect to the argument. If the matter source is a perfect fluid then the field equations (in view of Eq. (8)) becomes

$$R_{\mu\gamma} - \frac{1}{2}Rg_{\mu\gamma} = 8\pi T_{\mu\gamma} + 2f'(T)T_{\mu\gamma} + \left\{2pf'(T) + f(T)\right\}g_{\mu\gamma} \tag{11}$$

3 Model and Field Equations

Here we consider the higher dimensional spherical symmetric metric in the form

$$ds^2 = dt^2 - e^\gamma\left(dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2\right) - e^\mu dw^2 \tag{12}$$

where μ, γ are cosmic scale factors functions of time coordinate only. We define the expressions for the average scale factor and volume scale factor. Defining the generalized Hubble's parameter H is analogy with a flat FRW model. The mean scale factor 'a' of the cosmological model (12) is defined as

$$a = \left(e^{\frac{3\gamma+\mu}{2}}\right)^{\frac{1}{4}} \tag{13}$$

The spatial volume V is given by

$$V = a^4 = e^{\frac{3\gamma+\mu}{2}} \tag{14}$$

We define the average generalized Hubble's parameter H as

$$H = \frac{1}{4}\frac{V'}{V} = \frac{a'}{a} = \frac{1}{4}(H_1 + H_2 + H_3 + H_4) \tag{15}$$

where $H_1 = \frac{(e^{\frac{\gamma}{2}})'}{e^{\frac{\gamma}{2}}}$, $H_2 = \frac{(e^{\frac{\gamma}{2}})'}{e^{\frac{\gamma}{2}}}$, $H_3 = \frac{(e^{\frac{\gamma}{2}})'}{e^{\frac{\gamma}{2}}}$ and $H_4 = \frac{(e^{\frac{\mu}{2}})'}{e^{\frac{\mu}{2}}}$ are directional Hubble's parameters. An overhead prime denotes differentiation with respect to cosmic time 't' only. From Eqs. (13), (14) and (15), we obtain

$$H = \frac{1}{4}\left(\frac{3}{2}\gamma' + \frac{1}{2}\mu'\right) \tag{16}$$

Using co-moving co-ordinates and Eqs. (7) and (8), the $f(R,T)$ gravity field equations, with the particular choice of the function $f(T) = \lambda T$, where λ is an arbitrary constant [1].

The corresponding field equations (11) for the metric (12) can be written as

$$-\frac{3}{4}\left(\gamma'^2 + \mu'\gamma'\right) = (8\pi + 3\lambda)\rho - 2p\lambda \tag{17}$$

$$\gamma'' + \frac{3}{4}\gamma'^2 + \frac{1}{2}\mu'' + \frac{1}{4}\mu'^2 + \frac{1}{2}\mu'\gamma' = (8\pi + 4\lambda)p - \lambda\rho \tag{18}$$

$$\frac{3}{2}\left(\gamma'' + \gamma'^2\right) = (8\pi + 4\lambda)p - \lambda\rho \tag{19}$$

In the present paper, we use the natural system of units with $G = C = 1$. So that the Einstein gravitational constant is defined as $k^2 = 8\pi$.

Let us introduce the dynamical scalars such as expansion parameter (θ), shear scalar (σ^2) are defined as

$$\theta = U^{\mu}_{;\mu} = \frac{3}{2}\gamma' + \frac{1}{2}\mu' = 4H \tag{20}$$

$$\sigma^2 = \frac{1}{2}\sigma_{\mu\gamma}\sigma^{\mu\gamma} = \frac{3}{8}\gamma'^2 + \frac{1}{8}\mu'^2 - \frac{1}{2}\gamma' - \frac{1}{6}\mu' + \frac{2}{9} \tag{21}$$

The mean anisotropic parameter is

$$\Delta = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{\Delta H_i}{H}\right)^2 \tag{22}$$

where $\Delta H_i = H_i - H$, $(i = 1,2,3,4)$ represents the directional Hubble parameters. $\Delta = 0$ corresponds to isotropic expansion. The space approaches isotropy, in case of diagonal energy momentum tensor ($T^{0i} = 0$, where $i = 1,2,3,4$) if $\Delta \to 0$, $V \to \infty$ and $T^{00} > 0$ $(\rho > 0)$ as $t \to \infty$ (see [32] for details).

Here the projection tensor

$$p_{\mu\gamma} = g_{\mu\gamma} - U_\mu U_\gamma \tag{23}$$

The volumetric deceleration parameter

$$q = -\frac{aa''}{a'^2} \tag{24}$$

Since there are three field equations (17)–(19) are highly non linear nature and containing four unknowns viz. μ, γ, ρ and p respectively. For exact solutions of these equations, we use

$$\gamma = \alpha\mu(\alpha \neq 0) \tag{25}$$

Subtract Eq. (19) from Eq. (18) and use the condition (25), we obtain

$$\mu'' + \frac{3\alpha + 1}{2}\mu'^2 = 0 \tag{26}$$

4 Different Cosmological Solutions and Model

Here we like to discuss two different physically viable cosmologies for $\alpha = -\frac{1}{3}$ and $\alpha \neq 0$, $-\frac{1}{3}$ respectively, which have physical interests to describe the decelerating and accelerating phases of the universe.

Case-1. When $\alpha = -\frac{1}{3}$, the Eq. (26) reduces to

$$\mu'' = 0 \tag{27}$$

Solving the Eq. (27), we get

$$\mu = c_1 + c_2 t \tag{28}$$

where c_1 and $(c_2 \neq 0)$ are any arbitrary constants of integration. Using Eq. (25), we obtain

$$\gamma = -\frac{1}{3}(c_1 + c_2 t) \tag{29}$$

The metric (12) with the help of (28) and (29) can, now, be written as

$$ds^2 = dt^2 - e^{-\frac{1}{3}(c_1+c_2t)}\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right) - e^{(c_1+c_2t)}dw^2 \tag{30}$$

4.1 Physical Properties of the Model

Equation (30) represents a five dimensional cosmological model, with perfect fluid source, in $f(R,T)$ theory of gravity which is physically significant for the discussion of the early stages of evolution of the universe. The different physical and kinematical parameters which are important to discuss the physics of the five dimensional cosmological model (30) are given below.

The directional Hubble's parameters H_1, H_2, H_3 and H_4 have values given by

$$H_1 = H_2 = H_3 = -\frac{c_2}{6} \text{ (constant)} \tag{31}$$

$$H_4 = \frac{c_2}{2} \text{ (constant)} \tag{32}$$

From Eq. (15), the generalized Hubble's parameter 'H' has the value given by

$$H = 0 \text{ (constant)} \tag{33}$$

From Eqs. (20) and (21), the dynamical scalars are given by

$$\theta = 0 \text{ (constant)} \tag{34}$$

$$\sigma^2 = \frac{1}{6}c_2^2 + \frac{2}{9} \text{ (constant)} \tag{35}$$

The scalar curvature 'R' for general class of spherically symmetric five dimensional cosmological model is defined as

$$R = 3\gamma'' + \mu'' + 3\gamma'^2 + \frac{1}{2}\mu'^2 + \frac{3}{2}\mu'\gamma' \tag{36}$$

Using Eqs. (28) and (29) in (36), we obtain

$$R = \frac{c_2^2}{3} \text{ (constant)} \tag{37}$$

$$\rho = \frac{8\pi + 4\lambda}{(8\pi + 3\lambda)(8\pi + 4\lambda) - 2\lambda^2}\left(\frac{2\lambda c_2^2}{6(8\pi + 4\lambda)} + \frac{c_2^2}{6}\right) \text{ (constant)} \tag{38}$$

$$p = \left[\frac{c_2^2}{6} + \frac{\lambda(8\pi + 4\lambda)}{(8\pi + 3\lambda)(8\pi + 4\lambda) - 2\lambda^2}\left(\frac{2\lambda c_2^2}{6(8\pi + 4\lambda)} + \frac{c_2^2}{6}\right)\right]\frac{1}{(8\pi + 4\lambda)} \text{ (constant)} \tag{39}$$

From the above results, we observe that the perfect fluid does not survive in $f(R, T)$ theory of gravity for $\alpha = -\frac{1}{3}$. From Eq. (30), we observe that e^{γ} increases with the evolution of the universe and e^{μ} decreases for $c_2 < 0$. Thus the fifth dimension contracts after the creation of the universe while the other three standard dimensions continued to expand as the time proceeds.

Case-2. For $\alpha \neq 0, -\frac{1}{3}$, Eq. (26) reduces to

$$\mu'' + \frac{3\alpha + 1}{2}\mu'^2 = 0 \tag{40}$$

Solving Eq. (40), we obtain

$$\mu = \ln\left[\left(\frac{3\alpha + 1}{2}\right)(kt + k_1)\right]^{\frac{2}{3\alpha+1}} \tag{41}$$

where k ($\neq 0$), k_1 are any arbitrary constants of integration.

Using Eq. (41) in (25), we get

$$\mu = \ln\left[\left(\frac{3\alpha + 1}{2}\right)(kt + k_1)\right]^{\frac{2\alpha}{3\alpha+1}} \tag{42}$$

The metric (12) with the help of Eqs. (41) and (42) can, now, be written as

$$ds^2 = dt^2 - \left(\left(\frac{3\alpha + 1}{2}\right)(kt + k_1)\right)^{\frac{2\alpha}{3\alpha+1}}\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right) - \left(\left(\frac{3\alpha + 1}{2}\right)(kt + k_1)\right)^{\frac{2}{3\alpha+1}} dw^2 \tag{43}$$

4.2 Physical Properties of the Model

Equation (43) represents a five dimensional spherically symmetric cosmological universe, with perfect fluid source in $f(R,T)$ theory of gravity which is physically significant for the discussion of the early stage of evolution of the universe. The different physical and kinematical parameters which are important to discuss the physics of the five dimensional cosmological model (43) are given below.

The directional Hubble's parameters H_1, H_2, H_3 and H_4 have values given by

$$H_1 = H_2 = H_3 = \frac{\alpha k}{(3\alpha+1)(kt+k_1)} \tag{44}$$

$$H_4 = \frac{k}{(3\alpha+1)(kt+k_1)} \tag{45}$$

From Eq. (15), the average generalized Hubble's parameter 'H' has the value given by

$$H = \frac{k}{4(kt+k_1)} \tag{46}$$

From Eqs. (14), (20), (21), (22) and (24) the dynamical scalars are given by

$$V = \left(\left(\frac{3\alpha+1}{2}\right)(kt+k_1)\right) \tag{47}$$

$$\theta = \frac{k}{(kt+k_1)} \tag{48}$$

$$\sigma^2 = \frac{(3\alpha^2+1)k^2}{8((\frac{3\alpha+1}{2})(kt+k_1))^2} - \frac{(3\alpha+1)k}{6((\frac{3\alpha+1}{2})(kt+k_1))} + \frac{9}{2} \tag{49}$$

$$\Delta = 0 \tag{50}$$

The scalar curvature 'R' for five dimensional spherically symmetric cosmological model is defined as

$$R = 3\gamma'' + \mu'' + 3\gamma'^2 + \frac{1}{2}\mu'^2 + \frac{3}{2}\mu'\gamma' \tag{51}$$

Using Eqs. (41) and (42) in (51), we obtain

$$R = -\frac{3k^2\alpha(\alpha-1)}{2((\frac{3\alpha+1}{2})(kt+k_1))^2} \tag{52}$$

The physical parameters density (ρ) and pressure (p) in the model (43) are

$$\rho = \frac{3\alpha k^2(\alpha+1)(6\lambda+8\pi)}{4((\frac{3\alpha+1}{2})(kt+k_1))^2(2\alpha^2-(8\pi+3\lambda)(8\pi+4\lambda))} \tag{53}$$

$$p = \frac{3\alpha k^2(\alpha+1)(6\lambda+8\pi)}{4(8\pi+4\lambda)((\frac{3\alpha+1}{2})(kt+k_1))^2(2\lambda^2-(8\pi+3\lambda)(8\pi+4\lambda))} \tag{54}$$

From this case we observe that, the spatial volume V is zero at $t = -\frac{k_1}{k} = t_0$. The scalar curvature R, the energy density ρ and pressure p are infinity at $t = t_0$. Thus the universe starts evolving with zero volume at $t = t_0$ and expands with cosmic time. From Eq. (43), we observe that e^{γ} increases with the evolution of the universe and e^{μ} decreases for $\alpha < -\frac{1}{3}$. Thus the extra dimension contracts after the creation of the universe while the other three standard dimensions continued to expand as time proceeds. From Eqs. (44) and (45), we obtain the directional Hubble's parameters are dynamical. These are diverse at $t = t_0$ and approaches to zero as $t \to \infty$. From Eq. (50), one can observe that the universe expand isotropically at all time. From Eqs. (53) and (54) we get that the matter density (ρ) and pressure (p) diverge at $t = t_0$ and they vanish for large values of t. However, the volume scale factor increases with time showing the late time acceleration of the universe.

5 $H(z)$ and $\mu(z)$ Parameters

The relation between scale factor 'a' and red-shift 'z' is defined as

$$a = \frac{a_0}{1+z} \tag{55}$$

where a_0 is present value of the scale factor. And, the distance modulus (μ) is given by

$$\mu(z) = 5 \log d_L + 25 \tag{56}$$

where d_L is the luminosity distance and is defined as

$$d_L = r_1(1+z)a_0 \tag{57}$$

For the determination of r_1, we assume that a photon emitted by a source with co-ordinate $r = r_0$ and $t = t_0$ received at a time t by an observer located at $r = 0$, then we determine r_1 from following relation

$$r_1 = \int_t^{t_0} \frac{dt}{a} \tag{58}$$

Now, here we can solve the Eq. (55)–(58) by using the average scale factor 'a' obtained in case 2 for physically viable cosmological model (43). From Eqs. (41) and (42) by the help of Eqs. (13) and (55), we get

$$1 + z = \frac{a_0}{a} = \left(\frac{kt_0 + k_1}{kt + k_1}\right)^{\frac{1}{4}} \tag{59}$$

From the above equation easily obtain the expression for Hubble's parameter (H) in terms of redshift parameter 'z' as

$$H = H_0(1+z)^4 \tag{60}$$

Here H_0 is the present value of Hubble's parameter.

Now using Eqs. (41) and (42) in (13) and (58), we get

$$r_1 = \frac{1}{k((\frac{3\alpha+1}{2}))^{\frac{1}{4}}} H_0^{\frac{-3}{4}} \left[1 - (1+z)^{-3}\right] \tag{61}$$

From Eqs. (57) and (61) we have

$$d_L = H_0^{-1}(1+z)\left[1 - (1+z)^{-3}\right] \tag{62}$$

Thus the distance modulus μ in terms of redshift parameter 'z' is given by

$$\mu(z) = 5\log\bigl[(1+z)H_0^{-1}\bigl[1-(1+z)^{-3}\bigr]\bigr] + 25 \tag{63}$$

6 Conclusions

In this, paper we have studied higher dimensional spherical symmetric cosmological model in the presence of perfect fluid distribution in $f(R,T)$ theory of gravity. In case 1, we obtain, non-existence of five dimensional perfect fluid cosmological model in $f(R,T)$ gravity. In case 2, we obtain, the exact solutions to the corresponding field equations are obtained in quadratur form. The cosmological parameters have been discussed in detail. From Eq. (50) we conclude that, our model (43) is expanding isotropically at all time. For $\alpha < \frac{-1}{3}$, we observe that, the extra dimension contracts as t approaches to infinity while the other three standard dimensions expand isotropically as the time proceeds. For $\alpha = -1$, we get $\rho = 0$, $p = 0$ i.e. vacuum cosmological model in $f(R,T)$ gravity. We have also discussed the well known astrophysical phenomena, namely the Hubble parameter $H(z)$, luminosity distance d_L and distance modulus $\mu(z)$ with redshift. Also we obtained that, the luminosity distance increases linearly with redshift (z) for small value of redshift (z).

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