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Controlled Dense Coding Through a Genuine Five-Atom Entangled State in Cavity QED

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Abstract We present an experimentally feasible scheme for implementing controlled dense coding through a genuine five-atom entangled state in cavity QED. The scheme is insensitive to both the cavity decay and the thermal field. In the scheme that four-atom entangled states can be exactly distinguished with detecting the atomic state, and the controlled dense coding can be realized in a simple way.

Keywords Quantum information · Controlled dense coding · Genuine five-atom entangled state · Cavity QED

1 Introduction

Quantum dense coding [1] is one of the important applications of quantum entanglement in quantum communication. Some quantum dense coding schemes have been proposed by using GHZ state and the non-maximally two-particle entangled state [2–4]. Cavity quantum electrodynamics (QED) is considered to be one of the ideal systems for quantum information processing, such as quantum computing [5–8], quantum teleportation [9, 10], quantum cryptography [11, 12] and quantum state sharing [13–15]. In cavity QED systems, many controlled dense coding (CDC) protocols have been devised with the help of multi-atom entangled states such as the GHZ state [16], the W states [17], the genuine five-atom entangled states [18].

Recently, Brown et al. presents a genuine five-atom entangled state through an extensive numerical optimization procedure [19]. This state exhibits genuine multi-atom entanglement according to both negative partial transpose measure, as well as von Neumann entropy

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measure. This state is also genuinely entangled according to the recently proposed multiple entropy measures, and is more than the entanglement exhibited by the prototype-GHZ states [20], generalized W states [21] and cluster states. Even after tracing out one/two atoms from the state, entanglement sustains in the resulting subsystem and thus, is highly 'robust'. Also, the state is maximally mixed, after we trace out any possible number of atoms, which is an indication of genuine multi-atom entanglement for the five-atom state. Such a state has been shown useful in perfect teleportation, quantum state sharing and superdense coding [22, 23]. This gives us motivation to study the new application of the five-atom entangled state for implementing CDC protocol in cavity QED.

In this work, we present a simple scheme for implementing CDC by using a genuine five-atom entangled state in cavity QED. Here, we consider that two or more atoms interact with a thermal cavity with the assistance of the strong classical field, so the scheme is insensitive to both the cavity decay and the thermal field [24, 25].

2 The Model

We consider N identical two-level atoms simultaneously interacting with a single-mode cavity and being driven by a strong classical field. The Hamiltonian (we set $\hbar = 1$) in the rotating-wave approximation can be written as [26]

$$H = \omega_0 \sum_{j=1}^{N} \sigma_{z,j} + \omega_a a^\dagger a + \sum_{j=1}^{N} \left[g\left(a^\dagger \sigma_j^- + a\sigma_j^+\right) + \Omega\left(\sigma_j^+ e^{-i\omega t} + \sigma_j^- e^{i\omega t}\right)\right], \tag{1}$$

where $\sigma_j^+ = |1_j\rangle\langle 0_j|, \sigma_j^- = |0_j\rangle\langle 1_j|$ and $\sigma_{z,j} = \frac{1}{2}(|1_j\rangle\langle 1_j| - |0_j\rangle\langle 0_j|)$, with $|0_j\rangle$ and $|1_j\rangle$ being the ground and excited states of the j-th atom, a and $a^\dagger$ denote the annihilation and creation operators for the cavity mode, and g is the atom-cavity coupling strength, Ω is the Rabi frequency. ω_0 is atomic transition frequency, ω_a is cavity frequency and ω is the frequency of the classical field. Here we consider the case of $\omega_0 = \omega$. In the large detuning $\delta \gg g/2$ with δ is the detuning between ω_0 and ω_a, and strong driving field $\Omega \gg \delta \gg g$, there is no energy exchange between the atoms and the cavity. And then in the rotating-wave approximation, the effective Hamiltonian can be described as follows [26]

$$H_e = \lambda \left[\frac{1}{2} \sum_{j=1}^{N} \left(|0_j\rangle\langle 0_j| + |1_j\rangle\langle 1_j|\right) + \sum_{i,j=1}^{N} \left(\sigma_i^+ \sigma_j^+ + \sigma_i^+ \sigma_j^- + H.c.\right) \right], \quad i \neq j \tag{2}$$

where $\lambda = g^2/2\delta$. The Hamiltonian is independent of the cavity field state, allowing it to be in a thermal state. Then the evolution operator of the system is given by

$$U(t) = e^{-iH_0 t} e^{-iH_e t}, \tag{3}$$

where $H_0 = \sum_{j=1}^{N} \Omega(\sigma_j^+ + \sigma_j^-)$.

3 Generate the Genuine Five-Atom Entangled State

In order to generate the Brown state, we assume that five atoms are initially in the state $|00101\rangle_{12345}$. Firstly we let atoms 2, 3 and 5 interact simultaneously with a single-mode cavity and driven by a strong classical field. Choose the interaction time and the Rabi frequency appropriately so that $\lambda t_1 = \pi/4$ and $\Omega t_1 = 3\pi/4$. Secondly, we let atoms 1, 2 and 4 undergo the same evolutions. The state $|00101\rangle_{12345}$ will become the following state

$$|00101\rangle_{12345} \to |\varphi\rangle_{12345} = \frac{1}{2}\big(|00101\rangle + i|01000\rangle - |10010\rangle + i|11111\rangle\big)_{12345}. \quad (4)$$

Finally, we let atoms 4 and 5 interact with the single-mode cavity and driven by the classical field. Choose the interaction time appropriately so that $\lambda t_2 = \pi/4$ and $\Omega t_2 = \pi$ and then the state $|\varphi\rangle_{12345}$ becomes

$$\begin{aligned} |\varphi\rangle_{12345} \to \frac{1}{2}\bigg[&|001\rangle_{123}\frac{1}{\sqrt{2}}\big(|01\rangle - i|10\rangle\big)_{45} + i|010\rangle_{123}\frac{1}{\sqrt{2}}\big(|00\rangle - i|11\rangle\big)_{45} \\ &- |100\rangle_{123}\frac{1}{\sqrt{2}}\big(|10\rangle - i|01\rangle\big)_{45} + i|111\rangle_{123}\frac{1}{\sqrt{2}}\big(|11\rangle - i|00\rangle\big)_{45}\bigg], \end{aligned} \quad (5)$$

where we have discarded the common phase factor. After performing the transformations $|0\rangle_3 \to -i|0\rangle_3$ and $|1\rangle_4 \to -i|1\rangle_4$ on atoms 3 and 4, respectively, we can obtain the genuine five-atom entangled state [19]

$$\begin{aligned} |\psi\rangle_{12345} = \frac{\sqrt{2}}{4}\big[&|001\rangle_{123}\big(|01\rangle - |10\rangle\big)_{45} + |010\rangle_{123}\big(|00\rangle - |11\rangle\big)_{45} \\ &+ |100\rangle_{123}\big(|01\rangle + |10\rangle\big)_{45} + |111\rangle_{123}\big(|00\rangle + |11\rangle\big)_{45}\big]. \end{aligned} \quad (6)$$

Next, we implement CDC by using the above genuine five-atom entangled state in Cavity QED.

4 CDC Through the Genuine Five-Atom Entangled State in Cavity QED

Suppose that Alice, Bob and Charlie share the Brown state, where the atoms 1 and 2 belong to Alice, atom 4 belong to Charlie and atoms 3 and 5 belong to Bob, respectively. In order to encode four bits of classical information, Alice now can apply one of four local operators σ_i or σ_j $(i, j = 0, 1, 2, 3)$ (where $\sigma_0 = I$ is the identity operator and $\sigma_1 = \sigma_x$, $\sigma_2 = \sigma_z$, $\sigma_3 = -i\sigma_y$ are the Pauli operators) on her two atoms and then transform the Brown state into the following 16 orthogonal entangled states

$$\big|\psi^{ij}\big\rangle_{12345} = \big(\sigma_i^1 \otimes \sigma_j^2\big)\big|\psi^{00}\big\rangle_{12345} \quad (i, j = 0, 1, 2, 3), \quad (7)$$

where $|\psi^{00}\rangle_{12345}$ is just the Brown state as shown in (6). Then Alice sends atoms 1 and 2 to Bob. For assisting Bob to extract the encoded information, Charlie should make a measurement on atom 4 in the basis $\{|0\rangle_4, |1\rangle_4\}$ and then transmits the outcome to Bob over a classical communication channel. If the outcome of Charlie's measurement on atom 4 is $|0\rangle_4$, the atoms belong to Bob will collapse into the following states

$$\big|\eta^{ij}\big\rangle_{1235} = \big(\sigma_i^1 \otimes \sigma_j^2\big)\big|\eta^{00}\big\rangle_{1235} \quad (i, j = 0, 1, 2, 3), \quad (8)$$

where

$$\big|\eta^{00}\big\rangle_{1235} = \frac{1}{2}\big(|0011\rangle + |0100\rangle + |1001\rangle + |1110\rangle\big)_{1235}. \quad (9)$$

Otherwise, if the outcome of Charlie's measurement on atom 4 is $|1\rangle_4$, the atoms belong to Bob will collapse into

$$\big|\xi^{ij}\big\rangle_{1235} = \big(\sigma_i^1 \otimes \sigma_j^2\big)\big|\xi^{00}\big\rangle_{1235} \quad (i, j = 0, 1, 2, 3), \quad (10)$$

where

$$\left|\xi^{00}\right\rangle_{1235} = \frac{1}{2}\big(-|0010\rangle - |0101\rangle + |1000\rangle + |1111\rangle\big)_{1235}. \tag{11}$$

In order to implement CDC, Bob has to carry out four-atom entangled states measurement to obtain the message that Alice encodes in the state. The four-atom entangled states measurement is also the obstacle of CDC, it is difficult to realize in the present experiment. Luckily, the states $|\eta^{ij}\rangle_{1235}$ and $|\xi^{ij}\rangle_{1235}$ can be distinguished explicitly in cavity QED. Now, the procedure to distinguish the states $|\eta^{ij}\rangle_{1235}$ and $|\xi^{ij}\rangle_{1235}$ is given. Firstly we perform the rotation $|0\rangle_1 \to i|0\rangle_1$ on atom 1. Then we let atoms 1, 2 and 5 interact with the single-mode cavity and classical field, we choose the interaction time and the Rabi frequency appropriately so that $\lambda t = \pi/4$ and $\Omega t = 3\pi/4$. Subsequently, we perform the rotation $|0\rangle_2 \to i|0\rangle_2$ on atom 2. In the last step, atoms 2, 3 and 5 are simultaneously sent through another single-mode cavity and at the same time are driven by a classical field, and the time setting is $\lambda t = \pi/4$ and $\Omega t = 3\pi/4$. If $|\eta^{00}\rangle_{1235}$ is taken into account, according (3), it evolves as follows

$$\begin{aligned}\left|\eta^{00}\right\rangle_{1235} &\to \frac{1}{2}\big(i|0011\rangle + i|0100\rangle + |1001\rangle + |1110\rangle\big)_{1235}\\ &\to \frac{\sqrt{2}}{2}\big(i|0011\rangle + i|0100\rangle\big)_{1235}\\ &\to \frac{\sqrt{2}}{2}\big(-|0011\rangle + i|0100\rangle\big)_{1235}\\ &\to -|0011\rangle_{1235}.\end{aligned} \tag{12}$$

That is to say, through the four steps, the states $|\eta^{ij}\rangle_{1235}$ and $|\xi^{ij}\rangle_{1235}$ can be evolved as follows:

$$\begin{aligned}
&\left|\eta^{00}\right\rangle_{1235} \to -|0011\rangle_{1235}, \left|\eta^{01}\right\rangle_{1235} \to -|0000\rangle_{1235}, \left|\eta^{02}\right\rangle_{1235} \to -|1110\rangle_{1235},\\
&\left|\eta^{03}\right\rangle_{1235} \to |1101\rangle_{1235}, \left|\eta^{10}\right\rangle_{1235} \to -|0001\rangle_{1235}, \left|\eta^{11}\right\rangle_{1235} \to -|0010\rangle_{1235},\\
&\left|\eta^{12}\right\rangle_{1235} \to -|1100\rangle_{1235}, \left|\eta^{13}\right\rangle_{1235} \to |1111\rangle_{1235}, \left|\eta^{20}\right\rangle_{1235} \to -i|1001\rangle_{1235},\\
&\left|\eta^{21}\right\rangle_{1235} \to -i|1010\rangle_{1235}, \left|\eta^{22}\right\rangle_{1235} \to -i|0100\rangle_{1235}, \left|\eta^{23}\right\rangle_{1235} \to -|0111\rangle_{1235},\\
&\left|\eta^{30}\right\rangle_{1235} \to i|1011\rangle_{1235}, \left|\eta^{31}\right\rangle_{1235} \to i|1000\rangle_{1235}, \left|\eta^{32}\right\rangle_{1235} \to i|0110\rangle_{1235},\\
&\left|\eta^{33}\right\rangle_{1235} \to -i|0101\rangle_{1235}, \left|\xi^{00}\right\rangle_{1235} \to i|1000\rangle_{1235}, \left|\xi^{01}\right\rangle_{1235} \to i|1011\rangle_{1235},\\
&\left|\xi^{02}\right\rangle_{1235} \to i|0101\rangle_{1235}, \left|\xi^{03}\right\rangle_{1235} \to -i|0110\rangle_{1235}, \left|\xi^{10}\right\rangle_{1235} \to -i|1001\rangle_{1235},\\
&\left|\xi^{11}\right\rangle_{1235} \to -i|1010\rangle_{1235}, \left|\xi^{12}\right\rangle_{1235} \to -i|0111\rangle_{1235}, \left|\xi^{13}\right\rangle_{1235} \to i|0100\rangle_{1235},\\
&\left|\xi^{20}\right\rangle_{1235} \to -i|0010\rangle_{1235}, \left|\xi^{21}\right\rangle_{1235} \to -i|0001\rangle_{1235}, \left|\xi^{22}\right\rangle_{1235} \to |1111\rangle_{1235},\\
&\left|\xi^{23}\right\rangle_{1235} \to -|1100\rangle_{1235}, \left|\xi^{30}\right\rangle_{1235} \to -i|0000\rangle_{1235}, \left|\xi^{31}\right\rangle_{1235} \to -i|0011\rangle_{1235},\\
&\left|\xi^{32}\right\rangle_{1235} \to |1101\rangle_{1235}, \left|\xi^{33}\right\rangle_{1235} \to -i|1001\rangle_{1235}.
\end{aligned}$$

Hence, we can distinguish the four-atom entangled states $|\eta^{ij}\rangle_{1235}$ and $|\xi^{ij}\rangle_{1235}$ by detecting four atoms separately. Then Bob can distinguish what operations Alice has done on his atoms and extract four bits of classical information after receiving two atoms and a classical bit from Alice and Charlie respectively. Thus, the CDC is achieved.

For the Rydberg atoms with principal quantum numbers 49, 50, 51, the radiative time is $T_r = 3 \times 10^{-2}$ s and the coupling constant is $g = 2\pi \times 24$ kHz. Based on the coupling constant g, the required atom-cavity-field interaction time is on the order of $T = 10^{-4}$ s, so that the whole time to discriminate the multipartite measurement is lesser than 10^{-3} s, which is much shorter than the radiative time [27]. Thus, our scheme can be easily realized with present cavity QED techniques.

5 Conclusions

In this paper, we have presented a simple experimental scheme of CDC by using a genuine five-atom entangled state in cavity QED. In cavity QED systems, we have distinguished the four-atom entangled states $|\eta^{ij}\rangle_{1235}$ and $|\xi^{ij}\rangle_{1235}$ in detail. The scheme is insensitive to both the cavity decay and the thermal field. Based on present cavity QED techniques, our scheme might be realizable in the future.

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