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Holographic Dark Energy Density and JBP Parametrization

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Abstract In this article we consider the holographic dark energy density. We study dark energy density in Universe with arbitrary spatially curvature described by the Friedmann-Robertson-Walker metric. We use Jassal-Bagla-Padmanabhan parametrization to specify dark energy density.

Keywords Dark energy · Cosmological constant · Holography · Cold dark matter

1 Introduction

It is found that a large amount of the energy in Universe is of unknown nature to us which called dark energy. According to the latest studies and astronomical observations such as WMAP (Wilkinson microwave anisotropy probe [1]). Universe consists of about 23% dark matter, 3.6% intergalactic gas, 0.4% stars, etc., and 73% dark energy. Indeed Albert Einstein was the first person to realize that empty space is not nothing. The first property that Einstein discovered is that it is possible for more space to come into existence. Then one version of Einstein's gravity theory, the version that contains a cosmological constant, makes a second prediction: "empty space" can possess its own energy. As more space comes into existence, more of this energy-of-space would appear. As a result, this form of energy would cause the Universe to expand faster and faster. Nowadays it is clear that our universe expand with acceleration [2–6]. Therefore the study of the cosmological constant is one of the important problems in theoretical and experimental physics [7–11]. However there are many unsuccessful efforts to specify the nature of the dark energy. The reason is that the dark energy problem fallen in to an issue of quantum gravity which is unknown theory today. Just there is a nice candidate for the quantum gravity which is the string theory. Recent study [12] suggests that there exist a large number of consistent in string theory [13]. Recently, the weak gravity conjecture has been applied to explain the dark energy: holographic dark energy model [14–19]. One of the important ways to specify the nature of the dark

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energy is the study of the time-dependent dark energy density. For the similar reason the relation between dark matter density and temperature of the dark universe have already been studied [20]. Also the effect of dark matter on the solar system have already considered [21, 22] which extended to the case of the dark energy [23–25]. In [23] we investigated the relation between the time-dependent dark energy density and temperature of the dark universe. We found the temperature proportional to the inverse of the dark energy density. Therefore high density dark energy corresponds to low temperature universe. In [24] we considered the holographic dark energy model with interaction and space curvature. We calculated the cosmic scale factor by using the time-dependent dark energy density. Then we obtained the phenomenological interaction between the holographic dark energy and matter. In [25] we considered the cosmological model based on the holographic dark energy. We studied the dark energy density in Universe with arbitrary spatially curvature described by the Friedmann-Robertson-Walker metric. We used Chevallier-Polarski-Linder parametrization [26] to specify dark energy density. Now, in this paper we would like to use another parametrization so called Jassal-Bagla-Padmanabhan (JBP) parametrization [27].

This paper organized as the following. In Sect. 2 we recall the Friedmann equation for arbitrary space. Then in Sect. 3 we write holographic dark energy in terms of ω. In Sect. 4 we use JBP parametrization to specify the dark energy density for three different space. Finally in Sect. 5 we give conclusion.

2 Friedmann Equation

A natural generalization of the Einstein model is to allow the curvature radius to be a function of time. The universe is still homogeneous and isotropic on a constant surface, but it is no longer static. In the 1930s, Robertson and Walker (independently) showed that there are only three possible space-time metrics for a universe that is homogeneous and isotropic. While in the 1920s Friedmann used that metric in their cosmological models. Then Robertson and Walker proved that they are the only forms consistent with the cosmological Principle. It is commonly called the Friedmann-Robertson-Walker (FRW) metric which represented by the following line element,

$$ds^2 = -dt^2 + a(t)^2\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right), \tag{1}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. The angles θ and ϕ are the usual azimuthal and polar angles of spherical coordinates, with $0 \le \theta \le \pi$ and $0 \le \phi < 2\pi$. These coordinates (t, r, θ, ϕ) are called co-moving coordinates. The reason is because two objects at different spatial coordinates can remain at those coordinates at all times, while the proper distance between them changes with time according to how the scale factor $a(t)$ changes with time. One can image this phenomena as dots on a balloon whose coordinates are fixed, while the radius of the balloon changes with time. Also k is a constant representing the curvature of the space. The curvature k may be not only positive, corresponding to real finite radius, but also zero or negative, corresponding to infinite or imaginary radius. The possibilities are called closed ($k = 1$), flat ($k = 0$), and open ($k = -1$). The Friedmann's equation in unit of $8\pi G = 1$ is given by,

$$3\left(H^2 + \frac{k}{a^2}\right) = \rho_m + \rho_D, \tag{2}$$

where ρ_m is the density of matter and ρ_D is the dark energy density. $H \equiv \frac{\dot{a}}{a}$ gives the expansion rate of Universe, and it is called the Hubble parameter. Its present value H_0 is the Hubble constant. The dimension of H is $1/t$ (or velocity/distance).

3 Dark Energy Density

If we consider Universe dominated by the pressureless matter and dark energy, and assume that they interact with each other gravitationally only. Then one can write,

$$\rho_m = \rho_m(0)(1+z)^3, \tag{3}$$

and

$$\rho_D = \rho_D(0)e^{[3\int_0^z(\frac{1+\omega(z')}{1+z'})dz']}, \tag{4}$$

where the equation of state parameter of dark energy, ω is not constant. While in the holographic dark energy model the dark energy density is given by,

$$\rho_D = 3d^2H^2 \tag{5}$$

where d^2 is a dimensionless quantity that collects the uncertainties of the theory such as the number particle species. Usually this parameter is assumed constant. But in this paper we would like to consider non constant d. We should note that in relation (5) we used $8\pi G = 1$ unit.

Recalling Friedmann's equation (2) and using holographic dark energy density (5), we can obtain,

$$3H^2(1-d^2) + \frac{3k}{a^2} = \rho_m(0)(1+z)^3, \tag{6}$$

also by using relation (4) we can write,

$$3H^2 + \frac{3k}{a^2} = \rho_m(0)(1+z)^3 + \rho_D(0)e^{[3\int_0^z(\frac{1+\omega(z')}{1+z'})dz']}. \tag{7}$$

Combining (6) and (7) yields to the following relation,

$$r = \frac{1-d^2}{d^2} = \left(r_0(1+z)^3 - \rho_D(0)\frac{3k}{a^2}\right)e^{[-3\int_0^z(\frac{1+\omega(z')}{1+z'})dz']}, \tag{8}$$

where $r \equiv \frac{\rho_m}{\rho_D}$. The last equality tells us that the d parameter cannot assume constant but vary with expansion.

Before end of this section we use the conservation equation,

$$\dot{\rho}_D + 3H(1+\omega)\rho_D = 0, \tag{9}$$

where

$$\omega = -\frac{1}{3}\left(1 + 2\sqrt{\frac{\Omega_D}{d^2} + \Omega_k}\right), \tag{10}$$

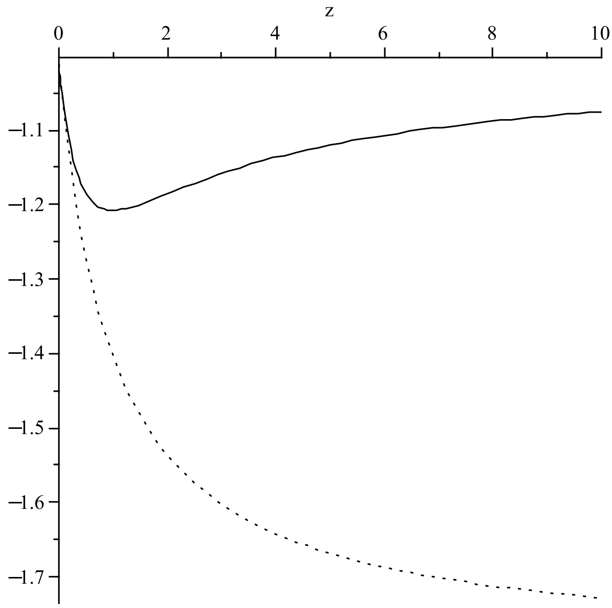

Fig. 1 The graphs of two different parametrization. *The solid line* drawn for Jassal-Bagla-Padmanabhan parametrization. *The dotted line* drawn for Chevallier-Polarski-Linder parametrization. We set our parameter with the lower limit of ω_0 and ω_1

and we defined $\Omega_D \equiv \frac{\rho_D}{3H^2}$ and $\Omega_k \equiv \frac{k}{\dot{a}^2}$. It yields us to obtain,

$$\frac{\dot{(d^2)}}{d^2} = -3H\omega\Omega_m, \tag{11}$$

where $\Omega_m \equiv \frac{\rho_m}{3H^2}$. According to relation (10) one can see that $\omega < 0$, therefore the d^2 parameter enhances with the expansion. An additional consequence of this point is that d^2 will vary sufficiently slow for $3|\omega|\Omega_m \leq 1$ only.

4 Jassal-Bagla-Padmanabhan Parametrization

The crucial point to specify dark energy density is type of ω. There are several parametrization which yield to different ω. Each of them have some advantage and some defect. It is important to choose appropriate parametrization in any theory. In [25] we used Chevallier-Polarski-Linder parametrization where $\omega(z) = \omega_0 + \omega_1 \frac{z}{1+z}$. Reference [27] suggested a new parametrization of dark energy,

$$\omega(z) = \omega_0 + \omega_1 \frac{z}{(1+z)^2}. \tag{12}$$

The constant parameters $\omega_0 = -0.90^{+0.11}_{-0.11}$ and $\omega_1 = -0.24^{+0.56}_{-0.55}$ are observationally constrained by supernovae, cosmic background radiation, and large scale structure. In Fig. 1 we draw plots of JBP parametrization and Chevallier-Polarski-Linder parametrization, which we used in the previous paper. It shows us that the JBP parametrization is more appropriate choice than Chevallier-Polarski-Linder parametrization to describe $\omega \approx 1$ behavior. In this case ω has a minimum about $z = 1$, and then get close to $\omega = -1$. We should note that for the special case of $\omega_1 = 0$ two parametrization reduce to one.

By using relation (12) in the exponential part of relation (8) one can obtain,

$$A \equiv e^{[-3\int_0^z(\frac{1+\omega(z')}{1+z'})dz']} = e^{\frac{3\omega_1(1+2z)}{2(1+z)^2}}(1+z)^{-3-3\omega_0}, \tag{13}$$

up to a constant. Now we try to obtain d in terms of z for three different cases of $k = 0, 1, -1$.

4.1 Flat Universe

Flat universe with infinite radius described by the FRW metric (1) with $k = 0$, so space geometry at constant time is Euclidean. We can rewrite the Friedmann's equation as the following,

$$\Omega_D + \Omega_m = 1. \tag{14}$$

In that case from (8) one can obtain,

$$d^2 = \frac{1}{1 + r_0 e^{\frac{3\omega_1(1+2z)}{2(1+z)^2}}(1+z)^{-3\omega_0}}. \tag{15}$$

For the short distance one can obtain $d^2 \approx 1 - r_0 e^{\frac{3\omega_1}{2}}$. On the other hand for large distance one can find $d^2 \approx 1 - r_0 z^{-3\omega_0}$. Therefore we find that for the short distance d^2 parameter depends to the ω_1 and for the large distance d^2 parameter depends to the ω_0.

4.2 AdS_3 Universe

AdS_3 universe with imaginary radius described by the FRW metric (1) with $k = -1$ and has constant negative curvature. We can rewrite the Friedmann's equation as the following,

$$\Omega_D + \Omega_m + \Omega_k = 1. \tag{16}$$

In that case one can obtain,

$$d^2 = \frac{a^2}{a^2(1 + Ar_0 + 3Ar_0 z + 3Ar_0 z^2 + Ar_0 z^3) - 3A\rho_D(0)}, \tag{17}$$

where A is given by relation (13). Therefore one can specify the holographic dark energy density as the following expression,

$$\rho_D = \frac{3\dot{a}^2}{a^2(1 + Ar_0 + 3Ar_0 z + 3Ar_0 z^2 + Ar_0 z^3) - 3A\rho_D(0)}. \tag{18}$$

4.3 Closed Universe

Closed universe with finite real radius described by the FRW metric (1) with $k = 1$ and has constant positive curvature. This metric corresponds to a three-sphere embedded in 4-dimensional Euclidean space. We can rewrite the Friedmann's equation as the following,

$$\Omega_D + \Omega_m + \Omega_k = 1. \tag{19}$$

In that case one can obtain,

$$d^2 = \frac{a^2}{a^2(1 + Ar_0 + 3Ar_0 z + 3Ar_0 z^2 + Ar_0 z^3) + 3A\rho_D(0)}, \tag{20}$$

where A is given by relation (13). Therefore one can specify the holographic dark energy density as the following expression,

$$\rho_D = \frac{3\dot{a}^2}{a^2(1 + Ar_0 + 3Ar_0 z + 3Ar_0 z^2 + Ar_0 z^3) + 3A\rho_D(0)}. \tag{21}$$

5 Conclusion

By using the general form of the Friedmann's equation we studied the holographic dark energy density. We have shown that the d^2 parameter in the dark energy density formula is no longer constant but vary with z. By choosing Jassal-Bagla-Padmanabhan parametrization we obtained d^2 parameter for three different space. Therefore we succeed to obtain holographic dark energy density.

References

1. Spergel, D., et al. (The WMAP Team): Astrophys. J. Suppl. Ser. **148**, 175 (2003)
2. Riess, A.G., et al.: Astron. J. **116**, 1009 (1998)
3. Perlmutter, S., et al.: Astrophys. J. **517**, 565 (1999)
4. Knop, R.A., et al.: Astrophys. J. **598**, 102 (2003)
5. Riess, A.G., et al.: Astrophys. J. **607**, 665 (2004)
6. Bennet, C.L., et al.: Astrophys. J. Suppl. Ser. **148**, 1 (2003)
7. Weinberg, S.: Rev. Mod. Phys. **61**, 1 (1989)
8. Carroll, S.M.: Living Rev. Relativ. **3**, 1 (2001). http://www.livingreviews.org/lrr-2001-1
9. Padmanabhan, T.: Phys. Rep. **380**, 235 (2003)
10. Peebles, P.J.E., Ratra, B.: Rev. Mod. Phys. **75**, 559 (2003)
11. Nobbenhuis, S.: Found. Phys. **36**, 613 (2006). arXiv:gr-qc/0411093
12. Kachru, S., Kallosh, R., Linde, A., Trivedi, S.P.: Phys. Rev. D **68**, 046005 (2003). hep-th/0301240
13. Susskind, L.: hep-th/0302219
14. Thomas, S.D.: Phys. Rev. Lett. **89**, 081310 (2002)
15. Horava, P., Minic, D.: Phys. Lett. **85**, 1610 (2000)
16. Zhao, W.: Phys. Lett. B **655**, 99 (2007)
17. Li, H., Guo, Z.K., Zhang, Y.Z.: Int. J. Mod. Phys. D **15**, 869 (2006)
18. Hu, B., Ling, Y.: Phys. Rev. D **73**, 123510 (2006)
19. Wang, B., Gong, Y., Abdalla, E.: Phys. Lett. **624**, 141 (2005)
20. Sadeghi, J., Saadat, H., Pourhassan, B.: Relation between dark matter density and temperature with power law. Chaos Solitons Fractals **42**, 1080 (2009)
21. Saadat, H.: Solar system and dark matter. Chaos Solitons Fractals **42**, 2236 (2009)
22. Saadat, H., et al.: The effect of dark matter on solar system and perihelion precession of Earth planet. Int. J. Theor. Phys. **49**(10), 2506 (2010)
23. Saadat, H.: Relation between the dark energy density and temperature. Int. J. Theor. Phys. **50**(1), 140 (2011)
24. Saadat, H., Saadat, A.M.: Time-dependent dark energy density and holographic DE model with interaction. Int. J. Theor. Phys. **50**, 1358–1366 (2011)
25. Saadat, H.: Holographic, dark energy density. Int. J. Theor. Phys. doi:10.1007/s10773-011-0685-2
26. Linder, E.V.: Exploring the expansion history of the universe. Phys. Rev. Lett. **90**, 091301 (2003). astro-ph/0208512
27. Jassal, H.K., Bagla, J.S., Padmanabhan, T.: Mon. Not. R. Astron. Soc. **356**, 11 (2005)