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Quantum Teleportation via *GHZ-like* State

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Abstract We propose an efficient teleportation scheme for an unknown state to either one of two receivers via *GHZ-like* states. We also discuss the fidelity of the quantum state when the control party is uncooperative.

Keywords Quantum teleportation · *GHZ-like* state · Fidelity · Qubit

1 Introduction

With the development of quantum physics, the information techniques based on quantum physics have been actively studied. Recently, more and more attentions are paid to the study of Quantum Teleportation. Quantum Teleportation is a technique of transmitting an unknown state to the receiver who has no quantum channel with the sender.

Bennett *et al.* first proposed the theoretical protocol for teleportation of a single qubit using an entangled pair of spin-1/2 particles in 1993 [1], after that many protocols of Quantum Teleportation are proposed [2–7]. And some of these protocols have already been realized in experiment. In these schemes, they used the two-particle entangled state-EPR state as the auxiliary state.

However, the quantum teleportation with three-particle entangled state is also attracted to many researchers. In [3], A. Karlsson *et al.* presented an scheme which could teleport an unknown state to either of two locations, but only one, can fully reconstruct the quantum state conditioned on the measurement result of the other. Two qubits teleportation was also considered by V.N. Gorbachev and A.I. Trubilko. In [2], they presented a protocol of teleporting two qubits state using *GHZ* state. In 2002, B.S. Shi *et al.* proposed an teleportation protocol via W state [6], which could teleportation an unknown state probabilistically. The probability of success is dependent on the coefficients of the unknown state. We present

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an new protocol in this paper, which is simpler than the protocol in [3] and more efficient compared to the protocol in [6].

The paper is outlined as follows. In this introduction we provide a brief review on Quantum Teleportation. In Sect. 2, we present the teleportation scheme via *GHZ-like* state. In Sect. 3, we consider the fidelity of the quantum state when the control party is uncooperative. Finally, in Sect. 4, we present some conclusions and discuss the results.

2 Teleportation Via *GHZ-like* State

2.1 Brief Introduction of *GHZ-like* State

Firstly, let us introduce the *GHZ-like* state $|\phi_G\rangle = \frac{1}{2}(|001\rangle + |010\rangle + |100\rangle + |111\rangle)$. As we can see, this *GHZ-like* state is quite like the W state. However, it belongs to the class of *GHZ* state but not W state. This state can be constructed as follows:

1. One prepares two states $|\varphi\rangle_1 = |0\rangle_1 + |1\rangle_1$ and $|\varphi\rangle_{23} = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)_{23}$, they make up an composite system as follows:

$$\begin{aligned}|\phi_{G1}\rangle_{123} &= |\varphi\rangle_1 \otimes |\varphi\rangle_{23} \\ &= (|0\rangle + |1\rangle)_1 \otimes \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)_{23} \\ &= \frac{1}{2}(|001\rangle + |010\rangle + |110\rangle + |101\rangle)_{123} \end{aligned} \tag{1}$$

2. He applies a quantum controlled-not gate to $|\phi_{G1}\rangle_{123}$, which the particle 2 is the target qubit and the particle 1 is the control qubit. It gives the output

$$|\phi_G\rangle_{123} = \frac{1}{2}(|001\rangle + |010\rangle + |100\rangle + |111\rangle)_{123} \tag{2}$$

Experimentally, the EPR states can be constructed already. As a result, it will also be possible to construct this *GHZ-like* state in practice.

2.2 Teleportation via *GHZ-like* State

Suppose the target state Alice wants to teleport is

$$|\varphi\rangle_1 = \alpha|0\rangle_1 + \beta|1\rangle_1 \tag{3}$$

As we said before, there are two receivers we called "Bob" and "Charlie" in our scheme. However, only one of the two receivers can obtain the unknown state finally, with the measurement outcome of the control party (we called the other receiver as the control party). But Alice can choose either one of the two as the real receiver. Here, we take an example that Bob is the person whom Alice wants to send the target unknown state to. Correspondingly, Charlie is as the control party in the process.

As shown in Fig. 1, Alice use a Teleportation "machine". To initiate the teleportation, the machine have a source of three-particle entangled state, here we use

$$|\phi_G\rangle_{234} = \frac{1}{2}(|001\rangle + |010\rangle + |100\rangle + |111\rangle)_{234} \tag{4}$$

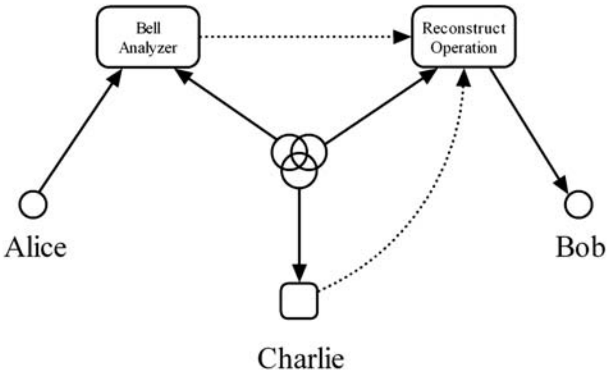

Fig. 1 Schematical picture of Teleportation protocol using *GHZ-like* state. In this case, Bob is the receiver and Charlie acts as the control party

from which one particle (particle 2) is kept by Alice, and particle 3 and particle 4 are sent to Bob and Charlie respectively. The joint product state of the target state and the *GHZ-like* state $|\varphi\rangle_1 \otimes |\phi_G\rangle_{234}$ can be written as

$$\begin{aligned}|\varphi\rangle_1 \otimes |\phi_G\rangle_{234} &= (\alpha|0\rangle + \beta|1\rangle)_1 \\ &\quad \otimes \frac{1}{2}(|001\rangle + |010\rangle + |100\rangle + |111\rangle)_{234} \\ &= \frac{1}{2}(\alpha|000\rangle + \beta|100\rangle + \alpha|011\rangle + \beta|111\rangle)_{123} \otimes |1\rangle_4 \\ &\quad + \frac{1}{2}(\alpha|001\rangle + \beta|101\rangle + \alpha|010\rangle + \beta|110\rangle)_{123} \otimes |0\rangle_4 \end{aligned} \tag{5}$$

In order to present the transformation clearly, we separate the equation above into two parts:

$$\begin{aligned}&\frac{1}{2}(\alpha|000\rangle + \beta|100\rangle + \alpha|011\rangle + \beta|111\rangle)_{123} \otimes |1\rangle_4 \\ &\quad = \frac{1}{4}[(\alpha|000\rangle + \alpha|110\rangle + \beta|001\rangle + \beta|111\rangle)_{123} \\ &\quad\quad + (\alpha|000\rangle - \alpha|110\rangle - \beta|001\rangle + \beta|111\rangle)_{123} \\ &\quad\quad + (\alpha|011\rangle + \alpha|101\rangle + \beta|010\rangle + \beta|100\rangle)_{123} \\ &\quad\quad + (\alpha|011\rangle - \alpha|101\rangle - \beta|010\rangle + \beta|100\rangle)_{123}] \otimes |1\rangle_4 \\ &\quad = \frac{1}{2\sqrt{2}}\left[\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle_{12} \otimes (\alpha|0\rangle + \beta|1\rangle)_3\right. \\ &\quad\quad + \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle_{12} \otimes (\alpha|0\rangle - \beta|1\rangle)_3 \\ &\quad\quad + \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle_{12} \otimes (\alpha|0\rangle + \beta|1\rangle)_3 \\ &\quad\quad \left. + \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle_{12} \otimes (\alpha|0\rangle - \beta|1\rangle)_3\right] \otimes |1\rangle_4 \end{aligned} \tag{6}$$

and

$$\begin{aligned}
&\frac{1}{2}(\alpha|001\rangle+\beta|101\rangle+\alpha|010\rangle+\beta|110\rangle)_{123}\otimes|0\rangle_4\\
&=\frac{1}{4}[(\beta|000\rangle+\beta|110\rangle+\alpha|001\rangle+\alpha|111\rangle)_{123}\\
&\quad+(-\beta|000\rangle+\beta|110\rangle+\alpha|001\rangle-\alpha|111\rangle)_{123}\\
&\quad+(\beta|011\rangle+\beta|101\rangle+\alpha|010\rangle+\alpha|100\rangle)_{123}\\
&\quad+(-\beta|011\rangle+\beta|101\rangle+\alpha|010\rangle-\alpha|100\rangle)_{123}]\otimes|0\rangle_4\\
&=\frac{1}{2\sqrt{2}}\left[\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle_{12}\otimes(\beta|0\rangle+\alpha|1\rangle)_3\right.\\
&\quad+\frac{1}{\sqrt{2}}(|00\rangle-|11\rangle_{12}\otimes(-\beta|0\rangle+\alpha|1\rangle)_3\\
&\quad+\frac{1}{\sqrt{2}}(|01\rangle+|10\rangle_{12}\otimes(\beta|0\rangle+\alpha|1\rangle)_3\\
&\quad\left.+\frac{1}{\sqrt{2}}(|01\rangle-|10\rangle_{12}\otimes(-\beta|0\rangle+\alpha|1\rangle)_3\right]\otimes|0\rangle_4
\end{aligned}\tag{7}$$

Alice measures the particles 1 and 2 using Bell state analyzers. Then, she asks Charlie to measure the particle 4 with the basis $\{|0\rangle, |1\rangle\}$. They sent their measurement outcomes to Bob respectively through the classical channel. That is why the quantum teleportation does not violate the causality [3]. Suppose that Alice obtain the result $|\phi^+\rangle_{12}$, and the measurement outcome of Charlie is $|1\rangle_4$, the particle 3 at the receiver Bob collapses to

$$|\varphi\rangle_3=\alpha|0\rangle_3+\beta|1\rangle_3 \tag{8}$$

that is just the target state $|\varphi\rangle$ which Alice want to teleport to Bob.

As we can see in (6) and (7), Alice may obtain four different outcomes, and Charlie will also get two different results. So, there are eight different groups of the measurement outcomes of Alice and Charlie. However, for some groups, Bob should do a simple unitary operation to reconstruct the desired quantum state. The different reconstruct operations will be shown in Table 1.

If Alice wants to choose Charlie as the receiver, then Bob will act as the control party. This case can be treated in the similar fashion.

3 Fidelity of the Quantum State

In Sect. 2, we proposed the teleportation scheme using *GHZ-like* state. Note that Bob must receive the classical message from Charlie, telling him what is the outcome. Generally, the receiver can obtain the desired quantum state successfully, when the control party is honest. Let us consider the situation: if the control party is not cooperative, whether it is possible that the receiver can still obtain the desired state?

The fidelity presents the distance between two quantum states. Here, it means the probability of the received state to pass as the desired state [3]. In the case in Sect. 2, there are two

Table 1 The unitary operation of Bob, according to the different measurement outcomes of Alice and Charlie. "None" presents that Bob does not need to do any operation

Outcomes of Alice	Outcomes of Charlie	Operation of Bob
$\|\phi^+\rangle_{12}$	$\|1\rangle_4$	None
$\|\phi^-\rangle_{12}$	$\|1\rangle_4$	$\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$
$\|\psi^+\rangle_{12}$	$\|1\rangle_4$	None
$\|\psi^-\rangle_{12}$	$\|1\rangle_4$	$\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$
$\|\phi^+\rangle_{12}$	$\|0\rangle_4$	$\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$
$\|\phi^-\rangle_{12}$	$\|0\rangle_4$	$\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}$
$\|\psi^+\rangle_{12}$	$\|0\rangle_4$	$\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$
$\|\psi^-\rangle_{12}$	$\|0\rangle_4$	$\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}$

different situation that Charlie is not cooperative. The first is he does not do the measurement on particle 4. What is more, he wants to get the target quantum state from particle 4. The other is Charlie tells Bob his measurement outcome in a reversed way.

Now let us compute the fidelity of the quantum state, if Charlie does not measure particle 4. Suppose that after the Bell state measurement, the particles 3 and 4 are in the state

$$|\varphi\rangle_{34} = (\alpha|01\rangle + \beta|11\rangle)_{34} \tag{9}$$

other cases can be treated in the same way. The target quantum state which Bob desires to receive is

$$|\varphi\rangle_3 = (\alpha|0\rangle + \beta|1\rangle)_3 \tag{10}$$

And the reduced density operator of the particle 3 is

$$\begin{aligned}\rho_3 &= Tr_4|\varphi\rangle_{3434}\langle\varphi| \\ &= |\alpha|^2|0\rangle_{33}\langle 0| + \alpha\beta^*|0\rangle_{33}\langle 1| \\ &\quad + \alpha^*\beta|1\rangle_{33}\langle 0| + |\beta|^2|1\rangle_{33}\langle 1|\end{aligned} \tag{11}$$

Then, we can obtain that

$$F_3 = {}_3\langle\varphi|\rho_3|\varphi\rangle_3 = (|\alpha|^2 + |\beta|^2)^2 \tag{12}$$

The fidelity varies between $\frac{1}{2}$(when $|\alpha| = |\beta| = \frac{1}{2}$) and 1 (when $|\alpha| = 1, |\beta| = 0$, or $|\alpha| = 0, |\beta| = 1$). It depends on the coefficients of the unknown state. Here, we compute the averaged fidelity: we can parametrize α and β as in [3] then (10) can be rewritten as

$$|\varphi\rangle_3 = (\cos(\upsilon/2)|0\rangle + e^{i\phi}\sin(\upsilon/2)|1\rangle)_3 \tag{13}$$

the averaged fidelity is

$$\overline{F}_3 \equiv \int_0^{2\pi} d\phi \int_0^{\pi} F_3 \sin(\upsilon) d\upsilon / 4\pi = \frac{1}{2} \tag{14}$$

It is obvious that the averaged fidelity of the quantum state, when Charlie attempts to get the target quantum state from his own particle, is the same with $\overline{F}_3$. We can also get the same result when Charlie tells Bob his measurement outcome in a reversed way. The average fidelity of quantum state is $\frac{1}{2}$, that means the particle of the receiver can not been reconstruct completed. As a result, all the participators in our teleportation protocol have important function, especially the control party. This can be used in quantum user authentication and other protocols.

4 Conclusion and Discussion

In this paper, we have proposed a protocol of quantum teleportation using the *GHZ-like* state. It is a simple and efficient protocol that can be used to teleport an unknown state to either one of two locations. Moreover, we discussed the fidelity of the quantum state. Although the fidelity is dependent on the unknown state, the averaged fidelity is $\frac{1}{2}$, that means if the control party fails to cooperate, the receiver will not get the target quantum state completely.

In our teleportation protocol, we can teleport one qubit state. How about two qubits state or more than two qubits state? It still needs to do more research on these points. We will continue to improve our scheme gradually. In terms of experimental implementations, the scheme requires the ability to generate the *GHZ-like* state.

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