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Accelerating Universe with a Special Form of Decelerating Parameter

Anup Kumar Singha · Ujjal Debnath

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Abstract In this article, we try to investigate the quintessence model with a minimally coupled scalar field by taking a special form of decelerating parameter q in such a way that which provides an early deceleration and late time acceleration for barotropic fluid and Chaplygin gas dominated models. We have shown that the potential function $V(\phi)$ is always decreases with the scalar field ϕ for barotropic and Chaplygin both models. The $\{r, s\}$ diagram shows the behaviour of the universe in different stages during the evolution.

Keywords Acceleration · Chaplygin gas · Statefinder parameters

From the supernova project [1–3], the Maxima [4], Boomerang [5] data on Cosmic Microwave Background (CMB) and also from very recent WMAP data [6] it is very much clear that the universe at present is expanding with ever acceleration. This is caused by an effective negative pressure and for this there are few possible candidates known as dark energy [7–10], which has the cosmological constant to be a strong candidate. Among this, the first one is dark energy governed by time varying cosmological parameter $\Lambda(t)$ but this has fallen from its consistent position [7, 9]. The next one is a scalar field with a positive definite potential which can provide to an effective negative pressure if the potential term dominates the kinetic term. This source of energy is called the quintessence matter (Q-matter). In this regard non-minimally coupled scalar fields had been investigated whether it could provide an accelerated expansion [11–17]. Such a candidate be Brans-Dicke scalar field which provides sufficient acceleration in later phase but it has some problem in early phase [18]. Also

A.K. Singha
Department of Mathematics, Calcutta Institute of Technology, Uluberia 711 316, India

U. Debnath (✉)
Department of Mathematics, Bengal Engineering and Science University, Shibpur, Howrah 711 103, India
e-mail: ujjaldebnath@yahoo.com

U. Debnath
e-mail: ujjal@iucaa.ernet.in

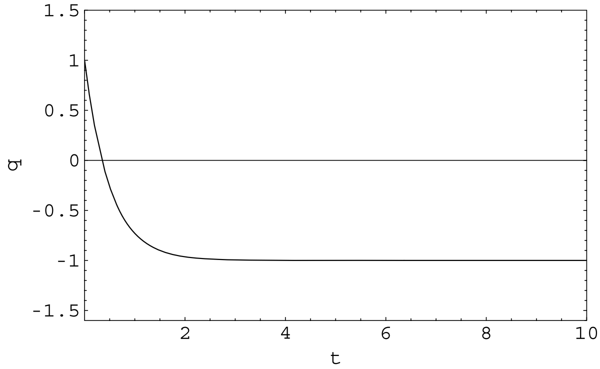


Fig. 1 Represents the variation of q against t for normalizing the constants $\alpha = 2$, $A = 1$

one can try Chaplygin gas model [19, 20] with equation of state (EOS), $p = -B/\rho$, as it generates negative pressure, where p and ρ are respectively the pressure and energy density and B is a positive constant.

In this work we consider the model Q-matter with barotropic fluid and Chaplygin gas which has a non-linear contribution of the energy density to the dynamics of the model. For dust dominated model this particular form of deceleration parameter provides a simple trigonometric potential which serves the good sign flip-flop of q [21].

Here we have chosen the form of deceleration parameter from [21] in such a way that it has the desired property for sign flip-flop. After that we find out the potential function $V(\phi)$ in terms of ϕ for both cases. From recently developed statefinder parameters, we have investigated the behaviour in different phases for evolution of the universe. But it is very clear that it is not a good way to investigate the dynamics of the universe but this reverse way of investigation was used by Ellis and Madson [22] earlier for finding out the potential driving inflation.

For a spatially flat FRW spacetime, the line element is

$$ds^2 = dt^2 - a^2(t)[dr^2 + r^2 d\Omega^2] \tag{1}$$

and the deceleration parameter q is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \tag{2}$$

where a is the scale factor of the universe and is a function of t alone. In order to get a model consistent with observations, we take special form of deceleration parameter [21] as

$$q = -1 + \frac{\alpha}{1 + a^{\alpha}} \tag{3}$$

where α (> 0) is constant. From Fig. 1, we have seen the q decreases from $+1$ to -1 for evolution of the universe.

From (3) we obtain the Hubble parameter as

$$H = \frac{\dot{a}}{a} = A(1 + a^{-\alpha}) \tag{4}$$

where A is an integrating constant.

Now we consider the universe is filled with scalar field and normal matter, so the Einstein field equations for the space-time given by (1) can be written as

$$3\frac{\dot{a}^2}{a^2}=\frac{1}{2}\dot{\phi}^2+V(\phi)+\rho \tag{5}$$

and

$$2\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}=-\frac{1}{2}\dot{\phi}^2+V(\phi)-p \tag{6}$$

where ρ is the density of matter, p is the pressure of matter, ϕ is scalar field and $V(\phi)$ is a scalar potential.

Also we have assumed that there is no interaction between scalar field and normal matter, so they are separately conserved. Now the energy conservation equations for normal matter and scalar field are

$$\dot{\rho}+3H(p+\rho)=0 \tag{7}$$

and

$$\ddot{\phi}+3H\dot{\phi}+\frac{dV}{d\phi}=0 \tag{8}$$

The barotropic equation of state for normal matter is

$$p=w\rho \tag{9}$$

Then from (7) we have the expression for density as

$$\rho=\rho_0 a^{-3(1+w)} \tag{10}$$

where ρ_0 is the integrating constant. And again from (4) we have by integration

$$a=(e^{A\alpha t}-1)^{\frac{1}{\alpha}} \tag{11}$$

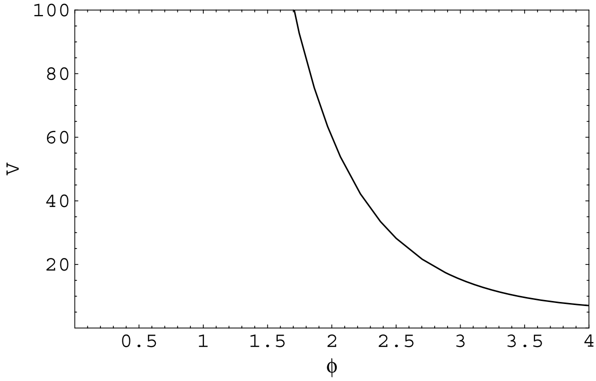
which gives the scale factor a in the explicit form of time t. Solving (5) and (6) and using the relations (9), (10) and (11), we have

$$\dot{\phi}=\left[2A^2\alpha(e^{A\alpha t}-1)^{-1}+2A^2\alpha(e^{A\alpha t}-1)^{-2}-(1+w)\rho_0(e^{A\alpha t}-1)^{-\frac{3(1+w)}{\alpha}}\right]^{\frac{1}{2}} \tag{12}$$

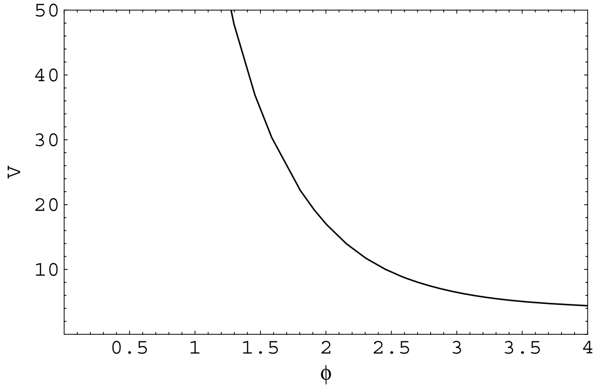
and

$$\begin{aligned}V(\phi)&=2A^2\alpha e^{A\alpha t}(e^{A\alpha t}-1)^{-1}+2A^2(3-\alpha)e^{2A\alpha t}(e^{A\alpha t}-1)^{-2}\\&\quad+\rho_0(w-1)(e^{A\alpha t}-1)^{-\frac{3(1+w)}{\alpha}}\end{aligned} \tag{13}$$

From (12), we have seen that it is not possible to find out the explicit form of ϕ in terms of t. So V can not be expressed in terms of ϕ explicitly. Now from the numerical investigations, we have plotted V against ϕ for some particular values of arbitrary constants ($\alpha=2$, $w=1/3$, $A=1$) in Fig. 2a. From the figure, we have seen that V is always decreases with ϕ.



(a)



(b)

Fig. 2 (**a**) and (**b**) show the variation of $V(\phi)$ against ϕ for barotropic fluid and Chaplygin gas respectively with normalizing the constants $\alpha=2$, $w=1/3$, $n=1/2$, $A=\gamma=B=1$, $C=D=0.1$

Now the equation of state for modified Chaplygin gas [23, 24] is given by

$$p=\gamma\rho-\frac{B}{\rho^n},\quad 0\le n\le 1 \tag{14}$$

where γ and B (>0) are constants. Then from (7) we have the expression for energy density as

$$\rho=\left[Ca^{-3(\gamma+1)(n+1)}+D\right]^{\frac{1}{n+1}} \tag{15}$$

where C (>0) and $D=\frac{B}{\gamma+1}>0$ are constants.

Solving (5) and (6) and using the relations (11), (14) and (15), we have

$$\dot{\phi}=\Bigg[2A^2\alpha(e^{A\alpha t}-1)^{-1}+2A^2\alpha(e^{A\alpha t}-1)^{-2}-\left\{C(e^{A\alpha t}-1)^{-\frac{3(1+A_0)(1+n)}{\alpha}}+D\right\}^{\frac{1}{n+1}}$$

$$\times \left\{1 + A_0 - B_0 \left(C(e^{A\alpha t} - 1)^{-\frac{3(1+A_0)(1+n)}{\alpha}} + D\right)^{-1}\right\}\Bigg]^{\frac{1}{2}} \tag{16}$$

and

$$\begin{aligned} V(\phi) &= 2A^2\alpha e^{A\alpha t}(e^{A\alpha t} - 1)^{-1} + 2A^2(3-\alpha)e^{2A\alpha t}(e^{A\alpha t} - 1)^{-2} \\ &\quad + \left\{C(e^{A\alpha t} - 1)^{-\frac{3(1+A_0)(1+n)}{\alpha}} + D\right\}^{\frac{1}{n+1}} \\ &\quad \times \left\{-1 + A_0 - B_0 \left(C(e^{A\alpha t} - 1)^{-\frac{3(1+A_0)(1+n)}{\alpha}} + D\right)^{-1}\right\} \end{aligned} \tag{17}$$

From (16), we have seen that it is not possible to find out the explicit form of ϕ in terms of t. So V can not be expressed in terms of ϕ explicitly. Now from the numerical investigations, we have plotted V against ϕ for some particular values of arbitrary constants ($\alpha = 2$, $n = 1/2$, $A = \gamma = B = 1$, $C = D = 0.1$) in Fig. 2b. From the figure, we have seen that V is always decreases with ϕ.

In 2003, V. Sahni et al. [25] have introduced a pair of parameters $\{r, s\}$, called statefinder parameters. In fact, trajectories in the $\{r, s\}$ plane corresponding to different cosmological models demonstrate qualitatively different behaviour. These parameters can effectively differentiate between different forms of dark energy and provide simple diagnostic regarding whether a particular model fits into the basic observational data. The above diagnostic pair has the following form:

$$r = \frac{\dddot{a}}{aH^3} \quad \text{and} \quad s = \frac{r-1}{3(q-\frac{1}{2})} \tag{18}$$

For our model, the parameters $\{r, s\}$ can be explicitly written in terms of a as

$$r = 1 + \frac{(\alpha^2 - 3a\alpha)}{(1+a^\alpha)} + \frac{\alpha^2}{(1+a^\alpha)^2}, \qquad s = \frac{2[\alpha^2 + (\alpha^2 - 3a\alpha)(1+a^\alpha)]}{3(1+a^\alpha)[2\alpha - 3(1+a^\alpha)]} \tag{19}$$

OR, the relation between r and s has the complicated form:

$$\frac{6\alpha s}{2(r-1)+9s} = 1 + \left[\frac{\alpha}{3} + \frac{2(r-1)+9s}{18s} - \frac{2s(r-1)}{2(r-1)+9s}\right]^\alpha \tag{20}$$

Fig. 3 Represents the variation of s against r for $\alpha = 2$

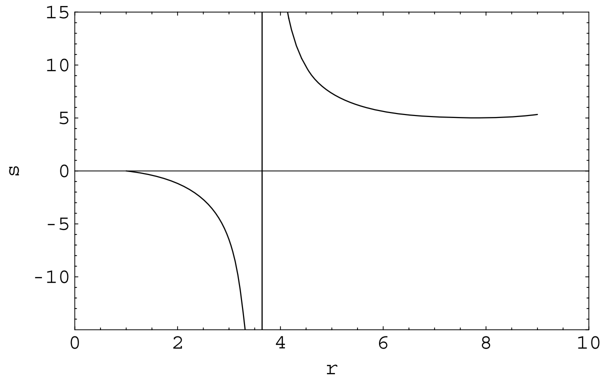


Figure 3 shows the variation of s with the variation of r for $\alpha = 2$. The curve in the positive side of s starts from radiation era and goes asymptotically to the dust model ($s \to +\infty$). But the portion in the negative side of s represents the evolution from dust state ($s \to -\infty$) to the ΛCDM ($r = 1, s = 0$) model. Thus the total curve represents the evolution of the universe starting from the radiation era to the ΛCDM model.

In this work, we have considered one type of form of deceleration parameter [21], so the quintessence problem can be solved in the presence scalar field with potential and barotropic fluid (or modified Chaplygin gas) when these dominate the Q-matter. For these models we have found that the potential function $V(\phi)$ always decreases with ϕ. The statefinder diagnostics generates that the universe starts from the radiation era to the ΛCDM model.

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