

Interacting Entropy-Corrected Agegraphic-Tachyon Dark Energy

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Abstract Motivated by recent work of Sheykhi (Phys. Lett. B 682:329, 2010), we generalize this work to agegraphic tachyon models of dark energy with entropy correction terms arising from loop quantum gravity. We establish a connection between the entropy-corrected agegraphic dark energy and the tachyon scalar field in a universe with spacial curvature and reconstruct the potential and the dynamics of the tachyon scalar field which describe the tachyon cosmology. The cosmological implications of the entropy-corrected agegraphic dark energy models are also discussed.

Keywords Agegraphic · Tachyon · Dark energy

1 Introduction

A mysterious force propelling the universe, is one of the deepest mysteries in all of science. This mysterious force now thought to account for about seventy percent of the energy density of the entire universe [2, 3] came to many as a surprise in 1998, when the Supernova Cosmology Project and the High-z Supernova Search teams [4–6] independently announced their discovery that the expansion of the universe is currently accelerating. Many theories have been proposed to explain the cosmic acceleration expansion. Although theories of trying to modify Einstein equations constitute a big part of these attempts, the mainstream explanation for this problem, however, is known as theories of dark energy. It is the most accepted idea that a mysterious dominant component, dark energy, with negative pressure, leads to this cosmic acceleration, though the nature of such dark energy is still much source of doubt.

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Many theoretical attempts towards understanding the dark energy problem are focused to shed light on it in the framework of a fundamental theory such as string theory or quantum gravity. Although a complete theory of quantum gravity has not established yet today, we still can make some attempts to investigate the nature of dark energy according to some principles of quantum gravity. The holographic dark energy (HDE) model and the agegraphic dark energy (ADE) model are just such examples, which are originated from some considerations of the features of the quantum theory of gravity. That is to say, the holographic and ADE models possess some significant features of quantum gravity. The former, that arose a lot of enthusiasm recently [7–35], is motivated from the holographic hypothesis [36]. In [37], a model of HDE with an interaction with matter fields had been investigated by choosing the future event horizon as an IR cutoff. It was shown that the ratio of energy densities can vary with time. Moreover, with the interaction between the two different constituents of the universe, they observed the evolution of the universe, from early deceleration to late time acceleration. In addition, they found that such an interacting dark energy model could accommodate a transition of the dark energy from a normal state where $w_D > -1$ to phantom regimes where $w_D < -1$.

The ADE is based on the uncertainty relation of quantum mechanics together with the gravitational effect in general relativity. The ADE model assumes that the observed dark energy comes from the spacetime and matter field fluctuations in the universe [38–40]. Following the line of quantum fluctuations of spacetime, Karolyhazy [41–43] proposed that the distance t in Minkowski spacetime cannot be known to a better accuracy than $\delta t = \lambda t_p^{2/3} t^{1/3}$, where λ is a dimensionless constant of order unity. Based on Karolyhazy relation, Maziashvili proposed that the energy density of metric fluctuations of Minkowski spacetime is given by [44, 45]

$$\rho_D \sim \frac{1}{t_p^2 t^2} \sim \frac{M_p^2}{t^2}, \quad (1)$$

where t_p and M_p are the reduced Planck time and mass, respectively. Since in ADE model the age of the universe is chosen as the length measure, instead of the horizon distance, the causality problem in the HDE is avoided [38]. The agegraphic models of dark energy have been examined and constrained by various astronomical observations [46–59].

It is important to note that the definition and derivation of holographic energy density depends on the entropy-area relationship of black holes in Einstein's gravity [7]. However, this definition can be modified from the inclusion of quantum effects, motivated from the loop quantum gravity (LQG). The quantum corrections provided to the entropy-area relationship leads to the curvature correction in the Einstein-Hilbert action and vice versa [60, 61]. The corrected entropy takes the form [62–67]

$$S = \frac{A}{4G} + \tilde{\alpha} \ln \frac{A}{4G} + \tilde{\beta}, \quad (2)$$

where $\tilde{\alpha}$ and $\tilde{\beta}$ are dimensionless constants of order unity. The exact values of these constants are not yet determined and still is an open issue in loop quantum cosmology. These corrections arise in the black hole entropy in LQG due to thermal equilibrium fluctuations and quantum fluctuations [68–72]. Taking the corrected entropy-area relation (2) into account, the energy density of the HDE will be modified as well. On this basis, Wei [73] proposed the energy density of the so-called “entropy-corrected holographic dark energy” (ECADE) as

$$\rho_D = 3n^2 M_p^2 L^{-2} + \alpha L^{-4} \ln(M_p^2 L^2) + \beta L^{-4}. \quad (3)$$

In this paper we would like to consider the so-called “entropy-corrected agegraphic dark energy” (ECADE) whose L in (3) is replaced with a time scale t of the universe. The energy density of ECADE is given by

$$\rho_D = 3n^2 M_p^2 t^{-2} + \alpha t^{-4} \ln(M_p^2 t^2) + \beta t^{-4}. \tag{4}$$

The motivation idea for taking the energy density of modified ADE in the form (4) comes from the fact that the origin of ADE and HDE are the same. Indeed, it has been shown that the ADE models are the HDE model with different IR length scales [74]. In the special case $\alpha = \beta = 0$, (4) yields the well-known agegraphic energy density [38]. Since the last two terms in (4) can be comparable to the first term only when t is very small, the corrections make sense only at the early stage of the universe. When the time scale t becomes large, ECADE reduces to the ordinary ADE.

On the other hand, among the various candidates to explain the accelerated expansion, the rolling tachyon condensates in a class of string theories may have interesting cosmological consequences. The tachyon is an unstable field which has become important in string theory through its role in the Dirac-Born-Infeld action which is used to describe the D-brane action [75–80]. It has been shown that the decay of D-branes produces a pressureless gas with finite energy density that resembles classical dust [81].

Our aim here is to establish a correspondence between the interacting ECADE scenarios and the tachyon scalar field in a non-flat universe. Although it is generally believed that our universe is flat, however, some experimental data has implied that our universe is not a perfectly flat universe and recent papers have favored the universe with spatial curvature [82–85]. We suggest the entropy-corrected agegraphic description of the tachyon dark energy and reconstruct the potential and the dynamics of the tachyon scalar field which describe the tachyon cosmology.

This paper is organized as follows. In the next section we associate the original ECADE with the tachyon field. In Sect. 3, we establish the correspondence between the new model of interacting ECADE and the tachyon dark energy. In Sect. 4 we study the cosmological implications of ECADE models. We summarize our results in Sect. 5.

2 Tachyon Reconstruction of Original ECADE

The effective Lagrangian for the tachyon field is described by

$$L = -V(\phi)\sqrt{1 - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}, \tag{5}$$

where $V(\phi)$ is the tachyon potential. The corresponding energy momentum tensor for the tachyon field can be written in a perfect fluid form

$$T_{\mu\nu} = (\rho_\phi + p_\phi)u_\mu u_\nu - p_\phi g_{\mu\nu}, \tag{6}$$

where ρ_ϕ and p_ϕ are, respectively, the energy density and pressure of the tachyon and the velocity u_μ is

$$u_\mu = \frac{\partial_\mu\phi}{\sqrt{\partial_\nu\phi\partial^\nu\phi}}. \tag{7}$$

A rolling tachyon has an interesting equation of state whose parameter smoothly interpolates between -1 and 0 [90]. Thus, tachyon can be realized as a suitable candidate for the

inflation at high energy [91–93] as well as a source of dark energy depending on the form of the tachyon potential [94–97]. Therefore it becomes meaningful to reconstruct tachyon potential $V(\phi)$ from some dark energy models possessing some significant features of the quantum gravity theory, such as holographic and ADE models. It was demonstrated that dark energy driven by tachyon, decays to cold dark matter in the late accelerated universe and this phenomenon yields a solution to cosmic coincidence problem [98]. The connection between tachyon field and the HDE [99, 100] and ADE [1, 101] have also been established.

We assume the background Friedmann-Robertson-Walker (FRW) universe is described by the line element

$$ds^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \tag{8}$$

where $a(t)$ is the scale factor, and k is the curvature parameter with $k = -1, 0, 1$ corresponding to open, flat, and closed universes, respectively. A closed universe with a small positive curvature ($\Omega_k \simeq 0.01$) is compatible with observations [82–85]. The first Friedmann equation takes the form

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2}(\rho_m + \rho_D), \tag{9}$$

where ρ_m and ρ_D are the energy densities of matter and dark energy. The dimensionless density parameters are

$$\Omega_m = \frac{\rho_m}{\rho_{cr}}, \quad \Omega_D = \frac{\rho_D}{\rho_{cr}}, \quad \Omega_k = \frac{k}{a^2 H^2}, \quad \rho_{cr} = \frac{3H^2}{8\pi G}. \tag{10}$$

The last expression is the critical energy density. Using (10) in (9), we get

$$1 + \Omega_k = \Omega_m + \Omega_D. \tag{11}$$

The energy density and pressure of the tachyon field are given by

$$\rho_\phi = -T_0^0 = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}, \tag{12}$$

$$p_\phi = T_i^i = -V(\phi)\sqrt{1 - \dot{\phi}^2}. \tag{13}$$

The equation of state parameter is

$$w_\phi = \frac{p_\phi}{\rho_\phi} = \dot{\phi}^2 - 1. \tag{14}$$

We assume the energy density of the original ECADE is of the form (4) where the time scale t is chosen to be the age of the universe,

$$\rho_D = 3n^2 M_p^2 T^{-2} + \alpha T^{-4} \ln(M_p^2 t^2) + \beta T^{-4}, \tag{15}$$

where T is defined by

$$T = \int_0^a \frac{da}{aH}. \tag{16}$$

Using (10) and (15), we can write

$$\Omega_D = \frac{n^2}{H^2 T^2} + \frac{\alpha}{3M_p^2 H^2 T^4} \ln(M_p^2 T^2) + \frac{\beta}{3M_p^2 H^2 T^4}. \tag{17}$$

The energy conservation equation is

$$\dot{\rho} + 3H(\rho + p) = 0, \quad \rho = \rho_m + \rho_D, \quad p = p_D. \tag{18}$$

Assuming an interaction of dark energy with dark matter

$$\dot{\rho}_m + 3H\rho_m = Q, \tag{19}$$

$$\dot{\rho}_D + 3H\rho_D(1 + w_D) = -Q, \tag{20}$$

where $Q = 3b^2 H\rho$ is an energy exchange term and b^2 is a coupling parameter [86, 87]. Dark energy interacting with dark matter is a promising model to solve the cosmic coincidence problem. In [88], the authors studied the signature of such interaction on large scale cosmic microwave background (CMB) temperature anisotropies. Based on the detail analysis in perturbation equations of dark energy and dark matter when they are in interaction, they found that the large scale CMB, especially the late Integrated Sachs Wolfe effect, is a useful tool to measure the coupling between dark sectors. It was deduced that in the 1σ range, the constrained coupling between dark sectors can solve the coincidence problem. In [89], a general formalism to study the growth of dark matter perturbations when dark energy perturbations and interactions between dark sectors were presented. They showed that the dynamical stability on the growth of structure depends on the form of coupling between dark sectors. Moreover due to the influence of the interaction, the growth index can differ from the value without interaction by an amount up to the observational sensibility, which provides an opportunity to probe the interaction between dark sectors through future observations on the growth of structure.

Differentiating (15) with respect to cosmic time t , we obtain

$$\dot{\rho}_D = -2H \left(\frac{3n^2 M_p^2 T^{-2} + 2\alpha T^{-4} \ln(M_p^2 T^2) + (2\beta - \alpha) T^{-4}}{\sqrt{3n^2 M_p^2 + \alpha T^{-2} \ln(M_p^2 T^2) + \beta T^{-2}}} \right) \sqrt{3M_p^2 \Omega_D}. \tag{21}$$

Using (21) in (20), we get

$$w_D = -1 + \frac{2}{3} \left(\frac{3n^2 M_p^2 T^{-2} + 2\alpha T^{-4} \ln(M_p^2 T^2) + (2\beta - \alpha) T^{-4}}{(3n^2 M_p^2 + \alpha T^{-2} \ln(M_p^2 T^2) + \beta T^{-2})^{3/2}} \right) \sqrt{3M_p^2 \Omega_D} - \frac{b^2(1 + \Omega_k)}{\Omega_D}. \tag{22}$$

Differentiating (17) with respect to cosmic time t and using $\dot{\Omega}_D = \Omega'_D H$, we obtain

$$\Omega'_D = -2 \frac{\dot{H}}{H^4} \left(\frac{n^2}{T^2} + \alpha \frac{\ln(M_p^2 T^2)}{3M_p^2 T^4} + \beta \frac{1}{3M_p^2 T^4} \right) - \frac{2}{(HT)^3} \left(n^2 - \alpha \frac{1}{3M_p^2 T^2} + \alpha \frac{2 \ln(M_p^2 T^2)}{3M_p^2 T^2} + \beta \frac{2}{3M_p^2 T^2} \right), \tag{23}$$

where the prime denotes differentiation w.r.t. $\ln a$, the e-folding time parameter. Differentiation (9) w.r.t. t and using (15), (21) and (22), we get

$$\frac{\dot{H}}{H^2} = \frac{-1}{3M_p^2 H^2} \left(\frac{3n^2 M_p^2 T^{-2} + 2\alpha T^{-4} \ln(M_p^2 T^2) + (2\beta - \alpha) T^{-4}}{\sqrt{3n^2 M_p^2 + \alpha T^{-2} \ln(M_p^2 T^2) + \beta T^{-2}}} \right) \sqrt{3M_p^2 \Omega_D} - \frac{\Omega_k}{2} - \frac{3}{2}(1 - \Omega_D) + \frac{3}{2}b^2(1 + \Omega_k). \tag{24}$$

Using (17) and (24) in (23), we obtain

$$\begin{aligned} \Omega'_D &= \frac{-2}{H^2} \left(\frac{n^2}{T^2} + \alpha \frac{\ln(M_p^2 T^2)}{3M_p^2 T^4} + \beta \frac{1}{3M_p^2 T^4} \right) \\ &\times \left[\frac{-1}{3M_p^2 H^2} \left(\frac{3n^2 M_p^2 T^{-2} + 2\alpha T^{-4} \ln(M_p^2 T^2) + (2\beta - \alpha) T^{-4}}{\sqrt{3n^2 M_p^2 + \alpha T^{-2} \ln(M_p^2 T^2) + \beta T^{-2}}} \right) \right. \\ &\times \left. \sqrt{3M_p^2 \Omega_D} - \frac{\Omega_k}{2} - \frac{3}{2}(1 - \Omega_D) + \frac{3}{2}b^2(1 + \Omega_k) \right] \\ &- 2 \left(\frac{3M_p^2 \Omega_D}{3n^2 M_p^2 + \alpha T^{-2} \ln(M_p^2 T^2) + \beta T^{-2}} \right)^{3/2} \\ &\times \left(n^2 - \alpha \frac{1}{3M_p^2 T^2} + \alpha \frac{2 \ln(M_p^2 T^2)}{3M_p^2 T^2} + \beta \frac{2}{3M_p^2 T^2} \right). \end{aligned} \tag{25}$$

To develop correspondence between original ECADE and tachyon field, we proceed

$$\begin{aligned} V(\phi) &= \rho_\phi \sqrt{1 - \dot{\phi}^2} \\ &= [3n^2 M_p^2 T^{-2} + \alpha T^{-4} \ln(M_p^2 T^2) + \beta T^{-4}] \\ &\times \sqrt{1 - \frac{2}{3} \left(\frac{3n^2 M_p^2 T^{-2} + 2\alpha T^{-4} \ln(M_p^2 T^2) + (2\beta - \alpha) T^{-4}}{(3n^2 M_p^2 + \alpha T^{-2} \ln(M_p^2 T^2) + \beta T^{-2})^{3/2}} \right) \sqrt{3M_p^2 \Omega_D} + \frac{b^2(1 + \Omega_k)}{\Omega_D}}. \end{aligned} \tag{26}$$

(27)

Also the kinetic energy gives

$$\begin{aligned} \dot{\phi} &= \sqrt{1 + w_D} \\ &= \sqrt{\frac{2}{3} \left(\frac{3n^2 M_p^2 T^{-2} + 2\alpha T^{-4} \ln(M_p^2 T^2) + (2\beta - \alpha) T^{-4}}{(3n^2 M_p^2 + \alpha T^{-2} \ln(M_p^2 T^2) + \beta T^{-2})^{3/2}} \right) \sqrt{3M_p^2 \Omega_D} - \frac{b^2(1 + \Omega_k)}{\Omega_D}}. \end{aligned} \tag{28}$$

Using $\dot{\phi} = \phi' H$, we can write

$$\phi' = \frac{1}{H} \sqrt{\frac{2}{3} \left(\frac{3n^2 M_p^2 T^{-2} + 2\alpha T^{-4} \ln(M_p^2 T^2) + (2\beta - \alpha) T^{-4}}{(3n^2 M_p^2 + \alpha T^{-2} \ln(M_p^2 T^2) + \beta T^{-2})^{3/2}} \right) \sqrt{3M_p^2 \Omega_D} - \frac{b^2(1 + \Omega_k)}{\Omega_D}}. \tag{29}$$

Integration yields

$$\begin{aligned} &\phi(a) - \phi(a_0) \\ &= \int_{a_0}^a \frac{1}{aH} \sqrt{\frac{2}{3} \left(\frac{3n^2 M_p^2 T^{-2} + 2\alpha T^{-4} \ln(M_p^2 T^2) + (2\beta - \alpha) T^{-4}}{(3n^2 M_p^2 + \alpha T^{-2} \ln(M_p^2 T^2) + \beta T^{-2})^{3/2}} \right) \sqrt{3M_p^2 \Omega_D} - \frac{b^2(1 + \Omega_k)}{\Omega_D}} da. \end{aligned} \tag{30}$$

Alternatively we can write (30) as

$$\begin{aligned} &\phi(t) - \phi(t_0) \\ &= \int_{t_0}^t \sqrt{\frac{2}{3} \left(\frac{3n^2 M_p^2 T^{-2} + 2\alpha T^{-4} \ln(M_p^2 T^2) + (2\beta - \alpha) T^{-4}}{(3n^2 M_p^2 + \alpha T^{-2} \ln(M_p^2 T^2) + \beta T^{-2})^{3/2}} \right) \sqrt{3M_p^2 \Omega_D} - \frac{b^2(1 + \Omega_k)}{\Omega_D}} dt'. \end{aligned} \tag{31}$$

Here a_0 is the value of the scale factor at the present time t_0 . Therefore, we have established an interacting entropy-corrected agegraphic tachyon dark energy model and reconstructed the potential and the dynamics of the tachyon field. It is interesting to note that in the limiting case $\alpha = 0 = \beta$, all the above expressions reduce to those presented in [1].

3 Tachyon Reconstruction of New ECADE

The energy density of the new ECADE is given by

$$\rho_D = 3n^2 M_p^2 \eta^{-2} + \alpha \eta^{-4} \ln(M_p^2 \eta^2) + \beta \eta^{-4}. \tag{32}$$

Here η is the conformal time, which is defined by

$$\eta = \int_0^a \frac{da}{a^2 H}. \tag{33}$$

Using (10) and (32), we can write

$$\Omega_D = \frac{n^2}{H^2 \eta^2} + \frac{\alpha}{3M_p^2 H^2 \eta^4} \ln(M_p^2 \eta^2) + \frac{\beta}{3M_p^2 H^2 \eta^4}. \tag{34}$$

Differentiating (32) with respect to cosmic time t , we obtain

$$\dot{\rho}_D = -\frac{2H}{a} \left(\frac{3n^2 M_p^2 \eta^{-2} + 2\alpha \eta^{-4} \ln(M_p^2 \eta^2) + (2\beta - \alpha) \eta^{-4}}{\sqrt{3n^2 M_p^2 + \alpha \eta^{-2} \ln(M_p^2 \eta^2) + \beta \eta^{-2}}} \right) \sqrt{3M_p^2 \Omega_D}. \tag{35}$$

Using (35) in (20), we get

$$\begin{aligned} w_D = -1 + &\frac{2}{3a} \left(\frac{3n^2 M_p^2 \eta^{-2} + 2\alpha \eta^{-4} \ln(M_p^2 \eta^2) + (2\beta - \alpha) \eta^{-4}}{(3n^2 M_p^2 + \alpha \eta^{-2} \ln(M_p^2 \eta^2) + \beta \eta^{-2})^{3/2}} \right) \sqrt{3M_p^2 \Omega_D} \\ &- \frac{b^2(1 + \Omega_k)}{\Omega_D}. \end{aligned} \tag{36}$$

Differentiating (34) with respect to cosmic time t and using $\dot{\Omega}_D = \Omega'_D H$, we obtain

$$\begin{aligned} \Omega'_D = & -2 \frac{\dot{H}}{H^4} \left(\frac{n^2}{\eta^2} + \alpha \frac{\ln(M_p^2 \eta^2)}{3M_p^2 \eta^4} + \beta \frac{1}{3M_p^2 \eta^4} \right) \\ & - \frac{2}{a(H\eta)^3} \left(n^2 - \alpha \frac{1}{3M_p^2 \eta^2} + \alpha \frac{2 \ln(M_p^2 \eta^2)}{3M_p^2 \eta^2} + \beta \frac{2}{3M_p^2 \eta^2} \right). \end{aligned} \tag{37}$$

Differentiation (5) w.r.t. t and using (16), (32) and (33), we get

$$\begin{aligned} \frac{\dot{H}}{H^2} = & \frac{-1}{3aM_p^2 H^2} \left(\frac{3n^2 M_p^2 \eta^{-2} + 2\alpha \eta^{-4} \ln(M_p^2 \eta^2) + (2\beta - \alpha) \eta^{-4}}{\sqrt{3n^2 M_p^2 + \alpha \eta^{-2} \ln(M_p^2 \eta^2) + \beta \eta^{-2}}} \right) \sqrt{3M_p^2 \Omega_D} \\ & - \frac{\Omega_k}{2} - \frac{3}{2}(1 - \Omega_D) + \frac{3}{2}b^2(1 + \Omega_k). \end{aligned} \tag{38}$$

Using (35) and (38) in (37), we obtain

$$\begin{aligned} \Omega'_D = & \frac{-2}{H^2} \left(\frac{n^2}{\eta^2} + \alpha \frac{\ln(M_p^2 \eta^2)}{3M_p^2 \eta^4} + \beta \frac{1}{3M_p^2 \eta^4} \right) \\ & \times \left[\frac{-1}{3aM_p^2 H^2} \left(\frac{3n^2 M_p^2 \eta^{-2} + 2\alpha \eta^{-4} \ln(M_p^2 \eta^2) + (2\beta - \alpha) \eta^{-4}}{\sqrt{3n^2 M_p^2 + \alpha \eta^{-2} \ln(M_p^2 \eta^2) + \beta \eta^{-2}}} \right) \right. \\ & \times \left. \sqrt{3M_p^2 \Omega_D} - \frac{\Omega_k}{2} - \frac{3}{2}(1 - \Omega_D) + \frac{3}{2}b^2(1 + \Omega_k) \right] \\ & - \frac{2}{a} \left(\frac{3M_p^2 \Omega_D}{3n^2 M_p^2 + \alpha \eta^{-2} \ln(M_p^2 \eta^2) + \beta \eta^{-2}} \right)^{3/2} \\ & \times \left(n^2 - \alpha \frac{1}{3M_p^2 \eta^2} + \alpha \frac{2 \ln(M_p^2 \eta^2)}{3M_p^2 \eta^2} + \beta \frac{2}{3M_p^2 \eta^2} \right). \end{aligned} \tag{39}$$

To develop correspondence between original ECADE and tachyon field, we proceed

$$\begin{aligned} V(\phi) = & [3n^2 M_p^2 \eta^{-2} + \alpha \eta^{-4} \ln(M_p^2 \eta^2) + \beta \eta^{-4}] \\ & \times \sqrt{1 - \frac{2}{3a} \left(\frac{3n^2 M_p^2 \eta^{-2} + 2\alpha \eta^{-4} \ln(M_p^2 \eta^2) + (2\beta - \alpha) \eta^{-4}}{(3n^2 M_p^2 + \alpha \eta^{-2} \ln(M_p^2 \eta^2) + \beta \eta^{-2})^{3/2}} \right) \sqrt{3M_p^2 \Omega_D} + \frac{b^2(1 + \Omega_k)}{\Omega_D}}. \end{aligned} \tag{40}$$

Also the kinetic energy gives

$$\dot{\phi} = \sqrt{\frac{2}{3a} \left(\frac{3n^2 M_p^2 \eta^{-2} + 2\alpha \eta^{-4} \ln(M_p^2 \eta^2) + (2\beta - \alpha) \eta^{-4}}{(3n^2 M_p^2 + \alpha \eta^{-2} \ln(M_p^2 \eta^2) + \beta \eta^{-2})^{3/2}} \right) \sqrt{3M_p^2 \Omega_D} - \frac{b^2(1 + \Omega_k)}{\Omega_D}}. \tag{41}$$

Using $\dot{\phi} = \phi' H$, we can write

$$\phi' = \frac{1}{H} \sqrt{\frac{2}{3a} \left(\frac{3n^2 M_p^2 \eta^{-2} + 2\alpha \eta^{-4} \ln(M_p^2 \eta^2) + (2\beta - \alpha) \eta^{-4}}{(3n^2 M_p^2 + \alpha \eta^{-2} \ln(M_p^2 \eta^2) + \beta \eta^{-2})^{3/2}} \right) \sqrt{3M_p^2 \Omega_D} - \frac{b^2(1 + \Omega_k)}{\Omega_D}}. \tag{42}$$

Integration yields

$$\begin{aligned} &\phi(a) - \phi(a_0) \\ &= \int_{a_0}^a \frac{1}{aH} \sqrt{\frac{2}{3a} \left(\frac{3n^2 M_p^2 \eta^{-2} + 2\alpha \eta^{-4} \ln(M_p^2 \eta^2) + (2\beta - \alpha) \eta^{-4}}{(3n^2 M_p^2 + \alpha \eta^{-2} \ln(M_p^2 \eta^2) + \beta \eta^{-2})^{3/2}} \right) \sqrt{3M_p^2 \Omega_D} - \frac{b^2(1 + \Omega_k)}{\Omega_D}} da. \end{aligned} \tag{43}$$

Alternatively we can write (43) as

$$\begin{aligned} &\phi(t) - \phi(t_0) \\ &= \int_{t_0}^t \sqrt{\frac{2}{3a} \left(\frac{3n^2 M_p^2 \eta^{-2} + 2\alpha \eta^{-4} \ln(M_p^2 \eta^2) + (2\beta - \alpha) \eta^{-4}}{(3n^2 M_p^2 + \alpha \eta^{-2} \ln(M_p^2 \eta^2) + \beta \eta^{-2})^{3/2}} \right) \sqrt{3M_p^2 \Omega_D} - \frac{b^2(1 + \Omega_k)}{\Omega_D}} dt'. \end{aligned} \tag{44}$$

Again, one can easily check that in the special case $\alpha = 0 = \beta$, all the above expressions reduce to those discussed in [1].

4 Cosmological Implications of ECNADE Models

As cosmological implications of the present model, we calculate the effective equation of state parameter of interacting ECADe and ECNADE. The effective equation of state parameter is defined by [86, 87]

$$w_D^{\text{eff}} = w_D + \frac{Q}{3H\rho_D} = w_D + b^2 \left(\frac{1 + \Omega_k}{\Omega_D} \right). \tag{45}$$

Making use of (22) and (36) in (45), we obtain

$$w_{\text{old}}^{\text{eff}} = -1 + \frac{2}{3} \left(\frac{3n^2 M_p^2 T^{-2} + 2\alpha T^{-4} \ln(M_p^2 T^2) + (2\beta - \alpha) T^{-4}}{(3n^2 M_p^2 + \alpha T^{-2} \ln(M_p^2 T^2) + \beta T^{-2})^{3/2}} \right) \sqrt{3M_p^2 \Omega_D}, \tag{46}$$

$$w_{\text{new}}^{\text{eff}} = -1 + \frac{2}{3a} \left(\frac{3n^2 M_p^2 \eta^{-2} + 2\alpha \eta^{-4} \ln(M_p^2 \eta^2) + (2\beta - \alpha) \eta^{-4}}{(3n^2 M_p^2 + \alpha \eta^{-2} \ln(M_p^2 \eta^2) + \beta \eta^{-2})^{3/2}} \right) \sqrt{3M_p^2 \Omega_D}. \tag{47}$$

For the sake of completeness we also calculate the deceleration parameter

$$q = -\frac{\ddot{a}}{aH^2} = -1 - \frac{\dot{H}}{H^2}. \tag{48}$$

Substituting (25) and (38) in (48) yields the expressions of deceleration parameter for original and new ECADE, respectively:

$$q_{\text{old}} = -1 + \frac{1}{3M_p^2 H^2} \left(\frac{3n^2 M_p^2 T^{-2} + 2\alpha T^{-4} \ln(M_p^2 T^2) + (2\beta - \alpha) T^{-4}}{\sqrt{3n^2 M_p^2 + \alpha T^{-2} \ln(M_p^2 T^2) + \beta T^{-2}}} \right) \sqrt{3M_p^2 \Omega_D} \\ + \frac{\Omega_k}{2} + \frac{3}{2}(1 - \Omega_D) - \frac{3}{2}b^2(1 + \Omega_k), \quad (49)$$

$$q_{\text{new}} = -1 + \frac{1}{3aM_p^2 H^2} \left(\frac{3n^2 M_p^2 \eta^{-2} + 2\alpha \eta^{-4} \ln(M_p^2 \eta^2) + (2\beta - \alpha) \eta^{-4}}{\sqrt{3n^2 M_p^2 + \alpha \eta^{-2} \ln(M_p^2 \eta^2) + \beta \eta^{-2}}} \right) \sqrt{3M_p^2 \Omega_D} \\ + \frac{\Omega_k}{2} + \frac{3}{2}(1 - \Omega_D) - \frac{3}{2}b^2(1 + \Omega_k). \quad (50)$$

5 Summary and Discussion

In summary, among the various candidates to play the role of the dark energy, tachyon has emerged as a possible source of dark energy for a particular class of potentials [94–97]. In this paper, we have associated the interacting ECADE models with a tachyon field which describe the tachyon cosmology in a non-flat universe. The addition of correction terms to the energy density of ADE models is a fundamental prediction of LQG and hence must be taken into account while studying the dynamics of dark energy in the universe. Using this modified energy density, we have demonstrated that the universe can be described completely by a tachyon scalar field in a certain way. We have adopted the viewpoint that the scalar field models of dark energy are effective theories of an underlying theory of dark energy. Thus, we should be capable of using the scalar field model to mimic the evolving behavior of the interacting ECADE and reconstructing this scalar field model. We have reconstructed the potential and the dynamics of the tachyon scalar field according to the evolutionary behavior of the interacting ECADE models. We have also studied the cosmological implications of ECADE scenarios. We would remark that numerical values of the above effective EoSs and deceleration parameters cannot be predicted from above due to ignorance of T and η .

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References

1. Sheykhi, A.: Phys. Lett. B **682**, 329 (2010)
2. Spergel, D.N., et al.: Astrophys. J. Suppl. **148**, 175 (2003)
3. Spergel, D.N., et al.: Astrophys. J. Suppl. **170**, 377 (2007)
4. Riess, A.G., et al.: Astron. J. **116**, 1009 (1998)
5. Perlmutter, S., et al.: Astrophys. J. **517**, 565 (1999)
6. Riess, A.G., et al.: Astrophys. J. **607**, 665 (2004)
7. Cohen, A., Kaplan, D., Nelson, A.: Phys. Rev. Lett. **82**, 4971 (1999)
8. Li, M.: Phys. Lett. B **603**, 1 (2004)
9. Huang, Q.G., Li, M.: J. Cosmol. Astropart. Phys. **08**, 013 (2004)
10. Hsu, S.D.H.: Phys. Lett. B **594**, 13 (2004)
11. Jamil, M., Farooq, M.U., Rashid, M.A.: Eur. Phys. J. C **61**, 471 (2009)
12. Jamil, M., Farooq, M.U.: Int. J. Theor. Phys. **49**, 42 (2010)

13. Jamil, M., Saridakis, E.N., Setare, M.R.: *Phys. Lett. B* **679**, 172 (2009)
14. Setare, M.R., Jamil, M.: *J. Cosmol. Astropart. Phys.* **02**, 010 (2010)
15. Jamil, M., Saridakis, E.N.: *J. Cosmol. Astropart. Phys.* **07**, 028 (2010)
16. Karami, K., Sorouri, A.: *Phys. Scr.* **82**, 025901 (2010)
17. Karami, K.: *J. Cosmol. Astropart. Phys.* **1001**, 015 (2010)
18. Karami, K., Khaledian, M.S., Felegary, F., Azarmi, Z.: *Phys. Lett. B* **686**, 216 (2010)
19. Karami, K., Fehri, J.: *Phys. Lett. B* **684**, 61 (2010)
20. Elizalde, E., Nojiri, S., Odintsov, S.D., Wang, P.: *Phys. Rev. D* **71**, 103504 (2005)
21. Guberina, B., Horvat, R., Nikolic, H.: *Phys. Lett. B* **636**, 80 (2006)
22. Li, H., Guo, Z.K., Zhang, Y.Z.: *Int. J. Mod. Phys. D* **15**, 869 (2006)
23. Huang, Q.G., Gong, Y.: *J. Cosmol. Astropart. Phys.* **0408**, 006 (2004)
24. Almeida, J.P.B., Pereira, J.G.: *Phys. Lett. B* **636**, 75 (2006)
25. Gong, Y.: *Phys. Rev. D* **70**, 064029 (2004)
26. Wang, B., Abdalla, E., Su, R.K.: *Phys. Lett. B* **611**, 21 (2005)
27. Shen, J.Y., Wang, B., Abdalla, E., Su, R.K.: *Phys. Lett. B* **609**, 200 (2005)
28. Setare, M.R.: *Phys. Lett. B* **642**, 1 (2006)
29. Setare, M.R., Vagenas, E.C.: *Phys. Lett. B* **666**, 111 (2008)
30. Sadjadi, H.M.: [arXiv:0902.2462](https://arxiv.org/abs/0902.2462)
31. Setare, M.R., Saridakis, E.N.: *Phys. Lett. B* **671**, 331 (2009)
32. Setare, M.R.: *Eur. Phys. J. C* **50**, 991 (2007)
33. Setare, M.R.: *Phys. Lett. B* **642**, 421 (2006)
34. Sheykhi, A.: *Phys. Lett. B* **681**, 205 (2009)
35. Sheykhi, A.: *Class. Quantum Gravity* **27**, 025007 (2010)
36. Susskind, L.: *J. Math. Phys.* **36**, 6377 (1995)
37. Wang, B., Gong, Y., Abdalla, E.: *Phys. Lett. B* **624**, 141 (2005)
38. Cai, R.G.: *Phys. Lett. B* **657**, 228 (2007)
39. Wei, H., Cai, R.G.: *Phys. Lett. B* **660**, 113 (2008)
40. Wei, H., Cai, R.G.: *Eur. Phys. J. C* **59**, 99 (2009)
41. Karolyhazy, F.: *Nuovo Cimento A* **42**, 390 (1966)
42. Karolyhazy, F., Frenkel, A., Lukacs, B.: In: Shimony, A., Feschbach, H. (eds.) *Physics as Natural Philosophy*. MIT Press, Cambridge (1982)
43. Karolyhazy, F., Frenkel, A., Lukacs, B.: In: Penrose, R., Isham, C.J. (eds.) *Quantum Concepts in Space and Time*. Clarendon Press, Oxford (1986)
44. Maziashvili, M.: *Int. J. Mod. Phys. D* **16**, 1531 (2007)
45. Maziashvili, M.: *Phys. Lett. B* **652**, 165 (2007)
46. Wei, H., Cai, R.G.: *Phys. Lett. B* **663**, 1 (2008)
47. Kim, Y.W., et al.: *Mod. Phys. Lett. A* **23**, 3049 (2008)
48. Wu, J.P., Ma, D.Z., Ling, Y.: *Phys. Lett. B* **663**, 152 (2008)
49. Kim, K.Y., Lee, H.W., Myung, Y.S.: *Phys. Lett. B* **660**, 118 (2008)
50. Zhang, J., Zhang, X., Liu, H.: *Eur. Phys. J. C* **54**, 303 (2008)
51. Neupane, I.P.: *Phys. Lett. B* **673**, 111 (2009)
52. Sheykhi, A.: *Phys. Lett. B* **680**, 113 (2009)
53. Sheykhi, A.: *Int. J. Mod. Phys. D* **18**(13), 2023 (2009)
54. Sheykhi, A.: *Phys. Rev. D* **81**, 023525 (2010)
55. Sheykhi, A., et al.: *J. Cosmol. Astropart. Phys.* **09**, 017 (2010)
56. Sheykhi, A.: *Int. J. Mod. Phys. D* **19**(3), 305 (2010)
57. Setare, M.R.: [arXiv:0907.4910](https://arxiv.org/abs/0907.4910)
58. Setare, M.R.: [arXiv:0908.0196](https://arxiv.org/abs/0908.0196)
59. Wei, H., Cai, R.G.: *Phys. Lett. B* **663**, 1 (2008)
60. Zhu, T., Ren, J.-R.: *Eur. Phys. J. C* **62**, 413 (2009)
61. Cai, R.-G., et al.: *Class. Quantum Gravity* **26**, 155018 (2009)
62. Jamil, M., Farooq, M.U.: *J. Cosmol. Astropart. Phys.* **03**, 001 (2010)
63. Banerjee, R., Majhi, B.R.: *Phys. Lett. B* **662**, 62 (2008)
64. Banerjee, R., Majhi, B.R.: *J. High Energy Phys.* **0806**, 095 (2008)
65. Majhi, B.R.: *Phys. Rev. D* **79**, 044005 (2009)
66. Banerjee, R., Modak, S.K.: *J. High Energy Phys.* **0905**, 063 (2009)
67. Modak, S.K.: *Phys. Lett. B* **671**, 167 (2009)
68. Rovelli, C.: *Phys. Rev. Lett.* **77**, 3288 (1996)
69. Ashtekar, A., Baez, J., Corichi, A., Krasnov, K.: *Phys. Rev. Lett.* **80**, 904 (1998)
70. Ghosh, A., Mitra, P.: *Phys. Rev. D* **71**, 027502 (2005)
71. Meissner, K.A.: *Class. Quantum Gravity* **21**, 5245 (2004)

72. Medved, A.J.M., Vagenas, E.C.: Phys. Rev. D **70**, 124021 (2004)
73. Wei, H.: Commun. Theor. Phys. **52**, 743 (2009)
74. Myung, Y.S., Seo, M.G.: Phys. Lett. B **671**, 435 (2009)
75. Sen, A.: J. High Energy Phys. **0204**, 048 (2002)
76. Sen, A.: Mod. Phys. Lett. A **17**, 1797 (2002)
77. Sen, A.: J. High Energy Phys. **9910**, 008 (1999)
78. Bergshoeff, E.A., de Roo, M., de Wit, T.C., Eyras, E., Panda, S.: J. High Energy Phys. **0005**, 009 (2000)
79. Kluson, J.: Phys. Rev. D **62**, 126003 (2000)
80. Kutasov, D., Niarchos, V.: Nucl. Phys. B **666**, 56 (2003)
81. Sen, A.: J. High Energy Phys. **0207**, 065 (2002)
82. Spergel, D.N.: Astrophys. J. Suppl. **148**, 175 (2003)
83. Bennett, C.L., et al.: Astrophys. J. Suppl. **148**, 1 (2003)
84. Seljak, U., Slosar, A., McDonald, P.: J. Cosmol. Astropart. Phys. **0610**, 014 (2006)
85. Spergel, D.N., et al.: Astrophys. J. Suppl. **170**, 377 (2007)
86. Jamil, M., Rashid, M.A.: Eur. Phys. J. C **58**, 111 (2008)
87. Jamil, M.: Int. J. Theor. Phys. **49**, 62 (2010)
88. He, J.-H., Wang, B., Zhang, P.: Phys. Rev. D **80**, 063530 (2009)
89. He, J.-H., Wang, B., Jing, Y.P.: J. Cosmol. Astropart. Phys. **0907**, 030 (2009)
90. Gibbons, G.W.: Phys. Lett. B **537**, 1 (2002)
91. Mazumdar, A., Panda, S., Perez-Lorezana, A.: Nucl. Phys. B **614**, 101 (2001)
92. Feinstein, A.: Phys. Rev. D **66**, 063511 (2002)
93. Piao, Y.S., Cai, R.G., Zhang, X.M., Zhang, Y.Z.: Phys. Rev. D **66**, 121301 (2002)
94. Padmanabhan, T.: Phys. Rev. D **66**, 021301 (2002)
95. Bagla, J.S., Jassal, H.K., Padmanabhan, T.: Phys. Rev. D **67**, 063504 (2003)
96. Guo, Z.K., Zhang, Y.Z.: J. Cosmol. Astropart. Phys. **0408**, 010 (2004)
97. Copeland, E.J., Garousi, M.R., Sami, M., Tsujikawa, S.: Phys. Rev. D **71**, 043003 (2005)
98. Srivastava, S.K.: [arXiv:gr-qc/0409074](https://arxiv.org/abs/gr-qc/0409074)
99. Setare, M.R., Sadeghi, J., Amani, A.R.: Phys. Lett. B **673**, 241 (2009)
100. Setare, M.R.: Phys. Lett. B **653**, 116 (2007)
101. Cui, J., Zhang, L., Zhang, J., Zhang, X.: Chin. Phys. B **19**, 019802 (2010)