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Bulk Viscous Bianchi Type V Cosmological Models with Decaying Cosmological Term Λ

J.P. Singh · P.S. Baghel

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Abstract We present bulk viscous Bianchi type V cosmological models with time-dependent cosmological term Λ. Exact solutions of Einstein field equations have been obtained by assuming shear scalar σ proportional to volume expansion θ. The coefficient of bulk viscosity is taken to be power function of energy density ρ or volume expansion θ. In these models cosmological term Λ come out to be negative. It is found that models obtained are expanding, shearing and non-rotating. They do not approach isotropy for large values of time t. Some observational parameters for the model have also been discussed.

Keywords Cosmological models · Bianchi space-time · Bulk viscosity · Variable cosmological term Λ

1 Introduction

A great deal of attention has recently been paid to cosmological models with nonzero cosmological term Λ [1, 2]. In modern cosmological theories, the cosmological constant remains a focal point of interest. The Λ-term arises naturally in general relativistic quantum field theory where it is interpreted as the energy density of the vacuum [3–5]. Linde [6] has suggested that Λ is a function of temperature and is related to the process of broken symmetries. Therefore, it could be a function of time in a spatially homogeneous, expanding universe [7]. It is widely believed that the value of Λ was large during the early stages of the evolution and strongly influenced its expansion, whereas its present value is incredibly small [8, 9]. Cosmological scenarios with a time-varying Λ were proposed by several researchers [10–14].

The investigation of relativistic cosmological models usually has the cosmic fluid as perfect fluid. But these models do not explain satisfactorily the early stages of evolution. Viscosity may be important in cosmology for a number of reasons. Dissipative mechanisms

J.P. Singh (✉) · P.S. Baghel
Department of Mathematical Sciences, A.P.S. University, Rewa 486003, India
e-mail: jpsinghmp@lycos.com

P.S. Baghel
e-mail: ps.baghel@yahoo.com

responsible for smoothing out initial anisotropies and the observed high entropy per baryon in the present stage of the universe can be explained by involving some kind of dissipative mechanism e.g. bulk viscosity [15, 16]. Dissipative effects including bulk viscosity are supposed to play a very important role in the early evolution of the universe. During the neutrino decoupling stage, apart from streaming neutrinos moving with fundamental velocity, there is a part behaving like a viscous fluid, comoving with matter. Decoupling of radiation and matter during the recombination era is also expected to give rise to viscous effects. Moreover a combination of cosmic fluid with bulk dissipative pressure can generate accelerated expansion [17]. Influence of viscosity on the nature of initial singularity and on the formation of galaxies have been investigated by Murphy [17], Collins and Stewart [18]. It has been shown that the coincidence problem can be solved by taking viscous effects into account [19, 20]. Bulk viscosity leading to an accelerated phase of the universe today has been studied by Fabris et al. [21]. Santos et al. [22] have derived exact solution with bulk viscosity by considering the bulk viscous coefficient as power function of mass density. Johri and Sudarshan [23] have investigated the effect of bulk viscosity on the evolution of Friedmann models. Cosmological models with bulk viscosity have also been studied by number of authors [24–30]. Recently Singh and Chaubey [31], Singh and Baghel [32, 33] have investigated some Bianchi cosmological models with bulk viscosity.

In this paper, we investigate Bianchi type V cosmological models with variable cosmological term Λ in the presence of bulk viscosity. The coefficient of bulk viscosity is assumed to be function of energy density ρ [25] or volume expansion θ:

$$\zeta \propto \rho^l$$

or

$$\zeta \propto \theta^m.$$

Here $l(\geq 0)$ and m are constant. ζ, ρ and θ are respectively the bulk viscous coefficient, matter energy density and volume expansion of the space-time. The observational parameters such as look-back time, proper distance, luminosity distance and angular diameter distance for the model have been discussed.

2 Metric and Field Equations

We consider the Bianchi type V space-time given by the line-element

$$ds^2 = -dt^2 + A^2(t)dx^2 + e^{2\alpha x}\left\{B^2(t)dy^2 + C^2(t)dz^2\right\}, \tag{1}$$

where α is a constant. We assume the cosmic matter consisting of bulk viscous fluid represented by the energy-momentum tensor

$$T_{ij} = (\rho + \bar{p})v_i v_j + \bar{p} g_{ij}, \tag{2}$$

where $\bar{p}$ is the effective pressure given by

$$\bar{p} = p - \zeta v^i{}_{;i}, \tag{3}$$

satisfying linear equation of state

$$p = \omega\rho, \quad 0 \leq \omega \leq 1. \tag{4}$$

Here p is the isotropic pressure and v^i, the flow vector of the fluid satisfying $v_i v^i = -1$. The Einstein field equations (in gravitational units $8\pi G = c = 1$) with time-dependent cosmological term $\Lambda(t)$ are

$$R_{ij} - \frac{1}{2}R_k^k g_{ij} = -T_{ij} + \Lambda(t) g_{ij}. \tag{5}$$

For the line-element (1), the field equations (5) in co-moving system of coordinates lead to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} = \Lambda - \bar{p}, \tag{6}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} - \frac{\alpha^2}{A^2} = \Lambda - \bar{p}, \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} = \Lambda - \bar{p}, \tag{8}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{3\alpha^2}{A^2} = \Lambda + \rho, \tag{9}$$

$$\frac{2\dot{A}}{A} = \frac{\dot{B}}{B} + \frac{\dot{C}}{C}. \tag{10}$$

Vanishing divergence of Einstein tensor $G_{ij} = R_{ij} - \frac{1}{2}R_k^k g_{ij}$ gives rise to

$$\dot{\rho} + (\rho + \bar{p})\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \dot{\Lambda} = 0. \tag{11}$$

We define average scale factor R for Bianchi V universe as

$$R^3 = ABC. \tag{12}$$

In analogy with FRW universe, we define generalized Hubble parameter H and generalized deceleration parameter q as

$$H = \frac{\dot{R}}{R} = \frac{1}{3}(H_1 + H_2 + H_3) \tag{13}$$

and

$$q = -\frac{\dot{H}}{H^2} - 1, \tag{14}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are directional Hubble's factors along x, y and z directions respectively.

We introduce volume expansion θ and shear scalar σ for the Bianchi V metric as

$$\theta = v^i{}_{;i} \tag{15}$$

and

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}, \tag{16}$$

where σ_{ij} is shear tensor given by

$$\sigma_{ij} = v_{i;j} + \frac{1}{2}\left(v_{i;k}v^k v_j + v_{j;k}v^k v_i\right) + \frac{1}{3}v^k{}_{;k}\left(g_{ij} + v_i v_j\right). \tag{17}$$

For the metric (1), expansion scalar θ and shear scalar σ come out to be

$$\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \tag{18}$$

and

$$\sigma^2 = \frac{1}{6}\left\{\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right)^2 + \left(\frac{\dot{C}}{C} - \frac{\dot{A}}{A}\right)^2\right\}. \tag{19}$$

Equations (6)–(9) and (11) can be expressed in terms of H, σ and q as

$$\bar{p} - \Lambda = (2q - 1)H^2 - \sigma^2 + \frac{\alpha^2}{R^2}, \tag{20}$$

$$\rho + \Lambda = 3H^2 - \sigma^2 - \frac{3\alpha^2}{R^2}, \tag{21}$$

$$\dot{\rho} + 3(\rho + \bar{p})H + \dot{\Lambda} = 0. \tag{22}$$

From (21), we observe that

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3} - \frac{\rho}{\theta^2} - \frac{3\alpha^2}{R^2\theta^2} - \frac{\Lambda}{\theta^2}. \tag{23}$$

Therefore, $0 < \frac{\sigma^2}{\theta^2} < \frac{1}{3}$ and $0 < \frac{\rho}{\theta^2} < \frac{1}{3}$ for $\Lambda \geq 0$. Thus a positive Λ puts restriction on the upper limit of anisotropy whereas a negative Λ will increase the anisotropy. From (20) and (21), we get

$$\frac{d\theta}{dt} = -\frac{1}{2}(\rho + 3\bar{p}) - \frac{\theta^2}{3} - 2\sigma^2 + \Lambda, \tag{24}$$

which is the Raychaudhuri equation for the given distribution.

3 Solution of the Field Equations

From (6)–(8) and (10), we obtain

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{1}{2}\left(\frac{\dot{B}^2}{B^2} - \frac{\dot{C}^2}{C^2}\right) = 0. \tag{25}$$

Thus we have one equation (25) in two unknowns B and C. We require one more condition to close the system. We assume that the expansion scalar θ in the model is proportional to the shear σ. This condition leads to

$$B = C^n, \tag{26}$$

where n is a constant. This assumption is in agreement with the observation of the velocity redshift relation for extragalactic sources suggesting that Hubble expansion of the universe is isotropic today within ≈30% [34, 35]. To put more precisely, the redshift studies place the limit $\frac{\sigma}{H} \leq 0.3$ on the ratio of shear σ to Hubble parameter H in the neighbourhood of our galaxy. Collins et al. [36] have pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous expansion satisfies the condition that σ/θ is constant.

Integrating (25), we obtain

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{a_1}{(BC)^{\frac{3}{2}}}, \tag{27}$$

where a_1 is a constant of integration. Using (26) in (27) and (10), we obtain

$$A = \left\{\frac{3}{2}(n+1)\left(a_2 + \frac{a_1 t}{n-1}\right)\right\}^{\frac{1}{3}}, \tag{28}$$

$$B = \left\{\frac{3}{2}(n+1)\left(a_2 + \frac{a_1 t}{n-1}\right)\right\}^{\frac{2n}{3(n+1)}}, \tag{29}$$

$$C = \left\{\frac{3}{2}(n+1)\left(a_2 + \frac{a_1 t}{n-1}\right)\right\}^{\frac{2}{3(n+1)}}, \tag{30}$$

where a_2 is an integration constant. This solution is tenable for $n \neq \pm 1$. For this solution the metric (1) assumes the following form after suitable transformation of coordinates

$$ds^2 = -dT^2 + T^{\frac{2}{3}} dX^2 + e^{\frac{2\alpha X}{k}} \left\{T^{\frac{4n}{3(n+1)}} dY^2 + T^{\frac{4}{3(n+1)}} dZ^2\right\}, \tag{31}$$

k being a constant.

4 Discussion

Matter density ρ and cosmological term Λ for the model (31) are given by

$$\rho = \frac{1}{1+\omega}\left\{\frac{4(n^2+4n+1)}{9(n+1)^2T^2} - \frac{2\alpha^2}{k^2 T^{\frac{2}{3}}} + \frac{\zeta}{T}\right\}, \tag{32}$$

$$\Lambda = \frac{1}{1+\omega}\left\{\frac{2(\omega-1)(n^2+4n+1)}{9(n+1)^2T^2} - \frac{(1+3\omega)\alpha^2}{k^2 T^{\frac{2}{3}}} - \frac{\zeta}{T}\right\}. \tag{33}$$

Scale factor R, expansion scalar θ and shear σ are obtained as

$$R = T^{\frac{1}{3}}, \tag{34}$$

$$\theta = \frac{1}{T}, \tag{35}$$

$$\sigma = \frac{n-1}{3(n+1)T}. \tag{36}$$

The model has point singularity at $T = 0$. We observe that the model starts with ρ, p, θ and σ all infinite and become zero at $T = \infty$, for constant value of ζ. The cosmological term Λ being infinite at the initial singularity becomes negligible for large times.

The integral

$$\int_{T_0}^{T} \frac{dt}{R(t)} = \frac{3}{2}\left[t^{\frac{2}{3}}\right]_{T_0}^{T}, \tag{37}$$

is finite. Therefore particle horizon exists in the model.

4.1 Some Observational Parameters

We investigate the consistency of the model (31) with the observational parameters. We measure the physical parameters such as redshift, look-back time, proper distance, luminosity distance, angular diameter etc.

4.1.1 Look-Back Time

The look-back time T_L is defined as the elapsed time between the present age of universe T_0 and the time T when the light from a cosmic source at a particular redshift z was emitted. In the context of the model (31), it is given by

$$T_L = T_0 - T = \int_{R}^{R_0} \frac{dR}{\dot{R}}, \tag{38}$$

where R_0 is the present day scale factor of the universe and

$$\frac{R_0}{R} = 1 + z. \tag{39}$$

For the model (31), we have

$$\frac{R_0}{R} = 1 + z = \left(\frac{T_0}{T}\right)^{\frac{1}{3}}, \tag{40}$$

implying

$$T = (1+z)^{-3} T_0. \tag{41}$$

Using (35), we obtain

$$T_0 - T = \theta_0{}^{-1}\left\{1 - (1+z)^{-3}\right\}. \tag{42}$$

Thus

$$\theta_0(T_0 - T) = \left\{1 - (1+z)^{-3}\right\}. \tag{43}$$

For small z, we get

$$\theta_0(T_0 - T) = 3\left\{z - \frac{4z^2}{2!} + \frac{20z^3}{3!} - \cdots\right\}. \tag{44}$$

For $z \to \infty$, from (43), we obtain $\theta_0 T_0 = 1$.

4.1.2 Proper Distance

The proper distance $d(z)$ is defined as the distance between a cosmic source emitting light at any instant $T = T_1$ located at $r = r_1$ with redshift z and an observer at $r = 0$ and $T = T_0$ receiving the light from the source emitted, i.e.

$$d(z) = r_1 R_0, \tag{45}$$

where

$$r_1 = \int_{T_1}^{T_0} \frac{dT}{R(T)} = \frac{3R_0^{-1}\theta_0^{-1}}{2}\{1-(1+z)^{-2}\}. \tag{46}$$

Hence

$$d(z) = r_1 R_0 = \frac{3\theta_0^{-1}}{2}\{1-(1+z)^{-2}\}. \tag{47}$$

For small z, we obtain

$$\theta_0 d(z) = 3\left\{z - \frac{3z^2}{2!} + \frac{12z^3}{3!} - \cdots\right\}. \tag{48}$$

For $z \to \infty$, we get

$$d(z=\infty) = \frac{3\theta_0^{-1}}{2}. \tag{49}$$

4.1.3 Luminosity Distance

The luminosity distance d_L of a light source is defined by

$$d_L = \left(\frac{L}{4\pi l}\right)^{\frac{1}{2}} = r_1 R_0(1+z), \tag{50}$$

where L is the absolute luminosity and l is the apparent luminosity of source. From (45) and (50), we get

$$d_L = d(z)(1+z). \tag{51}$$

Using (47), we obtain

$$d_L = \frac{3\theta_0^{-1}}{2}\left\{(1+z)-(1+z)^{-1}\right\}. \tag{52}$$

For small z, (52) gives

$$\theta_0 d_L = 3z - \frac{3z^2}{2}\left(1 - z + z^2 - \cdots\right). \tag{53}$$

4.1.4 Angular Diameter

The angular diameter of a light source of diameter D at $r = r_1$ and $T = T_1$ observed at $r = 0$ and $T = T_0$ is given by

$$\delta = \frac{D}{r_1 R(T_1)} = \frac{D(1+z)^2}{d_L}. \tag{54}$$

The angular diameter distance d_A is defined as the ratio of the source diameter to its angular diameter

$$d_A = \frac{D}{\delta} = r_1 R(T_1) = d_L(1+z)^{-2}. \tag{55}$$

Using (52), we get

$$d_A = \frac{3\theta_0^{-1}}{2}\left\{z(2+z)(1+z)^{-3}\right\}. \tag{56}$$

The maximum value of d_A occurs at

$$z_M = \sqrt{3} - 1 \approx 0.732. \tag{57}$$

4.2 Some Particular Cases

To determine the coefficient of bulk viscosity ζ is assumed to be power function of energy density ρ or volume expansion θ:

$$\zeta = \zeta_0 \rho^l \tag{58}$$

or

$$\zeta = \zeta_0 \theta^m, \tag{59}$$

where $\zeta_0(\geq 0)$, $l(\geq 0)$ and m are constants. We discuss some particular cases:

4.2.1 Model I ($\zeta = \zeta_0$)

When $l = 0$, $\zeta = \zeta_0$. From (32) and (33), we get

$$\rho = \frac{1}{1+\omega}\left\{\frac{4(n^2+4n+1)}{9(n+1)^2T^2} - \frac{2\alpha^2}{k^2T^{\frac{2}{3}}} + \frac{\zeta_0}{T}\right\}, \tag{60}$$

$$\Lambda = \frac{1}{1+\omega}\left\{\frac{2(\omega-1)(n^2+4n+1)}{9(n+1)^2T^2} - \frac{(1+3\omega)\alpha^2}{k^2T^{\frac{2}{3}}} - \frac{\zeta_0}{T}\right\}. \tag{61}$$

The model has singularity at $T = 0$. Matter density ρ, and vacuum density Λ are infinite at the initial singularity and decrease to become zero at late times. We observe that $\frac{\rho}{\theta^2}$ is constant at $T = 0$ and $\frac{\rho}{\theta^2} \to \infty$ as $T \to \infty$. Thus ρ and θ^2 are comparable at the initial epoch whereas matter dominates the expansion at late times.

4.2.2 Model II ($\zeta = \zeta_0\rho$)

When $l = 1$, we obtain $\zeta = \zeta_0\rho$. From (32) and (33), we get

$$\rho = \frac{1}{(1+\omega)T - \zeta_0}\left\{\frac{4(n^2+4n+1)}{9(n+1)^2} - \frac{2\alpha^2 T^{\frac{1}{3}}}{k^2}\right\}, \tag{62}$$

$$\Lambda = \frac{2(n^2+4n+1)}{9(n+1)^2T^2}\left\{\frac{(\omega-1)T-\zeta_0}{(1+\omega)T-\zeta_0}\right\} - \frac{\alpha^2}{k^2T^{\frac{2}{3}}}\left\{\frac{(1+3\omega)T+\zeta_0}{(1+\omega)T-\zeta_0}\right\}. \tag{63}$$

In this case also, ρ and Λ are infinite at $T = 0$ and become zero at $T = \infty$. Since $\frac{\rho}{\theta^2} = 0$ at $T = 0$ and $\frac{\rho}{\theta^2} \to \infty$ as $T \to \infty$. Thus initial epoch is dominated by the expansion whereas at late times matter dominates.

4.2.3 Model III ($\zeta = \zeta_0 \rho^{\frac{1}{2}}$)

For $l = \frac{1}{2}$, we obtain $\zeta = \zeta_0 \rho^{\frac{1}{2}}$. Using (32) and (33) we get

$$\rho = \frac{4(n^2+4n+1)}{9(1+\omega)(n+1)^2 T^2} + \frac{\zeta_0^2}{2(1+\omega)^2 T^2} - \frac{2\alpha^2}{(1+\omega)k^2 T^{\frac{2}{3}}} \pm \frac{\zeta_0^2}{2(1+\omega)^2 T^2}\left\{1 + \frac{16(1+\omega)(n^2+4n+1)}{9(n+1)^2\zeta_0^2} - \frac{2(1+\omega)\alpha^2 T^{\frac{4}{3}}}{k^2\zeta_0^2}\right\}^{\frac{1}{2}}, \tag{64}$$

$$\Lambda = \frac{2(\omega-1)(n^2+4n+1)}{9(1+\omega)(n+1)^2 T^2} - \frac{\zeta_0^2}{2(1+\omega)^2 T^2} - \frac{(1+3\omega)\alpha^2}{(1+\omega)k^2 T^{\frac{2}{3}}} \pm \frac{\zeta_0^2}{2(1+\omega)^2 T^2}\left\{1 + \frac{16(1+\omega)(n^2+4n+1)}{9(n+1)^2\zeta_0^2} - \frac{2(1+\omega)\alpha^2 T^{\frac{4}{3}}}{k^2\zeta_0^2}\right\}^{\frac{1}{2}}. \tag{65}$$

For this model ρ and Λ are infinite and the ratios ρ/θ^2 and Λ/ρ are constant at $T = 0$. $\frac{\rho}{\theta^2} \to \infty$ and Λ/ρ become constant at late times. Thus matter density dominates the expansion at late times. But Λ and ρ are comparable throughout the expansion.

4.2.4 Model IV ($\zeta = \zeta_0 \theta$)

In this case ρ and Λ are given by

$$\rho = \frac{1}{1+\omega}\left\{\frac{4(n^2+4n+1) + 9(n+1)^2\zeta_0}{9(n+1)^2 T^2} - \frac{2\alpha^2}{k^2 T^{\frac{2}{3}}}\right\}, \tag{66}$$

$$\Lambda = \frac{1}{1+\omega}\left\{\frac{2(\omega-1)(n^2+4n+1) - 9(n+1)^2\zeta_0}{9(n+1)^2 T^2} - \frac{(1+3\omega)\alpha^2}{k^2 T^{\frac{2}{3}}}\right\}. \tag{67}$$

We observe that ρ and Λ are infinite but ρ/θ^2 and Λ/ρ are constant at initial epoch. For $T \to \infty$, ρ and Λ become zero but $\frac{\rho}{\theta^2} \to \infty$ and $\frac{\Lambda}{\rho} \to$ *constant*. Thus ρ and θ^2 are comparable at initial epoch whereas ρ dominates θ^2 at late times. Vacuum density and energy density are comparable throughout the evolution.

4.2.5 Model V ($\zeta = \zeta_0/\theta$)

In this case ρ and Λ are given by

$$\rho = \frac{1}{1+\omega}\left\{\frac{4(n^2+4n+1)}{9(n+1)^2 T^2} - \frac{2\alpha^2}{k^2 T^{\frac{2}{3}}} + \zeta_0\right\}, \tag{68}$$

$$\Lambda = \frac{1}{1+\omega}\left\{\frac{2(\omega-1)(n^2+4n+1)}{9(n+1)^2 T^2} - \frac{(1+3\omega)\alpha^2}{k^2 T^{\frac{2}{3}}} + \zeta_0\right\}. \tag{69}$$

At $T = 0$, ρ and Λ are infinite but for large values of T they become finite. ρ and θ^2 are comparable at the initial singularity whereas matter dominates the expansion at late times.

5 Conclusion

Einstein's field equations with time-dependent cosmological term Λ have been considered in the context of Bianchi type V space-time and bulk viscous fluid source. The cosmic fluid is chosen to obey a linear equation of state. We have assumed the shear scalar σ proportional to volume expansion θ which gives rise to a relation between the metric potentials B and C. We observe that the model (31) starts with a big-bang at $T = 0$, the expansion in the model decreases as time increases and drops to zero as $T \to \infty$. We also observe that the presence of bulk viscosity is to increase the value of matter density ρ and to decrease the value of vacuum energy density Λ. The coefficient of bulk viscosity is taken to be power function of energy density ρ, or volume expansion θ. In these models cosmological term Λ is found to be negative. It is possible that models with negative cosmological constant represent very early universe. Models with negative cosmological constant have been investigated by Pedram et al. [37], Biswas and Mazumdar [38]. We find that $\frac{\sigma}{\theta} = constant$. Thus anisotropy is maintained throughout the evolution. However, if $n \sim 1$, it leads to quasi isotropic. The models represent an expanding, shearing and non-rotating universe. We have also taken an account of the consistency of our models with observational parameters such as look-back time, proper distance, luminosity distance and angular diameter distance.

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