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Perfect Teleportation of an Arbitrary Three-Qubit State by Using W-Class States

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Abstract A new application of the W-class state is investigated for teleportation of an arbitrary three-qubit state. We demonstrate that three sets of W-class states can be used to realize the perfect teleportation of an arbitrary three-qubit state by performing only the three-qubit von Neumann measurements and local unitary operations.

Keywords Quantum information · Quantum teleportation · W-class state · Arbitrary three-qubit state

1 Introduction

Quantum entanglement plays a central role in quantum information processing, such as quantum teleportation [1, 2], dense coding [3, 4], geometric and quantum computation [5–8], quantum information splitting [9–11] and so on. Quantum teleportation is one of the most striking features emerging from quantum entanglement which is inherent in quantum mechanics, and it has been experimentally realized by various groups [12–14]. Recently, many teleportation protocols have been devised with the help of multi-partite entangled states, such as the prototype-GHZ states [15], generalized W states [16] and cluster states [17, 18].

It is known that the W state has some interesting entanglement properties [19]. For example, it retains bipartite entanglement when any one of the three qubits is traced out and thus it is much more robust than the GHZ states, as demonstrated in the recent experiment [20]. In 2006, Agrawal et al. [16] have introduced a kind of W-class state which can be used for the quantum teleportation of single-qubit state via a three-particle von Neumann measurement. And the previous protocol can be generalized to deterministic teleportation of an arbitrary

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two-qubit state [21]. This gives us motivation to study the new application of the W-class state for teleportation of an arbitrary three-qubit state.

In this work, we demonstrate that three sets of W-class states can be used to realize the perfect teleportation of an arbitrary three-qubit state based on the three-qubit von Neumann measurements and local unitary operations. In our scheme, one only need to make some three-qubit von Neumann measurements of the W-class state, so this protocol can be realized in the experiment.

2 Teleportation of an Arbitrary Three-Qubit State

Our scheme can be described as follows. Suppose Alice has an arbitrary three-qubit state, which can be described as follows

$$|\psi\rangle_{ABC} = (a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle)_{ABC}, \tag{1}$$

where $|a|^2 + |b|^2 + |c|^2 + |d|^2 + |e|^2 + |f|^2 + |g|^2 + |h|^2 = 1$. For teleporting the arbitrary three-qubit state, suppose that Alice shares three sets of W-class entangled states $|W\rangle_{123}, |W\rangle_{456}, |W\rangle_{789}$ with Bob, where

$$|W\rangle_{123} = \frac{1}{2}(|100\rangle_{123} + |010\rangle_{123} + \sqrt{2}|001\rangle_{123}), \tag{2}$$

$$|W\rangle_{456} = \frac{1}{2}(|100\rangle_{456} + |010\rangle_{456} + \sqrt{2}|001\rangle_{456}), \tag{3}$$

$$|W\rangle_{789} = \frac{1}{2}(|100\rangle_{789} + |010\rangle_{789} + \sqrt{2}|001\rangle_{789}). \tag{4}$$

The qubits $A, B, C, 1, 2, 4, 5, 7$, and 8 belong to Alice, qubits 3, 6 and 9 belong to Bob, respectively.

At the beginning, the combined state of the quantum system composed of the twelve qubits $A, B, C, 1, 2, 3, 4, 5, 6, 7, 8$ and 9 can be written in the following form of direct product

$$|\Psi\rangle = |\psi\rangle_{ABC} \otimes |W\rangle_{123} \otimes |W\rangle_{456} \otimes |W\rangle_{789}. \tag{5}$$

To teleporting the original state $|\psi\rangle_{ABC}$ to Bob, Alice first performs three three-qubit von Neumann measurements on her qubits $(A, 1, 2)$, $(B, 4, 5)$ and $(C, 7, 8)$ in the basis of $\{|\eta^{\pm}\rangle_{ijk}, |\xi^{\pm}\rangle_{ijk}\}$, respectively. Here $|\eta^{\pm}\rangle_{ijk}$ and $|\xi^{\pm}\rangle_{ijk}$ are a set of orthogonal states in the W-state category given by

$$|\eta^{\pm}\rangle_{ijk} = \frac{1}{2}(|010\rangle_{ijk} + |001\rangle_{ijk} \pm \sqrt{2}|100\rangle_{ijk}), \tag{6}$$

$$|\xi^{\pm}\rangle_{ijk} = \frac{1}{2}(|110\rangle_{ijk} + |101\rangle_{ijk} \pm \sqrt{2}|000\rangle_{ijk}). \tag{7}$$

It is known that Alice may obtain one of the 64 kinds of possible measured results with equal probability, and the remaining qubits may collapse into one of the 64 states $|\phi^i\rangle_{369}$

$(i = 1, 2, \ldots, 64)$ after the measurement.

$$
\begin{aligned}
&{}_{C78}\langle\eta^{\pm}|_{B45}\langle\eta^{\pm}|_{A12}\langle\eta^{\pm}||\Psi\rangle \\
&\quad = \frac{1}{8}(a|000\rangle \pm + + b|001\rangle + \pm + c|010\rangle \pm \pm + d|011\rangle \\
&\qquad + + \pm e|100\rangle \pm + \pm f|101\rangle + \pm \pm g|110\rangle \pm \pm \pm h|111\rangle)_{369},
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
&{}_{C78}\langle\xi^{\pm}|_{B45}\langle\eta^{\pm}|_{A12}\langle\eta^{\pm}||\Psi\rangle \\
&\quad = \frac{1}{8}(\pm + +a|001\rangle + b|000\rangle \pm \pm + c|011\rangle + \pm + d|010\rangle \\
&\qquad \pm + \pm e|101\rangle + + \pm f|100\rangle \pm \pm \pm g|111\rangle + \pm \pm h|110\rangle)_{369},
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
&{}_{C78}\langle\eta^{\pm}|_{B45}\langle\xi^{\pm}|_{A12}\langle\eta^{\pm}||\Psi\rangle \\
&\quad = \frac{1}{8}(+ \pm +a|010\rangle \pm \pm + b|011\rangle + c|000\rangle \pm + + d|001\rangle \\
&\qquad + \pm \pm e|110\rangle \pm \pm \pm f|111\rangle + + \pm g|100\rangle \pm + \pm h|101\rangle)_{369},
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
&{}_{C78}\langle\xi^{\pm}|_{B45}\langle\xi^{\pm}|_{A12}\langle\eta^{\pm}||\Psi\rangle \\
&\quad = \frac{1}{8}(\pm \pm +a|011\rangle + \pm + b|010\rangle \pm + + c|001\rangle + d|000\rangle \\
&\qquad \pm \pm \pm e|111\rangle + \pm \pm f|110\rangle \pm + \pm g|101\rangle + + \pm h|100\rangle)_{369},
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
&{}_{C78}\langle\xi^{\pm}|_{B45}\langle\xi^{\pm}|_{A12}\langle\eta^{\pm}||\Psi\rangle \\
&\quad = \frac{1}{8}(+ + \pm a|100\rangle \pm + \pm b|101\rangle + \pm \pm c|110\rangle \pm \pm \pm d|111\rangle \\
&\qquad + e|000\rangle \pm + + f|001\rangle + \pm + g|010\rangle \pm \pm + h|011\rangle)_{369},
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
&{}_{C78}\langle\xi^{\pm}|_{B45}\langle\eta^{\pm}|_{A12}\langle\xi^{\pm}||\Psi\rangle \\
&\quad = \frac{1}{8}(\pm + \pm a|101\rangle + + \pm b|100\rangle \pm \pm \pm c|111\rangle + \pm \pm d|110\rangle \\
&\qquad \pm + + e|001\rangle + f|000\rangle \pm \pm + g|011\rangle + \pm + h|010\rangle)_{369},
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
&{}_{C78}\langle\eta^{\pm}|_{B45}\langle\xi^{\pm}|_{A12}\langle\xi^{\pm}||\Psi\rangle \\
&\quad = \frac{1}{8}(+ \pm \pm a|110\rangle \pm \pm \pm b|111\rangle + + \pm c|100\rangle \pm + \pm d|101\rangle \\
&\qquad + \pm + e|010\rangle \pm \pm + f|011\rangle + g|000\rangle \pm + + h|001\rangle)_{369},
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
&{}_{C78}\langle\xi^{\pm}|_{B45}\langle\xi^{\pm}|_{A12}\langle\xi^{\pm}||\Psi\rangle \\
&\quad = \frac{1}{8}(\pm \pm \pm a|111\rangle + \pm \pm b|110\rangle \pm + \pm c|101\rangle + + \pm d|100\rangle \\
&\qquad \pm \pm + e|011\rangle + \pm + f|010\rangle \pm + + g|001\rangle + h|000\rangle)_{369}.
\end{aligned} \tag{15}
$$

In (8)–(15), the notes "$\pm$" or "$+$" from right to left correspond to the three-qubit von Neumann measurements of qubits '$A12$', '$B45$' and '$C78$', respectively, and they mean multiplication of $\pm$ signs. To see this more explicitly, we assume the Alice's measured result is

Table 1 Alice's measurement results and unitary transformations corresponding to the states of qubits 3, 6 and 9

Alice's measurement results	Bob's unitary transformations
$\|\eta^{\pm}\rangle_{A12}\|\eta^{\pm}\rangle_{B45}\|\eta^{\pm}\rangle_{C78}$	$(\|0\rangle\langle 0\| \pm \|1\rangle\langle 1\|)_3 \otimes (\|0\rangle\langle 0\| \pm \|1\rangle\langle 1\|)_6 \otimes (\|0\rangle\langle 0\| \pm \|1\rangle\langle 1\|)_9$
$\|\eta^{\pm}\rangle_{A12}\|\eta^{\pm}\rangle_{B45}\|\xi^{\pm}\rangle_{C78}$	$(\|0\rangle\langle 0\| \pm \|1\rangle\langle 1\|)_3 \otimes (\|0\rangle\langle 0\| \pm \|1\rangle\langle 1\|)_6 \otimes (\|1\rangle\langle 0\| \pm \|0\rangle\langle 1\|)_9$
$\|\eta^{\pm}\rangle_{A12}\|\xi^{\pm}\rangle_{B45}\|\eta^{\pm}\rangle_{C78}$	$(\|0\rangle\langle 0\| \pm \|1\rangle\langle 1\|)_3 \otimes (\|1\rangle\langle 0\| \pm \|0\rangle\langle 1\|)_6 \otimes (\|0\rangle\langle 0\| \pm \|1\rangle\langle 1\|)_9$
$\|\eta^{\pm}\rangle_{A12}\|\xi^{\pm}\rangle_{B45}\|\xi^{\pm}\rangle_{C78}$	$(\|0\rangle\langle 0\| \pm \|1\rangle\langle 1\|)_3 \otimes (\|1\rangle\langle 0\| \pm \|0\rangle\langle 1\|)_6 \otimes (\|1\rangle\langle 0\| \pm \|0\rangle\langle 1\|)_9$
$\|\xi^{\pm}\rangle_{A12}\|\eta^{\pm}\rangle_{B45}\|\eta^{\pm}\rangle_{C78}$	$(\|1\rangle\langle 0\| \pm \|0\rangle\langle 1\|)_3 \otimes (\|0\rangle\langle 0\| \pm \|1\rangle\langle 1\|)_6 \otimes (\|0\rangle\langle 0\| \pm \|1\rangle\langle 1\|)_9$
$\|\xi^{\pm}\rangle_{A12}\|\eta^{\pm}\rangle_{B45}\|\xi^{\pm}\rangle_{C78}$	$(\|1\rangle\langle 0\| \pm \|0\rangle\langle 1\|)_3 \otimes (\|0\rangle\langle 0\| \pm \|1\rangle\langle 1\|)_6 \otimes (\|1\rangle\langle 0\| \pm \|0\rangle\langle 1\|)_9$
$\|\xi^{\pm}\rangle_{A12}\|\xi^{\pm}\rangle_{B45}\|\eta^{\pm}\rangle_{C78}$	$(\|1\rangle\langle 0\| \pm \|0\rangle\langle 1\|)_3 \otimes (\|1\rangle\langle 0\| \pm \|0\rangle\langle 1\|)_6 \otimes (\|0\rangle\langle 0\| \pm \|1\rangle\langle 1\|)_9$
$\|\xi^{\pm}\rangle_{A12}\|\xi^{\pm}\rangle_{B45}\|\xi^{\pm}\rangle_{C78}$	$(\|1\rangle\langle 0\| \pm \|0\rangle\langle 1\|)_3 \otimes (\|1\rangle\langle 0\| \pm \|0\rangle\langle 1\|)_6 \otimes (\|1\rangle\langle 0\| \pm \|0\rangle\langle 1\|)_9$

$|\eta^-\rangle_{A12}|\xi^+\rangle_{B45}|\eta^-\rangle_{C78}$, i.e.,

$$\begin{aligned}
{}_{C78}\langle\eta^-|_{B45}\langle\xi^+|_{A12}\langle\eta^-||\Psi\rangle &= \frac{1}{8}(+++a|010\rangle - ++b|011\rangle + c|000\rangle - ++d|001\rangle \\
&\quad ++-e|110\rangle - +-f|111\rangle ++-g|100\rangle - +-h|101\rangle)_{369} \\
&= \frac{1}{8}(a|010\rangle - b|011\rangle + c|000\rangle - d|001\rangle - e|110\rangle + f|111\rangle \\
&\quad - g|100\rangle + h|101\rangle)_{369}.
\end{aligned}$$

After these measurements, Alice informs Bob of her measured results via a classical channel. Next, Bob will try to reconstruct the original state with a unitary transformation on his qubits 3, 6 and 9. Table 1 shows all 64 kinds of different measurement results by Alice and Bob's relevant 64 kinds of unitary transformations. It will be mentioned that the note "±" of qubits 3, 6 and 9 in the right column is dependent on the three-qubit von Neumann measurements of qubits '$A12$', '$B45$' and '$C78$', respectively. The note "±" will be "+" if the three-qubit von Neumann measurement is "+", while in the other case it will be "−".

Now, let us take an example to demonstrate the principle of this teleportation protocol. Suppose Alice's three-qubit von Neumann measurements outcome is $|\eta^-\rangle_{A12}|\xi^+\rangle_{B45}|\eta^-\rangle_{C78}$, then the state of qubits 3, 6 and 9 would collapse into the state

$$\begin{aligned}
|\phi\rangle_{369} &= (a|010\rangle - b|011\rangle + c|000\rangle - d|001\rangle - e|110\rangle + f|111\rangle \\
&\quad - g|100\rangle + h|101\rangle)_{369},
\end{aligned} \tag{16}$$

Bob needs to apply the local unitary operation

$$U = (|0\rangle\langle 0| - |1\rangle\langle 1|)_3 \otimes (|1\rangle\langle 0| + |0\rangle\langle 1|)_6 \otimes (|0\rangle\langle 0| - |1\rangle\langle 1|)_9.$$

After doing the operation, Bob can successfully reconstruct the original three-qubit state $|\psi\rangle_{ABC}$.

3 Conclusion

In this paper, we have demonstrated that three sets of W-class states can be used as the quantum channel to realize the deterministic teleportation of an arbitrary three-qubit state based on the three-qubit von Neumann measurements and the local unitary operations. We hope that such a three-qubit teleportation scheme can be realized experimentally with photons. Work along an arbitrary N-qubit by using W-class states as the quantum channel is under way.

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