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Exact Bianchi Type-I Cosmological Models in a Scalar-Tensor Theory

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Abstract
In this paper, a spatially homogeneous and anisotropic Bianchi type-I space-time filled with perfect fluid is investigated within the framework of a scalar-tensor theory proposed by Saez and Ballester. Two different physically viable models of the universe are obtained by using a special law of variation for Hubble's parameter that yields a constant value of deceleration parameter. One of the models is found to generalize a model recently investigated by Reddy et al. (Astrophys. Space Sci. 306:171, 2006). The Einstein's field equations are solved exactly and the solutions are found to be consistent with the recent observations of type Ia supernovae. A detailed study of physical and kinematical properties of the models is carried out.

Keywords Bianchi space-time · Hubble's parameter · Deceleration parameter · Cosmological models · Scalar field

1 Introduction

At the present state of evolution, the universe is spherically symmetric and the matter distribution in it is on the whole isotropic and homogeneous. But in its early stages of evolution it could not have had such a smoothed out picture because the sorts of matter fields in the early universe are uncertain. Moreover there are no observational data that guarantee in an epoch prior to the recombination. Therefore anisotropy at early times is a very natural phenomenon to investigate, in order to sort out problems like the local anisotropies that we observe today in galaxies, clusters and superclusters. These anisotropies may have many possible sources; they could be associated with cosmological magnetic or electric fields, long wavelengh gravitational waves, Yang-Mills fields [1].

Friedmann-Robertson-Walker (FRW) models, being isotropic and homogeneous, best represent the large scale structure of the present universe. But to describe the early stages

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of the evolution of universe, models with anisotropic background are suitable. A Bianchi type-I model, being the straightforward generalization of the flat FRW model, is one of the simplest models of the universe with anisotropic background that describes a spatially homogeneous and flat universe. An interesting property of the Bianchi type-I universe is that even in the presence of matter, it behaves like a Kasner universe near the singularity and consequently falls within the general analysis of the singularity given by Belinskii et al. [2]. Also in a universe filled with matter for $p = \gamma\rho$, $\gamma < 1$, it has been shown that any initial singularity in Bianchi type-I universe quickly dies away and a Bianchi type-I universe eventually evolves into a FRW universe [5]. This feature of the Bianchi type-I universe makes it a prime candidate for studying the possible effects of anisotropy in the early universe on present-day observations. Several authors have investigated anisotropic Bianchi type-I models in different contexts (for references see [8]).

In last few decades, there has been a great deal of interest in the alternative theories of gravitation since the Einstein's theory of general relativity does not seem to resolve some of the important problems in cosmology such as dark matter or the missing matter problem. Saez and Ballester [19] have developed a theory in which the metric is coupled with a dimensionless scalar field ϕ in a simple manner. This ϕ-coupling (referred as by the authors Saez and Ballester) gives a satisfactory description of the weak fields. Inspite of the dimensionless character of the scalar field, an antigravity regime appears. This theory suggests a possible way to solve the missing matter problem in non-flat FRW cosmologies. Several authors have investigated cosmological models within the framework of Saez-Ballester's scalar-tensor theory of gravitation. Singh and Agrawal [26, 27] have studied Bianchi type-I to IX models in this theory. Shri Ram and Singh [20] have obtained spatially homogeneous and locally rotationally symmetric (LRS) solutions which admit a Bianchi-I group of motions on hypersurfaces $t = constant$. Shri Ram and Tiwari [21] have investigated inhomogeneous plane symmetric models. Singh and Shri Ram [25] have presented a spatially homogeneous and isotropic FRW model with zero-curvature. Reddy [14] has obtained an exact Bianchi type-I string cosmological model in the Saez Ballester's theory. Mohanty and Sahu [12, 13] have studied Bianchi type-VI_0 and Bianchi type-I models in this theory. Recently Reddy et al. [15] have obtained exact solutions for a spatially homogeneous and LRS Bianchi type-I space-time with constant deceleration parameter (DP) in the Saez-Ballester's scalar-tensor theory.

In this paper our intention is to construct physically realistic and exact Bianchi type-I models within the framework of the scalar-tensor theory proposed by Saez and Ballester [19]. Therefore we consider a spatially homogeneous and anisotropic Bianchi-I space-time filled with perfect fluid in the Saez-Ballester's theory. This work is organised as follows: In Sect. 2, the model and field equations have been presented. The field equations have been solved in Sect. 3 by using a special law of variation for Hubble's parameter that yields a constant value of DP. Two exact Bianchi type-I models have been obtained in Sects. 3.1 and 3.2. The physical behavior of the models has been discussed in detail in both the subsections. In the last section, i.e. Sect. 4, concluding remarks have been expressed.

2 Model and Field Equations

The spatially homogeneous and anisotropic Bianchi-I space-time is described by the line element

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2, \tag{1}$$

where $A(t)$, $B(t)$ and $C(t)$ are the metric functions of cosmic time t.

We define $a=(ABC)^{\frac{1}{3}}$ as the average scale factor so that the Hubble's parameter in anisotropic models may be defined as

$$H=\frac{\dot{a}}{a}=\frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right), \tag{2}$$

where an over dot denotes derivative with respect to the cosmic time t. Also we have

$$H=\frac{1}{3}(H_1+H_2+H_3), \tag{3}$$

where $H_1=\frac{\dot{A}}{A}$, $H_2=\frac{\dot{B}}{B}$ and $H_3=\frac{\dot{C}}{C}$ are directional Hubble's factors in the directions of x, y and z, respectively.

The field equations given by Saez and Ballester [19] for the combined scalar and tensor fields are

$$G_{ij}-\omega\phi^m\left(\phi_{,i}\phi_{,j}-\frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right)=-T_{ij} \tag{4}$$

and the scalar field ϕ satisfies the equation

$$2\phi^m\phi_{;i}^{;i}+m\phi^{m-1}\phi_{,k}\phi^{,k}=0, \tag{5}$$

where $G_{ij}=R_{ij}-\frac{1}{2}g_{ij}R$ is the Einstein tensor; ω and m are constants; T_{ij} is stress energy tensor of matter; comma and semicolon denote partial and covariant differentiation, respectively.

The energy momentum tensor T_{ij} for perfect fluid distribution has the form

$$T_{ij}=(\rho+p)u_iu_j-pg_{ij}, \tag{6}$$

where u^i is the four-velocity vector satisfying $u^iu_i=1$; ρ and p, respectively are the energy density and pressure of the fluid.

In a co-moving coordinate system, the field equations (4) and (5), for the anisotropic Bianchi type-I space-time (1), in case of (6), read as

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}=-p+\frac{\omega}{2}\phi^m\dot{\phi}^2, \tag{7}$$

$$\frac{\ddot{C}}{C}+\frac{\ddot{A}}{A}+\frac{\dot{C}\dot{A}}{CA}=-p+\frac{\omega}{2}\phi^m\dot{\phi}^2, \tag{8}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}=-p+\frac{\omega}{2}\phi^m\dot{\phi}^2, \tag{9}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}=\rho-\frac{\omega}{2}\phi^m\dot{\phi}^2, \tag{10}$$

$$\ddot{\phi}+\dot{\phi}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\frac{m}{2}\frac{\dot{\phi}^2}{\phi}=0. \tag{11}$$

The energy conservation equation

$$T^{ij}_{;j}=0 \tag{12}$$

leads to

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0. \tag{13}$$

The field equations (7–11) are five equations involving six unknowns A, B, C, p and ρ and ϕ. So for complete determinacy of the system, we need one more relation among the variables that we shall obtain in the following section by applying a special law of variation for Hubble's parameter recently proposed by the present authors [8]) and solve the field equations.

3 Solution of Field Equations

Einstein's field equations (7–11) are a coupled system of highly non-linear equations. In order to solve the field equations, we normally assume a form for the matter content or suppose that the space-time admits killing vector symmetries. Solutions to the field equations may also be generated by applying a law of variation for Hubble's parameter, which was first proposed by Berman [3] in FRW models and that yields a constant value of DP. Recently the present authors [8] have proposed a similar law of variation for Hubble's parameter in anisotropic Bianchi type-I space-time that also yields a constant value of DP. The variation for Hubble's parameter as assumed is not inconsistent with the observations.

Most of the well known models of Einstein's theory and Brans-Dicke theory for the FRW metric have been considered with constant DP. In literature several authors have considered cosmological models with constant DP (see [8] and references therein). Recently Reddy et al. [15, 16] have presented LRS Bianchi type-I models with constant DP in scalar tensor and scale covariant theories of gravitation. The present authors [22–24] have investigated LRS Bianchi type-II models with constant DP in general relativity, Guth's inflationary theory and self creation theory of gravitation.

Very recently the present authors [8] have obtained a class of exact solutions for a spatially homogeneous and anisotropic Bianchi type-I space-time with perfect fluid in general relativity by applying a special law of variation for Hubble's parameter that yields a constant value of DP. The law for variation of Hubble's parameter is

$$H = Da^{-n} = D(ABC)^{\frac{-n}{3}}, \tag{14}$$

where $D \geq 0$ and $n > 0$ are constants.

The DP q is defined by

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \tag{15}$$

From (2) and (14), we get

$$\frac{\dot{a}}{a} = Da^{-n}, \tag{16}$$

which on integration leads to

$$a = (nDt + C_1)^{\frac{1}{n}} \quad \text{for } n \neq 0, \tag{17}$$

and

$$a = C_2 e^{Dt} \quad \text{for } n = 0, \tag{18}$$

where C_1 and C_2 are constants of integration.

Substituting (17) into (15), we get

$$q = n - 1. \tag{19}$$

This shows that the law (14) leads to a constant value of DP.

Now subtracting (7) from (8), (7) from (9), (8) from (9) and taking second integral of each, we get the following three relations, respectively:

$$\frac{A}{B} = d_1 \exp\left(x_1 \int a^{-3} dt\right), \tag{20}$$

$$\frac{A}{C} = d_2 \exp\left(x_2 \int a^{-3} dt\right), \tag{21}$$

$$\frac{B}{C} = d_3 \exp\left(x_3 \int a^{-3} dt\right), \tag{22}$$

where d_1, x_1, d_2, x_2, d_3 and x_3 are constants of integration.

From (20–22), the metric functions can be explicitly written as

$$A(t) = a_1 a \exp\left(b_1 \int a^{-3} dt\right), \tag{23}$$

$$B(t) = a_2 a \exp\left(b_2 \int a^{-3} dt\right), \tag{24}$$

$$C(t) = a_3 a \exp\left(b_3 \int a^{-3} dt\right), \tag{25}$$

where

$$a_1 = \sqrt[3]{d_1 d_2}, \qquad a_2 = \sqrt[3]{d_1^{-1} d_3}, \qquad a_3 = \sqrt[3]{(d_2 d_3)^{-1}},$$
$$b_1 = \frac{x_1 + x_2}{3}, \qquad b_2 = \frac{x_3 - x_1}{3}, \qquad b_3 = \frac{-(x_2 + x_3)}{3}.$$

It deserves mention that these constants satisfy the following two relations:

$$a_1 a_2 a_3 = 1, \qquad b_1 + b_2 + b_3 = 0. \tag{26}$$

The second integral of (11) leads to

$$\phi(t) = \left[\frac{h(m+2)}{2} \int a^{-3} dt\right]^{\frac{2}{m+2}}, \tag{27}$$

where h is a constant due to first integral while the constant of second integral is taken as zero for simplicity.

In the following subsections we discuss cosmology for $n \neq 0$ and $n = 0$ with the help of (17) and (18).

3.1 Cosmology for $n \neq 0$

Using (17) in (23–25), we get the following expressions for scale factors:

$$A(t) = a_1 (nDt + C_1)^{\frac{1}{n}} \exp\left[\frac{b_1}{D(n-3)} (nDt + C_1)^{\frac{n-3}{n}}\right], \tag{28}$$

$$B(t) = a_2(nDt + C_1)^{\frac{1}{n}} \exp\left[\frac{b_2}{D(n-3)}(nDt + C_1)^{\frac{n-3}{n}}\right], \tag{29}$$

$$C(t) = a_3(nDt + C_1)^{\frac{1}{n}} \exp\left[\frac{b_3}{D(n-3)}(nDt + C_1)^{\frac{n-3}{n}}\right]. \tag{30}$$

Again substituting (17) in (27), the scalar field is given by

$$\phi(t) = \left[\frac{h(m+2)}{2D(n-3)}\right]^{\frac{2}{m+2}} (nDt + C_1)^{\frac{2(n-3)}{n(m+2)}}. \tag{31}$$

Substituting (28)–(31) in (9) and (10), the pressure and energy density of the model read as

$$p = D^2(2n-3)(nDt + C_1)^{-2} - \left(b_1^2 + b_2^2 + b_1b_2 - \frac{1}{2}\omega h^2\right)(nDt + C_1)^{\frac{-6}{n}}, \tag{32}$$

$$\rho = 3D^2(nDt + C_1)^{-2} + \left(b_1b_2 + b_2b_3 + b_3b_1 + \frac{1}{2}\omega h^2\right)(nDt + C_1)^{\frac{-6}{n}}. \tag{33}$$

In view of (26), one may observe that the solutions (28–33) satisfy the energy conservation equation (13) identically and hence represent exact solutions of the Einstein's field equations (7–11). Further if we assume $a_2 = a_3$ and $b_2 = b_3$, i.e. $B = C$, then the above solutions reduce to the solutions recently obtained by Reddy et al. [15]. Thus the above model generalizes the model investigated by Reddy et al. [15].

Now we find expressions for some other cosmological parameters of the model. The anisotropy parameter A is defined as

$$\mathrm{A} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2. \tag{34}$$

The directional Hubble factors H_i $(i = 1, 2, 3)$ as defined in (3) are given by

$$H_i = D(nDt + C_1)^{-1} + b_i(nDt + C_1)^{\frac{-3}{n}}. \tag{35}$$

The expansion scalar is given by

$$\Theta = 3H = 3D(nDt + C_1)^{-1}. \tag{36}$$

Using (35) and (36) in (34), we get

$$\mathrm{A} = \frac{1}{3D^2}(b_1^2 + b_2^2 + b_3^2)(nDt + C_1)^{\frac{2n-6}{n}}. \tag{37}$$

The volume and shear scalar of the model are given by

$$V^3 = (nDt + C_1)^{\frac{3}{n}}, \tag{38}$$

$$\sigma^2 = \frac{1}{3}[(b_1 - b_2)^2 + (b_2 - b_3)^2 + (b_3 - b_1)^2](nDt + C_1)^{\frac{-6}{n}}. \tag{39}$$

Physical Behavior of the Model It is observed that the spatial volume is zero at $t = t_0$ where $t_0 = -C_1/nD$ and expansion scalar is infinite, which shows that the universe starts evolving with zero volume at $t = t_0$ with an infinite rate of expansion. The scale factors also vanish at $t = t_0$ and hence the model has a point singularity at the initial epoch. The pressure, energy density, Hubble's factors and shear scalar diverge at the initial singularity. The scalar field and anisotropy parameter also tend to infinity at the initial epoch provided $n < 3$. The universe exhibits the power-law expansion after the big bang impulse. As t increases, the scale factors and spatial volume increase but the expansion scalar decreases. Thus the rate of expansion slows down with increase in time. Also ϕ, ρ, p, H_1, H_2, H_3, A and σ^2 decrease as t increases. As $t \to \infty$, scale factors and volume become infinite whereas ϕ, ρ, p, H_1, H_2, H_3, Θ, A and σ^2 tend to zero. Therefore the model would essentially give an empty universe for large time t. The ratio σ/Θ tends to zero as $t \to \infty$ provided $n < 3$. So the model approaches isotropy for large values of t. Thus the model represents shearing, non-rotating and expanding model of the universe with a big bang start approaching to isotropy at late times. The integral

$$\int_{t_0}^{t} [V(t')]dt' = \frac{1}{D(n-1)}[(nDt' + C_1)]_{t_0}^{t}$$

is finite provided $n \neq 1$. Therefore a horizon exists in this model.

Further it is observed that the above solutions are not valid for $n = 3$. For $n = 3$, the spatial volume grows linearly with cosmic time. For $n > 1$, $q > 0$; therefore the model represents a decelerating model of the universe. For $n \leq 1$, we get $-1 < q \leq 0$, which implies an accelerating model of the universe. Also recent observations of type Ia supernovae [6, 7, 9–11, 17, 18, 28] reveal that the present universe is accelerating and value of DP lies somewhere in the range $-1 < q \leq 0$. It follows that the solutions obtained in this model are consistent with the observations.

3.2 Cosmology for $n = 0$

Using (18) in (23–25), the scale factors of the model read as

$$A(t) = a_1 C_2 \exp\left(Dt - \frac{b_1}{3DC_2^3} e^{-3Dt}\right), \tag{40}$$

$$B(t) = a_2 C_2 \exp\left(Dt - \frac{b_2}{3DC_2^3} e^{-3Dt}\right), \tag{41}$$

$$C(t) = a_3 C_2 \exp\left(Dt - \frac{b_3}{3DC_2^3} e^{-3Dt}\right). \tag{42}$$

The scalar field is given by

$$\phi(t) = \left[\frac{h(m+2)}{6DC_2^3}\right]^{\frac{2}{m+2}} \exp\left(\frac{-6Dt}{m+2}\right). \tag{43}$$

The pressure and energy density are given by

$$p = -3D^2 - \left(b_1^2 + b_2^2 + b_1 b_2 - \frac{1}{2}\omega h^2 C_2^{-6}\right) e^{-6Dt}, \tag{44}$$

$$\rho = 3D^2 + \left(b_1 b_2 + b_2 b_3 + b_3 b_1 + \frac{1}{2}\omega h^2 C_2^{-6}\right) e^{-6Dt}. \tag{45}$$

The solutions (40–45) satisfy (13) identically and hence represent exact solutions of the field equations (7–11). The other cosmological parameters of the model have the following expressions:

$$H_i = D + b_i C_2^{-3} e^{-3Dt} \quad (i = 1, 2, 3), \tag{46}$$

$$\Theta = 3H = 3D, \tag{47}$$

$$\mathrm{A} = \frac{1}{3D^2}(b_1^2 + b_2^2 + b_3^2) C_2^{-6} e^{-6Dt}, \tag{48}$$

$$V^3 = C_2^3 e^{3Dt}, \tag{49}$$

$$\sigma^2 = C_2^{-6}[(b_1 - b_2)^2 + (b_2 - b_3)^2 + (b_3 - b_1)^2] e^{-6Dt}. \tag{50}$$

Physical Behavior of the Model The model has no initial singularity. The spatial volume, scale factors, scalar field, pressure, energy density and the other cosmological parameters are constants at $t = 0$. Thus the universe starts evolving with a constant volume and expands with exponential rate. As t increases, the scale factors and the spatial volume increase exponentially while the scalar field, pressure, energy density, anisotropy parameter and shear scalar decrease. It is interesting to note that the expansion scalar is constant throughout the evolution of universe and therefore the universe exhibits uniform exponential expansion in this model. As $t \to \infty$, the scale factors and volume of the universe become infinitely large whereas the scalar field, anisotropy parameter and shear scalar tend to zero. The pressure, energy density and Hubble's factors become constants such that $p = -\rho$. This shows that at late times the universe is dominated by vacuum energy, which drives the expansion of universe. The model approaches isotropy for large time t. Therefore the model represents a shearing, non-rotating and expanding universe with a finite start approaching to isotropy at late times.

It has also been observed that $\lim_{t\to 0} \rho/\Theta^2$ turns out to be a constant. Thus the model approaches homogeneity and matter is dynamically negligible near the origin; this agrees with a result already given by Collins [4]. Recent observations of type Ia supernovae [6, 7, 9–11, 17, 18, 28] suggest that the universe is accelerating in its present state of evolution. It is believed that the way universe is accelerating presently; it will expand at the fastest possible rate in future and forever. For $n = 0$, we get $q = -1$; incidentally this value of DP leads to $dH/dt = 0$, which implies the greatest value of Hubble's parameter and the fastest rate of expansion of the universe. Therefore the solutions presented in this model are consistent with the observations and may find applications in the analysis of late time evolution of the actual universe in Saez-Ballester's scalar-tensor theory of gravitation.

4 Conclusion

In this paper we have studied a spatially homogeneous and anisotropic Bianchi-I space-time within the framework of the scalar-tensor theory of gravitation proposed by Saez and Ballester [19]. The field equations have been solved exactly by using a special law of variation of Hubble's parameter that yields a constant value of DP. Two exact and physically viable Bianchi type-I models have been obtained in Sects. 3.1 and 3.2. The model obtained in Sect. 3.1 is found to generalize a model recently obtained by Reddy et al. [15]. Expressions for some important cosmological parameters have been obtained for both the models and physical behavior of the models is discussed in detail. It is found that for $n \neq 0$, all the matter and radiation is concentrated at the big bang epoch and the cosmic expansion is driven by the big bang impulse. The model has a point singularity at the initial epoch as the scale factors and volume vanish at this moment. The universe has singular origin and it exhibits power-law expansion after the big bang impulse. The rate of expansion slows down and finally drops to zero as $t \to \infty$. The pressure, energy density and scalar field become negligible whereas the scale factors and spatial volume become infinitely large as $t \to \infty$, which would give essentially an empty universe. For $n = 0$, the model has no real singularity, density being finite. Thus the universe has non-singular origin and cosmic expansion is driven by the creation of matter particles. The universe exhibits exponential expansion and expands uniformly. At late times in this model, the universe is dominated by vacuum energy, which is supposed to be responsible for the cosmic expansion.

Both the models represent shearing, non-rotating and expanding universe, which approaches to isotropy for large values of t. This is consistent with the behavior of the present universe as discussed in the introduction. The scalar field decreases to zero as $t \to \infty$ in both the models. If we assume $h = 0$, then the solutions reduce to the solutions in general relativity [8] and the Saez-Ballester's scalar-tensor theory of gravitation tends to standard general theory of relativity in all respects. The solutions obtained in the models are found to be consistent with the recent observations of type Ia supernovae as discussed in Sects. 3.1 and 3.2. Finally, the solutions presented in this paper are new and may be useful for better understanding of the evolution of universe in Bianchi-I space-time within the framework of Saez-Ballester's scalar-tensor theory of gravitation.

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