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Disentanglement of Three-qubit States Coupled to an XY Spin Chain with Multisite Interaction

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Abstract The entanglement dynamics of three-qubit states coupled to a general XY spin-chain with a three-site interaction environment is investigated. By using negativity as entanglement measure, we find that the entanglement evolution depends on not only the system-environment couplings but also the strength of three-site interaction. In the strong-coupling region, the three-site interaction can remarkably enhance the decay of the entanglement of three-qubit states. However, in the weak-coupling region, the process of disentanglement depends on the strength of the three-site interaction.

Keywords Disentanglement · Three-qubit states · Three-site interaction

1 Introduction

Entanglement is one of the most fascinating features of quantum mechanics and plays a central role in quantum information processing (QIP) [1–6]. Much progress concerning quantum entanglement both in theory and in experiment has been achieved. In the real world, QIP will always inevitably be affected by the decoherence induced by the external environment [7, 8]. In most of the previous studies, uncorrelated environments were usually considered, and modeled by a reservoir consisting of harmonic oscillators or spin chains. However, the environment systems always have interaction with each other [9–11]. What is the effect of decoherence induced by a correlated environment on quantum states? To answer this question, researchers have made much works. Quan et al. have studied the decoherence induced by the correlated environment [12]. It was shown that at the critical point of a quantum phase transition (QPT) the decoherence is enhanced. For a two-qubit case

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under a transverse Ising model, Sun et al. discovered that the concurrence decays exponentially with fourth power of time in the vicinity of the critical point of the environment system [13]. The multiqubit case was investigated by Ma et al. [14]. For a two-spin coupling to an XY chain with Dzyaloshinsky–Moriya (DM) interaction, Cheng et al. found that the DM interaction can enhance slightly the decay of the decoherence factor in the weak coupling region, and in the strong coupling region, the decoherence factor is super-sensitive to the DM interaction [15]. In addition, many theoretical studies paid attention not only to the nearest-neighbor spin-spin interactions but also to the next-nearest-neighbor ones, as well as multiple spin-exchange model. In comparison with the models with only nearest neighbor interactions, these more complicated models seem closer to a real situation [16]. For example, the additional terms involving the multisite interaction operators were recognized to be important for the description of many physical systems. In Ref. [17], Cheng et al. analyzed the disentanglement of a bipartite system in which the two parties coupled to the spin environment with multisite interaction. However, the effect of decoherence induced by the spin environment with multisite interaction on multipartite entanglement has not been investigated so far, and multipartite dynamics is an interesting problem. So in this paper, we will study the entanglement evolution of three-qubit states coupled to an XY spin chain environment with three-site interaction.

2 The Theoretical Model

We choose the environment to be a general *XY* spin chain with *XZY–YZX* type of three-site interaction, and the three spins are transversely coupled to the chain with inhomogeneous coupling parameters. The corresponding Hamiltonian reads

$$H = -\sum_{l}^{L} J\left(\frac{1+\gamma}{2}\sigma_l^x\sigma_{l+1}^x + \frac{1-\gamma}{2}\sigma_l^y\sigma_{l+1}^y + \lambda\sigma_l^z\right) - \sum_{l}^{L} \alpha J(\sigma_{l-1}^x\sigma_l^z\sigma_{l+1}^y - \sigma_{l-1}^y\sigma_l^z\sigma_{l+1}^x)$$
$$- \sum_{l}^{L} \frac{J}{2}(g_l^A\sigma_A^z + g_l^B\sigma_B^z + g_l^C\sigma_C^z)\sigma_l^z = H_E + H_I, \tag{1}$$

here, we set the spin-coupling interaction $J = 1.0$, α and λ are dimensionless, $g_l^{A,B,C(ABC)}$ denote the coupling strengths between the chain and the three spin-qubits. Noticing that $[\sigma_A^z + \sigma_B^z + \sigma_C^z, \sigma_l^\beta] = 0$, $\beta = x, y, z$, so (1) can be rewritten as $H = \sum_{\mu=1}^{\zeta} |\phi_\mu\rangle\langle\phi_\mu| \otimes H_E^{\lambda_\mu}$, where $|\phi_\mu\rangle(\mu = 1, \ldots, \zeta)$ is the μth eigenstate of operator $\delta = (g_l^A\sigma_A^z + g_l^B\sigma_B^z + g_l^C\sigma_C^z)$ corresponding to the μth eigenvalues g_μ, and λ_μ is given by $\lambda_\mu = \lambda + g_\mu$. $H_E^{\lambda_\mu}$ is given from H_E by replacing λ with λ_μ.

In order to get the time evolution of the density matrix of the system, we should follow the standard procedure by defining conventional Jordan-Wigner transformation which maps spin to one-dimensional spinless fermions with creation (annihilation) operators $a_l^+(a_l)$, and then by introducing the Fourier transforms of the fermionic operators described by $d_k = \frac{1}{\sqrt{L}}\sum_l a_l e^{-i2\pi lk/L}$, the Hamiltonian (1) can be diagonalized by transforming the fermion operators to momentum space and using the Bogoliubov transformation. The result is

$$H_E^{\lambda_\mu} = \sum_k \Lambda_k^{\lambda_\mu}\left(\gamma_{k,\lambda_\mu}^+\gamma_{k,\lambda_\mu} - \frac{1}{2}\right), \tag{2}$$

where the energy spectrum $\Lambda_k^{\lambda_\mu}$ is expressed by

$$\Lambda_k^{\lambda_\mu} = 2\left[\varepsilon_k + \alpha \sin\left(\frac{4\pi k}{L}\right)\right], \tag{3}$$

with $\varepsilon_k = \{[\cos(\frac{2\pi k}{L}) - \lambda_\mu]^2 + \gamma^2 \sin^2(\frac{2\pi k}{L})\}^{\frac{1}{2}}$. The corresponding Bogoliubov transformed fermion operators are defined by

$$\gamma_{k,\lambda_\mu} = \cos\frac{\theta_k^{\lambda_\mu}}{2} d_k - i \sin\frac{\theta_k^{\lambda_\mu}}{2} d_{-k}^+, \tag{4}$$

with angles $\theta_k^{\lambda_\mu} = \arctan[\gamma \sin(2\pi k/L)/(\lambda_\mu - \cos(2\pi k/L))]$.

In terms of these notations, we can obtain the time evolution of quantum states induced by the interaction between the system and the environment. Let the initial state of the composite system be $|\psi(0)\rangle = |\phi(0)\rangle_{ABC} \otimes |\phi(0)\rangle_E$, where $|\phi(0)\rangle_{ABC}$ and $|\phi(0)\rangle_E$ are the initial states of the three central spins and the environment spin chain, respectively. We have $|\psi(t)\rangle = U(t)|\psi(0)\rangle$, where $U(t) = \exp(-iHt)$. The reduced density matrix of system can be obtained as [13]

$$\rho_s(t) = Tr_E|\psi(t)\rangle\langle\psi(t)| = \sum_{\mu,\nu=1}^{\zeta} a_\mu a_\nu^* \langle\phi_E|U_E^{+(\lambda_\mu)}(t)U_E^{(\lambda_\nu)}(t)|\phi_E\rangle|\mu\rangle\langle\nu|, \tag{5}$$

where $a_\mu = \langle\phi_\mu|\phi_{ABC}\rangle$ and $U_E^{(\lambda_\mu)}(t) = \exp(-iH_E^{(\lambda_\mu)}t)$ is the projected time evolution operator for the spin chain dressed by the system-environment interaction parameter λ_μ . We define the decoherence factor $|F_{\mu\nu}(t)|$ as follows:

$$F_{\mu\nu}(t) = \langle\phi_E|U_E^{+(\lambda_\mu)}(t)U_E^{(\lambda_\nu)}(t)|\phi_E\rangle. \tag{6}$$

Here the initial state of the system is separable with the initial state of the environment, and the initial state of the environment $|\phi(0)\rangle$ is assumed to be

$$|G\rangle_\lambda = \prod_{k=1}^{M}(\cos(\theta_k^\lambda/2)|0\rangle_k|0\rangle_{-k} + i\sin(\theta_k^\lambda/2)|1\rangle_k|1\rangle_{-k}, \tag{7}$$

where $|0\rangle_k$ and $|1\rangle_k$ denote the vacuum and single excitation of the kth mode d_k, respectively. Noticing that

$$\gamma_{k,\lambda_\mu} = (\cos\Theta_k^{(\lambda_\mu)})\gamma_{k,\lambda} - i(\sin\Theta_k^{(\lambda_\mu)})\gamma_{-k,\lambda}^+, \tag{8}$$

where $\Theta_k^{(\lambda_\mu)} = (\theta_k^{(\lambda_\mu)} - \theta_k^{(\lambda)})/2$ and $\gamma_{k,\lambda_\mu}, \gamma_{k,\lambda}$ are the normal modes dressed by the system-environment interaction and the purely environment, respectively. The ground state $|G\rangle_\lambda$ of the Hamiltonian H_E can be obtained from the ground state $|G\rangle_{\lambda_\mu}$ of the qubit-dressed Hamiltonian $H_E^{\lambda_\mu}$ by the following relation:

$$|G\rangle_\lambda = \prod_{k=1}^{M}(\cos\Theta_k^{(\lambda_\mu)} + i\sin\Theta_k^{(\lambda_\mu)}\gamma_{k,\lambda_\mu}^+\gamma_{-k,\lambda_\mu}^+)|G\rangle_{\lambda_\mu}. \tag{9}$$

In this study, we assume the system is initially in a general three-qubit pure state

$$|\phi(0)\rangle_{ABC} = c_1|000\rangle + c_2|001\rangle + c_3|010\rangle + c_4|011\rangle + c_5|100\rangle + c_6|101\rangle + c_7|110\rangle + c_8|111\rangle, \tag{10}$$

where the coefficients satisfy the relation for normalization $|c_1|^2 + |c_2|^2 + \cdots + |c_8|^2 = 1$. After a careful calculation, we obtain the elements $M_{\mu\nu}$ of the reduced density matrix of the system, $\mu,\ \nu = 1, 2, \ldots, 8$. Here, we write the $M_{\mu\nu}$ in the following equations:

$$M_{\mu\nu} = c_\mu c_\nu^* F_{\mu\nu}(t), \quad \mu, \nu = 1, 2, \ldots, 8. \tag{11}$$

It should be noted that once all the moduli of $F_{\mu\nu}(t)$ are completely destroyed by decoherence, any state will become a separable one. The moduli of $F_{\mu\nu}(t)$ can be obtained in the following equation:

$$\begin{aligned} |F_{\mu\nu}(t)| = \prod_{k>0} [1 &+ 2\sin(2\Theta_k^{\lambda_\mu})\sin(2\Theta_k^{\lambda_\nu})\sin(\Lambda_k^{\lambda_\mu} t)\sin(\Lambda_k^{\lambda_\nu} t)\cos(\Lambda_k^{\lambda_\mu} t - \Lambda_k^{\lambda_\nu} t) \\ &- 4\sin^2(\Lambda_k^{\lambda_\mu} - \Lambda_k^{\lambda_\nu})\sin^2(\Lambda_k^{\lambda_\mu} t)\sin^2(\Lambda_k^{\lambda_\nu} t)\sin(2\Theta_k^{\lambda_\mu})\sin(2\Theta_k^{\lambda_\nu}) \\ &- \sin^2(2\Theta_k^{\lambda_\nu})\sin^2(\Lambda_k^{\lambda_\nu} t) - \sin^2(2\Theta_k^{\lambda_\mu})\sin^2(\Lambda_k^{\lambda_\mu} t)]^{\frac{1}{2}}, \end{aligned} \tag{12}$$

where $\lambda_{\mu,\nu}, \mu,\ \nu = 1, \ldots, 8$, take the following expressions:

$$\begin{aligned} &\lambda_1 = \lambda + \frac{g_A + g_B + g_C}{2}, \quad \lambda_2 = \lambda + \frac{g_A + g_B - g_C}{2}, \\ &\lambda_3 = \lambda + \frac{g_A - g_B + g_C}{2}, \quad \lambda_4 = \lambda + \frac{g_A - g_B - g_C}{2}, \\ &\lambda_5 = \lambda + \frac{-g_A + g_B + g_C}{2}, \quad \lambda_6 = \lambda + \frac{-g_A + g_B - g_C}{2}, \\ &\lambda_7 = \lambda + \frac{-g_A - g_B + g_C}{2}, \quad \lambda_8 = \lambda + \frac{-g_A - g_B - g_C}{2}. \end{aligned} \tag{13}$$

3 Dynamical Entanglement of Three Qubits

With the reduced density matrix of the system, we can evaluate the quantum entanglement dynamics of three-qubit states. Due to good operational and calculation properties, negativity has been used to measure the multipartite entanglement of quantum states with high dimensions. For a given quantum state with density matrix ρ, the negativity of ρ is defined by [18]

$$N(\rho) = \frac{\|\rho^{T_i}\| - 1}{2}, \tag{14}$$

where $\|\rho^{T_i}\|$ is the sum of the absolute values of the eigenvalues of ρ^{T_i}, and ρ^{T_i} denotes the partial transpose of density matrix ρ with respect to party i. The negativity is a measure of the degree of violation of the criterion of the positive partial transpose (PPT) in entangled states, and has been shown to be an entanglement monotone. Although the PPT criterion is only a necessary separability condition, sufficient just for the case of two spin-halves and the case of (1/2, 1) mixed spins [19, 20], it fulfills some fundamental properties of an entanglement measure. Here, we employ the negativity to measure the tripartite quantum state. For example, a tripartite quantum state ρ_{ABC} can be splitted into three bipartitions $AB-C$, $BC-A$, $AC-B$. Every bipartition gives a negativity, so there are three negativities N_{AB-C}, N_{BC-A}, N_{AC-B} to measure the quantum correlation between one group with two parties and the other group with one party. The residual entanglement of a reduced density matrix will be considered too, which can be obtained by tracing one party off the density matrix ρ_{ABC} to ρ^r_{AB}, ρ^r_{BC} or ρ^r_{CA}, and we can analyze the residual entanglement with negativities N_{A-B}, N_{B-C} and N_{C-A}, respectively. In practice, we are interested in some explicit examples. Here, firstly, we consider two known types of entangled states as initial states of the system. The GHZ class states and the W class states are considered as they bear incompatible quantum correlation in the sense that they cannot be transformed into each other by local operation and classical communication. Secondly, we discuss the entanglement evolution of the Werner-like state as an example of a mixed state under the environment.

3.1 The Case with an Initial GHZ State

We consider that the GHZ state is the initial state of the system and it reads

$$|\phi(0)\rangle_{ABC} = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle). \tag{15}$$

After a straightforward calculation, we obtain the time evolution of entanglement of the GHZ state,

$$N_{AB-C}(t) = \frac{1}{2}|F_{18}(t)|. \tag{16}$$

Tracing one party off the GHZ state, we find that the reduced density matrix is separable, so the residual entanglement of N_{A-B}, N_{B-C} and N_{C-A} are zero.

Firstly we consider the effect of the three-site interaction α and total site of the spin chain L on the time evolution of entanglement of the three qubits which are interacting with the environment in the weak-coupling region ($g \ll 1$). In Fig. 1, $N_{AB-C}(t)$ is plotted as a function of time t for different values of α, given parameters $L = 801$, $g = g_A + g_B + g_C = 0.05$, $\gamma =$

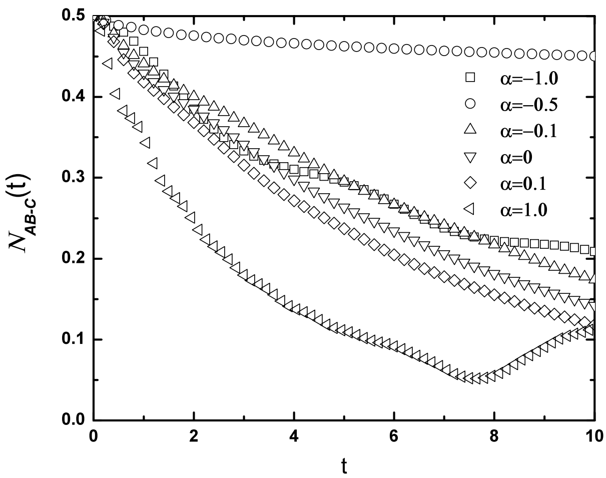

Fig. 1 Negativity of GHZ state as a function of time t and α in the weak-coupling region. $\lambda = 1.0$, $\gamma = 1.0$, and $L = 401$

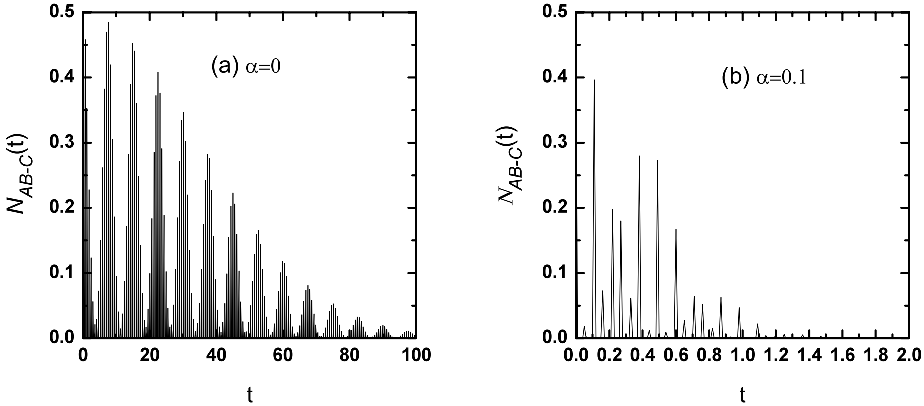

Fig. 2 Negativity of GHZ state as a function a time t and α in the strong-coupling region. $\lambda = 1.0$, $\gamma = 1.0$, and $L = 401$

1.0. At the critical point $\lambda = 1$, we take the value of α with $-1.0, -0.5, -0.1, 0.0, 0.1, 1.0$. In a short time, it is seen that the entanglement decays more sharply with increasing the intensity of three-site interaction in the interval $\alpha \geq 0$. However, the decay of the entanglement can be delayed remarkably with increasing the intensity of three-site interaction along the negative direction. To check this phenomenon, we carry out an analysis similar to Ref. [12]. In a short time, when $\lambda \to 1$, one has $|F_{\mu\nu}(t)| = e^{-(\tau_1+\tau_2+\tau_3)t^2}$, where τ_1, τ_2, and τ_3 are equal to $\frac{8E(K)\gamma^2 g^2}{(\lambda-1)^2}$, $\frac{32F(K)\gamma^2 g\alpha}{(\lambda-1)^2}$, and $\frac{32G(K)\gamma^2\alpha^2}{(\lambda-1)^2}$, respectively; $E(k)$, $F(k)$, and $G(k)$ are equal to $4\pi^2\sum(\frac{k}{L})^2$, $8\pi^3\sum(\frac{k}{L})^3$, and $16\pi^4\sum(\frac{k}{L})^4$, respectively. From the above approximation analysis we may expect that when the parameter λ is adjusted to the point $\lambda = 1$, the negativity $N_{AB-C}(t)$ will exponentially decay in the second power of time. It is easy to find that the negativity $N_{AB-C}(t)$ is determined by τ_1, τ_2, and τ_3, while we also can find that the expression $\tau_1 + \tau_2 + \tau_3$ is a second-order polynomial against variable α. This find indicates the existence of an extremum point at which the decay of the negativity can be delayed remarkably.

We now turn to study the effect of three-site interaction on the negativity in the strong coupling region ($g \gg 1$). Figure 2 is a plot of $N_{AB-C}(t)$ as a function of time t when $g = 500$. It is seen that the decay is characterized by an oscillatory Gaussian envelope and the width of the Gaussian envelope is very sensitive to the three-site interaction. To explain these features, we take a similar tactic employed by [12]. When ($g \gg 1$), we have the following from the angle of the Bogoliubov transformation: $\theta_k^{\lambda_1} \approx 0$ and $\theta_k^{\lambda_8} \approx \pi$. So we obtain $|F_{18}(t)| \approx \prod|\cos^2\Theta_k^{\lambda_1} e^{i\Lambda_k t} + \sin^2\Theta_k^{\lambda_8} e^{-i\Lambda_k t}|$, where $\Lambda_k = \Lambda_k^{\lambda_1} + \Lambda_k^{\lambda_8}$. This expression is similar to that given in [17]. We can follow the mathematical procedure and derive the approximate expression for $N_{AB-C}(t) \approx 0.5\exp(-s_L^2 t/2)|\cos(\Lambda t)|^{(L-1)/2}$, where Λ is the mean value of Λ_k and approximately equal to $4g + \gamma^2/g$; $s_L^2 = \sum\sin^2(2\Theta_k^{\lambda_1})\delta_k^2$, where the quantity δ_k describes the deviation of Λ_k from its mean values and is approximately equal to $-\frac{\gamma^2}{g}\cos\frac{4\pi k}{L} + 4\alpha\sin\frac{4\pi k}{L}$. This is the reason why the three-site interaction can enhance the decay of $N_{AB-C}(t)$ dramatically.

3.2 The Case with an Initial W State

Now, we consider that the initial state of the system is a three-qubit W state, $|\phi(0)\rangle = \frac{\sqrt{3}}{3}(|001\rangle + |010\rangle + |100\rangle)$. Similarly, we get the negativities of the W state in the following expression:

$$
\begin{aligned}
N_{AB-C} &= \frac{1}{3}\sqrt{|F_{23}|^2+|F_{25}|^2}, \quad N_{A-B} = \frac{1}{6}\left(\sqrt{4|F_{35}|^2+1}-1\right), \\
N_{AC-B} &= \frac{1}{3}\sqrt{|F_{23}|^2+|F_{35}|^2}, \quad N_{A-C} = \frac{1}{6}\left(\sqrt{4|F_{25}|^2+1}-1\right), \\
N_{BC-A} &= \frac{1}{3}\sqrt{|F_{25}|^2+|F_{35}|^2}, \quad N_{B-C} = \frac{1}{6}\left(\sqrt{4|F_{23}|^2+1}-1\right).
\end{aligned}
\tag{17}
$$

Contrary to the case with GHZ state, there are two obviously different behavior of entanglement evolution correspond to W state. Firstly, the residual entanglement of the W state is not zero; secondly, when all the coupling parameters g_i $(i = A, B, C)$ take the same value, we can find that the W state will not perceive the presence of the environment. In this sense, the W state is called as a decoherence-free quantum state and can be used to design noiseless quantum codes [21–23]. To examine the effect of three-site interaction on the entanglement evolution of the W state, we numerically calculate the quantity of N_{AB-C} both for the weak couplings region and strong ones. Because of symmetry, the other quantities of entanglement of the W state are omitted here. In Fig. 3(b), we plot the negativity $N_{AB-C}(t)$ as the function of time t with α equal to $-1.0, -0.5, -0.1, 0.0, 0.1, 1.0$, respectively. From Fig. 3(b), in the weak region, it can be seen that the three-site interaction can hasten the decay of entanglement by increasing the value of α in the interval $\alpha \geq 0$. However, in the region $\alpha < 0$, the decay of the negativity $N_{AB-C}(t)$ can be delayed remarkably with the increasing intensity along the negative direction. In Fig. 3(c) and (d), we display the evolution of $N_{AB-C}(t)$ in the strong region. It can be seen that the negativity $N_{AB-C}(t)$ presents a behavior of oscillation and the three-site interaction enhance the decay of entanglement remarkably.

3.3 The Case with an Initial Werner-like State

Having considered the pure quantum states above, the mixed states should also be discussed. Here, we employ a Werner-like state as the example to illustrate the effect of spin environment with three-site interaction on mixed states. A three-qubit Werner-like state reads [14]:

$$
\rho_{werner} = \frac{pI_{8\times 8}}{8} + (1-p)|\phi\rangle\langle\phi|, \tag{18}
$$

where p is a parameter ranging from 0 to 1 to characterize the extent to which the white noise exists in state $|\phi\rangle$, and $|\phi\rangle$ takes the form $|\phi\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$, operator $I_{8\times 8}$ is an identity matrix with rank of 8. Due to decoherence, the time evolution of the Werner-like state takes the following expression:

$$
N_{AB-C} = \frac{1-p}{2}|F_{18}| - \frac{p}{8}. \tag{19}
$$

Equation (19) holds under the condition that $\frac{1-p}{2}|F_{18}| > \frac{p}{8}$, otherwise, negativity $N_{AB-C}(t)$ takes a zero value. The entanglement evolution of the Werner-like state is illustrated in Fig. 4. From Fig. 4, we find that the more noise applied to the state $|\phi\rangle$, the shorter the time for entanglement persists. It is found that the entanglement vanishes abruptly for the mixed states corresponding to $p = 0.5$ with a finite time. Usually, researchers call this phenomenon of finite-time disentanglement as "entanglement sudden death" firstly observed by Yu and Eberly in the study of spontaneous emission [24]. From Fig. 4(a) and (b), we can find that the three-site interaction can also remarkably decay the process of disentanglement in the case of the mixed state as an initial state, if proper parameter values are chosen.

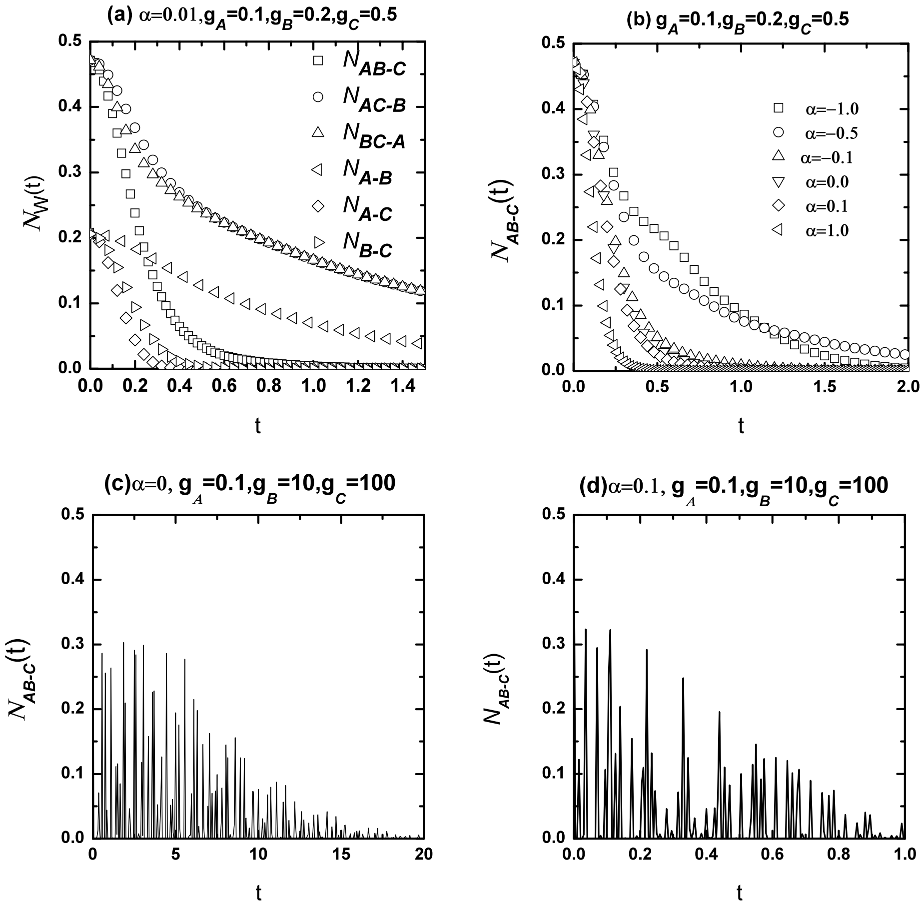

Fig. 3 Negativities of the W state as a function of time t. $\lambda = 1.0$, $\gamma = 1.0$, and $L = 401$

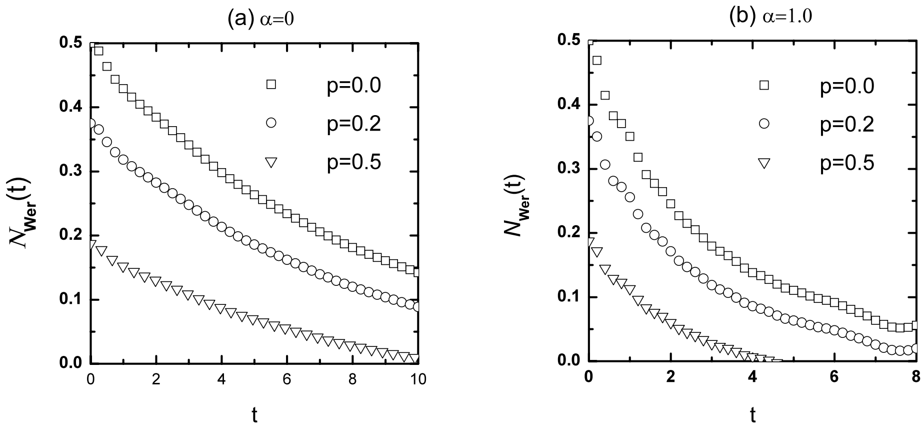

Fig. 4 Negativities of the Werner-like state as a function of time t are plotted for different p. $g = 0.05$, $\lambda = 1.0$, $\gamma = 1.0$, and $L = 401$. (**a**) $\alpha = 0$, (**b**) $\alpha = 1.0$

4 Conclusion

In summary, we have investigated the dynamical process of disentanglement of three qubits coupled to a general *XY* spin chain with three-site interaction environment. In the weak-coupling region, the three-site interaction can enhance the decay of the entanglement in most cases, and in some specific intervals the decay of the entanglement can be delayed remarkably both for the GHZ and W states. In the strong-coupling region, the decay of entanglement is very sensitive to the three-site interaction both for the GHZ and W states. When all the coupling constants take the same value,and the initial state is a W state, a linear decoherence-free quantum space is identified. Furthermore, when the three qubits are initially in the mixed states, although the complete disentanglement happens in a finite time, the additional three-site interaction can still delay the process of disentanglement remarkably.

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