Int J Theor Phys (2008) 47: 2203–2217
DOI 10.1007/s10773-008-9652-y

Charged Particles' Tunneling from Hot-NUT-Kerr-Newman-Kasuya Spacetime

M. Hossain Ali

Received: 29 October 2007 / Accepted: 8 January 2008 / Published online: 17 January 2008

Abstract We study the Hawking radiation as charged particles' tunneling across the horizons of the Hot-NUT-Kerr-Newman-Kasuya spacetime by considering the spacetime background as dynamical and incorporating the self-gravitation effect of the emitted particles when the energy conservation, the angular momentum conservation, and the electric charge conservation are taken into account. Our result shows that the tunneling rate is related to the change of Bekenstein-Hawking entropy and the radiant spectrum is not pure thermal, but is consistent with an underlying unitary theory. The emission process is a reversible one, and the information is preserved as a natural result of the first law of black hole thermodynamics.

Keywords Semi-classical tunneling · Self-gravitation · Energy and charge conservation · Bekenstein-Hawking entropy

1 Introduction

The classical "no hair" theorem stated that all information about the collapsing body was lost from the outside region except the three conserved quantities: the mass, the angular momentum, and the electric charge. This loss of information was not a serious problem in the classical theory. Because the information could be thought of as preserved inside the black hole but just not very accessible. However, the situation is changed under the consideration of the quantum effect. Black holes could shrink and eventually evaporate away completely by emitting quantum thermal spectrum [1, 2]. Since radiation with a pure thermal spectrum can carry no information, the information carried by a physical system falling toward black hole singularity has no way to be recovered after a black hole has disappeared completely.

To my teacher late Prof. Mainuddin Ahmed.

M.H. Ali (✉)
Department of Applied Mathematics, Rajshahi University, Rajshahi 6205, Bangladesh
e-mail: m_hossain_ali_bd@yahoo.com

M.H. Ali
The Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, 34014 Trieste, Italy

This is the so-called "information loss paradox" [3, 4]. It means that pure quantum states (the original matter that forms the black hole) can evolve into mixed states (the thermal spectrum at infinity). This type of evolution violates the fundamental principles of quantum theory which prescribe a unitary time evolution of basis states.

The information paradox can perhaps be ascribed to the semi-classical nature of the investigations of Hawking radiation. However, researches in string theory indeed support the idea that Hawking radiation can be described within a manifestly unitary theory, but it still remains a mystery regarding the recovery of information. Although a complete resolution of the information loss paradox might be within a unitary theory of quantum gravity or string/M-theory, Hawking argued that the information could come out if the outgoing radiation were not pure thermal but had subtle corrections [4].

Besides, there is some degree of mystery in the mechanism of black hole radiation. In the original derivation of black hole evaporation, Hawking described the thermal radiation as a quantum tunneling process created by vacuum fluctuations near the event horizon [5]. But in this theory, the created mechanism of the tunneling barrier is unclear for us. The related references do not use the language of quantum tunneling method to discuss Hawking radiation, and hence it is not the quantum tunneling method. In order to derive the radiant spectrum from the black hole horizon, one must solve the two difficulties: firstly, the formed mechanism of the potential hill; and secondly, the elimination of the coordinate singularity.

Recently, Kraus and Wilczek [6–12] did the pioneer work for a program that implemented Hawking radiation as a tunneling process. Parikh and Wilczek [13–15] developed the program by carrying out a dynamical treatment of black hole radiance in the static spherically symmetric black hole geometries. More specifically, they took into account the effects of a positive energy matter shell propagating outwards through the horizon of the Schwarzschild and Reissner-Nordström black holes, and incorporated the self-gravitation correction of the radiation. In particular, they considered the energy conservation and allowed the background geometry to fluctuate in their dynamical description of the black hole background. In this model, they allowed the black hole to lose mass while radiating, but maintained a constant energy for the total system (consisting of the black hole and the surrounding). The self-gravitation action among the particles creates the tunneling barrier with turning points at the location of the black hole horizon before and after the particle with energy emission. The radiation spectrum that they derived for the Schwarzschild and Reissner-Nordström black holes gives a leading-order correction to the emission rate arising from loss of mass of the black hole, which corresponds to the energy carried by the radiated quantum. This result displays that the radiant spectrum of the black hole is not pure thermal under the consideration of energy conservation and the unfixed spacetime background. This may be a correct amendment of the Hawking radiation spectrum.

In addition to the consideration of the energy conservation and the particle's self-gravitation, a crucial point in the analysis of Kraus-Parikh-Wilczek (KPW) is to introduce a coordinate system that is well-behaved at the event horizon in order to calculate the emission probability. In this regard, the so-called "Painlevé-Gullstrand coordinates" rediscovered by Kraus and Wilczek [16] are not only time independent and regular at the horizon, but for which time reversal is manifestly asymptotic, that is, the coordinates are stationary but not static. Following this method, a lot of people [17–30] investigated Hawking radiation as tunneling from various spherically symmetric black holes, and the derived results are very successful to support KPW's picture. However, all these studies are limited to the spherically symmetric black holes and most of them are confined only to investigate the tunneling effect of the uncharged massless particles. There are some attempts to extend this model to the case of the stationary axisymmetric geometries [31–39]. Recently, following KPW's approach, some authors investigated the massive charged particles' tunneling from the static spherically symmetric [40–44] as well as stationary axisymmetric (e.g., Kerr-Newman black hole [45, 46]) geometries. They all got a satisfying result.

In this paper we apply KPW's method to a more general spacetime. We calculate the emission rate of a charged massive particle from stationary axisymmetric Kerr-Newman black hole spacetime in the de Sitter universe endowed with NUT (magnetic mass) and magnetic monopole parameters. The metric of the spacetime can be written as

$$\begin{aligned} ds^2 = \frac{\Sigma}{\Delta_\theta} d\theta^2 + \frac{\Sigma}{\Delta_r} dr^2 - \frac{\Delta_r}{\Sigma}\left(dt_{HNK} - \frac{\mathcal{A}}{\chi} d\varphi\right)^2 \\ + \frac{\Delta_\theta \sin^2\theta}{\Sigma}\left(a\, dt_{HNK} - \frac{\rho}{\chi} d\varphi\right)^2, \end{aligned} \tag{1}$$

where

$$\begin{aligned} \Sigma = r^2 + (n + a\cos\theta)^2, \quad \Delta_\theta = 1 + \frac{a^2}{\ell^2}\cos^2\theta, \quad \ell^2 = \frac{3}{\Lambda}, \\ \Delta_r = \rho\left[1 - \frac{1}{\ell^2}(r^2 + 5n^2)\right] - 2(Mr + n^2) + Q^2 + P^2, \\ \rho = r^2 + a^2 + n^2, \quad \chi = 1 + \frac{a^2}{\ell^2}, \quad \mathcal{A} = a\sin^2\theta - 2n\cos\theta, \end{aligned} \tag{2}$$

t_{HNK} being the coordinate time of the spacetime. Beside the cosmological parameter Λ, the metric (1) possesses five parameters: M the mass parameter, a the angular momentum per unit mass parameter, n the NUT (magnetic mass) parameter, Q the electric charge parameter, and P the magnetic monopole parameter. The metric (1) solves the Einstein-Maxwell field equations with an electromagnetic vector potential

$$A = -\frac{Qr}{\sqrt{\Sigma\Delta_r}} e^0 - \frac{P\cos\theta}{\sqrt{\Sigma\Delta_\theta}\sin\theta} e^3, \tag{3}$$

and an associated field strength tensor given by

$$\begin{aligned} F = -\frac{1}{\Sigma^2}\left[Q(r^2 - a^2\cos^2\theta) + 2Pra\cos\theta\right] e^0 \wedge e^1 \\ + \frac{1}{\Sigma^2}\left[P(r^2 - a^2\cos^2\theta) - 2Qra\cos\theta\right] e^2 \wedge e^3, \end{aligned} \tag{4}$$

where we have defined the vierbein field

$$\begin{aligned} e^0 = \sqrt{\frac{\Delta_r}{\Sigma}}\left(dt_{HNK} - \frac{\mathcal{A}}{\chi} d\varphi\right), \qquad e^1 = \sqrt{\frac{\Sigma}{\Delta_r}}\, dr, \\ e^2 = \sqrt{\frac{\Sigma}{\Delta_\theta}}\, d\theta, \qquad e^3 = \sqrt{\frac{\Delta_\theta}{\Sigma}}\sin\theta\left(a\, dt_{HNK} - \frac{\rho}{\chi} d\varphi\right). \end{aligned} \tag{5}$$

The metric (1) describes the NUT-Kerr-Newman-Kasuya-de Sitter spacetime. Since the de Sitter spacetime has been interpreted as being hot [47], we call the spacetime a Hot-NUT-Kerr-Newman-Kasuya (H-NUT-KN-K, for briefness) spacetime.

There is a renewed interest in the cosmological parameter as it is found to be present in the inflationary scenario of the early universe. In this scenario the universe undergoes a stage where it is geometrically similar to de Sitter space [48]. Among other things inflation has led to the cold dark matter. If the cold dark matter theory proves correct, it would shed light on the unification of forces [49, 50]. The monopole hypothesis was propounded by Dirac relatively long ago. The ingenious suggestion by Dirac that magnetic monopole does exist was neglected due to the failure to detect such object. However, in recent years, the development of gauge theories has shed new light on it.

The H-NUT-KN-K spacetime includes, among others, the physically interesting black hole spacetimes as well as the NUT spacetime which is sometimes considered as unphysical. The curious properties of the NUT spacetime induced Misner [51] to consider it "as a counter example to almost anything". This spacetime plays a significant role in exhibiting the type of effects that can arise in strong gravitational fields.

If we set $\ell \to \infty$, $a = 0$, $Q = 0 = P$ in (1), it then results the NUT metric which is singular along the axis of symmetry $\theta = 0$ and $\theta = \pi$. Because of the axial singularities the metric admits different physical interpretations. Misner [52] introduced a periodic time coordinate to remove the singularity, but this makes the metric an uninteresting particle-like solution. To avoid a periodic time coordinate, Bonnor [53] removed the singularity at $\theta = 0$ and related the singularity at $\theta = \pi$ to a semiinfinite massless source of angular momentum along the axis of symmetry. This is analogous to representing the magnetic monopole in electromagnetic theory by semiinfinite solenoid [54]. The singularity along z-axis is analogous to the Dirac string.

McGuire and Ruffini [55] suggested that the spaces endowed with the NUT parameter should never be directly physically interpreted. To make a physically reasonable solution Ahmed [56] used Bonnor's interpretation of the NUT parameter, i.e., the NUT parameter n is due to the strength of the physical singularity on $\theta = \pi$, and further considered that $n = a$. That means, the angular momentum of the mass M and the angular momentum of massless rod coalesce, and in this case, the metric (1) gives a new black hole solution which poses to solve an outstanding problem of thermodynamics and black hole physics.

In view of all the above considerations the work of this paper is interesting. Since we are investigating charged particles' tunneling from the charged H-NUT-KN-K spacetime, not only should the energy conservation but also the electric charge conservation be considered. In particular, two significant points of this paper are as follows. The first is that we need to find the equation of motion of a charged massive tunneling particle. We can treat the massive charged particle as a de Broglie wave, and then its equation of motion can be obtained by calculating the phase velocity of the de Broglie wave corresponding to the outgoing particle. Secondly, we should also consider the effect of the electromagnetic field outside the H-NUT-KN-K spacetime when a charged particle tunnels out. The Lagrangian function of the electromagnetic field corresponding to the generalized coordinates described by A_μ is $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$. But these are ignorable coordinates in dragged coordinate system. To eliminate the freedoms corresponding to these coordinates, we modify the Lagrangian function. Using WKB method we then derive the emission rate for a charged massive particle with revised Lagrangian function.

We organize the paper as follows. In Sect. 2 we introduce the Painlevé-H-NUT-KN-K coordinate system, and obtain the radial geodesic equation of a charged massive particle. In Sect. 3 we use KPW's tunneling framework to calculate the emission spectrum. Finally, in Sect. 4 we present our concluding remarks. Throughout the paper, the geometrized units $(G = c = \hbar = 1)$ have been used.

2 Painlevé-like Coordinate Transformation and the Radial Geodesics

The null surface equation $g^{\mu\nu}\partial_\mu f\partial_\nu f=0$ gives

$$r^4+(a^2+6n^2-\ell^2)r^2+2M\ell^2 r-\Xi=0, \tag{6}$$

where

$$\Xi=\left\{(a^2-n^2+Q^2+P^2)\ell^2-5n^2(a^2+n^2)\right\}. \tag{7}$$

Equation (6) has four real roots: one is negative root r_- and three roots r_0, r_H, r_C corresponding to the inner, outer (event) and cosmological horizon of the H-NUT-KN-K spacetime respectively, can be given by [57]

$$\begin{aligned} r_0&=-t_1+t_2+t_3,\\ r_H&=t_1-t_2+t_3,\\ r_C&=t_1+t_2-t_3, \end{aligned} \tag{8}$$

where

$$\begin{aligned} t_1&=\left[\frac{1}{6}(\ell^2-6n^2-a^2)+\frac{1}{6}\sqrt{\{(\ell^2-6n^2-a^2)^2-12\Xi\}}\cos\frac{\psi}{3}\right]^{\frac{1}{2}},\\ t_2&=\left[\frac{1}{6}(\ell^2-6n^2-a^2)-\frac{1}{6}\sqrt{\{(\ell^2-6n^2-a^2)^2-12\Xi\}}\cos\frac{\psi+\pi}{3}\right]^{\frac{1}{2}},\\ t_3&=\left[\frac{1}{6}(\ell^2-6n^2-a^2)-\frac{1}{6}\sqrt{\{(\ell^2-6n^2-a^2)^2-12\Xi\}}\cos\frac{\psi-\pi}{3}\right]^{\frac{1}{2}}, \end{aligned} \tag{9}$$

$$\cos\psi=-\frac{(\ell^2-6n^2-a^2)\{(\ell^2-6n^2-a^2)^2+36\Xi\}-54M^2\ell^4}{\{(\ell^2-6n^2-a^2)^2-12\Xi\}^{3/2}}, \tag{10}$$

under the conditions

$$\begin{aligned} &\{(\ell^2-6n^2-a^2)^2-12\Xi\}^3>\{(\ell^2-6n^2-a^2)^3+36\Xi(\ell^2-6n^2-a^2)-54M^2\ell^4\}^2,\\ &\qquad(\ell^2-6n^2-a^2)>0, \end{aligned} \tag{11}$$

Ξ being given by (7).

To investigate the tunneling process, we should adopt the dragged coordinate system. The line element in the dragging coordinate system is [57]

$$ds^2=\hat{g}_{00}dt_d^2+\frac{\Sigma}{\Delta_r}dr^2+\frac{\Sigma}{\Delta_\theta}d\theta^2, \tag{12}$$

where

$$\hat{g}_{00}=g_{00}-\frac{(g_{03})^2}{g_{33}}=-\frac{\Delta_\theta\Delta_r(\rho-a\mathcal{A})^2\sin^2\theta}{\Sigma(\Delta_\theta\rho^2\sin^2\theta-\Delta_r\mathcal{A}^2)}. \tag{13}$$

In fact, the line element (12) represents a three-dimensional hypersurface in the four-dimensional H-NUT-KN-K spacetime. The components of the electromagnetic potential in the dragged coordinate system can be given by

$$A_0'=A_a(\partial_{t_d})^a=-\frac{Qr}{\Sigma}\left(1-\frac{\mathcal{A}}{\chi}\Omega\right)-\frac{P\cos\theta}{\Sigma}\left(a-\frac{\rho}{\chi}\Omega\right),\qquad A_1'=0=A_2', \tag{14}$$

where

$$(\partial_{t_d})^a = (\partial_{t_{HNK}})^a + \Omega(\partial_\varphi)^a, \tag{15}$$

$\Omega = -g_{03}/g_{33}$ being the dragged angular velocity. The metric (12) has a coordinate singularity at the horizon, which brings us inconvenience to investigate the tunneling process across the horizon.

In order to eliminate the coordinate singularity from the metric (12), we perform general Painlevé coordinate transformation [58]

$$\mathrm{d}t_d = \mathrm{d}t + F(r,\theta)\mathrm{d}r + G(r,\theta)\mathrm{d}\theta, \tag{16}$$

which reduces the metric in the Painlevé-H-NUT-KN-K coordinate system [57]

$$\begin{aligned} \mathrm{d}s^2 = {} & \hat{g}_{00}\mathrm{d}t^2 + \mathrm{d}r^2 \pm 2\sqrt{\hat{g}_{00}(1-g_{11})}\,\mathrm{d}t\mathrm{d}r + \left[\hat{g}_{00}\{G(r,\theta)\}^2 + g_{22}\right]\mathrm{d}\theta^2 \\ & + 2\sqrt{\hat{g}_{00}(1-g_{11})}G(r,\theta)\mathrm{d}r\mathrm{d}\theta + 2\hat{g}_{00}G(r,\theta)\mathrm{d}t\mathrm{d}\theta, \end{aligned} \tag{17}$$

where $F(r,\theta)$ satisfies

$$g_{11} + \hat{g}_{00}\{F(r,\theta)\}^2 = 1, \tag{18}$$

and $G(r,\theta)$ is determined by

$$G(r,\theta) = \int \frac{\partial F(r,\theta)}{\partial\theta}\mathrm{d}r + C(\theta), \tag{19}$$

where $C(\theta)$ is an arbitrary analytic function of θ. The plus (minus) sign in (17) denotes the spacetime line element of the charged massive outgoing (ingoing) particles at the horizon.

According to Landau's theory of the coordinate clock synchronization [59] in a spacetime decomposed in $(3+1)$, the coordinate time difference of two events which take place simultaneously in different locations, is

$$\Delta T = -\int \frac{g_{0i}}{g_{00}}\mathrm{d}x^i \quad (i = 1,2,3). \tag{20}$$

If the simultaneity of coordinate clocks can be transmitted from one location to another and has nothing to do with the integration path, then

$$\frac{\partial}{\partial x^i}\left(-\frac{g_{0j}}{\hat{g}_{00}}\right) = \frac{\partial}{\partial x^j}\left(-\frac{g_{0i}}{\hat{g}_{00}}\right) \quad (i,j = 1,2,3). \tag{21}$$

Condition (21) with the metric (17) gives $\partial_\theta F(r,\theta) = \partial_r G(r,\theta)$, which is the condition (19). Thus the Painlevé-H-NUT-KN-K metric (17) satisfies the condition of coordinate clock synchronization. Apart from that, the new line element has many other attractive features: firstly, the metric is regular at the horizons; secondly, the infinite red-shift surface and the horizons are coincident with each other; thirdly, spacetime is stationary; and fourthly, constant-time slices are just flat Euclidean space in radial. All these characteristics would provide superior condition to the quantum tunneling radiation.

The component of the electromagnetic potential in the Painlevé-H-NUT-KN-K coordinate system is

$$A_0 = -\frac{Qr}{\Sigma}\left(1 - \frac{\mathcal{A}}{\chi}\Omega\right) - \frac{P\cos\theta}{\Sigma}\left(a - \frac{\rho}{\chi}\Omega\right), \quad A_1 = 0 = A_2, \tag{22}$$

which on the event horizon becomes

$$A_0|_{r_H} = -V_0 = -\frac{Qr_H}{r_H^2 + a^2 + n^2}, \quad A_1 = 0 = A_2. \tag{23}$$

Now let us derive the geodesics for the charged massive particles. Since the world-line of a massive charged quanta is not light-like, it does not follow radial light-like geodesics when it tunnels across the horizon. For the sake of simplicity, we consider the outgoing massive charged particle as a massive charged shell (de Broglie s-wave). According to the WKB formula, the approximative wave function is

$$\phi(r,t) = N\exp\left[\mathrm{i}\left(\int_{r_i-\varepsilon}^{r} p_r \mathrm{d}r - \omega t\right)\right], \tag{24}$$

where $r_i - \varepsilon$ denotes the initial location of the particle. If

$$\int_{r_i-\varepsilon}^{r} p_r \mathrm{d}r - \omega t = \phi_0, \tag{25}$$

then, we have

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \dot{r} = \frac{\omega}{k}, \tag{26}$$

where k is the de Broglie wave number. By definition, $\dot{r}$ in (26) is the phase velocity of the de Broglie wave. Unlike the electromagnetic wave, the phase velocity v_p of the de Broglie wave is not equal to the group velocity v_g. They have the following relationship

$$v_p = \frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\omega}{k}, \qquad v_g = \frac{\mathrm{d}r_c}{\mathrm{d}t} = \frac{\mathrm{d}\omega}{\mathrm{d}k} = 2v_p, \tag{27}$$

where r_c denotes the location of the tunneling particle. Since tunneling across the barrier is an instantaneous process, there are two events that take place simultaneously in different places during the process. One is the particle tunneling into the barrier, and the other is the particle tunneling out the barrier. In terms of Landau's condition of coordinate clock synchronization, the coordinate time difference of these two simultaneous events is

$$\mathrm{d}t = -\frac{g_{0i}}{g_{00}}\mathrm{d}x^i = -\frac{g_{01}}{g_{00}}\mathrm{d}r_c \quad (\mathrm{d}\theta = 0 = \mathrm{d}\varphi). \tag{28}$$

So the group velocity is

$$v_g = \frac{\mathrm{d}r_c}{\mathrm{d}t} = -\frac{g_{00}}{g_{01}},$$

and therefore, using (17), the phase velocity (the radial geodesics) can be expressed as

$$\dot{r} = v_p = \frac{1}{2}v_g = \mp\frac{\Delta_r}{2}\left[\frac{\Delta_\theta(\rho - a\mathcal{A})^2\sin^2\theta}{\Sigma(\Sigma-\Delta_r)(\Delta_\theta\rho^2\sin^2\theta - \Delta_r\mathcal{A}^2)}\right]^{\frac{1}{2}}, \tag{29}$$

where the upper sign corresponds to the geodesic of the outgoing particle near the event horizon, and the lower sign corresponds to that of the ingoing particle near the cosmological horizon. Moreover, if we take into account the self-gravitation of the tunneling particle with energy ω and electric charge q, then M and Q should be replaced with $M \mp \omega$ and $Q \mp q$ in

(17) and (29), respectively, with the upper (lower) sign corresponding to outgoing (ingoing) motion of the particle.

In the subsequent section, we shall discuss Hawking radiation from the event and cosmological horizons, and calculate the emission rate from each horizon by tunneling process. Sine the overall picture of tunneling radiation for the metric is very involved, we shall consider for simplification the outgoing radiation from the event horizon and ignore the incoming radiation from the cosmological horizon, when we deal with the event horizon. In the similar manner, we shall only consider the incoming radiation from the cosmological horizon and ignore the outgoing radiation from the event horizon for the moment when we deal with the cosmological horizon. Of course, this assumption is reasonable as long as the two horizons separate away very large from each other. The radius of the cosmological horizon is very large due to a very small cosmological constant Λ, while the event horizon considered here is relatively very small because the Hawking radiation can take an important effect only for tiny black hole typical of $1 \sim 10$ TeV energy in the brane-world scenario. Hawking radiation is a kind of quantum effect. It can be neglected and may not be observed for an astronomical black hole with typical star mass about 10 $M_\odot$.

3 Tunneling Process of Massive Charged Particles from H-NUT-KN-K Spacetime

In the investigation of charged massive particles' tunneling, the effect of the electromagnetic field outside the black hole should be taken into consideration. So the matter-gravity system consists of the black hole and the electromagnetic field outside the black hole. The Lagrangian of the matter-gravity system can be written as

$$L = L_m + L_e, \tag{30}$$

where $L_e = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ is the Lagrangian function of the electromagnetic field corresponding to the generalized coordinate described by $A_\mu = (A_t, 0, 0)$ in the Painlevé-H-NUT-KN-K coordinate system. In the case of a charged particle's tunneling out, the system transits from one state to another. But the expression of L_e tells us that $A_\mu = (A_t, 0, 0)$ is an ignorable coordinate. Furthermore, in the dragging coordinate system, the coordinate φ does not appear in the metric expressions (12) and (17). That is, φ is also an ignorable coordinate in the Lagrangian function L. In order to eliminate these two degrees of freedom completely, the action for the classically forbidden trajectory should be written as

$$S = \int_{t_i}^{t_f} (L - p_{A_t} \dot{A}_t - p_\varphi \dot{\varphi}) \mathrm{d}t, \tag{31}$$

which is related to the tunneling rate of the emitted particle by

$$\Gamma \sim e^{-2 \operatorname{Im} S}. \tag{32}$$

The imaginary part of the action is

$$\begin{aligned} \operatorname{Im} S &= \operatorname{Im} \left\{ \int_{r_i}^{r_f} \left[p_r - \frac{\dot{A}_t}{\dot{r}} p_{A_t} - \frac{\dot{\varphi}}{\dot{r}} p_\varphi \right] \mathrm{d}r \right\} \\ &= \operatorname{Im} \left\{ \int_{r_i}^{r_f} \left[\int_{(0,0,0)}^{(p_r, p_{A_t}, p_\varphi)} \mathrm{d}p'_r - \frac{\dot{A}_t}{\dot{r}} \mathrm{d}p'_{A_t} - \frac{\dot{\varphi}}{\dot{r}} \mathrm{d}p'_\varphi \right] \mathrm{d}r \right\}, \end{aligned} \tag{33}$$

where p_{A_t} and p_φ are the canonical momenta conjugate to A_t and φ, respectively.

3.1 Tunneling from the Event Horizon

If the black hole is treated as a rotating sphere and the particle self-gravitation is taken into account, one then finds

$$\dot{\varphi} = \Omega'_H, \tag{34}$$

and

$$J' = (M - \omega')a = p'_{\varphi}, \tag{35}$$

where Ω'_H is the dragged angular velocity of the event horizon. The imaginary part of the action for the charged massive particle can be written as

$$\mathrm{Im}\, S = \mathrm{Im}\left\{\int_{r_{Hi}}^{r_{Hf}} \left[\int_{(0,0,J)}^{(p_r, p_{A_t}, J-\omega a)} \mathrm{d}p'_r - \frac{\dot{A}_t}{\dot{r}}\mathrm{d}p'_{A_t} - \frac{\Omega'_H}{\dot{r}}\mathrm{d}J'_{\varphi}\right]\mathrm{d}r\right\}, \tag{36}$$

where r_{Hi} and r_{Hf} represent the locations of the event horizon before and after the particle with energy ω and charge q tunnels out. We now eliminate the momentum in favor of energy by applying Hamilton's canonical equations of motion

$$\dot{r} = \left.\frac{\mathrm{d}H}{\mathrm{d}p_r}\right|_{(r;A_t,p_{A_t};\varphi,p_{\varphi})} = \frac{1}{\chi^2}\frac{\mathrm{d}(M-\omega')}{\mathrm{d}p_r} = \frac{1}{\chi^2}\frac{\mathrm{d}M'}{\mathrm{d}p_r}, \tag{37}$$

$$\dot{A}_t = \left.\frac{\mathrm{d}H}{\mathrm{d}p_{A_t}}\right|_{(A_t;r,p_r;\varphi,p_{\varphi})} = \frac{1}{\chi}V'_0\frac{\mathrm{d}(Q-q')}{\mathrm{d}p_{A_t}} = \frac{1}{\chi^2}\frac{(Q-q')r_H}{r_H^2+a^2+n^2}\frac{\mathrm{d}(Q-q')}{\mathrm{d}p_{A_t}}, \tag{38}$$

where $\frac{M}{\chi^2}$ is the total energy and $\frac{Q}{\chi}$ is the total electric charge of the H-NUT-KN-K space-time, and we have treated the black hole as a charged conducting sphere to derive (38) [60].

It follows, similar to [9, 10], directly that a massive charged particle tunneling across the event horizon also sees the effective metric (17), but with the replacements $M \to M - \omega'$ and $Q \to Q - q'$. We have to perform the same substitutions in (8), (23) and (29). Equation (29) then gives the desired expression of $\dot{r}$ as a function of ω' and q'. Equation (36) can now be written explicitly as follows:

$$\begin{aligned}\mathrm{Im}\, S = \mathrm{Im}\int_{r_{Hi}}^{r_{Hf}} \Bigg[\int -\frac{1}{\chi^2}\frac{2\sqrt{\Sigma(\Sigma-\Delta'_r)(\Delta_\theta\rho^2\sin^2\theta - \Delta'_r\mathcal{A}^2)}}{\Delta'_r\sqrt{\Delta_\theta(\rho-a\mathcal{A})^2\sin^2\theta}} \\ \times\left(\mathrm{d}M' - \frac{Q'r'_H}{r'^2_H+a^2+n^2}\mathrm{d}Q' - \Omega'_H\mathrm{d}J'\right)\Bigg]\mathrm{d}r,\end{aligned} \tag{39}$$

where

$$\begin{aligned}\Delta'_r &= (r^2+a^2+n^2)\left[1-\frac{1}{\ell^2}(r^2+5n^2)\right] - 2(M'r+n^2) + Q'^2 + P^2 \\ &= \frac{1}{\ell^2}(r-r'_-)(r-r'_0)(r-r'_H)(r-r'_C).\end{aligned} \tag{40}$$

The above integral can be evaluated by deforming the contour around the single pole at $r = r'_H$ so as to ensure that positive energy solution decay in time. In this way, we finish the

r integral and obtain

$$\operatorname{Im} S = -\frac{1}{2}\int_{(\frac{M}{\chi^2},\frac{Q}{\chi})}^{(\frac{M-\omega}{\chi^2},\frac{Q-q}{\chi})} \frac{1}{\chi}\frac{4\pi\ell^2(r_H'^2+a^2+n^2)}{(r_H'-r_-')(r_H'-r_0')(r_C'-r_H')} \times\left(\mathrm{d}M' - \frac{Q'r_H'}{r_H'^2+a^2+n^2}\mathrm{d}Q' - \Omega_H'\mathrm{d}J'\right). \tag{41}$$

Completing this integration and using the entropy expression $S_{EH} = \pi(r_H^2+a^2+n^2)/\chi$, we obtain

$$\operatorname{Im} S = -\frac{1}{2}\Delta S_{EH}, \tag{42}$$

where $\Delta S_{EH} = S_{EH}' - S_{EH}$ is the difference of Bekenstein-Hawking entropies of the H-NUT-KN-K spacetime before and after the emission of the particle. In fact, if one bears in mind that

$$T' = \frac{(r_H'-r_-')(r_H'-r_0')(r_C'-r_H')}{4\pi\ell^2(r_H'^2+a^2+n^2)}, \tag{43}$$

one can get

$$\frac{1}{T'}(\mathrm{d}M' - V_0'\mathrm{d}Q' - \Omega_H'\mathrm{d}J') = \mathrm{d}S'. \tag{44}$$

That means, (42) is a natural result of the first law of black hole thermodynamics. Therefore, the emission rate of the tunneling particle is

$$\Gamma \sim e^{-2\operatorname{Im} S} = e^{\Delta S_{EH}}. \tag{45}$$

Obviously, the emission spectrum (45) deviates from the thermal spectrum.

In quantum mechanics, the tunneling rate is obtained by

$$\Gamma(i \to f) \sim |a_{if}|^2 \cdot \alpha_n, \tag{46}$$

where a_{if} is the amplitude for the tunneling action and $\alpha_n = n_f/n_i$ is the phase space factor with n_i and n_f being the number of the initial and final states, respectively. Since $S_j \sim \ln n_j$, i.e., $n_j \sim e^{S_j}$ $(j = i, f)$, then

$$\Gamma \sim \frac{e^{S_f}}{e^{S_i}} = e^{S_f - S_i} = e^{\Delta S}. \tag{47}$$

Equation (47) is consistent with our result obtained by applying the KPW's semi-classical quantum tunneling process. Hence (45) satisfies the underlying unitary theory in quantum mechanics, and takes the same functional form as that of uncharged massless particles [57].

3.2 Tunneling at the Cosmological Horizon

The particle is found tunneled into the cosmological horizon differently from the particle's tunneling behavior of the event horizon. When the particle with energy ω and charge q tunnels into the cosmological horizon, (8), (17), (23) and (29) should have to modify by replacing M with $(M+\omega)$ and Q with $(Q+q)$ after taking the self-gravitation action into account. Thus, after tunneling the particle with energy ω and charge q into the cosmological horizon, the radial geodesic takes the form

$$\dot{r} = \frac{\Delta_r''}{2}\left[\frac{\Delta_\theta(\rho - a\mathcal{A})^2\sin^2\theta}{\Sigma(\Sigma - \Delta_r'')(\Delta_\theta\rho^2\sin^2\theta - \Delta_r''\mathcal{A}^2)}\right]^{\frac{1}{2}}, \tag{48}$$

where

$$\Delta_r'' = (r^2+a^2+n^2)\left[1 - \frac{1}{\ell^2}(r^2+5n^2)\right] - 2\left\{(M+\omega)r + n^2\right\} + (Q+q)^2 + P^2.$$

Different from the event horizon, $(-\frac{M}{\chi^2}, -\frac{Q}{\chi})$ and $(-\frac{M+\omega}{\chi^2}, -\frac{Q+q}{\chi})$ are, respectively, the total mass and electric charge of the H-NUT-KN-K spacetime before and after the particle with energy ω and charge q tunnels into. We treat the spacetime as a charged conducting sphere.

The imaginary part of the action for the charged massive particle incoming from the cosmological horizon can be expressed as follows:

$$\begin{aligned}\operatorname{Im} S = \operatorname{Im}\int_{r_{Ci}}^{r_{Cf}}\Bigg[\int -\frac{1}{\chi^2}\frac{2\sqrt{\Sigma(\Sigma - \tilde{\Delta}_r'')(\Delta_\theta\rho^2\sin^2\theta - \tilde{\Delta}_r''\mathcal{A}^2)}}{\tilde{\Delta}_r''\sqrt{\Delta_\theta(\rho - a\mathcal{A})^2\sin^2\theta}} \\ \times\left(\mathrm{d}M' - \frac{Q'r_C'}{r_C'^2 + a^2 + n^2}\mathrm{d}Q' - \Omega_C'\mathrm{d}J'\right)\Bigg]\mathrm{d}r,\end{aligned} \tag{49}$$

where

$$\begin{aligned}\tilde{\Delta}_r'' &= (r^2+a^2+n^2)\left[1 - \frac{1}{\ell^2}(r^2+5n^2)\right] - 2(M'r+n^2) + Q'^2 + P^2 \\ &= \frac{1}{\ell^2}(r - r_-')(r - r_0')(r - r_H')(r - r_C'),\end{aligned}$$

r_{Ci} and r_{Cf} are the locations of the cosmological horizon before and after the particle of energy ω and charge q is tunneling into. There exists a single pole at the cosmological horizon in (49). Carrying out the r integral, we have

$$\begin{aligned}\operatorname{Im} S &= -\frac{1}{2}\int_{(-\frac{M}{\chi^2}, -\frac{Q}{\chi})}^{(-\frac{M+\omega}{\chi^2}, -\frac{Q+q}{\chi})}\frac{1}{\chi}\frac{4\pi\ell^2(r_C'^2 + a^2 + n^2)}{(r_C' - r_-')(r_C' - r_0')(r_C' - r_H')} \\ &\quad\times\left(\mathrm{d}M' - \frac{Q'r_C'}{r_C'^2 + a^2 + n^2}\mathrm{d}Q' - \Omega_C'\mathrm{d}J'\right) \\ &= -\frac{1}{2}\Delta S_{CH},\end{aligned} \tag{50}$$

where ΔS_{CH} is the change in Bekenstein-Hawking entropy during the process of emission. The tunneling rate from the cosmological horizon is therefore

$$\Gamma \sim e^{-2\operatorname{Im} S} = e^{\Delta S_{CH}}. \tag{51}$$

It also deviates from the pure thermal spectrum and is consistent with the underlying unitary theory, and takes the same functional form as that of uncharged massless particles [57].

4 Concluding Remarks

In this paper we present our investigation of tunneling radiation characteristics of massive charged particles from a more general spacetime, namely, the NUT-Kerr-Newman-Kasuya-de Sitter spacetime, which we call the Hot-NUT-Kerr-Newman-Kasuya (H-NUT-KN-K, for briefness) spacetime, since the de Sitter spacetime has the interpretation of being hot [47]. We apply KPW's framework [6–15] to calculate the emission rate at the event/cosmological horizon. We first introduce a simple but useful Painlevé coordinate [58] which transforms the line element in a convenient form with having many superior features in favor of our study. Secondly, we treat the charged massive particle as a de Broglie wave, and derive the equation of motion by computing the phase velocity. Thirdly, we take into account the particle's self-gravitation and treat the background spacetime as dynamical. Then the energy conservation and the angular momentum conservation as well as the electric charge conservation are enforced in a natural way. Adapting this tunneling picture we were able to compute the tunneling rate and the radiant spectrum for a massive charged particle with revised Lagrangian function and WKB approximation. The result displays that tunneling rate is related to the change of Bekenstein-Hawking entropy and depends on the emitted particle's energy and electric charge. Meanwhile, this implies that the emission spectrum is not perfect thermal but is in agreement with an underlying unitary theory.

The result obtained by us reduces to the Kerr-Newman black hole case for $\ell \to \infty$, $P = 0 = n$, and gives the result of Zhang et al. [45]. For $\ell \to \infty$, $a = 0 = n$, our result reduces to that of the Reissner-Nordström black hole, as was obtained in [29]. Moreover, by suitably choosing the parameters of the spacetime, the result of this paper can be specialized for all the interesting black hole spacetimes, de Sitter spacetimes as well as the NUT spacetime which has curious properties [51]. The NUT spacetime is a generalization of the Schwarzschild spacetime and plays an important role in the conceptual development of general relativity and in the construction of brane solutions in string theory and M-theory [61–63]. The NUT spacetime has been of particular interest in recent years because of the role it plays in furthering our understanding of the AdS-CFT correspondence [64–66]. Solutions of Einstein equations with negative cosmological constant Λ and a nonvanishing NUT charge have a boundary metric that has closed timelike curves. The behavior of quantum field theory is significantly different in such spacetimes. It is of interest to understand how AdS-CFT correspondence works in these sorts of cases [67]. Our result can directly be extended to the AdS case (as was obtained in Taub-NUT-AdS spacetimes in [68]) by changing the sign of the cosmological parameter ℓ^2 to a negative one. In view of these attractive features, the study of this paper is interesting.

Our study indicates that the emission process satisfies the first law of black hole thermodynamics, which is, in fact, a combination of the energy conservation law: $\mathrm{d}M - V_0\mathrm{d}Q - \Omega_H \mathrm{d}J = \mathrm{d}Q_h$ and the second law of thermodynamics: $\mathrm{d}S = \mathrm{d}Q_h/T$, Q_h being the heat quantity. Indeed, the equation of energy conservation is suitable for any process, reversible or irreversible, but $\mathrm{d}S = \mathrm{d}Q_h/T$ is only reliable for the reversible process; for an irreversible process, $\mathrm{d}S > \mathrm{d}Q_h/T$. The emission process in KPW tunneling framework is thus an reversible one. In this picture, the background spacetime and the outside approach an thermal equilibrium by the process of entropy flux $\mathrm{d}S = \mathrm{d}Q_h/T$. As the H-NUT-KN-K spacetime radiates, its entropy decreases but the total entropy of the system remains constant, and the information is preserved. But in fact, the existence of the negative heat capacity makes an evaporating black hole a highly unstable system, and the thermal equilibrium between the black hole and the outside becomes unstable, there will exist difference in temperature. Then the process is irreversible, $\mathrm{d}S > \mathrm{d}Q_h/T$, and the underlying unitary theory is not satisfied.

There will be information loss during the evaporation and the KPW's tunneling framework will fail to prove the information conservation. Further, the preceding study is still a semi-classical analysis—the radiation is treated as point particles. The validity of such an approximation can only exist in the low energy regime. To properly address the information loss problem, a better understanding of physics at the Planck scale is a necessary prerequisite, especially that of the last stages or the endpoint of Hawking evaporation.

Using the interesting method of complex paths Shankaranarayanan et al. [69–71] investigated the Hawking radiation by tunneling approach, considering the amplitude for pair creation both inside and outside the horizon. In their formalism the tunneling of particles produced just inside the horizon also contributes to the Hawking radiation.

Akhmedov et al. [72, 73] investigated Hawking radiation in the quasi-classical tunneling picture by the Hamilton-Jacobi equations, using $\Gamma \propto \exp\{\mathrm{Im}(\oint p\,\mathrm{d}r)\}$ [74], rather than $\Gamma \propto \exp\{2\,\mathrm{Im}(\int p\,\mathrm{d}r)\}$, and argued that the former expression for Γ is correct since $\oint p\,\mathrm{d}r$ is invariant under canonical transformation, while $\int p\,\mathrm{d}r$ is not. According to their argument the temperature of the Hawking radiation should be twice as large as originally calculated.

Wu et al. [75] studied the tunneling effect near a weakly isolated horizon [76] by applying the null geodesic method of KPW and the Hamilton-Jacobi method [31], both lead to the same result. However, there are subtle differences, e.g., in KPW's method, only the canonical time direction can define the horizon mass and lead to the first law of black hole mechanics, while the thermal spectrum exists for any choice of time direction in the Hamilton-Jacobi method. Berezin et al. [77] used a self-consistent canonical quantization of self-gravitating spherical shell to describe Hawking radiation as tunneling. Their work is analogous to KPW but due to the fact that they took into account back reaction of the shell on the metric they did not have a singular potential at r_g (it is smoothed in their case between r_{in} and r_{out}) and the use of the semi-classical approximation to describe tunneling seems more justified.

Since the discovery of the first exact solution of Einstein's field equations, the studying property of black holes is always a highlight of gravitational physics. Providing mechanisms to fuel the most powerful engines in the cosmos, black holes are playing a major role in relativistic astrophysics. Indeed, the famous Hawking radiation from the event horizon of black holes is one of the most important achievements of quantum field theory in curved space-times. In fact, due to Hawking evaporation classical general relativity, statistical physics, and quantum field theory are connected in quantum black hole physics. It is generally believed that the deep investigation of black hole physics would be helpful to set up a satisfactory quantum theory of gravity. In view of this, tunneling process of Hawking radiation deserves more investigations in a wider context.

Acknowledgement I am thankful to SIDA as well as the Abdus Salam International Centre for Theoretical Physics, Trieste, Italy, where this paper was produced during my Associateship visit.

References

1. Hawking, S.W.: Nature (Lond.) **248**, 30 (1974)
2. Hawking, S.W.: Commun. Math. Phys. **43**, 199 (1975)
3. Hawking, S.W.: Phys. Rev. D **14**, 2460 (1976)
4. Hawking, S.W.: Phys. Rev. D **72**, 084013 (2005)
5. Hartle, J.B., Hawking, S.W.: Phys. Rev. D **13**, 2188 (1976)
6. Kraus, P.: Nucl. Phys. B **425**, 615 (1994)
7. Kraus, P., Wilczek, F.: Nucl. Phys. B **433**, 403 (1995)
8. Kraus, P., Wilczek, F.: Nucl. Phys. B **437**, 231 (1995)
9. Keski-Vakkuri, E., Kraus, P.: Phys. Rev. D **54**, 7407 (1996)

10. Keski-Vakkuri, E., Kraus, P.: Nucl. Phys. B **491**, 249 (1997)
11. Parikh, M.K., Wilczek, F.: Phys. Rev. D **58**, 064011 (1998)
12. Zaidi, H., Gegenberg, J.: Phys. Rev. D **57**, 1112 (1998)
13. Parikh, M.K., Wilczek, F.: Phys. Rev. Lett. **85**, 5042 (2000). hep-th/9907001
14. Parikh, M.K.: Int. J. Mod. Phys. D **13**, 2351 (2004). hep-th/0405160
15. Parikh, M.K.: Gen. Relativ. Gravit. **36**, 2419 (2004). hep-th/0402166
16. Kraus, P., Wilczek, F.: Mod. Phys. Lett. A **9**, 3713 (1994)
17. Hemming, S., Keski-Vakkuri, E.: Phys. Rev. D **64**), 044006 (2001)
18. Medved, A.J.M.: Phys. Rev. D **66**, 124009 (2002)
19. Parikh, M.K.: Phys. Lett. B **546**, 189 (2002)
20. Arzano, M., Medved, A.J.M., Vagenas, E.C.: J. High Energy Phys. **09**, 037 (2005). hep-th/0505266
21. Vagenas, E.C.: Phys. Lett. B **559**, 65 (2003). hep-th/0209185
22. Radinschi, I.: The KKW Generalized Analysis for a Magnetic Stringy Black Hole, gr-qc/0412111
23. Arzano, M.: Mod. Phys. Lett. A **21**, 41 (2006). hep-th/0504188
24. Vagenas, E.C.: Phys. Lett. B **503**, 399 (2001). hep-th/0012134
25. Vagenas, E.C.: Mod. Phys. Lett. A **17**, 609 (2002). hep-th/0108147
26. Medved, A.J.M.: Class. Quantum Gravity **19**, 589 (2002)
27. Medved, A.J.M., Vagenas, E.C.: Mod. Phys. Lett. A **20**, 1723 (2005). gr-qc/0505015
28. Medved, A.J.M., Vagenas, E.C.: Mod. Phys. Lett. A **20**, 2449 (2005). gr-qc/0504113
29. Zhang, J.Y., Zhao, Z.: J. High Energy Phys. **10**, 055 (2005)
30. Jiang, Q.Q., Wu, S.Q.: Phys. Lett. B **635**, 151 (2006). hep-th/0511123
31. Angheben, M., Nadalini, M., Vanzo, L., Zerbini, S.: J. High Energy Phys. **05**, 014 (2005). hep-th/0503081
32. Nadalini, M., Vanzo, L., Zerbini, S.: J. Phys. A **39**, 6601 (2006). hep-th/0511250
33. Vagenas, E.C.: Phys. Lett. B **533**, 302 (2002). hep-th/0109108
34. Liu, W.B.: Phys. Lett. B **634**, 541 (2006). gr-qc/0512099
35. Zhang, J.Y., Zhao, Z.: Phys. Lett. B **618**, 14 (2005)
36. Zhang, J.Y., Zhao, Z.: Mod. Phys. Lett. A **20**, 1673 (2005)
37. Yang, S.Z.: Chin. Phys. Lett. **22**, 2492 (2005)
38. Yang, S.Z., Jiang, Q.Q., Li, H.L.: Chinese Phys. **14**, 2411 (2005)
39. Yang, S.Z., Jiang, Q.Q., Li, H.L.: Int. J. Theor. Phys. **46**, 625 (2007)
40. Liu, M.Q., Yang, S.Z.: Int. J. Theor. Phys. **46**, 65 (2007)
41. Jiang, Q.Q., Yang, S.Z., Wu, S.Q.: Int. J. Theor. Phys. **45**, 2311 (2006)
42. Li, H.L., Qi, D.J., Jiang, Q.Q., Yang, S.Z.: Int. J. Theor. Phys. **45**, 2471 (2006)
43. Wu, S.Q., Jiang, Q.Q.: Hawking radiation of charged particles as tunneling from higher dimensional Reissner-Nordström-de Sitter black holes, hep-th/0603082
44. Hu, Y., Zhang, J., Zhao, Z.: Int. J. Mod. Phys. D **16**, 847 (2007). gr-qc/0611085
45. Zhang, J., Zhao, Z.: Phys. Lett. B **638**, 110 (2006)
46. Jiang, Q.Q., Wu, S.Q., Cai, X.: Phys. Rev. D **73**, 064003 (2006), 069902(E). hep-th/0512351
47. Gasperini, M.: Class. Quantum Gravity **5**, 521 (1988)
48. Guth, A.H.: Phys. Rev. D **23**, 347 (1981)
49. Turner, M.S.: Fermilab-Conf-95-125A (1995)
50. Turner, M.S.: Fermilab-Conf-95-126A (1998)
51. Misner, C.W.: Taub-NUT space as a counter example to almost anything. In: Ehlers, J. (ed.) Relativity Theory and Astrophysics 1. Lectures in Applied Mathematics, vol. 8, p. 160. Am. Math. Soc., Providence (1967)
52. Misner, C.W.: J. Math. Phys. **4**, 924 (1963)
53. Bonnor, W.B.: Proc. Camb. Philos. Soc. **66**, 145 (1969)
54. Dowker, J.S.: Gen. Relat. Gravit. **5**, 603 (1974)
55. Mcguire, P., Ruffini, R.: Phys. Rev. D **12**, 3019 (1975)
56. Ahmed, M.: Int. J. Theor. Phys. **46**, 445 (2007)
57. Ali, M.H.: Class. Quantum Gravity **24**, 5849 (2007)
58. Painlevé, P.: C. R. Acad. Sci. Ser. 1 Math. **173**, 677 (1921)
59. Landau, L.D., Lifshitz, E.M.: Classical Theory of Fields, 4th edn., vol. 2, Pergamon, New York (1987)
60. Damour, T.: Phys. Rev. D **18**, 18 (1978)
61. Cherkis, S., Hashimoto, A.: J. High Energy Phys. **11**, 036 (2002)
62. Clarkson, R., Ghezelbash, A.M., Mann, R.B.: J. High Energy Phys. **04**, 063 (2004)
63. Clarkson, R., Ghezelbash, A.M., Mann, R.B.: J. High Energy Phys. **08**, 025 (2004)
64. Hawking, S.W., Hunter, C.J., Page, D.N.: Phys. Rev. D **59**, 044033 (1999)
65. Chamblin, A., Emparan, R., Johnson, C.V., Myers, R.C.: Phys. Rev. D **59**, 064010 (1999)
66. Mann, R.B.: Phys. Rev. D **60**, 104047 (1999)

67. Astefanesei, D., Mann, R.B., Radu, E.: J. High Energy Phys. **01**, 049 (2005)
68. Kerner, R., Mann, R.B.: Phys. Rev. D **73**, 104010 (2006). gr-qc/0603019
69. Shankaranarayanan, S., Padmanabhan, T., Srinivasan, K.: Class. Quantum Gravity **19**, 2671 (2002). gr-qc/0010042
70. Shankaranarayanan, S., Srinivasan, K., Padmanabhan, T.: Mod. Phys. Lett. A **19**, 571 (2001). gr-qc/0007022
71. Shankaranarayanan, S.: Phys. Rev. D **67**, 084026 (2003). gr-qc/0301090.
72. Akhmedov, E.T., Akhmedova, V., Singleton, D.: Phys. Lett. B **642**, 124 (2006). hep-th/0608098
73. Akhmedov, E.T., Akhmedova, V., Singleton, D., Pilling, T.: Int. J. Mod. Phys. A **22**, 1705 (2007). hep-th/0605137
74. Chowdhury, B.D.: Tunneling of Thin Shells from Black Holes: An Ill Defined Problem, hep-th/0605197
75. Wu, X., Gao, S.: Phys. Rev. D **75**, 044027 (2007). gr-qc/0702033
76. Lewandowski, J.: Class. Quantum Gravity **17**, L53 (2000)
77. Berezin, V.A., Boyarsky, A.M., Neronov, A.Yu.: Gravit. Cosmol. **5**(00), 1–10 (1999). gr-qc/0605099