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A Dark Energy Model in a Scale Covariant Theory of Gravitation

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Abstract An axially symmetric Bianchi type-I space time with variable equation of state (EoS) parameter and constant deceleration parameter has been investigated in scale covariant theory of gravitation formulated by Canuto et al. (Phys. Rev. Lett. 39:429, 1977). With the help of special law of variation for Hubble's parameter proposed by Bermann (Nuovo Cimento 74B:182, 1983) a dark energy cosmological model is obtained in this theory. Some physical and kinematical properties of the model are also discussed.

Keywords Bianchi type-I metric · Dark energy · Scale covariant theory

1 Introduction

In recent years there has been a lot of interest in the study of alternative theories of gravitation (Brans and Dicke [1] Nordtvedt [2], Sen [3], Sen and Dunn [4] and Seaz and Ballester [5]). Canuto et al. [6] formulated scale-covariant theory of gravitation which is a viable alternative to general relativity (Wesson [7]; Will [8]). In Brans-Dicke theory there exists a variable gravitational parameter G. In the scalecovariant theory Einstein's field equations are valid in gravitational units where as physical quantities are measured in atomic units. The metric tensors in the two systems of units are related by a conformal transformation

$$\overline{g_{ij}} = \phi^2\left(x^k\right) g_{ij} \tag{1}$$

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where in Latin indices take values 1, 2, 3, 4, bar denote gravitational units and unbar denotes atomic quantities. The gauge function ϕ $(0 < \phi < \infty)$ in its most general formulation is function of all space-time coordinates. Thus using the conformal transformation of the type given by (1), Canuto et al. [6] transformed the usual Einstein equations into

$$R_{ij} - \frac{1}{2} R g_{ij} + f_{ij}(\phi) = -8\pi G(\phi) T_{ij} + \Lambda(\phi) g_{ij} \tag{2}$$

where

$$\phi^2 f_{ij} = 2\phi\phi_{i;j} - 4\phi_i\phi_j - g_{ij}\left(\phi\phi_{ik}^{\prime k} - \phi^k\phi_k\right) \tag{3}$$

Here R_{ij} is the Ricci tensor, R is the Ricci scalar, Λ is the cosmological 'constant', G is the gravitational 'constant' and T_{ij} is the energy momentum tensor. A semi colon denotes covariant derivative and ϕ_i denotes ordinary derivative with respect to x^i. A particular feature of this theory is that no independent equation for ϕ exists. The possibilities that have been considered for gauge function ϕ (Canuto et al. [6]) are

$$\phi(t) = \left(\frac{t_0}{t}\right)^{\varepsilon}, \quad \varepsilon = \pm 1, \ \pm\frac{1}{2} \tag{4}$$

where t_0 is a constant. The form

$$\phi \sim t^{\frac{1}{2}} \tag{5}$$

is the one most favored to fit observations (Canuto and Goldman [9]). Reddy and Rao [10], Reddy et al. [11], Reddy and Naidu [12], Beesham [13–15], Reddy and Venkateshwarlu [16, 17] and Singh and Devi [18] are some of the authors who have investigated several aspects of the scale covariant theory of gravitation.

The discovery of the accelerated expansion of the universe supposedly driven by exotic dark energy (Perlmutter et al. [19], Reiss et al. [20], Spergel et al. [21, 22], Copeland et al. [23]) has lead, in recent years, to the investigation of dark energy models both in general relativity and in alternative theories of gravitation. The nature and composition of dark energy is still an open problem. Dark energy is usually characterized by the EoS parameter $\omega(t) = \frac{p}{\rho}$ which is not necessarily constant, where p is the fluid pressure and ρ is the energy density (Carroll and Hoffman [24]). A lucid introduction and nice review of the work done on dark energy model in general relativity is given by Farooq et al. [25] and Pradhan et al. [26]. Pradhan and Amihaschi [27, 28], Amirhashchi et al. [29], Pradhan et al. [30] have discussed dark energy models in anisotropic Bianchi type space times with variable EOS parameter. Recently Naidu et al. [31, 32] have obtained Bianchi type-II and V dark energy models in the scalar-tensor theory of gravitation proposed by Saez and Ballester [5].

Spatially homogeneous and anisotropic cosmological models play a vital role in the study of the early stages of evolution of the universe. Here we have investigated axially symmetric Bianchi type-I dark energy model in the scale-covariant theory of gravitation formulated by Canuto et al. [6].

2 Metric and Field Equations

We consider a spatially homogeneous axially symmetric Bianchi type-I metric in the form

$$ds^2 = dt^2 - A^2 dx^2 - B^2\left(dy^2 + dz^2\right) \tag{6}$$

where A and B are functions of cosmic time t only.

One may determine EoS parameter of perfect fluid separately on each spatial axis by preserving the diagonal form of the energy momentum tensor in a consistent way with the metric (6). Therefore, the energy momentum tensor of the fluid is taken as

$$T_j^i = diag\big[T_0^0, T_1^1, T_2^2, T_3^3\big] \tag{7}$$

We can parameterize as follows

$$\begin{aligned} T_j^i &= diag[p_i, -p_x-, p_y-, p_z] \\ &= diag[1, -\omega_x, -\omega_y, -\omega_z]\rho \\ &= diag[1, -\omega, -(\omega+\delta), -(\omega+\gamma)]\rho \end{aligned} \tag{8}$$

where ρ is the energy density of the fluid, p_x, p_y, p_z are the pressures and ω_x, ω_y, ω_z are the directional EoS parameters along the X, Y, and Z axes respectively. ω is the deviation free EoSparameter of the fluid. We have parameterized the deviation from isotropy by setting $\omega_x = \omega$ and then introducing skewness parameters δ and γ, i.e. deviation from ω along the Y and Z axes respectively. Since in axially symmetric Bianchi type-I space time $T_2^2 = T_3^3$, we obtain

$$\delta = \gamma \tag{9}$$

In a comoving coordinate system the field equations (2) and (3) of the scale-covariant theory for the metric (6) with help of (8) and (9) take the form

$$2\frac{\dot{A}\dot{B}}{AB} + \left(\frac{\dot{B}}{B}\right)^2 - \frac{\ddot{\phi}}{\phi} + \frac{\dot{A}\dot{\phi}}{A\phi} + 2\frac{\dot{B}\dot{\phi}}{B\phi} + 3\left(\frac{\dot{\phi}}{\phi}\right)^2 = 8\pi Gp \tag{10}$$

$$2\frac{\ddot{B}}{B} + \left(\frac{\dot{B}}{B}\right)^2 - \frac{\ddot{\phi}}{\phi} + \frac{\dot{A}\dot{\phi}}{A\phi} + 2\frac{\dot{B}\dot{\phi}}{B\phi} - \left(\frac{\dot{\phi}}{\phi}\right)^2 = -8\pi G\omega p \tag{11}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{A}\dot{\phi}}{A\phi} - \left(\frac{\phi}{\phi}\right)^2 = -8\pi Gp(\omega+\gamma) \tag{12}$$

where an overhead dot denotes ordinary differentiation with respect to t.

The spatial volume for the metric (6) is given by

$$V^3 = AB^2 \tag{13}$$

We define the average scale factor of the metric (6) as

$$R = \left(AB^2\right)^{\frac{1}{3}} \tag{14}$$

so that the Hubble's parameter is given by

$$H = \frac{\dot{R}}{R} = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) \tag{15}$$

The deceleration parameter q is conventionally defined by

$$q = -\frac{R\dot{R}}{(\dot{R})^2} \tag{16}$$

The scale expansion θ, shear scalar σ^2 and the average anisotropy parameter are defined by

$$\theta = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \tag{17}$$

$$\sigma^2 = \frac{1}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2 \tag{18}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \tag{19}$$

where

$$\Delta H_i = H_i - H \quad (i = 1, 2, 3)$$

$$H = \frac{1}{3}(H_x + H_y + H_z) \tag{20}$$

and $H_x = \frac{A}{A}$, $H_y = \frac{B}{B}$, and $H_y = H_z$ are the directional Hubble's parameters in the directions of x, y, z respectively.

3 Solutions of the Field Equations

The field equations (10)–(12) are a system of three independent equations in five unknowns $A, B, \phi, \omega, \gamma$ [in view of the fact $G = G(\phi)$ and (5) and (9)]. Two additional constraints relating these parameters are required to obtain explicit solutions of the system.

Firstly, we use the special law of variation of Hubble's parameter proposed by Bermann [33]. We consider the constant deceleration parameter defined by (16) and integrating it we get

$$R = (at + b)^{\frac{1}{1+q}} \tag{21}$$

where $a \neq 0$ and b are constants of integration. This equation implies that the condition of accelerated expansion is $1 + q > 0$.

Secondly, we assume that the scalar of expansion θ in the model is proportional to the shear scalar σ. This condition leads to

$$A = B^m \tag{22}$$

where m is a constant (Collins et al. [34])

After solving the field equations (10)–(12) with the help of (14), (21) and (22) we obtain the expressions for metric coefficients as

$$B = (at + b)^{\frac{3}{(m+2)(1+q)}} \tag{23}$$

$$A = (at + b)^{\frac{3m}{(m+2)(1+q)}} \tag{24}$$

Hence the metric (6) through a proper choice of integration constants (i.e. $a = 1$, $b = 0$) can be written as

$$ds^2 = dt^2 - t^{\frac{6m}{(m+2)(1+q)}}\,dx^2 - t^{\frac{6m}{(m+2)(1+q)}}\left(dy^2 + dz^2\right) \tag{25}$$

4 Some Physical and Kinematical Properties of the Model

The metric given by (25) represents an axially symmetric Bianchi type-I dark energy cosmological model with the following physical and kinematical parameters in scale covariant theory of gravitation.

The energy density

$$p = \frac{1}{8\pi G}\left[\frac{9(2m+1)}{(1+q)^2(m+2)^2}\frac{1}{t^2} - \left\{\left(\phi_0 - 3\phi_0^2\right)\varepsilon^2 - \phi_0\varepsilon\right\}\frac{1}{t^2} + \frac{3\phi_0\varepsilon}{1+q}\frac{1}{t^2}\right] \tag{26}$$

EoS parameter

$$\begin{aligned}\omega = \frac{1}{8\pi Gp}\left[\frac{6}{(1+q)(m+2)}\frac{1}{t} + \frac{9}{(1+q)^2(m+2)^2}\frac{1}{t^2}\right.\\ \left. + \frac{3\varepsilon(2-m)}{(1+q)(m+2)}\frac{1}{t^2} - \frac{\varepsilon}{t^2}\right]\end{aligned} \tag{27}$$

Skewness parameter

$$\begin{aligned}\gamma = \frac{1}{8\pi Gp}\left[\frac{3}{(1+q)^2(m+2)^2}\left\{(1+q)(m+2)(m+1) - 3\left(m^2+m+2\right)\right\}\frac{1}{t^2}\right.\\ \left. + \frac{6}{(1+q)(m+2)}\frac{1}{t} - \frac{6\phi_0\varepsilon(m-1)}{(1+q)(m+2)t^2}\right]\end{aligned} \tag{28}$$

Scalar field

$$\phi = \phi_0 t^{\varepsilon}, \quad \varepsilon = \pm 1, \pm\frac{1}{2} \tag{29}$$

The spatial volume

$$V^3 = t^{\frac{3}{t^{1+q}}} \tag{30}$$

Scalar expansion

$$\theta = \frac{3}{1+q}\frac{1}{t} \tag{31}$$

Shear scalar

$$\sigma^2 = \frac{3(m-1)}{(m+2)(1+q)^2}\frac{1}{t^2} \tag{32}$$

Average anisotropy parameter

$$A_m = \frac{4}{3}\frac{1}{(1+q)^2} \tag{33}$$

Hubble's parameter

$$H = \frac{1}{1+q}\frac{1}{t} \tag{34}$$

Also

$$\frac{\sigma^2}{\theta^2} = \frac{(m-1)}{3(m+2)^2} \tag{35}$$

It can be observed that the model (25) has no initial singularity, i.e. at $t=0$. Physical quantities p, ω, γ diverge at $t=0$ while they vanish for large values of t. The scalar field ϕ tends to infinity for large t when $\varepsilon = 1, \frac{1}{2}$ while it vanishes when $\varepsilon = -1, -\frac{1}{2}$. The spatial volume increases as t increases (since $1+q>0$) which shows that the universe is expanding. The scalar of expansion θ, shear scalar σ^2 and the Hubble's parameter H diverge at $t=0$ and vanish for large t. The mean anisotropic parameter is uniform throughout the evolution of the universe, since it does not depend on the cosmic time t. Since $\frac{\sigma^2}{\theta^2} = constant$, the model does not approach isotropy for large values of t.

However, since $1+q>0$ the universe is expanding with the increase of cosmic time but the rate of expansion and shear scalar decrease to zero and tend to isotropic. Thus this case implies an accelerating model of universe. Hence it follows that our model represents physical dark energy model.

5 Conclusions

Dark energy cosmological models are, recently, playing a vital role in the discussion of accelerated expansion of the universe in general relativity. With the advent of alternative theories of gravitation study of these models is gaining importance. Here we have investigated axially symmetric, anisotropic Bianchi type-I dark energy model with variable EoS parameter in a scale-covariant theory of gravitation formulated by Canuto et al. [6]. It is observed that the model has no initial singularity and all the physical parameters are infinite at the initial epoch, $t=0$ and tend to zero for large t. It is also observed that the model does not approach isotropy through the whole evolution of the universe. This model, definitely, throws some light on the understanding of dark energy model in scale covariant theory of gravitation.

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