

Bianchi Type-III Magnetized Wet Dark Fluid Cosmological Model in General Relativity

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Abstract Bianchi type-III cosmological model of universe filled with dark energy from a wet dark fluid (WDF) in presence and absence of magnetic field is investigated in general theory of relativity. We assume that F_{12} is the only non-vanishing component of F_{ij} . We obtain exact solutions to the field equations using the condition that expansion is proportional to the shear scalar i.e. ($B = C^n$). The physical behavior of the model is discussed with and without magnetic field. We conclude that universe model do not approach isotropy through the evolution of the universe.

Keywords Bianchi type-III space-time · Magnetic field · Wet dark fluid · Dark energy

1 Introduction

Type Ia supernovae observational data suggest that the universe is dominated by two dark components containing dark matter (DM) and dark energy (DE). Dark matter, a matter without pressure, is mainly used to explain galactic curves and large-scale structure formation, while dark energy, an exotic energy with negative pressure, is used to explain the present cosmic accelerating expansion. To understand the origin of dark matter & dark energy and its nature is one of the greatest astronomical/cosmological problems of the 21st century. The nature of both components remains unknown, and in the near future we can hope that the Large Hadron Collider (LHC) will be able to provide hints on the nature of DM & DE.

The understanding of cosmology has been revolutionized by recently observed astronomical phenomena. Consequences of combined analysis of Ia Supernovae (SNeIa) observations [20, 53–55, 57, 59, 60, 65, 67, 78], galaxy cluster measurements [5] and cosmic microwave background (CMB) data [77] have shown that the expansion of our present universe is accelerating rather than slowing down. This late time cosmic acceleration cannot be explained by the four known fundamental interactions in the standard models. Within the framework of Einstein's general relativity, an exotic component with negative pressure called as dark

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energy is invoked to explain this observed phenomena. The simplest dark energy candidate is the vacuum energy which is mathematically equivalent to cosmological constant Λ which has the equation of state $\omega = -1$. As per Copeland et al. [19] “fine-tuning” and the “cosmic coincidence” are the two well known difficulties of the cosmological constant problems. There are different alternative theories for the dynamical dark energy scenario which have been proposed by scientists to interpret the accelerating universe. The dynamical scalar field models of dark energy includes quintessence [58, 86], k-essence based on earlier work of k-inflation [3, 16], phantom (ghost) field [13, 51] and quintom field [2, 22], tachyon field [1, 52, 68], dilatonic ghost condensate [25, 56]. The interacting dark energy models includes Chaplygin gas [9, 35], Holographic dark energy models [17, 24, 30, 42, 43, 80, 88, 89] and Braneworld models [21, 66]. Freeman and Turner [23], Wang and Tegmark [83] established firmly that universe is actually undergoing an acceleration, with repulsive gravity of some strange energy-form i.e. dark energy at work. Dark energy, a “mysterious substance” pressure of which is “negative” and accounts for nearly 70% of total matter-energy budget of universe, but has no clear explanation. Setare [69–72]. Setare and Saridakis [73, 74] have developed the idea of holographic dark energy. Recently the original agegraphic dark energy (OADE) and new agegraphic dark energy (NADE) models were proposed by Cai [12] and Wei and Cai [84, 85]. Karami et al. [37] introduced a polytropic gas model of DE as an alternative model to explain the accelerated expansion of the universe. Gupta and Pradhan [27] proposed new candidate known as cosmological nuclear-energy as a possible suspect (candidate) for the dark energy. Mukhopadhyay [50] studied higher dimensional dark energy investigation with variable Λ and G . However, so far, the nature of dark energy is still unclear.

Bianchi type cosmological models are important in the sense that these are homogeneous and anisotropic, from which the process of isotropization of the universe is studied through the passage of time. Moreover, from the theoretical point of view anisotropic universe have a greater generality than isotropic models. The simplicity of the field equations made Bianchi space time useful in constructing models of spatially homogeneous and anisotropic cosmologies. Bianchi type-III models in the presence of dark energy have been studied in general relativity in the last thirty years. Moussiaux et al. [49] has given an exact particular solution of the Einstein field equations for vacuum with a cosmological constant. Lorentz [44] has presented a model with dust and a cosmological constant. Chakraborty and Chakraborty [15]; Singh et al. [76]; Tiwari [82]; Bali and Tinkar [7] have studied various cosmological models with variable Λ and G .

The magnetic field has important role at the cosmological scale and is present in galactic and intergalactic spaces. The importance of the magnetic field for various astrophysical phenomenon has been studied by many researchers. Melvin [46] has pointed out that during the evolution of the universe, the matter is in a highly ionized state and is smoothly coupled with the field, subsequently forming neutral matter as a result of universe expansion. Thorne [81], Jacobs [32, 33], Collins [18], Roy and Prakash [63] have investigated magnetized cosmological models for perfect fluid distribution in general relativity. Misra and Radhakrishna [48] have obtained the analogous relation between some components of metric and electromagnetic potentials for source free Einstein-Maxwell field described by Einstein-Rosen metric. Milaneschi and Fabbri [47] studied the anisotropy and polarization properties of CMB radiation in homogeneous Bianchi type-I cosmological model. Roy et al. [64] investigated Bianchi type-I cosmological models containing perfect fluid and magnetic field directed along x -axis. RajBali and Singh [6] also discussed some cylindrically symmetric inhomogeneous cosmological models for perfect fluid distribution with electromagnetic field. Madsen [45] have shown that primordial magnetic fields can have a significant impact

on the CMB anisotropy depending on the direction of the field lines. Also Iwazaki [31] and Cea and Tedesco [14] gives the interesting phenomena as the magnetization of domain walls and the dynamical generation of massive ferromagnetic domain walls. The models of dark energy based on vector fields are studied by Kiselev [39]; Boehmer and Harko [10]; Koivisto and Mota [40]; Bamba et al. [8]. Jimenez and Maroto [34] explored the possibility of understanding dark energy from the standard electromagnetic field. King and Coles [38] used the magnetized perfect fluid energy-momentum tensor to discuss the effects of magnetic field on the evolution of the universe.

Zeldovich and Novikov [87] explained that the early stage of expansion of universe exhibits substantially non-Friedmannian behavior. Holman and Naidu [29] studied the homogeneous, isotropic FRW case by using the wet dark fluid (WDF) as dark energy (DE). Recently, Singh and Chaubey [75] have studied Bianchi type I universe with wet dark fluid. This motivates the authors to study Bianchi type-III wet dark fluid cosmological model in the presence and absence of a magnetic field in general relativity.

2 Wet Dark Fluid

Holman and Naidu [29] studied the homogeneous, isotropic FRW case by using the wet dark fluid (WDF) as dark energy (DE). This model was in the spirit of the Generalized Chaplygin Gas (GCG) [26], where a physically motivated equation of state was offered with properties relevant for the dark energy problem. This was stemmed from an empirical equation of state proposed by Tait [79] and Hayward [28] to treat water and aqueous solution.

The equation of state for WDF is given in simple form as

$$p_{WDF} = \gamma (\rho_{WDF} - \rho^*) \tag{2.1}$$

It is motivated by the fact that it is a good approximation for many fluids, including water, in which initial attraction of the molecules makes negative pressure possible.

The parameters γ and ρ^* are taken to be positive and we restrict ourselves to $0 \leq \gamma \leq 1$. Note that if c_s denotes the adiabatic sound speed in WDF, then $\gamma = c_s^2$ (refer, Babichev et al. [4]).

To find the WDF energy density, we use the energy conservation equation

$$\rho_{WDF}^* + 3H (\rho_{WDF} + p_{WDF}) = 0 \tag{2.2}$$

From equation of state (2.1) and using $3H = \frac{\dot{v}}{v}$ in (2.2), we get

$$\rho_{WDF} = \frac{\gamma}{1 + \gamma} \rho^* + \frac{D}{v(1 + \gamma)}, \tag{2.3}$$

where D is the constant of integration and v is the volume expansion.

WDF naturally includes two components, a piece that behaves as a cosmological constant as well as a piece that red shifts as a standard fluid with an equation of state $p = \gamma\rho$.

We can show that if we take $D > 0$, this fluid will not violate the strong energy condition $p + \rho \geq 0$. Thus, we get

$$\begin{aligned} p_{WDF} + \rho_{WDF} &= (1 + \gamma)\rho_{WDF} - \gamma\rho^* \\ &= (1 + \gamma)\frac{D}{v^{(1+\gamma)}} \geq 0 \end{aligned} \tag{2.4}$$

Holman and Naidu [29] observed that their model is consistent with the most recent SNIa data, the WMAP results as well as the constraints coming from measurements of the power spectrum. Here, they have considered both, the case where the dark fluid is smooth (i.e. only the CDM component clusters gravitationally) as well as the case where the dark fluid also clusters.

3 The Metric and the Field Equations

The spatially homogeneous and anisotropic Bianchi type-III line element is given by

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{-2ax} dy^2 - C^2 dz^2, \tag{3.1}$$

where a is non zero constant and A, B, C are functions of time ‘ t ’.

The energy momentum tensor of the source is given by

$$T_{ij} = {}^{WDF}T_{ij} + {}^{EM}T_{ij}, \tag{3.2}$$

where

$${}^{WDF}T_{ij} = (\rho_{WDF} + p_{WDF})u_i u_j - p_{WDF}, \tag{3.3}$$

where u^i is the flow vector satisfying

$$g_{ij}u^i u^j = 1 \tag{3.4}$$

In commoving system of coordinates, from (3.3) and (3.4), we find

$${}^{WDF}T_1^1 = {}^{WDF}T_2^2 = {}^{WDF}T_3^3 = -p_{WDF}, \quad {}^{WDF}T_4^4 = \rho_{WDF}. \tag{3.5}$$

${}^{EM}T_{ij}$ is the electromagnetic field tensor which is given by

$${}^{EM}T_{ij} = \frac{1}{4\pi} \left[-F_{is} F_{jp} g^{sp} + \frac{1}{4} g_{ij} F_{sp} F^{sp} \right] \tag{3.6}$$

We assume that F_{12} is the only non-vanishing component of F_{ij} .

The Maxwell’s equation is

$$\frac{\partial}{\partial x^j} (F_{ij} \sqrt{-g}) = 0$$

which leads to

$$\frac{\partial}{\partial y} (F^{12} ABC e^{-ax}) = 0 \tag{3.7}$$

$$F_{12} = H e^{-ax}, \quad \text{where } H \text{ and } a \text{ are constants.}$$

We take F_{12} as the only non-vanishing component of F_{ij} because a cosmological model which contains a global magnetic field is necessarily anisotropic since the magnetic field vector specifies a preferred spatial direction [11]. We assume that the current is flowing along z -axis, so magnetic field is in the xy -plane. Thus F_{12} is the only non-vanishing component of F_{ij} . In the same way, other components F_{13}, F_{23} of F_{ij} may be assumed non-zero but it will create more complexity. Thus, if $F_{12} \neq 0$ then $F_{13} = F_{23} = 0$ and $F_{14} = F_{24} = F_{34} = 0$

due to the assumption of infinite electrical conductivity [61]. If finite conductivity is assumed then $F_{14} \neq 0, F_{24} \neq 0, F_{34} \neq 0$. In these situations, the problem becomes more complex to solve Einstein’s field equations. The reference in the text involves such type of assumptions to avoid the complexity of the problem. Hence, to get realistic model of the universe, we take F_{12} different from zero.

The non-vanishing components of ${}^{EM}T_{ij}$ corresponding to the line element (3.1) are as follows:

$${}^{EM}T_1^1 = \frac{-H^2}{8\pi A^2 B^2} = {}^{EM}T_2^2 = -{}^{EM}T_3^3 = -{}^{EM}T_4^4 \quad \text{and} \quad {}^{EM}T_1^4 = 0 \tag{3.8}$$

The Einstein field equations are

$$R_i^j - \frac{1}{2}Rg_i^j = -8\pi T_i^j \tag{3.9}$$

The field equations (3.9) for the line element (3.1) with (3.2), to (3.8) will be

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4}{B} \frac{C_4}{C} = -8\pi p_{WDF} - \frac{H^2}{A^2 B^2} \tag{3.10}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4}{A} \frac{C_4}{C} = -8\pi p_{WDF} - \frac{H^2}{A^2 B^2} \tag{3.11}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4}{A} \frac{B_4}{B} - \frac{a^2}{A^2} = -8\pi p_{WDF} + \frac{H^2}{A^2 B^2} \tag{3.12}$$

$$\frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4}{B} \frac{C_4}{C} - \frac{a^2}{A^2} = 8\pi \rho_{WDF} + \frac{H^2}{A^2 B^2} \tag{3.13}$$

$$\frac{A_4}{A} - \frac{B_4}{B} = 0, \tag{3.14}$$

where the subscript ‘4’ after A, B and C denote ordinary differentiation with respect to t .

From (3.14), we have

$$A = \mu B, \quad \text{where } \mu \text{ is a constant of integration.} \tag{3.15}$$

Using (3.15), the set of (3.10) to (3.13) becomes

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4}{B} \frac{C_4}{C} = -8\pi p_{WDF} - \frac{H^2}{\mu^2 B^4} \tag{3.16}$$

$$\frac{2B_{44}}{B} + \left(\frac{B_4}{B}\right)^2 - \frac{a^2}{\mu^2 B^2} = -8\pi p_{WDF} + \frac{H^2}{\mu^2 B^4} \tag{3.17}$$

$$\left(\frac{B_4}{B}\right)^2 + \frac{2B_4}{B} \frac{C_4}{C} - \frac{a^2}{\mu^2 B^2} = 8\pi \rho_{WDF} + \frac{H^2}{\mu^2 B^4} \tag{3.18}$$

From (3.16) and (3.17), we have

$$\frac{B_{44}}{B} - \frac{C_{44}}{C} + \left(\frac{B_4}{B}\right)^2 - \frac{B_4 C_4}{BC} - \frac{a^2}{\mu^2 B^2} = 2 \frac{H^2}{\mu^2 B^2} \tag{3.19}$$

In order to solve the above equation, we use a physical condition that expansion scalar is proportional to shear scalar. According to Thorne [81], observations of velocity red shift

relation for extragalactic sources suggest that Hubble expansion of the universe is isotropic within about 30% range approximately (Kantowski and Sachs [36], Kristian and Sachs [41]) and red shift studies place the limit $\frac{\sigma}{H} \leq 0.30$, where σ is shear and H is Hubble constant. Collins [18] discussed the physical significance of this condition for perfect fluid and barotropic equation of state in a more general case. In many papers Roy and Prakash [63], Roy and Banerjee [62], Bali and Singh [6] have proposed this condition to find exact solutions of cosmological models.

Thus, we use the condition

$$B = C^n \tag{3.20}$$

From (3.19) and (3.20), we have

$$\frac{C_{44}}{C} + 2n \left(\frac{C_4}{C} \right)^2 = \frac{2H^2}{(n-1)\mu^2 C^{4n}} + \frac{a^2}{(n-1)\mu^2 C^{2n}},$$

which again leads to

$$C C_{44} + 2n C_4^2 = \frac{2H^2}{(n-1)\mu^2 C^{4n-2}} + \frac{a^2}{(n-1)\mu^2 C^{2n-2}} \tag{3.21}$$

Let us consider

$$\begin{aligned} C_4 &= f(C) \\ \therefore C_{44} &= f f' \quad \text{where } f' = \frac{df}{dC} \end{aligned} \tag{3.22}$$

With the help of (3.22), (3.21) reduces to

$$2 f f' + 4n \frac{f^2}{C} = \frac{4H^2}{(n-1)\mu^2 C^{4n-1}} + \frac{2a^2}{(n-1)\mu^2 C^{2n-1}} \tag{3.23}$$

On simplifying (3.23), we have

$$f^2 = \frac{2H^2}{(n-1)\mu^2} C^{2-4n} + \frac{a^2}{(n^2-1)\mu^2} \frac{C^{2-2n}}{C} + k_1 C^{-4n} \tag{3.24}$$

where k_1 is constant of integration.

$$\text{But } f = C_4 \tag{3.25}$$

Using (3.25), (3.24) becomes

$$\frac{C^{2n-1}}{\sqrt{\beta^2 + \frac{a^2}{(n^2-1)\mu^2} C^{2n} + k_1 C^{-2}}} dC = dt \tag{3.26}$$

where

$$\beta^2 = \frac{2H^2}{(n-1)\mu^2} \tag{3.27}$$

To get determinate solution, we assume $k_1 = 0$.

Therefore (3.26) reduces to

$$\beta^2 + \frac{a^2 C^{2n}}{(n^2 - 1)\mu^2} = \frac{n^2 (t + k_2)^2 a^4}{(n^2 - 1)^2 \mu^4} \tag{3.28}$$

$$C^{2n} = (t + k_2)^2 \frac{a^2 n^2}{\mu^2 (n^2 - 1)} - \beta^2 \frac{(n^2 - 1)}{a^2} \mu^2$$

From (3.15), (3.20) and (3.28), we have

$$A = \mu [k_3^2 (T)^2 - \beta^2 k_4^2]^{\frac{1}{2}}$$

$$B = [k_3^2 (T)^2 - \beta^2 k_4^2]^{\frac{1}{2}} \tag{3.29}$$

$$C = [k_3^2 (T)^2 - \beta^2 k_4^2]^{\frac{1}{2n}}$$

where $T = t + k_2$, $k_3^2 = \frac{n^2}{(n^2-1)} \frac{a^2}{\mu^2}$ and $k_4^2 = \frac{n^2}{k_3^2}$.

Using (3.29), the line element (3.1) becomes

$$ds^2 = dt^2 - \mu^2 [k_3^2 T^2 - \beta^2 k_4^2] dx^2 - [k_3^2 T^2 - \beta^2 k_4^2] e^{-2ax} dy^2 - [k_3^2 T^2 - \beta^2 k_4^2]^{\frac{1}{n}} dz^2 \tag{3.30}$$

The physical properties that are important in cosmology are proper volume v , expansion scalar θ , shear scalar σ^2 , pressure and density. For the model (3.30), these are given by

$$v = \sqrt{-g} = \mu [k_3^2 T^2 - \beta^2 k_4^2]^{\frac{2n+1}{2n}} e^{-ax}$$

$$\theta = \frac{2n + 1}{3n} \left[\frac{k_3^2 T}{k_3^2 T^2 - \beta^2 k_4^2} \right]$$

$$\sigma^2 = \frac{1}{6} \left(\frac{2n + 1}{3n} \right)^2 \left[\frac{k_3^2 T}{k_3^2 T^2 - \beta^2 k_4^2} \right]^2$$

$$p_{WDF} = \frac{-1}{16\pi} \left\{ \frac{(3k_3^2 + \frac{k_3^2}{n} - \frac{a^2}{\mu^2})}{(k_3^2 T^2 - \beta^2 k_4^2)} + \frac{(2n^2 + n + 1)}{n^2} \frac{T^2 k_3^4}{(k_3^2 T^2 - \beta^2 k_4^2)^2} \right\}$$

$$\rho_{WDF} = \frac{1}{8\pi} \left\{ \frac{k_3^4 T^2}{(k_3^2 T^2 - \beta^2 k_4^2)^2} \left(\frac{2n^2 + 5n + 1}{2n^2} \right) - \left(\frac{1}{(k_3^2 T^2 - \beta^2 k_4^2)} \right) \right.$$

$$\left. \times \left(\frac{n - 1}{2n} \right) k_3^2 + \frac{1}{2} \frac{a^2}{\mu^2} \right\}$$

$$H^2 = \mu^2 (k_3^2 T^2 - \beta^2 k_4^2) \left[\left(\frac{n - 1}{2n} \right) k_3^2 - \frac{a^2}{2\mu^2} \right] - \mu^2 T^2 k_3^4 \left(\frac{1 + n}{2n^2} \right)$$

We know that initially a scalar field candidate for dark energy was the quintessence scenario which was based on a fluid with equation of state lying in range $-1 < \omega < -\frac{1}{3}$.

$$\omega = \frac{p}{\rho} > -1$$

Here, we get

$$= -\frac{1}{2} > -1$$

This is very satisfactory value obtained for WDF.

We observe that pressure of WDF is negative.

We note that spatial volume is zero at initial epoch and increases as $T \rightarrow \infty$.

The expansion and shear scalar are infinite at $T = 0$ and decreases with the increase in cosmic time.

Thus, the universe starts evolving with the zero volume at the initial epoch with infinite rate of expansion which slows down for the later times of the universe.

The component of magnetic field reduces the expansion and shear scalar.

The anisotropy parameter of the expansion $\lim_{T \rightarrow \infty} (\frac{\sigma}{\theta}) \neq 0$ is found to be constant. Thus, the model does not approach to isotropy for the future evolution of the universe.

4 In Absence of Magnetic Field

In this case, when $\beta \rightarrow 0$ then the corresponding metric reduces to the following form:

$$ds^2 = dt^2 - \mu^2 k_3^2 T^2 dx^2 - k_3^2 T^2 e^{-2ax} dy^2 - (k_3^2 T^2)^{\frac{1}{n}} dz^2 \tag{4.1}$$

By using the transformations

$$k_3 \mu x = X, \quad k_3 y = Y, \quad k_3^{\frac{1}{n}} z = Z, \quad t + k_2 = T$$

the metric (4.1) reduces to the form

$$ds^2 = dT^2 - T^2 dX^2 - T^2 e^{-\frac{2ax}{k_3 \mu}} dY^2 - T^{\frac{2}{n}} dZ^2 \tag{4.2}$$

The physical properties (proper volume, scalar expansion and shear scalar) for the model (4.2) are given by

$$v = \sqrt{-g} = T^{\frac{2n+1}{2n}} e^{-\frac{ax}{k_3 \mu}}$$

$$\theta = \left(\frac{2n+1}{3n} \right) \frac{1}{T}$$

$$\sigma^2 = \frac{1}{6} \left(\frac{2n+1}{3n} \right)^2 \frac{1}{T^2}$$

In this case also, we note that spatial volume is zero at initial epoch and increases as $T \rightarrow \infty$.

The expansion and shear scalar are infinite at $T = 0$ and decreases with the increase in cosmic time.

Thus, the universe starts evolving with the zero volume at the initial epoch with infinite rate of expansion which slows down for the later times of the universe.

The anisotropy parameter of the expansion $\lim_{T \rightarrow \infty} (\frac{\sigma}{\theta}) \neq 0$ is found to be constant. Thus, the model does not approach to isotropy for the future evolution of the universe.

5 Conclusion

In this paper we have constructed Bianchi type-III cosmological model with dark energy in the form of Wet Dark Fluid in presence and absence of magnetic field. The component

of magnetic field reduces the energy density, expansion, shear scalar. The expansion in the universe is found to be infinite at the initial epoch which decreases with the increase in time. In absence of magnetic field, a similar behavior of expansion is observed. The universe model does not approach to isotropy with or without magnetic field.

Wet dark fluid with magnetic field and space times associated with them has cosmological interest due to their important applications in the structure formation of the universe.

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