Int J Theor Phys (2010) 49: 2592–2599
DOI 10.1007/s10773-010-0450-y

Quantum Teleportation and Quantum Information Splitting by Using a Genuinely Entangled Six-Qubit State

Yuan-Hua Li · Jun-Chang Liu · Yi-You Nie

Received: 23 April 2010 / Accepted: 27 July 2010 / Published online: 12 August 2010

Abstract A new application of the genuinely entangled six-qubit state introduced recently by Tapiador et al. (J. Phys. A 42:415301, 2009) is investigated for the quantum teleportation of an arbitrary three-qubit state and for quantum information splitting (QIS) of an arbitrary two-qubit state. For QIS, we have shown that it can be completed perfectly with two distinct measurement methods. In our scheme, the joint Bell-state measurement and the joint multi-qubit measurement are needed. This quantum teleportation and QIS schemes are deterministic.

Keywords Quantum information · Quantum teleportation · Quantum information splitting · Genuinely entangled six-qubit state · Bell-state measurement

1 Introduction

Entanglement is one of the peculiar properties of quantum physics and plays a crucial role in quantum information processing. Quantum teleportation is an important technique for transfer of information between two or more parties. Since Bennett et al. [1] presented the first protocol of quantum teleportation through an entangled channel of EPR pair in 1993, many teleportation protocols have been devised by using multi-partite entangled states, such as the prototype-GHZ states [2], generalized W states [3] and cluster states [4–7]. The first scheme for quantum information splitting (QIS) was confirmed in 1999 using an entangled three-qubit GHZ state [8]. The basic idea of QIS is to share of quantum information among a group of participants such that the original information cannot be completely reconstructed by any one of the parties by themselves, and its investigations are well articulated in the

Y.-H. Li · J.-C. Liu · Y.-Y. Nie (✉)
Department of Physics, Jiangxi Normal University, Nanchang 330022, China
e-mail: nieyiyou@jxnu.edu.cn

Y.-Y. Nie
Key Laboratory of Optoelectronic & Telecommunication of Jiangxi Province, Nanchang 330022, China
e-mail: nieyiyou@163.com

literatures [9–14]. Recently, attention has turned towards the investigation of the efficacy of a number of multipartite entangled channels for the teleportation of an arbitrary two-qubit state [15–18]. The QIS of an arbitrary two-qubit state was initially realized using four Bell pairs [19], and the following schemes included cluster states [7] and a genuinely entangled five-qubit state [17, 20]. In 2007, Borras et al. [21], introduced a genuinely entangled six-qubit state which is not decomposable into pairs of Bell states. The state is robust against decoherence and its entanglement still prevails after particle loss. Choudhury et al. [18] proved that the Borras et al.'s $|\psi_6\rangle$ state can be used for teleportation of an arbitrary three-qubit state and QIS of an arbitrary two-qubit state deterministically. In the case of six qubits, a new state with an algebraic structure arguably simpler than Borras et al.'s $|\psi_6\rangle$ state [21] has been discovered in [22], this state exhibits genuine multipartite entanglement according to negative partial transpose measure, all the partial density matrices are completely mixed for this state, i.e. $\mathrm{Tr}(\rho^2)$ being $1/2$, $1/4$ and $1/8$ for all the 1-quibit, 2-qubit and 3-qubit marginal density matrices, respectively. Hence, this state may play an important role in quantum information processing. This motivate us to study the application of the new six-qubit state for new teleportation and QIS of an arbitrary two-qubit state and thus it is expected to be more interesting.

The organization of the paper is as follows. In Sect. 2, the new six-qubit state is shown to be used for the teleportation of an arbitrary three-qubit state under a deterministic way. In Sect. 3, we investigate in detail QIS of an arbitrary two-qubit state in two distinct ways and compare their properties. In Sect. 4, at last, conclusions are given.

2 Teleportation of an Arbitrary Three-Qubit State

Our scheme can be described as follows. Suppose the sender Alice has an unknown arbitrary three-qubit state

$$|\psi\rangle_{ABC} = \sum_{i_1,i_2,i_3=0}^{1} \alpha_{i_1 i_2 i_3} |i_1 i_2 i_3\rangle_{ABC}, \tag{1}$$

with $\sum |\alpha_{i_1 i_2 i_3}|^2 = 1$ and she wants to teleport to Bob. She prepares a genuinely entangled six-qubit state with six qubits 1, 2, 3, 4, 5 and 6 [22]. We note that the state can also be given by

$$\begin{aligned}|\psi\rangle_{123456} = \frac{\sqrt{2}}{4}(&|000\rangle_{123}|\varsigma_{000}\rangle_{456} + |001\rangle_{123}|\varsigma_{001}\rangle_{456} + |010\rangle_{123}|\varsigma_{010}\rangle_{456} + |011\rangle_{123}|\varsigma_{011}\rangle_{456} \\ &+ |100\rangle_{123}|\varsigma_{100}\rangle_{456} + |101\rangle_{123}|\varsigma_{101}\rangle_{456} + |110\rangle_{123}|\varsigma_{110}\rangle_{456} + |111\rangle_{123}|\varsigma_{111}\rangle_{456}),\end{aligned}$$

where the states $|\varsigma_{i_1 i_2 i_3}\rangle_{456}$ of qubits 4, 5 and 6 are given by

$$\begin{aligned}
|\varsigma_{000}\rangle_{456} &= \frac{1}{\sqrt{2}}(|000\rangle_{456} - |011\rangle_{456}), & |\varsigma_{001}\rangle_{456} &= \frac{1}{\sqrt{2}}(|100\rangle_{456} + |111\rangle_{456}),\\
|\varsigma_{010}\rangle_{456} &= \frac{1}{\sqrt{2}}(|101\rangle_{456} + |110\rangle_{456}), & |\varsigma_{011}\rangle_{456} &= \frac{1}{\sqrt{2}}(|001\rangle_{456} - |010\rangle_{456}),\\
|\varsigma_{100}\rangle_{456} &= \frac{1}{\sqrt{2}}(|101\rangle_{456} - |110\rangle_{456}), & |\varsigma_{101}\rangle_{456} &= \frac{1}{\sqrt{2}}(|001\rangle_{456} + |010\rangle_{456}),\\
|\varsigma_{110}\rangle_{456} &= \frac{1}{\sqrt{2}}(|000\rangle_{456} + |011\rangle_{456}), & |\varsigma_{111}\rangle_{456} &= \frac{1}{\sqrt{2}}(|100\rangle_{456} - |111\rangle_{456}).
\end{aligned}$$

The combined state of the nine qubits can be expressed as

$$|\Psi\rangle_{ABC123456} = |\psi\rangle_{ABC} \otimes |\psi\rangle_{123456}. \tag{2}$$

To achieve the purpose of teleportation, Alice firstly sends the qubits 4, 5 and 6 to the receiver Bob, and then performs a six-qubit von-Neumann measurement on her qubits A, B, C, 1, 2, 3. Then she tells her measured result to Bob via a classical channel. For instance, if Alice's measure result is $\sum_{i_1,i_2,i_3=0}^{1} |i_1 i_2 i_3\rangle_{ABC} |i_1 i_2 i_3\rangle_{123}$, then Bob's system collapses to $\sum_{i_1,i_2,i_3=0}^{1} \alpha_{i_1 i_2 i_3} |\varsigma_{i_1 i_2 i_3}\rangle_{456}$. Now, Bob can apply an appropriately unitary operation on his qubits and obtain the original state $|\psi\rangle_{ABC}$. In the following, we shall furthermore investigate the application of this state for QIS of an arbitrary two-qubit state.

3 QIS of an Arbitrary Two-Qubit State

3.1 Protocol 1

In this protocol, we let Alice possesses qubits 1, 2, Bob possesses qubits 5, 6 and Charlie possesses qubits 3, 4 in Tapiador et al.'s $|\psi_6\rangle$ state, respectively. Alice possesses an arbitrary two-qubit state $|\psi\rangle_{ab} = \alpha|00\rangle + \mu|10\rangle + \gamma|01\rangle + \beta|11\rangle$, where $|\alpha|^2 + |\mu|^2 + |\gamma|^2 + |\beta|^2 = 1$. To achieve the purpose of QIS, Alice firstly performs Bell-state measurements on her qubit pairs $(a, 1)$ and $(b, 2)$, respectively, and Alice has 16 kinds of possible measure results with equal probability $1/16$. There are also 16 kinds of corresponding collapse states $|\phi_{3456}\rangle^i$ ($i = 1, 2, \ldots, 16$) of qubits 3, 4, 5 and 6 after the measurement by Alice. The outcome of the measurement performed by Alice and the corresponding collapse state of Bob-Charlie's system are shown in Table 1, where the states $|\eta_i\rangle_{3456}$ of qubits 3, 4, 5 and 6 are given by

$$|\eta_1\rangle_{3456} = \frac{1}{\sqrt{2}}(|00\rangle_{34}|\Phi^-\rangle_{56} + |11\rangle_{34}|\Phi^+\rangle_{56}),$$

$$|\eta_2\rangle_{3456} = \frac{1}{\sqrt{2}}(|11\rangle_{34}|\Phi^-\rangle_{56} + |00\rangle_{34}|\Phi^+\rangle_{56}),$$

$$|\eta_3\rangle_{3456} = \frac{1}{\sqrt{2}}(|10\rangle_{34}|\Psi^-\rangle_{56} + |01\rangle_{34}|\Psi^+\rangle_{56}),$$

$$|\eta_4\rangle_{3456} = \frac{1}{\sqrt{2}}(|01\rangle_{34}|\Psi^-\rangle_{56} + |10\rangle_{34}|\Psi^+\rangle_{56}),$$

with $|\Phi^\pm\rangle_{a1} = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)_{a1}$ and $|\Psi^\pm\rangle_{a1} = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)_{a1}$.

Then Alice tells the result of her measurement to Bob and Charlie via a classical channel. Neither Bob nor Charlie can recover the original state only by local operations on their own qubits. If the Charlie allows Bob to get the initial state, then Charlie makes a two-qubit projective measurement on qubits 3 and 4 under the basis of $\{|\varphi_{34}\rangle_i\}$ ($i = 1, 2, \ldots, 4$), where $|\varphi_{34}\rangle_i$ are given by

$$|\varphi_{34}\rangle_1 = \frac{1}{\sqrt{2}}(|00\rangle + |01\rangle), \qquad |\varphi_{34}\rangle_2 = \frac{1}{\sqrt{2}}(|00\rangle - |01\rangle),$$

$$|\varphi_{34}\rangle_3 = \frac{1}{\sqrt{2}}(|11\rangle + |10\rangle), \qquad |\varphi_{34}\rangle_4 = \frac{1}{\sqrt{2}}(|11\rangle - |10\rangle).$$

Table 1 The outcome of the measurement performed by Alice and the corresponding state obtained by Bob and Charlie, where the normalization factors have been omitted for convenience

Alice's result	State obtained by Charlie and Bob
$\|\Phi^+\rangle_{a1}\|\Phi^+\rangle_{b2}$	$\|\phi_{3456}\rangle^1=\alpha\|\eta_1\rangle+\mu\|\eta_4\rangle+\gamma\|\eta_3\rangle+\beta\|\eta_2\rangle$
$\|\Phi^+\rangle_{a1}\|\Phi^-\rangle_{b2}$	$\|\phi_{3456}\rangle^2=\alpha\|\eta_1\rangle+\mu\|\eta_4\rangle-\gamma\|\eta_3\rangle-\beta\|\eta_2\rangle$
$\|\Phi^-\rangle_{a1}\|\Phi^+\rangle_{b2}$	$\|\phi_{3456}\rangle^3=\alpha\|\eta_1\rangle-\mu\|\eta_4\rangle+\gamma\|\eta_3\rangle-\beta\|\eta_2\rangle$
$\|\Phi^-\rangle_{a1}\|\Phi^-\rangle_{b2}$	$\|\phi_{3456}\rangle^4=\alpha\|\eta_1\rangle-\mu\|\eta_4\rangle-\gamma\|\eta_3\rangle+\beta\|\eta_2\rangle$
$\|\Phi^+\rangle_{a1}\|\Psi^+\rangle_{b2}$	$\|\phi_{3456}\rangle^5=\alpha\|\eta_3\rangle+\mu\|\eta_2\rangle+\gamma\|\eta_1\rangle+\beta\|\eta_4\rangle$
$\|\Phi^+\rangle_{a1}\|\Psi^-\rangle_{b2}$	$\|\phi_{3456}\rangle^6=\alpha\|\eta_3\rangle+\mu\|\eta_2\rangle-\gamma\|\eta_1\rangle-\beta\|\eta_4\rangle$
$\|\Phi^-\rangle_{a1}\|\Psi^+\rangle_{b2}$	$\|\phi_{3456}\rangle^7=\alpha\|\eta_3\rangle-\mu\|\eta_2\rangle+\gamma\|\eta_1\rangle-\beta\|\eta_4\rangle$
$\|\Phi^-\rangle_{a1}\|\Psi^-\rangle_{b2}$	$\|\phi_{3456}\rangle^8=\alpha\|\eta_3\rangle-\mu\|\eta_2\rangle-\gamma\|\eta_1\rangle+\beta\|\eta_4\rangle$
$\|\Psi^+\rangle_{a1}\|\Phi^+\rangle_{b2}$	$\|\phi_{3456}\rangle^9=\alpha\|\eta_4\rangle+\mu\|\eta_1\rangle+\gamma\|\eta_2\rangle+\beta\|\eta_3\rangle$
$\|\Psi^+\rangle_{a1}\|\Phi^-\rangle_{b2}$	$\|\phi_{3456}\rangle^{10}=\alpha\|\eta_4\rangle+\mu\|\eta_1\rangle-\gamma\|\eta_2\rangle-\beta\|\eta_3\rangle$
$\|\Psi^-\rangle_{a1}\|\Phi^+\rangle_{b2}$	$\|\phi_{3456}\rangle^{11}=\alpha\|\eta_4\rangle-\mu\|\eta_1\rangle+\gamma\|\eta_2\rangle-\beta\|\eta_3\rangle$
$\|\Psi^-\rangle_{a1}\|\Phi^-\rangle_{b2}$	$\|\phi_{3456}\rangle^{12}=\alpha\|\eta_4\rangle-\mu\|\eta_1\rangle-\gamma\|\eta_2\rangle+\beta\|\eta_3\rangle$
$\|\Psi^+\rangle_{a1}\|\Psi^+\rangle_{b2}$	$\|\phi_{3456}\rangle^{13}=\alpha\|\eta_2\rangle+\mu\|\eta_3\rangle+\gamma\|\eta_4\rangle+\beta\|\eta_1\rangle$
$\|\Psi^+\rangle_{a1}\|\Psi^-\rangle_{b2}$	$\|\phi_{3456}\rangle^{14}=\alpha\|\eta_2\rangle+\mu\|\eta_3\rangle-\gamma\|\eta_4\rangle-\beta\|\eta_1\rangle$
$\|\Psi^-\rangle_{a1}\|\Psi^+\rangle_{b2}$	$\|\phi_{3456}\rangle^{15}=\alpha\|\eta_2\rangle-\mu\|\eta_3\rangle+\gamma\|\eta_4\rangle-\beta\|\eta_1\rangle$
$\|\Psi^-\rangle_{a1}\|\Psi^-\rangle_{b2}$	$\|\phi_{3456}\rangle^{16}=\alpha\|\eta_2\rangle-\mu\|\eta_3\rangle-\gamma\|\eta_4\rangle+\beta\|\eta_1\rangle$

Table 2 The outcome of the measurement performed by Charlie and the corresponding state obtained by Bob, where the normalization factors have been omitted for convenience

Charlie's result		States obtained by Bob	U_i
$\|\varphi_{34}\rangle_1$	$\|00\rangle+\|01\rangle$	$\|\phi_{56}\rangle^1=\alpha\|\Phi^-\rangle+\mu\|\Psi^-\rangle+\gamma\|\Psi^+\rangle+\beta\|\Phi^+\rangle$	U_1
$\|\varphi_{34}\rangle_2$	$\|00\rangle-\|01\rangle$	$\|\phi_{56}\rangle^2=\alpha\|\Phi^-\rangle-\mu\|\Psi^-\rangle-\gamma\|\Psi^+\rangle+\beta\|\Phi^+\rangle$	U_2
$\|\varphi_{34}\rangle_3$	$\|11\rangle+\|10\rangle$	$\|\phi_{56}\rangle^3=\alpha\|\Phi^+\rangle+\mu\|\Psi^+\rangle+\gamma\|\Psi^-\rangle+\beta\|\Phi^-\rangle$	U_3
$\|\varphi_{34}\rangle_4$	$\|11\rangle-\|10\rangle$	$\|\phi_{56}\rangle^4=\alpha\|\Phi^+\rangle-\mu\|\Psi^+\rangle-\gamma\|\Psi^-\rangle+\beta\|\Phi^-\rangle$	U_4

According to Alice's and Charlie's measured results, Bob can apply an appropriately unitary transformation on his qubits to reconstruct the original state $|\psi\rangle_{ab}$. For instance, if Bob-Charlie's system had evolved into the first state given in Table 1, i.e. Bob-Charlie's system collapses to $|\phi_{3456}\rangle^1$, then the outcome of the measurement performed by Charlie will be one of the states $|\varphi_{34}\rangle_i$ $(i=1,2,\ldots,4)$. The outcome of the measurement performed by Charlie, the corresponding state obtained by Bob and Bob's operation are shown in Table 2.

In Table 2, the unitary transformations U_i $(i=1,2,\ldots,4)$ are given by

$$U_1=|00\rangle\langle\Phi^-|+|10\rangle\langle\Psi^-|+|01\rangle\langle\Psi^+|+|11\rangle\langle\Phi^+|,$$

$$U_2=|00\rangle\langle\Phi^-|-|10\rangle\langle\Psi^-|-|01\rangle\langle\Psi^+|+|11\rangle\langle\Phi^+|,$$

$$U_3=|00\rangle\langle\Phi^+|+|10\rangle\langle\Psi^+|+|01\rangle\langle\Psi^-|+|11\rangle\langle\Phi^-|,$$

$$U_4=|00\rangle\langle\Phi^+|-|10\rangle\langle\Psi^+|-|01\rangle\langle\Psi^-|+|11\rangle\langle\Phi^-|.$$

It is evident that there are 64 kinds of possibly collapsed states $|\phi_{56}\ \rangle^i(i=1,2,\ldots,64)$ of qubits 5 and 6 after the measurement by Charlie. For instance, if Bob get the state $|\phi_{56}\rangle^1$, then he performs the unitary transformation U_1 on his two qubits to obtain $|\psi\rangle_{ab}$.

Another possible scenario is that, Alice can send her measure result to Bob by four classical bits of information so that Bob and Charlie can co-operate and apply four-qubit joint unitary transformations on their qubits and convert their all possible states $|\phi_{3456}\rangle^i$ $(i = 1, 2, \ldots, 16)$ into $|\phi_{3456}\rangle^1$. After performing the unitary transformation, Bob and Charlie can be spatially separated. Then Charlie can perform a two-qubit projective measurement on his qubits and Bob can obtain the original state by applying an appropriately unitary operator on own qubits. The measured result performed by Charlie, the corresponding state obtained by Bob and Bob's operation are shown in Table 2.

3.2 Protocol 2

In this protocol, we let Alice possesses qubits 1, 2 and 3, Charlie possesses qubit 4 and Bob possesses qubits 5 and 6 in Tapiador et al. 's $|\psi_6\rangle$ state, respectively. To achieve the purpose of QIS, Alice firstly performs a five-qubit von-Neumann measurement on her qubits a, b, 1, 2 and 3 under a basis of five-qubit entangled states $|\zeta_{ab123}\rangle_i$ $(i = 1, 2, \ldots, 32)$, which has 32 kinds of possible measure results with equal probability 1/32. There are also 32 kinds of corresponding collapse states $|\varepsilon_{456}\rangle^i (i = 1, 2, \ldots, 32)$ of qubits 4, 5 and 6 after the measurement by Alice. The possible result of the measurement performed by Alice and the corresponding state obtained by Charlie and Bob are shown in Table 3, where the states $|\chi_i\rangle_{456}$ of qubits 4, 5 and 6 are given by

$$|\chi_1\rangle_{456} = \frac{1}{\sqrt{2}}(|000\rangle_{456} - |011\rangle_{456}), |\chi_2\rangle_{456} = \frac{1}{\sqrt{2}}(|100\rangle_{456} + |111\rangle_{456}),$$

$$|\chi_3\rangle_{456} = \frac{1}{\sqrt{2}}(|100\rangle_{456} - |111\rangle_{456}), |\chi_4\rangle_{456} = \frac{1}{\sqrt{2}}(|000\rangle_{456} + |011\rangle_{456}),$$

$$|\chi_5\rangle_{456} = \frac{1}{\sqrt{2}}(|001\rangle_{456} - |010\rangle_{456}), |\chi_6\rangle_{456} = \frac{1}{\sqrt{2}}(|10\rangle_{456} + |110\rangle_{456}),$$

$$|\chi_7\rangle_{456} = \frac{1}{\sqrt{2}}(|101\rangle_{456} - |110\rangle_{456}), |\chi_8\rangle_{456} = \frac{1}{\sqrt{2}}(|001\rangle_{456} + |010\rangle_{456}).$$

Then Alice tells the result of her measurement to Bob and Charlie via a classical channel. Neither Bob nor Charlie can recover the original state only by local operations on their own particles. If the Charlie allows Bob to get the initial state, then Charlie makes a single-qubit projective measurement on qubit 4 under the basis $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$. According to Alice's and Charlie's measured results, Bob can apply an appropriately unitary transformation on his qubits to reconstruct the original state $|\psi\rangle_{ab}$. For instance, if Bob-Charlie's system had evolved into the first state given in Table 3, i.e. Bob-Charlie's system collapses to $|\varepsilon_{456}\rangle^1$, then the outcome of the measurement performed by Charlie will be one of the $|\pm\rangle_4$. The outcome of the measurement performed by Charlie, the corresponding state obtained by Bob and Bob's operation are shown in Table 4.

In Table 4, the unitary transformations U'_j $(j = 1, 2)$ are given by

$$U'_1 = |00\rangle\langle\Phi^-| + |10\rangle\langle\Phi^+| + |01\rangle\langle\Psi^-| + |11\rangle\langle\Psi^+|,$$
$$U'_2 = |00\rangle\langle\Phi^-| - |10\rangle\langle\Phi^+| + |01\rangle\langle\Psi^-| - |11\rangle\langle\Psi^+|.$$

It is evident that there are 64 kinds of possibly collapsed states $|\phi_{56}\rangle^i$ $(i = 1, 2, \ldots, 64)$ of qubits 5 and 6 after the measurement by Charlie. For instance, if Bob get the state $|\phi_{56}\rangle^1$, then he performs the unitary transformation U'_1 on his two qubits to obtain $|\psi\rangle_{ab}$.

Table 3 The outcome of the measurement performed by Alice and the state obtained by Bob and Charlie, where the normalization factors have been omitted for convenience

Alice's measure result		State obtained by Charlie and Bob
$\|\zeta_{ab123}\rangle_1$	$\|00000\rangle + \|10001\rangle + \|01011\rangle + \|11010\rangle$	$\|\varepsilon_{456}\rangle^1 = \alpha\|\chi_1\rangle + \mu\|\chi_2\rangle + \gamma\|\chi_5\rangle + \beta\|\chi_6\rangle$
$\|\zeta_{ab123}\rangle_2$	$\|00000\rangle - \|10001\rangle - \|01011\rangle + \|11010\rangle$	$\|\varepsilon_{456}\rangle^2 = \alpha\|\chi_1\rangle - \mu\|\chi_2\rangle - \gamma\|\chi_5\rangle + \beta\|\chi_6\rangle$
$\|\zeta_{ab123}\rangle_3$	$\|00000\rangle - \|10001\rangle + \|01011\rangle - \|11010\rangle$	$\|\varepsilon_{456}\rangle^3 = \alpha\|\chi_1\rangle - \mu\|\chi_2\rangle + \gamma\|\chi_5\rangle - \beta\|\chi_6\rangle$
$\|\zeta_{ab123}\rangle_4$	$\|00000\rangle + \|10001\rangle - \|01011\rangle - \|11010\rangle$	$\|\varepsilon_{456}\rangle^4 = \alpha\|\chi_1\rangle + \mu\|\chi_2\rangle - \gamma\|\chi_5\rangle - \beta\|\chi_6\rangle$
$\|\zeta_{ab123}\rangle_5$	$\|00111\rangle + \|10110\rangle + \|01100\rangle + \|11101\rangle$	$\|\varepsilon_{456}\rangle^5 = \alpha\|\chi_3\rangle + \mu\|\chi_4\rangle + \gamma\|\chi_7\rangle + \beta\|\chi_8\rangle$
$\|\zeta_{ab123}\rangle_6$	$\|00111\rangle - \|10110\rangle - \|01100\rangle + \|11101\rangle$	$\|\varepsilon_{456}\rangle^6 = \alpha\|\chi_3\rangle - \mu\|\chi_4\rangle - \gamma\|\chi_7\rangle + \beta\|\chi_8\rangle$
$\|\zeta_{ab123}\rangle_7$	$\|00111\rangle - \|10110\rangle + \|01100\rangle - \|11101\rangle$	$\|\varepsilon_{456}\rangle^7 = \alpha\|\chi_3\rangle - \mu\|\chi_4\rangle + \gamma\|\chi_7\rangle - \beta\|\chi_8\rangle$
$\|\zeta_{ab123}\rangle_8$	$\|00111\rangle + \|10110\rangle - \|01100\rangle - \|11101\rangle$	$\|\varepsilon_{456}\rangle^8 = \alpha\|\chi_3\rangle + \mu\|\chi_4\rangle - \gamma\|\chi_7\rangle - \beta\|\chi_8\rangle$
$\|\zeta_{ab123}\rangle_9$	$\|00001\rangle + \|10000\rangle + \|01010\rangle + \|11011\rangle$	$\|\varepsilon_{456}\rangle^9 = \alpha\|\chi_2\rangle + \mu\|\chi_1\rangle + \gamma\|\chi_6\rangle + \beta\|\chi_5\rangle$
$\|\zeta_{ab123}\rangle_{10}$	$\|00001\rangle - \|10000\rangle - \|01010\rangle + \|11011\rangle$	$\|\varepsilon_{456}\rangle^{10} = \alpha\|\chi_2\rangle - \mu\|\chi_1\rangle - \gamma\|\chi_6\rangle + \beta\|\chi_5\rangle$
$\|\zeta_{ab123}\rangle_{11}$	$\|00001\rangle - \|10000\rangle + \|01010\rangle - \|11011\rangle$	$\|\varepsilon_{456}\rangle^{11} = \alpha\|\chi_2\rangle - \mu\|\chi_1\rangle + \gamma\|\chi_6\rangle - \beta\|\chi_5\rangle$
$\|\zeta_{ab123}\rangle_{12}$	$\|00001\rangle + \|10000\rangle - \|01010\rangle - \|11011\rangle$	$\|\varepsilon_{456}\rangle^{12} = \alpha\|\chi_2\rangle + \mu\|\chi_1\rangle - \gamma\|\chi_6\rangle - \beta\|\chi_5\rangle$
$\|\zeta_{ab123}\rangle_{13}$	$\|00110\rangle + \|10111\rangle + \|01101\rangle + \|11100\rangle$	$\|\varepsilon_{456}\rangle^{13} = \alpha\|\chi_4\rangle + \mu\|\chi_3\rangle + \gamma\|\chi_8\rangle + \beta\|\chi_7\rangle$
$\|\zeta_{ab123}\rangle_{14}$	$\|00110\rangle - \|10111\rangle - \|01101\rangle + \|11100\rangle$	$\|\varepsilon_{456}\rangle^{14} = \alpha\|\chi_4\rangle - \mu\|\chi_3\rangle - \gamma\|\chi_8\rangle + \beta\|\chi_7\rangle$
$\|\zeta_{ab123}\rangle_{15}$	$\|00110\rangle - \|10111\rangle + \|01101\rangle - \|11100\rangle$	$\|\varepsilon_{456}\rangle^{15} = \alpha\|\chi_4\rangle - \mu\|\chi_3\rangle + \gamma\|\chi_8\rangle - \beta\|\chi_7\rangle$
$\|\zeta_{ab123}\rangle_{16}$	$\|00110\rangle + \|10111\rangle - \|01101\rangle - \|11100\rangle$	$\|\varepsilon_{456}\rangle^{16} = \alpha\|\chi_4\rangle + \mu\|\chi_3\rangle - \gamma\|\chi_8\rangle - \beta\|\chi_7\rangle$
$\|\zeta_{ab123}\rangle_{17}$	$\|00011\rangle + \|10010\rangle + \|01000\rangle + \|11001\rangle$	$\|\varepsilon_{456}\rangle^{17} = \alpha\|\chi_5\rangle + \mu\|\chi_6\rangle + \gamma\|\chi_1\rangle + \beta\|\chi_2\rangle$
$\|\zeta_{ab123}\rangle_{18}$	$\|00011\rangle - \|10010\rangle - \|01000\rangle + \|11001\rangle$	$\|\varepsilon_{456}\rangle^{18} = \alpha\|\chi_5\rangle - \mu\|\chi_6\rangle - \gamma\|\chi_1\rangle + \beta\|\chi_2\rangle$
$\|\zeta_{ab123}\rangle_{19}$	$\|00011\rangle - \|10010\rangle + \|01000\rangle - \|11001\rangle$	$\|\varepsilon_{456}\rangle^{19} = \alpha\|\chi_5\rangle - \mu\|\chi_6\rangle + \gamma\|\chi_1\rangle - \beta\|\chi_2\rangle$
$\|\zeta_{ab123}\rangle_{20}$	$\|00011\rangle + \|10010\rangle - \|01000\rangle - \|11001\rangle$	$\|\varepsilon_{456}\rangle^{20} = \alpha\|\chi_5\rangle + \mu\|\chi_6\rangle - \gamma\|\chi_1\rangle - \beta\|\chi_2\rangle$
$\|\zeta_{ab123}\rangle_{21}$	$\|00010\rangle + \|10011\rangle + \|01001\rangle + \|11000\rangle$	$\|\varepsilon_{456}\rangle^{21} = \alpha\|\chi_6\rangle + \mu\|\chi_5\rangle + \gamma\|\chi_2\rangle + \beta\|\chi_1\rangle$
$\|\zeta_{ab123}\rangle_{22}$	$\|00010\rangle - \|10011\rangle - \|01001\rangle + \|11000\rangle$	$\|\varepsilon_{456}\rangle^{22} = \alpha\|\chi_6\rangle - \mu\|\chi_5\rangle - \gamma\|\chi_2\rangle + \beta\|\chi_1\rangle$
$\|\zeta_{ab123}\rangle_{23}$	$\|00010\rangle - \|10011\rangle + \|01001\rangle - \|11000\rangle$	$\|\varepsilon_{456}\rangle^{23} = \alpha\|\chi_6\rangle - \mu\|\chi_5\rangle + \gamma\|\chi_2\rangle - \beta\|\chi_1\rangle$
$\|\zeta_{ab123}\rangle_{24}$	$\|00010\rangle + \|10011\rangle - \|01001\rangle - \|11000\rangle$	$\|\varepsilon_{456}\rangle^{24} = \alpha\|\chi_6\rangle + \mu\|\chi_5\rangle - \gamma\|\chi_2\rangle - \beta\|\chi_1\rangle$
$\|\zeta_{ab123}\rangle_{25}$	$\|00100\rangle + \|10101\rangle + \|01111\rangle + \|11110\rangle$	$\|\varepsilon_{456}\rangle^{25} = \alpha\|\chi_7\rangle + \mu\|\chi_8\rangle + \gamma\|\chi_3\rangle + \beta\|\chi_4\rangle$
$\|\zeta_{ab123}\rangle_{26}$	$\|00100\rangle - \|10101\rangle - \|01111\rangle + \|11110\rangle$	$\|\varepsilon_{456}\rangle^{26} = \alpha\|\chi_7\rangle - \mu\|\chi_8\rangle - \gamma\|\chi_3\rangle + \beta\|\chi_4\rangle$
$\|\zeta_{ab123}\rangle_{27}$	$\|00100\rangle - \|10101\rangle + \|01111\rangle - \|11110\rangle$	$\|\varepsilon_{456}\rangle^{27} = \alpha\|\chi_7\rangle - \mu\|\chi_8\rangle + \gamma\|\chi_3\rangle - \beta\|\chi_4\rangle$
$\|\zeta_{ab123}\rangle_{28}$	$\|00100\rangle + \|10101\rangle - \|01111\rangle - \|11110\rangle$	$\|\varepsilon_{456}\rangle^{28} = \alpha\|\chi_7\rangle + \mu\|\chi_8\rangle - \gamma\|\chi_3\rangle - \beta\|\chi_4\rangle$
$\|\zeta_{ab123}\rangle_{29}$	$\|00101\rangle + \|10100\rangle + \|01110\rangle + \|11111\rangle$	$\|\varepsilon_{456}\rangle^{29} = \alpha\|\chi_8\rangle + \mu\|\chi_7\rangle + \gamma\|\chi_4\rangle + \beta\|\chi_3\rangle$
$\|\zeta_{ab123}\rangle_{30}$	$\|00101\rangle - \|10100\rangle - \|01110\rangle + \|11111\rangle$	$\|\varepsilon_{456}\rangle^{30} = \alpha\|\chi_8\rangle - \mu\|\chi_7\rangle - \gamma\|\chi_4\rangle + \beta\|\chi_3\rangle$
$\|\zeta_{ab123}\rangle_{31}$	$\|00101\rangle - \|10100\rangle + \|01110\rangle - \|11111\rangle$	$\|\varepsilon_{456}\rangle^{31} = \alpha\|\chi_8\rangle - \mu\|\chi_7\rangle + \gamma\|\chi_4\rangle - \beta\|\chi_3\rangle$
$\|\zeta_{ab123}\rangle_{32}$	$\|00101\rangle + \|10100\rangle - \|01110\rangle - \|11111\rangle$	$\|\varepsilon_{456}\rangle^{32} = \alpha\|\chi_8\rangle + \mu\|\chi_7\rangle - \gamma\|\chi_4\rangle - \beta\|\chi_3\rangle$

Another possible scenario is that, Alice can send her measure result to Bob by five classical bits of information so that Bob and Charlie can co-operate and apply three-qubit joint unitary transformations on their qubits and convert their all possible states $|\varepsilon_{456}\rangle^i$ $(i = 1, 2, \ldots, 32)$ into $|\varepsilon_{456}\rangle^1$. After performing the unitary transformation, Bob and Charlie can be spatially separated. Then Charlie can make a single-qubit projective measurement on his qubit and Bob can obtain the original state by applying an appropriately unitary operator on own qubits. The measured result performed by Charlie, the corresponding state obtained by Bob and Bob's operation are shown in Table 4.

Table 4 The outcome of the measurement performed by Charlie and the corresponding state obtained by Bob, where the normalization factors have been omitted for convenience

Charlie's measure results		States obtained by Bob	U'_j
$\|+\rangle_4$	$\|0\rangle+\|1\rangle$	$\|\phi_{56}\rangle^1=\alpha\|\Phi^-\rangle+\mu\|\Phi^+\rangle+\gamma\|\Psi^-\rangle+\beta\|\Psi^+\rangle$	U'_1
$\|-\rangle_4$	$\|0\rangle-\|1\rangle$	$\|\phi_{56}\rangle^2=\alpha\|\Phi^-\rangle-\mu\|\Phi^+\rangle+\gamma\|\Psi^-\rangle-\beta\|\Psi^+\rangle$	U'_2

3.3 Comparison

In the paper, we have devised two distinct protocols for QIS of an arbitrary two-qubit state using a genuinely entangled six-qubit state [22]. In the two protocols, the first protocol performs joint Bell-state measurement, whereas the second protocol makes a five-qubit von-Neumann measurement under 32 orthogonal basis. It is evident that both of the two protocols have 64 kinds of possible results after the measurement by Charlie, and their successful probability are 100%. In general, the difficulty of implementing joint multi-qubit measurement is increasing with the number of qubits. In experiment, the joint Bell-state measurement is easier to be realized than the multi-qubit measurement. Considering this fact, we think the first protocol is more advantageous and we think that Alice will make the joint Bell-state measurement instead of the multi-qubit measurement.

4 Conclusion

In this paper, we proposed a new scheme for teleportation of an arbitrary three-qubit state and for QIS of an arbitrary two-qubit state using a genuinely entangled six-qubit state discovered by Tapiador et al. as the quantum channel. For QIS, we have shown that it can be completed perfectly with two distinct measurement methods and both of them successful probability are 100%. Hence, this state will play an important role in quantum information processing. In our two schemes for QIS, the first protocol is simple and easy to be fulfilled in the experiment. These property of the schemes can be utilized to construct a controlled quantum channel, which may be useful in the future quantum computation.

Acknowledgements This work is supported by the National Natural Science Foundation of China (Grant No. 60807014), the Natural Science Foundation of Jiangxi Province, China (Grant No. 2009GZW0005), the Research Foundation of state key laboratory of advanced optical communication systems and networks, and the Research Foundation of the Education Department of Jiangxi Province (Grant No. GJJ09153).

References

1. Bennett, C.H., Brassard, G., Crepeau, C., et al.: Phys. Rev. Lett. **70**, 1895 (1993)
2. Karlsson, A., Bourennane, M.: Phys. Rev. A **58**, 4394 (1998)
3. Agrawal, P., Pati, A.: Phys. Rev. A **74**, 062320 (2006)
4. Briegel, H.J., Raussendorf, R.: Phys. Rev. Lett. A **86**, 910 (2001)
5. Nie, Y.Y., Hong, Z.H., Huang, Y.B., et al.: Int. J. Theor. Phys. **48**, 1485 (2009)
6. Zhang, B.B., Liu, Y.: Int. J. Theor. Phys. **48**, 2644 (2009)
7. Muralidharan, S., Panigrahi, P.K.: Phys. Rev. A **78**, 062333 (2008)
8. Hillery, M., Buzek, V., Berthiaume, A.: Phys. Rev. A **59**, 1829 (1999)
9. Karlsson, A., Koashi, M., Imoto, N.: Phys. Rev. A **59**, 162 (1999)
10. Cleve, R., Gottesman, D., Lo, H.K.: Phys. Rev. Lett. **83**, 648 (1999)
11. Murao, M., Jonathan, D., Plenio, M.B., Vedral, V.: Phys. Rev. A **59**, 156 (1999)

12. Yan, F.L., Wang, D.: Phys. Lett. A **316**, 297 (2003)
13. Yang, C.P., et al.: Phys. Rev. A **70**, 022329 (2004)
14. Man, Z.X., et al.: Phys. Rev. A **75**, 052306 (2007)
15. Rigolin, G.: Phys. Rev. A **71**, 032303 (2005)
16. Yeo, Y., Chua, W.K.: Phys. Rev. Lett. **96**, 060502 (2006)
17. Muralidharan, S., Panigrahi, P.K.: Phys. Rev. A **77**, 032321 (2008)
18. Choudhury, S., Muralidharan, S., Panigrahi, P.K.: J. Phys. A **42**, 115303 (2009)
19. Deng, F.G., et al.: Phys. Rev. A **72**, 044301 (2005)
20. Hou, K., Li, Y.B., Shi, S.H.: Opt. Commun. **283**, 1961 (2010)
21. Borras, A., Plastino, A.R., Batle, J., et al.: J. Phys. A **40**, 13407 (2007)
22. Tapiador, J.E., et al.: J. Phys. A **42**, 415301 (2009)