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Multi-photon Entanglement Concentration Protocol for Partially Entangled W States with Projection Measurement

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Abstract We propose a novel entanglement concentration protocol (ECP) for nonlocal N-photon systems in a partially entangled pure W-class state, following some ideas in the work by Deng (Phys. Rev. A 85:022311, 2012). Our ECP resorts to an ancillary single photon and the projection measurement on it. Compared with other ECPs for W-class states, our ECP requires only one copy of the multipartite entangled system in a partially entangled W-class pure state, not two copies, which reduces the difficulty of its implementation largely. Moreover, it requires only one of the N parties in quantum communication to perform the local unitary operation for reconstructing the standard W state from the W-class state and it only resorts to linear optical elements for its implementation. All these advantages make our ECP more feasible than others in practical applications in quantum communication.

Keywords Entanglement concentration · W state · Quantum communication · Projection measurement · Ancillary single photon

1 Introduction

Quantum entanglement plays an important role in quantum computation [1] and quantum communication. It is the information carries in quantum key distribution [2–6], quantum teleportation [7], controlled teleportation [8–11], quantum dense coding [12, 13], quantum secret sharing [14–22], quantum state sharing [23–30], quantum secure direct communication [31–43], and so on. In particular, the photon systems in a maximally entangled state are usually required in many practical quantum communication protocols for their security. Usually, entanglement is produced locally and then be distributed to the remote parties in quantum communication. In experiment, it is difficult for users to obtain an ideal entanglement source which can emit a photon pair in a maximally entangled state in each time. Moreover, entanglement is very fragile in the process of transmission and that of storage,

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due to the influence of decoherence and the imperfection at the source. These factors lead to the success probability of the implementation less than 100 %. That is, sharing maximally entangled states between distant parties is still a challenge in experiment with current techniques. In a long-distance quantum communication, the users should exploit quantum repeaters to connect the neighboring nodes with entanglement for improving the efficient distance. In a quantum repeater, entanglement swapping of two nonlocal entanglements with a high fidelity is necessary. However, after the transmission of entanglement over a practical channel with noise and the storage of entangled systems, the initially maximally entangled state usually become a mixed entangled state or a partially entangled pure state.

If the initially maximally entangled state becomes a mixed entangled one, the two remote users can exploit entanglement purification [44–61] to improve its fidelity. If the photon systems are still in a pure state with a less entanglement, i.e., a partially entangled pure state, the user can exploit entanglement concentration [62–74] to reconstruct a subset of systems in a maximally entangled state. Entanglement purification is more general for distilling high-entanglement from nonmaximally entangled states, but entanglement concentration is more efficient in the distillation of the maximally entangled state from a partially entangled pure state as entanglement purification can only be used to improve the fidelity of an ensemble in a mixed entangled state by consuming a great deal of quantum resource [67, 74].

Generally speaking, entanglement concentration is a useful tool with which the two legitimate users in quantum communication, say, Alice and Bob, convert a large set of copies of a known (or unknown) pure state into as a subset of copies of the Bell state as possible by applying local operations and classical communication (LOCC). It does not require that they succeed exactly but only with the fidelity 100 % in theory [75]. By far, there are many interesting entanglement concentration protocols [62–74] in theory, such as the entanglement concentration protocol (ECP) based on the Schmidt projection method [62], that on entanglement swapping of two copies of partially entangled photon systems [63], that on a collective unitary evolution with an ancillary qubit [64], those based on line optical elements [65, 66], those with cross-Kerr nonlinearities [67, 68], those for atom systems [69, 70], and those for electronic systems [71, 72]. More recently, Sheng et al. [73] proposed an interesting ECP for photon systems in a Bell state, resorting to ancillary single photons. In 2012, Deng [74] presented another interesting ECP for photon systems in a Bell state, resorting to an ancillary single photon and the projection measurement on the ancillary photon.

Although there are many interesting ECPs existing, almost all of them are used to distill some maximally entangled Bell states or Greenberger-Horne-Zeilinger (GHZ) states [62–74]. There are few schemes [75, 76] for concentrating the non-maximally entangled pure W-class states. It is well known that W states are inequivalent to GHZ states in the sense that they cannot be converted to each other under stochastic local operations and classical communication (SLOCC) and they are highly robust against the loss of one or two qubits. W states have important applications in quantum information processing. That is, it is of practical significance to concentrate on the partially entangled W state on the demand for long-distance quantum communication and quantum information processing. In 2010, Wang, Zhang, and Yeon [75] proposed an interesting ECP for two copies of photon systems in a partially entangled W-class pure state with only linear optical elements, following some ideas in Refs. [65, 66]. In 2011, Xiong and Ye proposed an interesting ECP for two copies of photon systems in a partially entangled W-class pure state with cross-Kerr nonlinearity, following some ideas in Ref. [67].

In this paper, we propose a novel ECP for nonlocal N-photon systems in a partially entangled pure W-class state, following some ideas in the work by Deng [74]. Our ECP resorts to an ancillary single photon and the projection measurement on it. Compared with

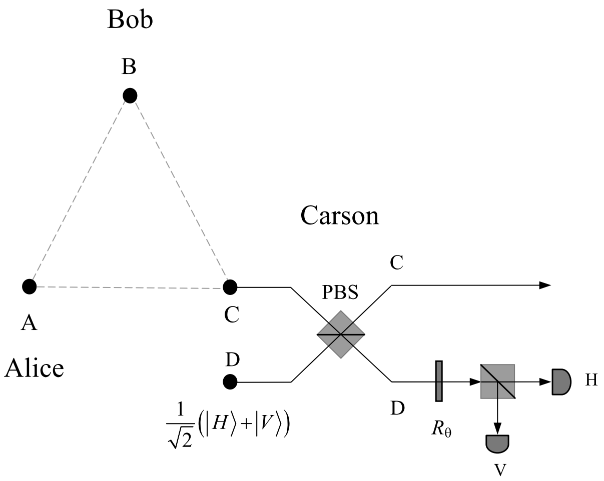

Fig. 1 Schematic diagram of our ECP for three-photon W states with an ancillary single photon and a projection measurement. PBS denotes a polarization beam splitter. R_θ represents a unitary rotation operation on the polarization of the ancillary single photon D. Carson uses a PBS and two detectors to detect the polarization states of the ancillary single photon. When Carson obtain the outcome of measurement on the photon D $|H\rangle_D$, the three parties obtain a standard three-photon W state. When he obtains $|V\rangle_D$, Carson should tell the other parties to discard the three-photon system

other ECPs for W-class states, our protocol has some advantages. First, it requires only one copy of the multipartite entangled system in a partially entangled W-class pure state, not two copies, which reduces the difficulty of its implementation largely. Second, the users exploit only linear optical elements to accomplish the entanglement concentration. Third, it requires only one of the N parties in quantum communication to perform the local unitary operation for reconstructing the standard W state from the W-class state. All these advantages make our ECP more feasible and more convenient than others.

2 Entanglement Concentration of Partially Entangled Three-Photon W States

In the following, we will demonstrate how to concentrate entanglement of a multi-photon system in a partially entangled W state with an ancillary single photon and a projection measurement. We first discuss the principle of our ECP for a three-photon system and then generalize it to the case with a multi-photon system.

Assume that the three-photon system composed of the three photons ABC is in the following polarization entangled states:

$$|\phi\rangle_{ABC} = \alpha|H\rangle_A|H\rangle_B|V\rangle_C + \beta\big(|H\rangle_A|V\rangle_B|H\rangle_C + |V\rangle_A|H\rangle_B|H\rangle_C\big), \tag{1}$$

where the subscripts A, B, and C represent the three photons shared by three remote parties in quantum communication, say Alice, Bob, and Carson. α and β are two known real numbers and satisfy the relation

$$\alpha^2 + 2\beta^2 = 1. \tag{2}$$

The principle of our ECP is shown in Fig. 1. In order to concentrate each three-photon system in this W-class state, Carson prepares a single photon D whose initial state is

$$|\phi'\rangle_D = \frac{1}{\sqrt{2}}\big(|H\rangle_D + |V\rangle_D\big). \tag{3}$$

Then the state of the composite system $ABCD$ is

$$\begin{aligned}|\Psi\rangle_{ABCD} &= |\phi\rangle_{ABC}\otimes|\phi'\rangle_D \\ &= \frac{1}{\sqrt{2}}\big\{\alpha|H\rangle_A|H\rangle_B|V\rangle_C|H\rangle_D+\beta\big(|H\rangle_A|V\rangle_B+|V\rangle_A|H\rangle_B\big)|H\rangle_C|V\rangle_D \\ &\quad+\alpha|H\rangle_A|H\rangle_B|V\rangle_C|V\rangle_D+\beta\big(|H\rangle_A|V\rangle_B+|V\rangle_A|H\rangle_B\big)|H\rangle_C|H\rangle_D\big\}. \end{aligned} \tag{4}$$

The first polarization beam splitter (PBS) in Carson's laboratory will make the two photons C and D emit from two outputs when they have the identical polarization states $|H\rangle_C|H\rangle_D$ or $|V\rangle_C|V\rangle_D$. When they have different polarization states $|H\rangle_C|V\rangle_D$ or $|V\rangle_C|H\rangle_D$, the two photons will emit from the same output of the first PBS. Carson only picks up the two-mode instances for obtaining the standard W state. That is, each of the spatial modes of the PBS has one and only one photon in this time. The polarization state $|H\rangle_C|V\rangle_D$ will lead to the fact that the two photons emit from the same spatial mode, i.e., the upper mode, while the state $|V\rangle_C|V\rangle_D$ will lead the two photons emit from the down spatial mode. That is, when the two photons emit from different spatial modes, Carson can pick up the state

$$\begin{aligned}|\Psi'\rangle_{ABCD} &= \frac{1}{\sqrt{\alpha^2+2\beta^2}}\big[\alpha|H\rangle_A|H\rangle_B|V\rangle_C|V\rangle_D \\ &\quad+\beta\big(|H\rangle_A|V\rangle_B+|V\rangle_A|H\rangle_B\big)|H\rangle_C|H\rangle_D)\big]. \end{aligned} \tag{5}$$

R_θ represents a rotation operation on the polarizations of the ancillary single photon D and it is used to accomplish the transformation as following

$$\begin{aligned}|H\rangle &\to \frac{1}{\sqrt{\alpha^2+\beta^2}}\big(\alpha|H\rangle-\beta|V\rangle\big), \\ |V\rangle &\to \frac{1}{\sqrt{\alpha^2+\beta^2}}\big(\beta|H\rangle+\alpha|V\rangle\big). \end{aligned} \tag{6}$$

After the rotation R_θ, the state of the system becomes

$$\begin{aligned}|\Psi''\rangle_{ABCD} &= \frac{1}{\sqrt{(\alpha^2+\beta^2)(\alpha^2+2\beta^2)}}\Big\{\alpha\beta\big[|H\rangle_A|H\rangle_B|V\rangle_C \\ &\quad+\big(|H\rangle_A|V\rangle_B+|V\rangle_A|H\rangle_B\big)|H\rangle_C\big]|H\rangle_D \\ &\quad+\big[\alpha^2|H\rangle_A|H\rangle_B|V\rangle_C-\beta^2\big(|H\rangle_A|V\rangle_B+|V\rangle_A|H\rangle_B\big)|H\rangle_C\big]|V\rangle_D\Big\}. \end{aligned} \tag{7}$$

When Carson obtains the state $|H\rangle_D$, the system ABC is in the standard three-photon W state

$$|W^+\rangle_{ABC} = \frac{1}{\sqrt{3}}\big(|H\rangle|H\rangle|V\rangle+|H\rangle|V\rangle|H\rangle+|V\rangle|H\rangle|H\rangle\big)_{ABC}. \tag{8}$$

When Carson obtains the state $|V\rangle_D$, the system ABC is in a three-photon W-class state with less entanglement

$$\begin{aligned}|\Psi_L\rangle_{ABC} &= \frac{1}{\sqrt{\alpha^4+2\beta^4}}\big[\alpha^2|H\rangle_A|H\rangle_B|V\rangle_C \\ &\quad-\beta^2\big(|H\rangle_A|V\rangle_B+|V\rangle_A|H\rangle_B\big)|H\rangle_C\big]. \end{aligned} \tag{9}$$

Carson should tell the other parties to discard the three-photon system when he obtain the state $|V\rangle_D$.

In our ECP for three-photon W states, Carson can accomplish the concentration by picking up the two-mode instances and performing a project measurement on the ancillary single photon with a unitary rotation operation and a PBS. With linear optical elements only, the success probability of our ECP for three-photon W-class states is

$$P_3 = \frac{3\alpha^2\beta^2}{2(\alpha^2+\beta^2)}. \tag{10}$$

3 Entanglement Concentration of Partially Entangled N-Photon W States

It is straightforward to extend our ECP to reconstruct standard multipartite entangled W states from partially entangled W-class states. Let us assume that the partially entangled N-photon W-class states are described as follows:

$$\begin{aligned} |\phi_N\rangle_{ABC\cdots Z} = {} & \alpha|V\rangle_A|H\rangle_B|H\rangle_C\cdots|H\rangle_Z \\ & + \beta\big(|H\rangle_A|V\rangle_B|H\rangle_C\cdots|H\rangle_Z + |H\rangle_A|H\rangle_B|V\rangle_C\cdots|H\rangle_Z + \cdots \\ & + |H\rangle_A|H\rangle_B|H\rangle_C\cdots|V\rangle_Z\big). \end{aligned} \tag{11}$$

The subscript $A, B, C, \ldots,$ and Z represent the photons in W-class states shared by the N parties in quantum communication, i.e., Alice, Bob, Carson, ..., and Zenger, respectively. The parameters α and β satisfy the following relation

$$\alpha^2 + (N-1)\beta^2 = 1. \tag{12}$$

Similar to the case with three-photon systems, Alice prepares an ancillary photon D in the state $|\phi\rangle_D = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)_D$ for reconstructing the standard N-photon W state. The state of the composite system can be written as

$$\begin{aligned} |\Psi\rangle_{DABC\cdots Z} &= |\phi\rangle_D \otimes |\phi_N\rangle_{ABC\cdots Z} \\ &= \frac{1}{\sqrt{2}}\big(|H\rangle + |V\rangle\big)_D \otimes \big[\alpha|V\rangle_A|H\rangle_B|H\rangle_C\cdots|H\rangle_Z \\ &\quad + \beta|H\rangle_A\big(|V\rangle_B|H\rangle_C\cdots|H\rangle_Z + |H\rangle_B|V\rangle_C\cdots|H\rangle_Z \\ &\quad + \cdots + |H\rangle_B|H\rangle_C\cdots|V\rangle_Z\big)\big] \\ &= \frac{1}{\sqrt{2}}\big[\alpha|H\rangle_D|V\rangle_A|H\rangle_B|H\rangle_C\cdots|H\rangle_Z \\ &\quad + \beta|H\rangle_D|H\rangle_A\big(|V\rangle_B|H\rangle_C\cdots|H\rangle_Z \\ &\quad + |H\rangle_B|V\rangle_C\cdots|H\rangle_Z + \cdots + |H\rangle_B|H\rangle_C\cdots|V\rangle_Z\big) \\ &\quad + \alpha|V\rangle_D|V\rangle_A|H\rangle_B|H\rangle_C\cdots|H\rangle_Z + \beta|V\rangle_D|H\rangle_A\big(|V\rangle_B|H\rangle_C\cdots|H\rangle_Z \\ &\quad + |H\rangle_B|V\rangle_C\cdots|H\rangle_Z + \cdots + |H\rangle_B|H\rangle_C\cdots|V\rangle_Z\big)\big]. \end{aligned} \tag{13}$$

Alice performs the rotation R_θ on the ancillary single photon D and then detects its polarization states with a PBS and two single-photon detectors. After the rotation R_θ, the state of the $(N-1)$-photon becomes

$$\begin{aligned} |\Psi'\rangle_{DABC\cdots Z} = \frac{1}{\sqrt{\alpha^2+\beta^2}} \big\{ & \alpha\beta|H\rangle_D\big[|V\rangle_A|H\rangle_B|H\rangle_C\cdots|H\rangle_Z \\ & + |H\rangle_A\big(|V\rangle_B|H\rangle_C\cdots|H\rangle_Z + |H\rangle_B|V\rangle_C\cdots|H\rangle_Z \\ & + \cdots + |H\rangle_B|H\rangle_C\cdots|V\rangle_Z\big)\big] \\ & + |V\rangle_D\big[\alpha^2|V\rangle_A|H\rangle_B|H\rangle_C\cdots|H\rangle_Z \\ & - \beta^2|H\rangle_A\big(|V\rangle_B|H\rangle_C\cdots|H\rangle_Z + |H\rangle_B|V\rangle_C\cdots|H\rangle_Z \\ & + \cdots + |H\rangle_B|H\rangle_C\cdots|V\rangle_Z\big)\big]\big\}. \end{aligned} \tag{14}$$

When Alice obtains the polarization state $|H\rangle_D$, the N-photon system is in the standard W state

$$\begin{aligned} |W^+\rangle_{ABC\cdots Z} = \frac{1}{\sqrt{N}}\big[& |H\rangle_A\big(|V\rangle_B|H\rangle_C\cdots|H\rangle_Z \\ & + |H\rangle_B|V\rangle_C\cdots|H\rangle_Z + \cdots + |H\rangle_B|H\rangle_C\cdots|V\rangle_Z\big) \\ & + |V\rangle_A|H\rangle_B|H\rangle_C\cdots|H\rangle_Z\big] \end{aligned} \tag{15}$$

When Alice obtains the polarization state $|V\rangle_D$, the N-photon system is in a W-class state with less entanglement

$$\begin{aligned} |W_L\rangle_{ABC\cdots Z} = {} & \alpha^2|V\rangle_A|H\rangle_B|H\rangle_C\cdots|H\rangle_Z \\ & - \beta^2|H\rangle_A\big(|V\rangle_B|H\rangle_C\cdots|H\rangle_Z + |H\rangle_B|V\rangle_C\cdots|H\rangle_Z \\ & + \cdots + |H\rangle_B|H\rangle_C\cdots|V\rangle_Z\big). \end{aligned} \tag{16}$$

With linear optical elements only, the success probability of our ECP for N-photon systems is

$$P_N = \frac{N\alpha^2\beta^2}{2(\alpha^2+\beta^2)}. \tag{17}$$

4 Discuss and Summary

Compared with other existing ECPs for W-class states [75, 76], the present ECS has some advantages. First, our ECP requires only an N-photon system and an ancillary photon, not two N-photon entangled systems. Second, only one of the N parties in quantum communication perform the local unitary operation for reconstructing the standard W state from the W-class state and she need only communicate the classical information to other parties for retaining or discarding their photons, which greatly simplifies the complication of classical communication. Third, it has a higher efficiency than others. Moreover, our ECP only requires linear optical elements and an ancillary single photon in a known polarization state, which makes our ECP more feasible than other ECPs.

In summary, we have proposed an ECP for nonlocal N-photon systems in a partially entangled pure W-class state, resorting to an ancillary single photon and the projection measurement on it [74]. Only one of the N parties in quantum communication prepares an ancillary photon and operates the entanglement concentration process for obtaining the standard

N-photon W state from each partially entangled pure W-class state by choosing the two-mode instances and detecting the ancillary single photon. Compared with other ECPs for W-class states [75, 76], our ECP has some advantages. It requires an N-photon system and an ancillary photon for each round of concentration, not two systems. It has a higher success efficiency. Moreover, it requires only linear optical elements and it requires only one of the N parties in quantum communication to perform the local unitary operation for reconstructing the standard W state from the W-class state. All these advantages make our ECP more feasible than others. It maybe have good applications in quantum communication, especially in quantum secret sharing.

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References

1. Nielsen, M.A., Chuang, I.L.: Quantum Computation and Quantum Information. Cambridge Univ. Press, Cambridge (2000)
2. Ekert, A.K.: Phys. Rev. Lett. **67**, 661 (1991)
3. Bennett, C.H., Brassard, G., Mermin, N.D.: Phys. Rev. Lett. **68**, 557 (1992)
4. Long, G.L., Liu, X.S.: Phys. Rev. A **65**, 032302 (2002)
5. Deng, F.G., Long, G.L.: Phys. Rev. A **68**, 042315 (2003)
6. Li, X.H., Deng, F.G., Zhou, H.Y.: Phys. Rev. A **78**, 022321 (2008)
7. Bennett, C.H., Brassard, G., Crepeau, C., Jozsa, R., Peres, A., Wootters, W.K.: Phys. Rev. Lett. **70**, 1895 (1993)
8. Karlsson, A., Bourennane, M.: Phys. Rev. A **58**, 4394 (1998)
9. Yan, F.L., Wang, D.: Phys. Lett. A **316**, 297 (2003)
10. Yang, C.P., Chu, S.I., Han, S.: Phys. Rev. A **70**, 022329 (2004)
11. Deng, F.G., Li, C.Y., Li, Y.S., Zhou, H.Y., Wang, Y.: Phys. Rev. A **72**, 022338 (2005)
12. Bennett, C.H., Wiesner, S.J.: Phys. Rev. Lett. **69**, 2881 (1992)
13. Liu, X.S., Long, G.L., Tong, D.M., Feng, L.: Phys. Rev. A **65**, 022304 (2002)
14. Hillery, M., Bužek, V., Berthiaume, A.: Phys. Rev. A **59**, 1829 (1999)
15. Karlsson, A., Koashi, M., Imoto, N.: Phys. Rev. A **59**, 162 (1999)
16. Xiao, L., Long, G.L., Deng, F.G., Pan, J.W.: Phys. Rev. A **69**, 052307 (2004)
17. Gu, B., Li, C.Q., Xu, F., Chen, Y.L.: Chin. Phys. B **18**, 4690 (2009)
18. Deng, F.G., Long, G.L., Zhou, H.Y.: Phys. Lett. A **337**, 329 (2005)
19. Zhang, Z.J., Li, Y., Man, Z.X.: Phys. Rev. A **71**, 044301 (2005)
20. Deng, F.G., Li, X.H., Zhou, H.Y., Zhang, Z.J.: Phys. Rev. A **72**, 044302 (2005)
21. Yan, F.L., Gao, T.: Phys. Rev. A **72**, 012304 (2005)
22. Gu, B., Mu, L.L., Ding, L.G., Zhang, C.Y., Li, C.Q.: Opt. Commun. **283**, 3099 (2010)
23. Cleve, R., Gottesman, D., Lo, H.K.: Phys. Rev. Lett. **83**, 648 (1999)
24. Lance, A.M., Symul, T., Bowen, W.P., Sanders, B.C., Lam, P.K.: Phys. Rev. Lett. **92**, 177903 (2004)
25. Deng, F.G., Li, X.H., Li, C.Y., Zhou, P., Zhou, H.Y.: Phys. Rev. A **72**, 044301 (2005)
26. Deng, F.G., Li, X.H., Li, C.Y., Zhou, P., Zhou, H.Y.: Eur. Phys. J. D **39**, 459 (2006)
27. Li, X.H., Zhou, P., Li, C.Y., Zhou, H.Y., Deng, F.G.: J. Phys. B **39**, 1975 (2006)
28. Man, Z.X., Xia, Y.J., An, N.B.: Eur. Phys. J. D **42**, 333 (2007)
29. Wang, Z.Y., Yuan, H., Shi, S.H., Zhang, Z.J.: Eur. Phys. J. D **41**, 371 (2007)
30. Sheng, Y.B., Deng, F.G., Zhou, H.Y.: Eur. Phys. J. D **48**, 279 (2008)
31. Deng, F.G., Long, G.L., Liu, X.S.: Phys. Rev. A **68**, 042317 (2003)
32. Wang, C., Deng, F.G., Li, Y.S., Liu, X.S., Long, G.L.: Phys. Rev. A **71**, 044305 (2005)
33. Yan, F.L., Zhang, X.: Eur. Phys. J. B **41**, 75 (2004)
34. Gao, T., Yan, F.L., Wang, Z.X.: J. Phys. A **38**, 5761 (2005)
35. Gao, T., Yan, F.L., Wang, Z.X.: J. Phys. A, Math. Gen. **38**, 5761 (2005)
36. Gao, T., Yan, F.L., Wang, Z.X.: Chin. Phys. **14**, 893 (2005)
37. Man, Z.X., Zhang, Z.J., Li, Y.: Chin. Phys. Lett. **22**, 18 (2005)
38. Zhu, A.D., Xia, Y., Fan, Q.B., Zhang, S.: Phys. Rev. A **73**, 022338 (2006)
39. Li, X.H., Deng, F.G., Zhou, H.Y.: Phys. Rev. A **74**, 054302 (2006)

40. Wang, J., Zhang, Q., Tang, C.J.: Phys. Lett. A **358**, 256 (2006)
41. Wang, C., Deng, F.G., Long, G.L.: Opt. Commun. **253**, 15 (2005)
42. Gu, B., Pei, S.X., Song, B., Zhong, K.: Sci. China Ser. G **52**, 1913 (2009)
43. Gu, B., Zhang, C.Y., Cheng, G.S., Huang, Y.G.: Sci. China, Phys. Mech. Astron. **54**, 942 (2011)
44. Bennett, C.H., Brassard, G., Popescu, S., Schumacher, B., Smolin, J.A., Wootters, W.K.: Phys. Rev. Lett. **76**, 722 (1996)
45. Deutsch, D., Ekert, A., Jozsa, R., Macchiavello, C., Popescu, S., Sanpera, A.: Phys. Rev. Lett. **77**, 2818 (1996)
46. Pan, J.W., Simon, C., Brukner, C., Zeilinger, A.: Nature **410**, 1067 (2001)
47. Simon, C., Pan, J.W.: Phys. Rev. Lett. **89**, 257901 (2002)
48. Yang, M., Song, W., Cao, Z.L.: Phys. Rev. A **71**, 012308 (2005)
49. Sheng, Y.B., Deng, F.G., Zhou, H.Y.: Phys. Rev. A **77**, 042308 (2008)
50. Czechlewski, M., Grudka, A., Ishizaka, S., Wójcik, A.: Phys. Rev. A **80**, 014303 (2009)
51. Sheng, Y.B., Deng, F.G.: Phys. Rev. A **81**, 032307 (2010)
52. Sheng, Y.B., Deng, F.G.: Phys. Rev. A **82**, 044305 (2010)
53. Li, X.H.: Phys. Rev. A **82**, 044304 (2010)
54. Deng, F.G.: Phys. Rev. A **83**, 062316 (2011)
55. Deng, F.G.: Phys. Rev. A **84**, 052312 (2011)
56. Wang, C., Zhang, Y., Jin, G.S.: Quantum Inf. Comput. **11**, 988 (2011)
57. Sheng, Y.B., Deng, F.G., Zhao, B.K., Wang, T.J., Zhou, H.Y.: Eur. Phys. J. D **55**, 235 (2009)
58. Feng, X.L., Kwek, L.C., Oh, C.H.: Phys. Rev. A **71**, 064301 (2005)
59. Sheng, Y.B., Deng, F.G., Long, G.L.: Phys. Lett. A **375**, 396 (2011)
60. Sheng, Y.B., Long, G.L., Deng, F.G.: Phys. Lett. A **376**, 314 (2012)
61. Gu, B., Chen, Y.L., Zhang, C.Y., Huang, Y.G.: Chin. Phys. Lett. **27**, 100304 (2010)
62. Bennett, C.H., Bernstein, H.J., Popescu, S., Schumacher, B.: Phys. Rev. A **53**, 2046 (1996)
63. Bose, S., Vedral, V., Knight, P.L.: Phys. Rev. A **60**, 194 (1999)
64. Shi, B.S., Jiang, Y.K., Guo, G.C.: Phys. Rev. A **62**, 054301 (2000)
65. Yamamoto, T., Koashi, M., Imoto, N.: Phys. Rev. A **64**, 012304 (2001)
66. Zhao, Z., Pan, J.W., Zhan, M.S.: Phys. Rev. A **64**, 014301 (2001)
67. Sheng, Y.B., Deng, F.G., Zhou, H.Y.: Phys. Rev. A **77**, 062325 (2008)
68. Sheng, Y.B., Deng, F.G., Zhou, H.Y.: Quantum Inf. Comput. **10**, 272 (2010)
69. Yang, M., Zhao, Y., Song, W., Cao, Z.L.: Phys. Rev. A **71**, 044302 (2005)
70. Cao, Z.L., Zhang, L.H., Yang, M.: Phys. Rev. A **73**, 014303 (2006)
71. Sheng, Y.B., Deng, F.G., Zhou, H.Y.: Phys. Lett. A **373**, 1823 (2009)
72. Wang, C., Zhang, Y., Jin, G.S.: Phys. Rev. A **84**, 032307 (2011)
73. Sheng, Y.B., Zhou, L., Zhao, S.M., Zheng, B.Y.: Phys. Rev. A **85**, 012307 (2012)
74. Deng, F.G.: Phys. Rev. A **85**, 022311 (2012)
75. Wang, H.F., Zhang, S., Yeon, K.H.: J. Opt. Soc. Am. B **27**, 2159 (2010)
76. Xiong, W., Ye, L.: J. Opt. Soc. Am. B **28**, 2030 (2011)