A Class of Charged Relativistic Superdense Star Models

S.K. Maurya · Y.K. Gupta · Pratibha

Abstract In this article we obtain a new class of well behaved charged solutions by using particular forms of the metric potential g_{44} and electric intensity, which involves a parameter K. The metric describing the superdense stars joins smoothly with the Reissner-Nordstrom metric at the pressure free boundary. This class of solutions describes well behaved charged fluid balls. The class of solutions gives range of parameter K ($0.13 \leq K \leq 1.9999$) for which the solution is well behaved hence, suitable for modeling of super dense star. The interior of the stars possess there energy density, pressure, pressure-density ratio and velocity of sound to be monotonically decreasing towards the pressure free interface. In view of the surface density 2×10^{14} gm/cm^3, the maximum mass of the charged fluid balls and corresponding radius are $0.4711 M_{\Theta}$ and 7.0122 km. The red shift at the centre and boundary are found to be 0.1640 and 0.1100 respectively.

1 Introduction

Exact interior solutions of the Einstein-Maxwell field equations joining smoothly to the Nordstrom solution at the pressure free interface are gathering big applause due to some of the following reasons:

– Gravitational collapse of a charged fluid sphere to a point singularity may be avoided.
– Charged solutions of Einstein-Maxwell equations are useful in the study of cosmic matter.
– Charged-dust models and electromagnetic mass models are expected to provide some clue about structure of an electron.
– Several fluid spheres which do not satisfy some or all the relevant physical condition i.e. reality conditions, become relevant when they are charge.

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Charged fluid model are normally of three kinds:

– The charged fluid reduces to a neutral fluid in the absence of charge.
– There is several charged fluid can not be uncharged or more correctly charge cannot vanish.
– Some charge fluid models reduces to flat space after the removal of charge. The later is said to be charged fluid with electromagnetic mass.

Many of workers have charged the well known neutral solutions to obtain the charged fluids such as Schwarzschild interior solution [2, 7], Whittaker's interior solution by [6], charged analogue of Durgapal–Fuloria [5] solution by Gupta and Maurya [8]. Some more solutions in this context are published recently [1, 9, 10, 13–19].

In the present article our aim is to obtain a class of regular and well behaved solution. To achieve the same we consider a spherically symmetric metric with its metric potential $g_{44} = (1 - Cr^2)^{-1/3}$ and using a particular electric intensity, which involves a parameter K. The metric so obtained is joined smoothly with the Reissner-Nordstrom metric and then utilized to describe a class of super-dense star models with the surface density 2×10^{14} gm/cm^3. The charged models so obtained are found regular and well behaved throughout the interior of the star.

2 Field Equations

Let us consider a spherically symmetric metric curvature coordinates as

$$ds^2 = -e^\lambda dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + e^\nu dt^2 \tag{1}$$

where the functions $\lambda(r)$ and $\nu(r)$ satisfy the Einstein-Maxwell equations

$$-\kappa T_j^i = R_j^i - \frac{1}{2}R\delta_j^i = -\kappa \left[(c^2\rho + p)v^i v_j - p\delta_j^i + \frac{1}{4\pi}\left(-F^{im}F_{jm} + \frac{1}{4}\delta_j^i F_{mn}F^{mn}\right)\right] \tag{2}$$

with $\kappa = \frac{8\pi G}{c^4}$ while ρ, p, v^i, F_{ij} denote energy density, fluid pressure, flow vector and skew-symmetric electromagnetic field tensor respectively. The resulting field equations [4] are

$$\frac{\lambda'}{r}e^{-\lambda} + \frac{(1 - e^{-\lambda})}{r^2} = \kappa c^2 \rho + \frac{q^2}{r^4} \equiv \kappa T_4^4 \tag{3}$$

$$\frac{\nu'}{r}e^{-\lambda} - \frac{(1 - e^{-\lambda})}{r^2} = \kappa p - \frac{q^2}{r^4} = \kappa T_1^1 \tag{4}$$

$$\left[\frac{\nu''}{2} - \frac{\lambda'\nu'}{4} + \frac{\nu'^2}{4} + \frac{\nu' - \lambda'}{2r}\right]e^{-\lambda} = \kappa p + \frac{q^2}{r^4} = \kappa T_2^2 = \kappa T_3^3 \tag{5}$$

Let us take the transformation,

$$e^\nu = B(1 - Cr^2)^{-1/3}, \tag{6}$$

where B and C are positive constant.

Substitution of (6) in to (3)–(5) leads to

$$\frac{2Y}{3(1-x)} - \frac{(1-Y)}{x} + \frac{Cq^2}{x^2} = \frac{\kappa p}{C} \tag{7}$$

$$\frac{(1-Y)}{x} - 2\frac{dY}{dx} - \frac{Cq^2}{x^2} = \frac{\kappa c^2 \rho}{C} \tag{8}$$

and

$$\frac{dY}{dx} + P(x)Y = f(x) \tag{9}$$

where

$$x = Cr^2, \qquad e^{-\lambda} = Y, \qquad P(x) = -\frac{(2x^2 - 18x + 9)}{3x(1-x)(3-2x)}$$

$$f(x) = \frac{3[(2q^2 C/x) - 1](1-x)}{x(3-2x)}$$

The solution of (9) is given by

$$Y = FA + FI \tag{10}$$

where

$$F = -\int P(x)dx = \frac{x(1-x)^{7/3}}{(3-2x)^3}, \qquad I = 3\int \frac{[(2q^2 C/x) - 1](3-2x)^2 dx}{x^2(1-x)^{4/3}}$$

and A is arbitrary constant of integration.

3 New Class of Solutions

In order to solve the integral I for obtaining the new class of solution, we consider the electric intensity E of the form

$$\frac{E^2}{C} = \frac{Cq^2}{x^2} = \frac{Kx}{2(3-2x)^2} \tag{11}$$

where $K \geq 0$ is constant. For $0 < x < 2/3$ and $K \geq 0$, the electric field intensity given by (10) is physically palatable since E^2 remains regular and positive throughout the sphere. In addition, the electric field given by (11) vanishes at the centre of the star.

With reference of (11), (9) yields following solution

$$Y = \left[\frac{9Kx(1-x)^2}{(3-2x)^3} + \frac{9(3-4x)(1-x)^2}{(3-2x)^3} + \frac{Ax(1-x)^{7/3}}{(3-2x)^3} \right] \tag{12}$$

Using (12), into (7) and (8), we get the following expressions for pressure and energy density

$$\frac{\kappa p}{C} = \left[\frac{K(18 - 21x + 4x^2)}{2(3-2x)^3} + \frac{A(3-x)(1-x)^{4/3}}{3(3-2x)^3} - \frac{(18 - 21x + 4x^2)}{(3-2x)^3} \right] \tag{13}$$

$$\frac{\kappa c^2 \rho}{C} = \left[\frac{324 - 999x + 462x^2 - 56x^3}{(3-2x)^4} - \frac{K \cdot B(x)}{(3-2x)^4} - \frac{A \cdot (1-x)^{4/3} \cdot H(x)}{3 \cdot (3-2x)^4} \right] \tag{14}$$

where

$$B(x) = (162 - 423x + 294x^2 - 32x^3), \qquad H(x) = (27 - 51x + 10x^2)$$

Consequently the expressions for pressure and density gradients read as

$$\frac{\kappa}{C} \cdot \frac{dp}{dx} = \left[\frac{K \cdot (45 - 60x + 8x^2)}{2 \cdot (3 - 2x)^4} + \frac{A \cdot (9 - 21x + 4x^2)}{9 \cdot (3 - 2x)^4} - \frac{(45 - 60x + 8x^2)}{(3 - 2x)^4} \right] \tag{15}$$

$$\frac{\kappa c^2}{C} \cdot \frac{d\rho}{dx} = \left[-\frac{(405 + 1620x - 1344x^2 + 112x^3)}{(3 - 2x)^5} - \frac{K \cdot D(x)}{2 \cdot (3 - 2x)^5} + \frac{A \cdot E(x)}{9 \cdot (3 - 2x)^5} \right] \tag{16}$$

where

$$D(x) = (27 - 744x + 928x^2 - 64x^3), \qquad E(x) = (135 + 99x - 330x^2 + 40x^3)$$

4 Conditions for Regular and Well Behaved Model

For well behaved nature of the solution implies in curvature coordinates, the following conditions should be satisfied (augmentation of Delgaty-Lake [3] and Pant et al. [16] conditions).

  (i) The solution should be free from physical and geometric singularities and non zero positive values of e^λ and e^v i.e. $(e^\lambda)_{r=0} = 1$ and $e^v > 0$.
 (ii) Pressure p should be zero at boundary $r = a$.
(iii) $c^2\rho \geq p > 0$ or $c^2\rho \geq 3p > 0$, $0 \leq r \leq a$, where former inequality denotes weak energy condition (WEC), while the later inequality implies strong energy condition (SEC).
 (iv) $(dp/dr)_{r=0} = 0$ and $(d^2p/dr^2)_{r=0} < 0$ so that pressure gradient dp/dr is negative for $0 < r \leq a$.
  (v) $(d\rho/dr)_{r=0} = 0$ and $(d^2\rho/dr^2)_{r=0} < 0$ so that density gradient $d\rho/dr$ is negative for $0 < r \leq a$.
      The condition (iii) and (iv) imply that pressure and density should be maximum at the centre and monotonically decreasing towards the surface.
 (vi) The casualty condition $(dp/c^2 d\rho)^{1/2} < 1$ i.e. velocity of sound should be less than that of light throughout the model. In addition to the above the velocity of sound should be decreasing towards the surface i.e. $\frac{d}{dr}(\frac{dp}{d\rho}) < 0$ or $(\frac{d^2p}{d\rho^2}) > 0$ for $0 \leq r \leq a$ i.e. the velocity of sound is increasing with the increase of density.
(vii) The ratio of pressure to the density $(p/c^2\rho)$ should be monotonically decreasing with the increase of r i.e. $\frac{d}{dr}(\frac{p}{c^2\rho})_{r=0} = 0$ and $\frac{d^2}{dr^2}(\frac{p}{c^2\rho})_{r=0} < 0$ and $\frac{d}{dr}(\frac{p}{c^2\rho})$ is negative valued function for $r > 0$.
(viii) The central red shift Z_0 and surface red shift Z_a should be positive and finite i.e. $Z_0 = [(e^{-v/2} - 1)_{r=0}] > 0$ and $Z_a = [e^{\lambda(a)/2} - 1] > 0$ and both should be bounded.
 (ix) Electric intensity E, such that $E(0) = 0$ is taken to be monotonically increasing i.e. $(dE/dr) > 0$ for $0 < r < a$.

5 Properties of New Class of Solution

$$\left[\frac{\kappa p}{C} \right]_{r=0} = \frac{K}{3} + \frac{A}{27} - \frac{2}{3} \tag{17}$$

$$\left[\frac{\kappa c^2 \rho}{C}\right]_{r=0} = -K - \frac{A}{9} + 4 \tag{18}$$

$$\left[\frac{\kappa}{C}\frac{dp}{dr}\right]_{r=0} = 2Cr\left[\frac{\kappa}{C}\frac{dp}{dx}\right]_{x=0} = 0 \tag{19}$$

$$\left[\frac{\kappa c^2}{C}\frac{d\rho}{dr}\right]_{r=0} = 2Cr\left[\frac{\kappa}{C}\frac{d\rho}{dx}\right]_{x=0} = 0 \tag{20}$$

$$\left[\frac{\kappa}{C}\frac{d^2 p}{dr^2}\right]_{r=0} = -\frac{K}{18} + \frac{5A}{81} - \frac{5}{3} \tag{21}$$

$$\left[\frac{\kappa c^2}{C}\frac{d^2 \rho}{dr^2}\right]_{r=0} = \frac{5K}{18} + \frac{A}{81} - \frac{5}{9} \tag{22}$$

For $p_{r=0}$ and $\rho_{r=0}$ must be positive, $\frac{p_{r=0}}{c^2 \rho_{r=0}} \leq 1$ and $(d^2 p/dr^2)_{r=0} < 0$, $(d^2 \rho/dr^2)_{r=0} < 0$. Consequently we have

$$-9K - 18 < A < \frac{9K}{10} + 27 \tag{23}$$

Hence velocity of sound at the centre given by

$$\left[\frac{dp}{c^2 d\rho}\right]_{r=0} = \frac{-9K + 10A - 270}{45K + 2A - 90} \tag{24}$$

which should be $[\frac{dp}{c^2 d\rho}]_{r=0} < 1$, for all values of $K \geq 0$ and A.

Using (13) and (14)

$$\left[\frac{p}{c^2 \rho}\right] = \frac{I(x)}{J(x)} \tag{25}$$

where

$$I(x) = \left[\frac{K \cdot (18 - 21x + 4x^2)}{2(3 - 2x)^3} + \frac{A \cdot (3 - x)(1 - x)^{4/3}}{3(3 - 2x)^3} - \frac{(18 - 21x + 4x^2)}{2(3 - 2x)^3}\right]$$

$$J(x) = \left[\frac{324 - 999x + 462x^2 - 56x^3}{(3 - 2x)^4} - \frac{K \cdot B(x)}{(3 - 2x)^4} - \frac{A \cdot (1 - x)^{4/3} \cdot H(x)}{3 \cdot (3 - 2x)^4}\right]$$

Differentiating (25) w.r.t. x

$$\frac{d}{dx}\left[\frac{p}{c^2 \rho}\right] = \frac{J(x)\frac{dI(x)}{dx} - I(x)\frac{dJ(x)}{dx}}{\{J(x)\}^2} \tag{26}$$

$$\left[\frac{d^2}{dr^2}\left(\frac{p}{c^2 \rho}\right)\right]_{r=0} = 2C \cdot \frac{\alpha \cdot \beta - \gamma \cdot \delta}{\alpha^2} \tag{27}$$

where

$$\alpha = 3(36 - 9K - A), \qquad \beta = (-270 - 9K + 10A)$$

$$\gamma = (-18 + 9K + A), \qquad \delta = (-90 + 45K + 2A)$$

The expression of right hand side of (27) is negative for all values of $K \geq 0$ and A. Then pressure-density ratio $\frac{p}{c^2 \rho}$ is maximum at the centre.

The expression for gravitational red-shift Z is given by

$$Z = \frac{(1-x)^{1/6}}{\sqrt{B}} - 1 \tag{28}$$

The central value of gravitational red-shift to be non zero positive finite, we have

$$1 > \sqrt{B} > 0$$

Differentiating (28) w.r.t. x, we get,

$$\left[\frac{d^2 Z}{dr^2}\right]_{r=0} = -\frac{C}{3\sqrt{B}} < 0 \tag{29}$$

The expression of right hand side of (29) is negative, and then the gravitational red shift is the maximum at centre and monotonically decreasing towards the surface.

Differentiating (11) w.r.t. x, we get,

$$\frac{d}{dx}\left(\frac{E^2}{C}\right) = \frac{K}{2}\left[\frac{(3+2x)}{(3-2x)^3}\right]$$

$\frac{d}{dr}(\frac{E^2}{C}) = Cr \cdot \frac{K}{2}[\frac{(3+2x)}{(3-2x)^3}] = +ve$ for $0 < x < 2/3$.

Thus the electric intensity is zero at the centre and monotonically increasing towards the pressure free interface for all values of $K > 0$.

6 Boundary Conditions

Besides the above, the charged fluid spheres is expected to join smoothly with the Reissner-Nordstrom metric at the pressure free boundary $r = a$

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + \left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right) dt^2 \tag{30}$$

which requires the continuity of e^λ, e^ν and q across the boundary

$$e^{-\lambda(a)} = 1 - \frac{2M}{a} + \frac{e^2}{a^2} \tag{31}$$

$$e^{\nu(a)} = y^2_{(r=a)} = 1 - \frac{2M}{a} + \frac{e^2}{a^2} \tag{32}$$
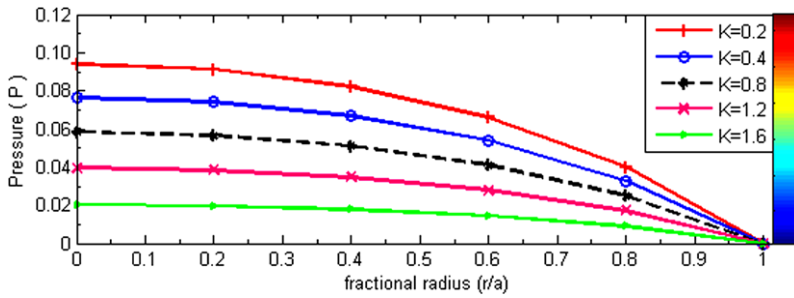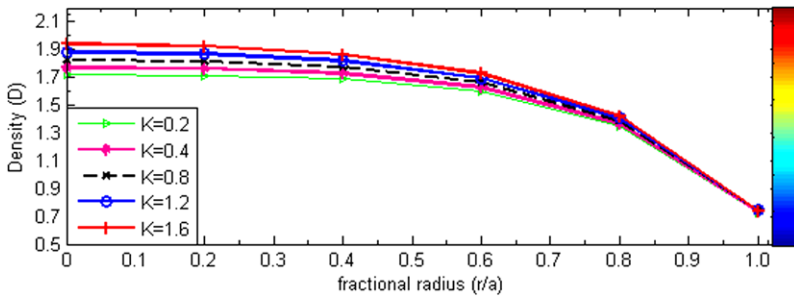
$$q(a) = e \tag{33}$$

$$p_{(r=a)} = 0 \tag{34}$$

The condition (34) can be utilized to compute the values of arbitrary constants A as follows:

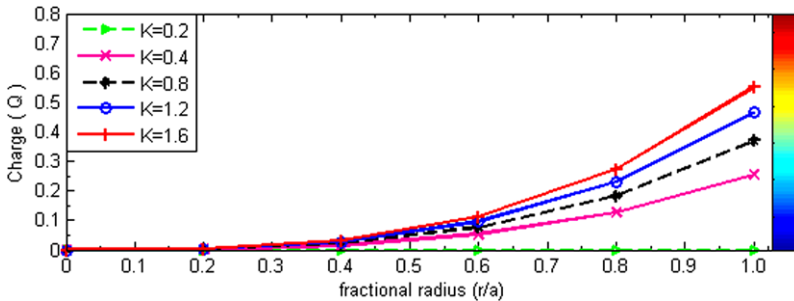On setting $x_{r=a} = X = Ca^2$ (a being the radius of the charged sphere). Pressure at $p_{(r=a)} = 0$ gives

$$A = \left[\frac{3 \cdot (18 - 21X + 4X^2)}{(3 - X) \cdot (1 - X)^{4/3}} \cdot \frac{(K - 2)}{2}\right] \tag{35}$$

Fig. 1 Behavior of pressure (P) versus radius

Fig. 2 Behavior of density versus radius

Fig. 3 Behavior of charge (Q) versus radius

The expression for mass can be written as

$$m(a) = \frac{a}{2}\left[1 - \frac{KX \cdot (18 - 39X + 20X^2)}{2 \cdot (3 - 2X)^3} - \frac{9(3 - 4X)(1 - X)^2}{(3 - 2X)^3} - \frac{AX(1 - X)^{7/3}}{(3 - 2X)^3}\right]$$

such that $e^{-\lambda(a)} = 1 - \frac{2M}{a} + \frac{e^2}{a^2}$, where $M = m(a)$ and $y^2_{(r=a)} = 1 - \frac{2M}{a} + \frac{e^2}{a^2}$ gives

$$B = \left[\frac{9K \cdot X(1 - X)^{7/3}}{(3 - 2X)^3} + \frac{9 \cdot (3 - 4X)(1 - X)^{7/3}}{(3 - 2X)^3} + \frac{A \cdot X \cdot (1 - X)^{8/3}}{(3 - 2X)^3}\right]$$
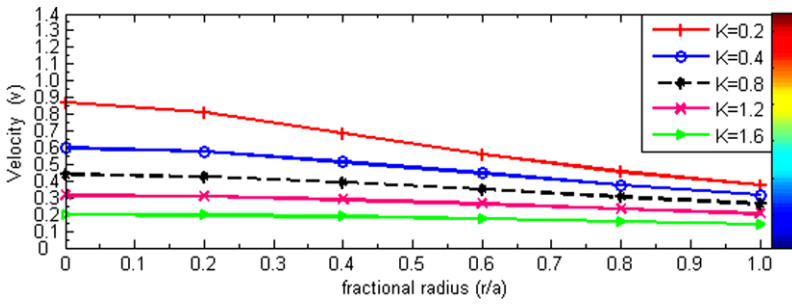
Fig. 4 Behavior of velocity of sound versus radius
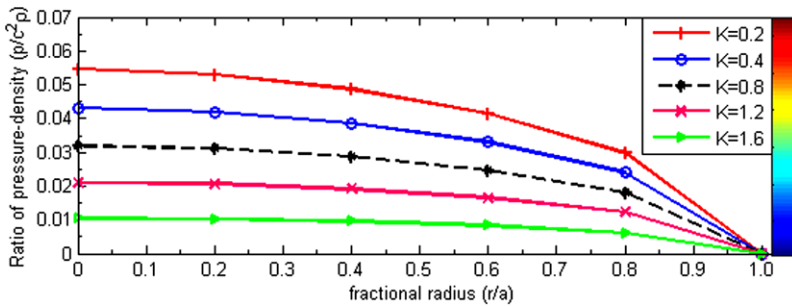
Fig. 5 Behavior of ratio of pressure and density versus radius
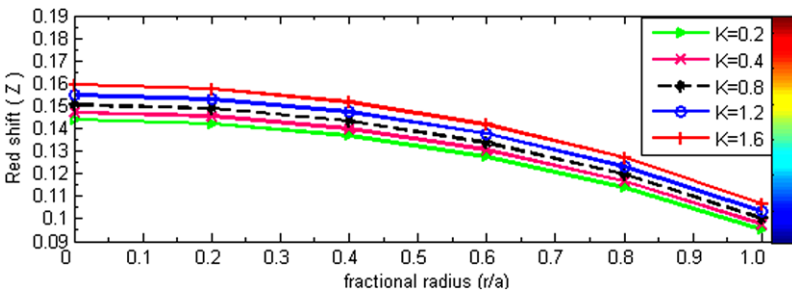
Fig. 6 Behavior of red shift (Z) versus radius

Also, if the surface density ρ_a is prescribed as 2×10^{14} g cm^{-3} (super dense star case) then value of constant C can be calculated for a given $X (= Ca^2)$, using the following expression

$$\kappa c^2 \rho_a = C \left[\frac{324 - 999X + 462X^2 - 56X^3}{(3 - 2X)^4} - \frac{K \cdot B(X)}{(3 - 2X)^4} - \frac{A \cdot (1 - X)^{4/3} \cdot H(X)}{3 \cdot (3 - 2X)^4} \right]$$

where

$$B(X) = (162 - 423X + 294X^2 - 32X^3), \qquad H(X) = (27 - 51X + 10X^2)$$

7 Conclusions

In this article, we have derived a class of regular and well behaved charge superdense star models using a particular electric field, which depend upon a parameter K, vanishing of which leads to the neutral case. The charged superdense star models satisfy the energy conditions $c^2\rho \geq 3p > 0$, $(dp/dr) < 0$, $(d\rho/dr) < 0$, causality condition $(dp/c^2d\rho) < 1$ and adiabatic index $\gamma = ((p + c^2\rho)/p)(dp/(c^2d\rho)) > 1$ for $0.13 \leq K \leq 1.9999$. The velocity of sound and the ratio $p/c^2\rho$ are seen monotonically decreasing towards the surface for $0.13 \leq K \leq 1.9999$. The red shift is also decreasing monotonically from the centre to the pressure free interface for $0.13 \leq K \leq 1.9999$. The maximum mass and the corresponding radius of the model are computed to $0.4711 M_\Theta$ and 7.0122 km respectively for $K = 1.9999$ and $Ca^2 = 0.2470$, with the red shift $Z_0 = 0.1640$ and $Z_a = 0.1100$.

The proposed plan for future includes the charging of the fluid spheres and plates recently derived by two of the authors of the present paper [11, 12]. The process may provide pressure free interface which is not available in the uncharged case.

8 Tables for Numerical Values of Physical Quantities

In Tables 1–7: $Z_0 =$ red shift at the centre, $Z_a =$ red shift at the surface, Solar mass $M_\Theta = 1.475$ km, $G = 6.673 \times 10^{-8}$ cm^3/gs^2, $c = 2.997 \times 10^{10}$ cm/s, $D = (8\pi G/c^2)\rho a^2$,

$$P = (8\pi G/c^4)pa^2, \qquad \gamma = \frac{p + c^2\rho}{p}\frac{dp}{c^2d\rho},$$

$$R = \frac{p}{c^2\rho}, \qquad v = (dp/c^2d\rho)^{1/2}, \qquad Q = q$$

Table 1 Values of various parameters

K	$0 \leq p \leq c^2\rho$ (WEC)			$0 \leq 3p \leq c^2\rho$ (SEC)		
	Ca^2	Radius (Km)	Max(M/M_Θ)	Ca^2	Radius (Km)	Max(M/M_Θ)
0.2	0.2300	6.6967	0.3770	0.2300	6.6967	0.3770
0.4	0.2330	6.7743	0.3939	0.2330	6.7743	0.3939
0.8	0.2360	6.8548	0.4116	0.2360	6.8548	0.4116
1.2	0.2400	6.9056	0.4304	0.2400	6.9056	0.4304
1.6	0.2440	6.9592	0.4502	0.2440	6.9592	0.4502
1.9999	0.2470	7.0122	0.4711	0.2470	7.0122	0.4711

Table 2 $K = 0.2$, $Ca^2 = 0.23$, Radius (a) $= 6.6967$ Km, Maximum mass $M = 0.3770 M_\Theta$

x	P	D	γ	Q	$\sqrt{dp/c^2d\rho}$	$p/c^2\rho$	Z
0.0	0.0939	1.7183	14.6083	0.0000	0.8700	0.0546	0.1438
0.2	0.0911	1.7137	13.0500	0.0000	0.8115	0.0531	0.1420
0.4	0.0823	1.6880	10.1331	0.0000	0.6862	0.0487	0.1367
0.6	0.0661	1.5989	7.9043	0.0000	0.5603	0.0414	0.1274
0.8	0.0402	1.3499	7.1966	0.0000	0.4563	0.0298	0.1138
1.0	0.0000	0.7277	∞	0.0000	0.3733	0.0000	0.0950

Table 3 $K = 0.4$, $Ca^2 = 0.2330$, Radius $(a) = 6.7743$ Km, Maximum mass $M = 0.3939 M_\Theta$

x	P	D	γ	Q	$\sqrt{dp/c^2 d\rho}$	$p/c^2\rho$	Z
0.0	0.0763	1.7710	8.6785	0.0000	0.5988	0.0431	0.1471
0.2	0.0740	1.7639	8.1954	0.0019	0.5746	0.0420	0.1453
0.4	0.0669	1.7308	7.1373	0.0152	0.5154	0.0387	0.1399
0.6	0.0538	1.6307	6.1871	0.0523	0.4447	0.0330	0.1305
0.8	0.0328	1.3690	6.0800	0.1268	0.3771	0.0239	0.1167
1.0	0.0000	0.7350	∞	0.2551	0.3170	0.0000	0.0975

Table 4 $K = 0.8$, $Ca^2 = 0.2360$, Radius $(a) = 6.8548$ Km, Maximum mass $M = 0.4116 M_\Theta$

x	P	D	γ	Q	$\sqrt{dp/c^2 d\rho}$	$p/c^2\rho$	Z
0.0	0.0582	1.8255	6.1744	0.0000	0.4366	0.0319	0.1505
0.2	0.0564	1.8157	5.9716	0.0027	0.4243	0.0311	0.1487
0.4	0.0510	1.7751	5.5007	0.0221	0.3920	0.0287	0.1432
0.6	0.0411	1.6636	5.0708	0.0758	0.3495	0.0247	0.1336
0.8	0.0250	1.3889	5.2502	0.1839	0.3049	0.0180	0.1196
1.0	0.0000	0.7430	∞	0.3702	0.2620	0.0000	0.1001

Table 5 $K = 1.2$, $Ca^2 = 0.2400$, Radius $(a) = 6.9056$ Km, Maximum mass $M = 0.4304 M_\Theta$

x	P	D	γ	Q	$\sqrt{dp/c^2 d\rho}$	$p/c^2\rho$	Z
0.0	0.0396	1.8812	4.7667	0.0000	0.3134	0.0210	0.1549
0.2	0.0384	1.8686	4.6693	0.0034	0.3067	0.0206	0.1530
0.4	0.0347	1.8200	4.4405	0.0277	0.2884	0.0191	0.1474
0.6	0.0280	1.6961	4.2533	0.0952	0.2629	0.0165	0.1376
0.8	0.0171	1.4058	4.5598	0.2309	0.2341	0.0122	0.1232
1.0	0.0000	0.7415	∞	0.4651	0.2046	0.0000	0.1032

Table 6 $K = 1.6$, $Ca^2 = 0.2440$, Radius $(a) = 6.9592$ Km, Maximum mass $M = 0.4502 M_\Theta$

x	P	D	γ	Q	$\sqrt{dp/c^2 d\rho}$	$p/c^2\rho$	Z
0.0	0.0202	1.9394	3.8757	0.0000	0.1999	0.0104	0.1593
0.2	0.0196	1.9239	3.8263	0.0041	0.1965	0.0102	0.1574
0.4	0.0177	1.8669	3.7136	0.0328	0.1870	0.0095	0.1517
0.6	0.0143	1.7301	3.6510	0.1126	0.1731	0.0083	0.1417
0.8	0.0088	1.4237	4.0153	0.2734	0.1567	0.0061	0.1270
1.0	0.0000	0.7407	∞	0.5511	0.1389	0.0000	0.1065

Detailed physically behavior of the models for various K are displayed by means of graphs and tables as below, where Figs. 1–6 are given with reference of Tables 2–6.

Table 7 $K = 1.9999$, $Ca^2 = 0.2470$, Radius $(a) = 7.0122$ Km, Maximum mass $M = 0.4711M_\Theta$

x	P	D	γ	Q	$\sqrt{dp/c^2 d\rho}$	$p/c^2\rho$	Z
0.0	5.1580×10^{-6}	2.0000	3.2602	0.0000	0.0029	2.5790×10^{-6}	0.1640
0.2	5.0078×10^{-6}	1.9815	3.2352	0.0047	0.0029	2.5272×10^{-6}	0.1621
0.4	4.5344×10^{-6}	1.9159	3.1831	0.0376	0.0027	2.3668×10^{-6}	0.1562
0.6	3.6626×10^{-6}	1.7656	3.1874	0.1291	0.0026	2.0744×10^{-6}	0.1460
0.8	2.2425×10^{-6}	1.4424	3.5722	0.3135	0.0024	1.5547×10^{-6}	0.1309
1.0	0.0000	0.7396	∞	0.6322	0.0021	0.0000	0.1100

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