

The Role of Pressure During Shearing, Dissipative Collapse

M. Govender · K.S. Govinder · D. Fleming

Received: 21 March 2012 / Accepted: 25 May 2012 / Published online: 15 June 2012
© Springer Science+Business Media, LLC 2012

Abstract We analyse a new family of solutions to the Einstein field equations describing the collapse of a fluid sphere in the presence of heat flux and shear. These solutions ensure that the collapsing fluid is accelerating and provide a generalisation of the geodesic fluid models studied in earlier treatments. In particular, we demonstrate the role played by pressure in the dynamics of the collapse process.

Keywords Radiating stars · General relativity · Thermodynamics

1 Introduction

The Cosmic Censorship Hypothesis asserts that any reasonable matter distribution undergoing continued gravitational collapse should form a black hole. The singularity theorems of Hawking and Penrose formed the bedrock of all investigations into the endstates of gravitational collapse [1]. One of the earliest attempts at understanding the contraction of a fluid sphere under the action of gravity was due to Oppenheimer and Snyder in which they studied a dust sphere undergoing continued gravitational collapse [2]. This model has subsequently been generalised to include charge, the cosmological constant and it has been extended to higher dimensions. The Oppenheimer-Snyder model is an idealised one as dissipative effects are ignored during the collapse process. It was only until 1985 when Santos presented the junction conditions [3] for a radiating sphere matched to Vaidya's outgoing solution that researchers began to take an active role in determining the role played by dissipation in the final outcome of gravitational collapse.

M. Govender (✉) · K.S. Govinder · D. Fleming
Astrophysics and Cosmology Research Unit, School of Mathematical Sciences, University of
KwaZulu-Natal, Private Bag X54001, Durban 4000, South Africa
e-mail: govenderm43@ukzn.ac.za

K.S. Govinder
e-mail: govinder@ukzn.ac.za

D. Fleming
e-mail: 211555531@ukzn.ac.za

The earliest models of dissipative collapse included radiating spheres dissipating energy to the exterior spacetime in the form of radial heat flux, the presence of bulk viscosity, electromagnetic field and even the cosmological constant. These models were restricted to the shear-free case. Banerjee et al. presented a model of shear-free collapse [4] in which the rate of collapse is balanced by the rate of energy emission thus leading to the avoidance of the formation of the horizon. This is one of the first models of a reasonable matter distribution undergoing gravitational collapse leading to the formation of a naked singularity. The first exact model of a dissipative sphere undergoing collapse in the presence of shear was provided by Naidu et al. [5]. Fluid trajectories within the stellar fluid were assumed to be geodesics. This model was subsequently generalised by Thirukannesh and Maharaj [6] and later by Rajah and Maharaj [7]. Another interesting class of shearing models is the Euclidean stars first proposed by Herrera and Santos [8]. These models exhibit the special feature that the proper radius equals the areal radius throughout the collapse process.

A complete description of a radiating stellar model requires the smooth matching of the interior spacetime with Vaidya's [9] solution. These junction conditions are well-known and have been widely utilised in modeling radiating stars with vanishing shear. The conservation of momentum flux across the matching hypersurface requires that the isotropic pressure at the boundary be proportional to the magnitude of the heat flux. This condition leads to a nonlinear ordinary differential equation which governs the temporal behaviour of the model. In the case of nonvanishing shear, the momentum balance at the stellar surface demands that the *radial* pressure be proportional to the magnitude of the heat flux. In this case there are a limited number of exact solutions to the boundary condition with nearly all of them describing acceleration-free collapse or geodesic flows within the stellar fluid. The first exact model of a Euclidean star was obtained by Govender et al. [10]. Another interesting application of shearing fluids is the so-called expansion-free collapse models studied by Herrera and coworkers in which they show the emergence of a cavity after a central explosion within the stellar interior [11]. It has been shown that various impositions on the Weyl tensor, local anisotropy of pressure and/or the presence of dissipative fluxes generate energy-density inhomogeneities within the stellar fluid.

It is the purpose of this paper to seek solutions to the Einstein field equations describing a spherically symmetric matter distribution with shear and nonvanishing acceleration. We show that the boundary condition for a general spherically symmetric interior matter distribution matched to Vaidya's outgoing solution leads to a Riccati equation. We solve this equation for particular combinations of the metric functions. We further investigate the thermal behaviour of a particular model and relate our results to the acceleration-free models in the literature.

2 Shearing Spacetimes

The interior spacetime of the collapsing sphere is described by the general spherically symmetric, shearing metric in comoving coordinates

$$ds^2 = -A^2 dt^2 + B^2 dr^2 + R^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (1)$$

where the metric functions $A = A(t, r)$, $B = B(t, r)$ and $R = R(t, r)$ are to be determined. The matter content for the interior is described by

$$T_{\alpha\beta} = (\mu + P_{\perp})V_{\alpha}V_{\beta} + P_{\perp}g_{\alpha\beta} + (P_r - P_{\perp})\chi_{\alpha}\chi_{\beta} + q_{\alpha}V_{\beta} + q_{\beta}V_{\alpha} \quad (2)$$

where μ represents the energy density, P_r the radial pressure, P_\perp the tangential pressure and q^α is the heat flux vector. The fluid four-velocity V is comoving and is given by

$$V^\alpha = \frac{1}{A} \delta_0^\alpha. \tag{3}$$

The heat flow vector assumes the form

$$q^\alpha = (0, q, 0, 0), \tag{4}$$

since $q^\alpha V_\alpha = 0$ ensuring radial heat dissipation. We further have

$$\chi^\alpha \chi_\alpha = 1, \quad \chi^\alpha V_\alpha = 0. \tag{5}$$

The expansion scalar and the fluid four acceleration are given by

$$\Theta = V_{;\alpha}^\alpha, \quad a_\alpha = V_{\alpha;\beta} V^\beta, \tag{6}$$

and the shear tensor by

$$\sigma_{\alpha\beta} = V_{(\alpha;\beta)} + a_{(\alpha} V_{\beta)} - \frac{1}{3} \Theta (g_{\alpha\beta} + V_\alpha V_\beta). \tag{7}$$

For the comoving line element (1) the kinematical quantities take the following forms

$$a_1 = \frac{A'}{A} \tag{8}$$

$$\Theta = \frac{1}{A} \left(\frac{\dot{B}}{B} + 2 \frac{\dot{R}}{R} \right) \tag{9}$$

$$\sigma = \frac{1}{A} \left(\frac{\dot{A}}{A} - \frac{\dot{R}}{R} \right), \tag{10}$$

where dots and primes denote differentiation with respect to t and r respectively. The nonzero components of the Einstein’s field equations for the line element (1) and the energy-momentum (2) are

$$\mu = \frac{1}{A^2} \left(2 \frac{\dot{B}}{B} + \frac{\dot{R}}{R} \right) \frac{\dot{R}}{R} - \frac{1}{B^2} \left[2 \frac{R''}{R} + \left(\frac{R'}{R} \right)^2 - 2 \frac{B'}{B} \frac{R'}{R} - \left(\frac{B}{R} \right)^2 \right], \tag{11}$$

$$P_r = -\frac{1}{A^2} \left[2 \frac{\ddot{R}}{R} - \left(2 \frac{\dot{A}}{A} - \frac{\dot{R}}{R} \right) \frac{\dot{R}}{R} \right] + \frac{1}{B^2} \left(2 \frac{A'}{A} + \frac{R'}{R} \right) \frac{R'}{R} - \frac{1}{R^2}, \tag{12}$$

$$P_\perp = -\frac{1}{A^2} \left[\frac{\ddot{B}}{B} + \frac{\ddot{R}}{R} - \frac{\dot{A}}{A} \left(\frac{\dot{B}}{B} + \frac{\dot{R}}{R} \right) + \frac{\dot{B}}{B} \frac{\dot{R}}{R} \right] + \frac{1}{B^2} \left[\frac{A''}{A} + \frac{R''}{R} - \frac{A'}{A} \frac{B'}{B} + \left(\frac{A'}{A} - \frac{B'}{B} \right) \frac{R'}{R} \right], \tag{13}$$

$$q = \frac{2}{AB} \left(\frac{\dot{R}'}{R} - \frac{\dot{B}}{B} \frac{R'}{R} - \frac{\dot{R}}{R} \frac{A'}{A} \right). \tag{14}$$

This is an underdetermined system of four coupled partial differential equations in seven unknowns viz., $A, B, R, \mu, P_r, P_\perp$ and q .

3 Exterior Spacetime and Junction Conditions

The exterior spacetime is taken to be the Vaidya solution given by [9]

$$ds^2 = -\left(1 - \frac{2m(v)}{R}\right)dv^2 - 2dvdr + R^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (15)$$

where $m(v)$ represents the Newtonian mass of the gravitating body as measured by an observer at infinity. The necessary conditions for the smooth matching of the interior spacetime (1) to the exterior spacetime (15) have been extensively investigated. We present the main results that are necessary for modeling a radiating star. The continuity of the intrinsic and extrinsic curvature components of the interior and exterior spacetimes across a time-like boundary are

$$m(v)_{\Sigma} = \left\{ \frac{R}{2} \left[\left(\frac{\dot{R}}{A} \right)^2 - \left(\frac{R'}{B} \right)^2 + 1 \right] \right\}_{\Sigma} \quad (16)$$

$$(P_r)_{\Sigma} = q_{\Sigma}. \quad (17)$$

Relation (17) determines the temporal evolution of the collapsing star.

4 Temporal Evolution

The junction condition $(P_R)_{\Sigma} = q_{\Sigma}$ yields

$$\dot{B} = \left(\frac{R}{2AR'} \right) \left[2 \frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R} \right)^2 - 2 \frac{\dot{A}}{A} \frac{\dot{R}}{R} + \frac{A^2}{R^2} \right] B^2 + \left[\frac{\dot{R}'}{R'} - \frac{A'}{A} \frac{\dot{R}}{R'} \right] B - \frac{A}{2} \left[\frac{R'}{R} + 2 \frac{A'}{A} \right] \quad (18)$$

which is of the form

$$\dot{B} = C_0(t)B^2 + C_1(t)B + C_2(t) \quad (19)$$

This is a Riccati equation which, in general, is difficult to solve. It can always be transformed to a second order linear equation, but that equation will also be difficult to solve given the forms of the C_i . We now attempt to integrate (18) in a special case.

If we set

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} - \frac{2\dot{A}}{A} \frac{\dot{R}}{R} + \frac{A^2}{R^2} = 0$$

then (18) reduces to a linear equation. We now need to solve

$$\dot{A} - \left[\frac{\ddot{R}}{R} + \frac{\dot{R}}{2R} \right] A = A^3 \left(\frac{1}{2R\dot{R}} \right)$$

which is a Bernoulli equation in the variable A . This equation can be integrated in general to yield

$$A^2 = \frac{R\dot{R}^2}{f(r) - R} \quad (20)$$

where $f(r)$ is a function of integration. With the result (20), we find that (18) becomes

$$\dot{B} - \left[\frac{\dot{R}'}{R'} - \frac{A'}{A} \frac{\dot{R}}{R'} \right] B + \frac{A}{2} \left[\frac{R'}{R} + 2 \frac{A'}{A} \right] = 0 \tag{21}$$

which is linear in B . We are in a position to integrate (21) which then allows us to obtain a particular solution of the junction condition (18) given by

$$A = \sqrt{\frac{R\dot{R}^2}{f(r) - R}} \tag{22}$$

$$B = \frac{g(r) - \int (A' + \frac{AR'}{2R}) \exp[\int (\frac{A'\dot{R}}{AR'} - \frac{\dot{R}'}{R'}) dt] dt}{\exp[\int (\frac{A'\dot{R}}{AR'} - \frac{\dot{R}'}{R'}) dt] dt} \tag{23}$$

$$R = R(t, r) \tag{24}$$

where $g(r)$ is the second function of integration. We believe that (22)–(24) is a new solution to the boundary condition (18) which allows for nonzero acceleration of the fluid particles. We note that once the arbitrary function R is specified, we can then obtain explicit forms for $A(r, t)$ and $B(r, t)$ which completely specifies the gravitational behaviour of our model. (Note: This solution was also independently obtained by Thirukkanesh et al. [12].)

Other particular solutions for (18) have also been found by Fleming [13] and Thirukkanesh et al. [12] and we do not list them here.

5 A Particular Radiating Model

In order to study the physical viability of the class of solutions describing dissipative collapse we take a closer look at solutions (22)–(24). To this end we choose

$$R(r, t) = a + cr - bt \tag{25}$$

which subsequently allows us to obtain

$$A(r, t) = 2\sqrt{\frac{b(a + cr - bt)}{t}} \tag{26}$$

$$B(r, t) = \frac{a\sqrt{b(a + cr - bt)} - 2ct\sqrt{\frac{b(a+cr-bt)}{t}}}{a + cr - bt} \tag{27}$$

where we have set $f(r) = a + cr$. The temporal and spatial behaviour of $R(r, t)$ are chosen so as to avoid any singular behaviour in the kinematical and dynamical quantities. The Einstein field equations (11)–(14) yield

$$\begin{aligned} \rho = & \left(4bc^3(cr - bt)^2 + a^3 \left(-4bc + \sqrt{a + cr - bt} \sqrt{\frac{b(a + cr - bt)}{t}} \right) \right. \\ & - 2a^2 \left(-2bc^2(c - r) - 2b^2ct + c^2\sqrt{a + cr - bt} \sqrt{\frac{b(a + cr - bt)}{t}} \right) \\ & \left. - 2a \left(-4bc^4r + 4b^2c^3t + c^3r\sqrt{a + cr - bt} \sqrt{\frac{b(a + cr - bt)}{t}} \right) \right) \end{aligned}$$

$$\begin{aligned}
& -3bc^2t\sqrt{a+cr-bt}\sqrt{\frac{b(a+cr-bt)}{t}}) \\
& \times \left(\sqrt{\frac{b(a+cr-bt)}{t}}(-a-cr+bt) \left(-a\sqrt{a+cr-bt} + 2ct\sqrt{\frac{b(a+cr-bt)}{t}} \right)^3 \right)
\end{aligned} \quad (28)$$

$$P_r = qB = \frac{2c^2}{(a\sqrt{a+cr-bt} - 2ct\sqrt{\frac{b(a+cr-bt)}{t}})^2} \quad (29)$$

The thermodynamical quantities at the center of star become

$$\begin{aligned}
\rho_0 = 59 & \left(4b^3c^3t^2 + a^3 \left(-4bc + \sqrt{a-bt}\sqrt{\frac{b(a-bt)}{t}} \right) \right. \\
& - 2a^2 \left(-2bc^3 - 2b^2ct + c^2\sqrt{a-bt}\sqrt{\frac{b(a-bt)}{t}} \right) \\
& - 2a \left(4b^2c^3t - 3bc^2t\sqrt{a-bt}\sqrt{\frac{b(a-bt)}{t}} \right) \\
& \left. / \left(\sqrt{\frac{b(a-bt)}{t}}(-a+bt) \left(-a\sqrt{a-bt} + 2ct\sqrt{\frac{b(a-bt)}{t}} \right)^3 \right) \right)
\end{aligned} \quad (30)$$

$$P_{r0} = Q_0 = \frac{2c^2}{(a\sqrt{a-bt} - 2ct\sqrt{\frac{b(a-bt)}{t}})^2} \quad (31)$$

The kinematical quantities yield

$$\sigma = \frac{a+cr+2(a+cr)t-2bt^2}{2t\sqrt{\frac{bR}{t}}(-R)} \quad (32)$$

$$\Theta = \frac{-2ac\sqrt{R} - 2c(cr-4bt)\sqrt{R} - 3at\sqrt{\frac{bR}{t}}}{2R^{(3/2)}(a\sqrt{R} - 2ct\sqrt{\frac{bR}{t}})} \quad (33)$$

$$a_1 = \frac{c}{2(a+cr-bt)} \quad (34)$$

6 Energy Conditions and Stability

In order to test the physical viability of our solution it is imperative to impose the energy conditions on the interior matter distribution. We firstly require that the energy density, radial pressure and tangential pressure all be positive within the stellar core, i.e.,

$$\rho \geq 0, \quad P_r \geq 0, \quad P_{\perp} \geq 0$$

We further require that the energy density and pressure decrease outwards from the center to the stellar surface, i.e.,

$$\rho' < 0, \quad P_r' < 0$$

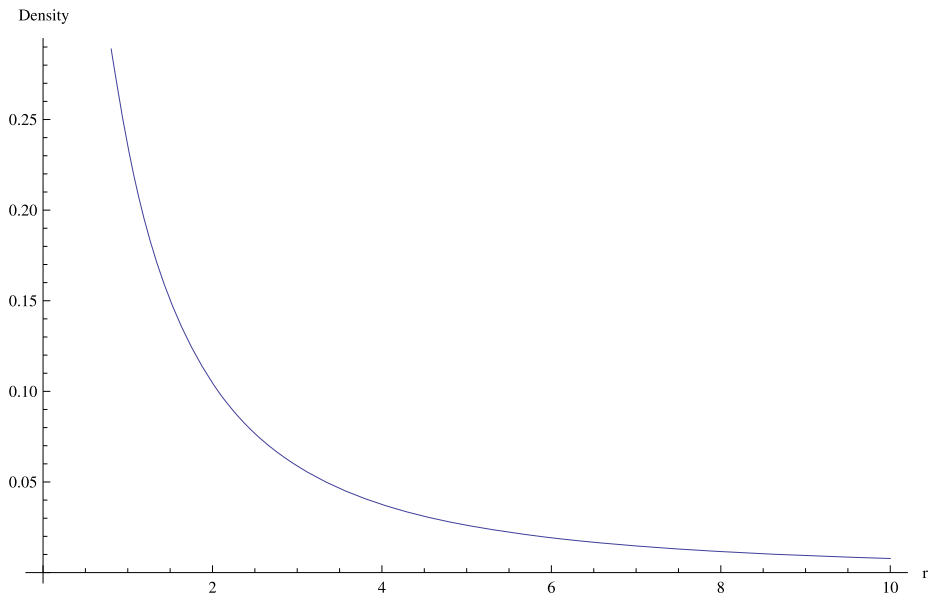


Fig. 1 Density vs. radial coordinate

We expect this as the density of the star at the center is the highest and gradually drops off towards the surface (Fig. 1). From Fig. 2 we note that the radial pressure also drops off as one moves from the center of the star to the stellar surface. Since $p = qB$ for all time, the heat flux is highest at the center of the core and falls off radially outwards leading to cooling of the surface layers. An interesting feature of this model is that the tangential pressure becomes negative for this particular epoch of the star's evolution (Fig. 3). This suggests that the tension on each concentric shell is such that it tends to cause the shell to expand while the sphere is collapsing under gravity. We can think of the negative tangential pressure as contributing to the outward radial pressure thus slowing down the collapse process. Graphical analysis of the energy conditions confirm: $(\rho + P_r)^2 - 4q^2 > 0$ and $\rho - P_r - 2P_\perp + [(\rho + P_r)^2 - 4q^2]^{1/2} > 0$ to always hold within the stellar interior. These requirements ensure that the energy conditions (weak, strong and dominant) are satisfied at all interior points of our stellar model.

7 Thermal Behaviour

The behaviour of the temperature during dissipative collapse has been extensively studied in shear-free models and shearing models with geodesic flows. Up to this point, the only nonzero accelerating models with shear are the so-called Euclidean stars which were first investigated by Herrera et al. [8]. The first exact model of a Euclidean star was presented by Govender et al. [10]. This model has been generalised by Govender and Govinder [14] in which the thermal behaviour and stability of Euclidean stars were investigated. The thermal behaviour of these models were well studied within the context of extended irreversible thermodynamics. Just as in the cases of shear-free models and shearing, geodesic models it was shown that relaxational effects due to heat dissipation leads to higher interior temperatures

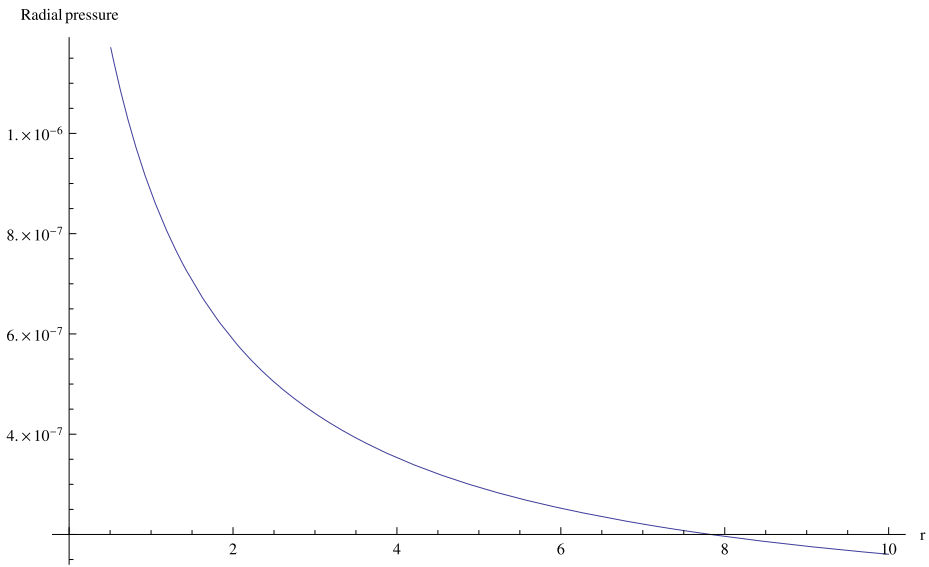


Fig. 2 Radial pressure vs. radial coordinate

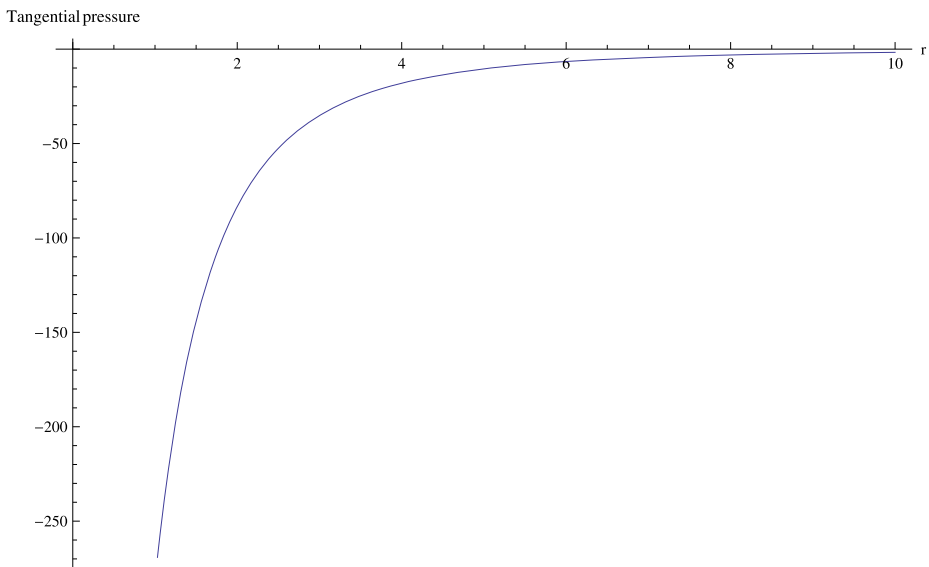


Fig. 3 Tangential pressure vs. radial coordinate

of the Euclidean stellar core. To this end we investigate the evolution of the temperature profiles in our class of accelerating models with nonzero shear. We employ a causal transport equation for the heat flux in order to display relaxational effects during the collapse process [15]. The truncated causal transport equation for the heat flux for the line element (1) is

given by

$$\tau(qB) \dot{A}(qB) + A(qB) = -\kappa \frac{(AT)'}{B} \tag{35}$$

where τ is the relaxation time and κ is the thermal conductivity. Switching ‘off’ τ leads to the noncausal Fourier heat transport equation

$$A(qB) = -\kappa \frac{(AT)'}{B} \tag{36}$$

which suffers various pathologies in terms of causality and stability of the equilibrium states. Adopting the thermodynamic coefficients for radiative transfer, we set

$$\kappa = \gamma T^3 \tau_c, \quad \tau_c = \left(\frac{\alpha}{\gamma}\right) T^{-\sigma} \tag{37}$$

where $\alpha \geq 0$, $\gamma \geq 0$ and $\sigma \geq 0$ are constants. We further assume that the velocity of thermal dissipative signals is comparable to the adiabatic sound speed which is ensured if the relaxation time is proportional to the collision time:

$$\tau = \left(\frac{\beta\gamma}{\alpha}\right) \tau_c \tag{38}$$

where $\beta (\geq 0)$ is a constant. Using the above definitions for τ and κ , (35) takes the form

$$\beta(qB) \dot{T}^{-\sigma} + A(qB) = -\alpha \frac{T^{3-\sigma} (AT)'}{B}. \tag{39}$$

We are in a position to integrate (39) for the special case of constant collision time corresponding to $\sigma = 0$:

$$\begin{aligned} T(r, t) = & \left(\frac{1}{X_2^2} \left(X_1^2 \left(c^2 X_1^3 \left(a^2 c + 2ac^2 d + c^3 d^2 - b^2 ct^2 - at\sqrt{X_1} \sqrt{\frac{bX_1}{t}} \right) \right. \right. \right. \\ & \times \frac{(-ab\sqrt{X_1} + ac\sqrt{\frac{bX_1}{t}} + c\sqrt{\frac{bX_1}{t}}(cd + bt))}{d^2 \sqrt{\frac{bX_1}{t}} (a\sqrt{X_1} - 2ct\sqrt{\frac{bX_1}{t}})^6} \\ & + \frac{1}{b^2 X_1^2 \psi} 4t^2 \left(\frac{4b^2 c X_1 (a\sqrt{X_1} + 2ct\sqrt{\frac{bX_1}{t}})}{3t^2 (a^2 - 4bc^2 t)} \right. \\ & - \left(2b^2 c \beta \left(-2c^2 d \left(4abc\sqrt{X_1} + a^2 \sqrt{\frac{bX_1}{t}} + 4bc^2 t \sqrt{\frac{bX_1}{t}} \right) \right. \right. \\ & - \left. \left. b(a^2 - 4bc^2 t) \left(a\sqrt{X_1} \log X_1 + 2ct\sqrt{\frac{bX_1}{t}} \log \frac{bX_1}{t} \right) \right) \right) \\ & \left. \left. \left. / \left(t^2 \sqrt{\frac{bX_1}{t}} (a^2 - 4bc^2 t)^2 \right) \right) \right) \\ & - \frac{4t^2}{(b^2 \psi)} \left(\frac{4b^2 c (X_2) (a\sqrt{X_2} + 2ct\sqrt{\frac{bX_2}{t}})}{3t^2 (a^2 - 4bc^2 t)} \right) \end{aligned}$$

this equation under various assumptions to generate several classes of radiating solutions with shear. We further studied the physical viability of one these classes of solutions by imposing the energy conditions. An interesting feature of this particular model is that the tangential pressure becomes negative for a particular period of the collapse. We point out that such a scenario would lead to the slowing down of the collapse as opposed to case of positive tangential pressure where the collapsing sphere is squeezed into a smaller volume. We also showed that the causal temperature is everywhere higher within the stellar core as compared to the Eckart temperature. Up to this point, the shearing radiating solutions that exist in the literature are all acceleration-free [5–7]. We have investigated the evolution of the temperature for a shearing collapsing fluid with nonzero acceleration. We have found that in the presence of acceleration and shear, the causal temperature within the stellar core is higher than the Eckart temperature. This reinforces the belief that relaxational effects cannot be ignored during dissipative gravitational collapse, even when the stellar fluid is close to hydrostatic equilibrium.

References

1. Hawking, S., Penrose, R.: *The Nature of Space and Time*. Princeton University Press, Princeton (1996)
2. Oppenheimer, J.R., Snyder, H.: On continued gravitational contraction. *Phys. Rev. D* **56**, 455 (1939)
3. Santos, N.O.: Non-adiabatic radiating collapse. *Mon. Not. R. Astron. Soc.* **216**, 403 (1985)
4. Banerjee, A., Chatterjee, S., Dadhich, N.: Spherical collapse with heat flow and without horizon. *Mod. Phys. Lett. A* **35**, 2335 (2002)
5. Naidu, N.F., Govender, M., Govinder, K.S.: Thermal evolution of a radiating anisotropic star with shear. *Int. J. Mod. Phys. D* **15**, 1053 (2006)
6. Govender, M., Thirukkanesh, S.: Dissipative collapse in the presence of Λ . *Int. J. Theor. Phys.* **48**, 3558 (2009)
7. Rajah, S.S., Maharaj, S.D.: A Riccati equation in radiative stellar collapse. *J. Math. Phys.* **49**, 012501 (2008)
8. Herrera, L., Santos, N.O.: Collapsing spheres satisfying an “Euclidean condition”. *Gen. Relativ. Gravit.* **42**, 2383 (2010)
9. Vaidya, P.C.: The gravitational field of a radiating star. *Proc. Indian Acad. Sci., Sect. A, Phys. Sci.* **33**, 264 (1951)
10. Govender, G., Govender, M., Govinder, K.S.: Thermal behaviour of Euclidean stars. *Int. J. Mod. Phys. D* **19**, 1773 (2010)
11. Di Prisco, A., Herrera, L., Ospino, J., Santos, N.O., Viña-Cervantes, V.M.: Expansion-free cavity evolution: some exact analytical models. *Int. J. Mod. Phys. D* **20**, 2351 (2011)
12. Thirukkanesh, S., Rajah, S.S., Maharaj, S.D.: Shearing radiative collapse with expansion and acceleration. *J. Math. Phys.* **53**, 032506 (2012)
13. Fleming, D.: Dissipative gravitating systems. Unpublished MSc Thesis, UKZN, South Africa (2012)
14. Govinder, K.S., Govender, M.: A general class of Euclidean stars. *Gen. Relativ. Gravit.* **44**, 147 (2012)
15. Maartens, R.: Causal thermodynamics in relativity (1996). [arXiv:astro-ph/9609119v1](https://arxiv.org/abs/astro-ph/9609119v1)