Int J Thermophys (2017) 38:14
DOI 10.1007/s10765-016-2154-x

Nonlinear Radiative Heat Transfer in Blasius and Sakiadis Flows Over a Curved Surface

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Received: 24 November 2015 / Accepted: 9 November 2016 / Published online: 17 November 2016

Abstract This study investigates the heat transfer characteristics for Blasius and Sakiadis flows over a curved surface coiled in a circle of radius R having constant curvature. Effects of thermal radiation are also analyzed for nonlinear Rosseland approximation which is valid for all values of the temperature difference between the fluid and the surface. The considered physical situation is represented by a mathematical model using curvilinear coordinates. Similar solutions of the developed partial differential equations are evaluated numerically using a shooting algorithm. Fluid velocity, skin-friction coefficient, temperature and local Nusselt number are the quantities of interest interpreted for the influence of pertinent parameters. A comparison of the present and the published data for a flat surface validates the obtained numerical solution for the curved geometry.

Keywords Blasius/Sakiadis flows · Curved surface · Numerical solution · Thermal radiation

1 Introduction

Heat transfer analysis for the viscous flow has gained a significant attention due to its wide range of practical applications in the fields of engineering and industry. Such

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applications include the extrusion of rubber and plastic surface, continuous casting of metals, spinning of fiber, drawing of plastic films, cooling of continues strips or filaments, crystal growing, glass blowing, paper production [1–4]. In these processes, the drag force and rate of surface heat transfer play a critical role for the produced quality of the final product as discussed in detail by Karwe and Jaluria [5]. One of the typical problems in the theory of boundary layer flows is the flow past on a stationary sheet caused by a uniform free stream velocity famously known as Blasius flow [6]. Abussita [7] analyzed the Blasius flow past on a flat plate and discussed the existence of the solution. Asaithambi [8] used finite-difference technique to solve the Falkner–Skan's flow equation. For some recent investigations for the Blasius flow readers are referred to the articles [9–15] and references therein.

Flow on a moving flat surface is theoretically investigated by Sakiadis [16] which is another famous problem in the theory of boundary layer flows. The reduction of governing partial differential equations for both Blasius and Sakiadis flows through the respective similarity variables results in the same ordinary differential equation with different boundary conditions. The discussion regarding the flow and heat transfer on a moving sheet for experimental and theoretical results was presented by Tsou et al. [17]. The results of [17] provided an experimental setup for the existence of Sakiadis flow and authenticated the numerical results in [16]. Based on the fact that a single ordinary differential equation (namely, Blasius equation together with two set of boundary conditions) governs both Blasius and Sakiadis flows, an inventive way of analysis for these two types of boundary layer flows discusses both together. A literature survey reveals that the literature is scarce in this direction [18–23]. However, different aspects of flow phenomena have been discussed by several researchers separately. For details, the interested readers are referred to the articles [24–33].

Influence of radiation on convective heat transfer process is significant in processes involving high temperature. Examples of such processes include solar power technology, nuclear power plant, gas turbine, electrical power generation, storage of thermal energy, space vehicle re-entry. Raptis et al. [34] investigated the influence of radiation on a hydromagnetic flow of a Newtonian fluid. Cortell [35,36] investigated the influence of thermal radiation in the Sakiadis and Blasius flows, respectively. The influence of thermal radiation in the case of nonlinear Rosseland approximation in Sakiadis flow was studied by Pantokratoras and Fang [37]. In another paper, Pantokratoras and Fang [38] extended the same situation to discuss the Blasius flow. The study of heat transfer in the flow of nanofluid by considering nonlinear thermal radiation was carried out by Mushtaq et al. [39]. Naveed et al. [40] analyzed radiation effects for a viscous fluid through a curved channel. Recently, Abbas et al. [41] discussed the Hall effects on a viscous fluid through a semi-porous curved channel by invoking the nonlinear Rosseland approximation.

Much attention in the above studies is given to the situation when fluid is flowing on a flat plate and governing equations are given in Cartesian coordinates. Sajid et al. [42] in a recent study proposed a mathematical model to study the flow problem over a curved surface of constant curvature. Abbas et al. [43] studied the effects of magnetic field and heat transfer through a curved stretching surface. Recently, Naveed et al. [44] studied the effects of radiation in MHD micropolar fluid due to curved stretching sheet. Rosca and Pop [45] have discussed the unsteady flow past a porous curved

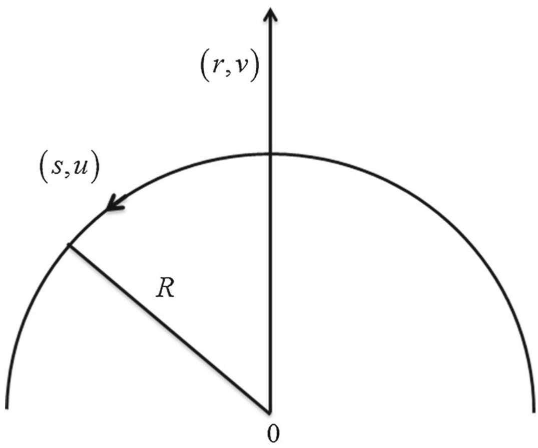

Fig. 1 Sketch diagram of the curved surface

stretching/shrinking wall. The hydromagnetic flow over an unsteady curved stretching wall was carried out by Naveed et al. [46]. Very recently, Abbas et al. [47] discussed the hydromagnetic flow and heat transfer of nanofluid on a curved stretching wall by considering the radiation and heat generation in the energy equation. The objective of this study is to investigate the influence of nonlinear thermal radiation for Blasius and Sakiadis flows over a curved surface. Numerical solutions are obtained by using shooting method for the fluid velocity and temperature distribution. Numerical results are in the form of graphs and tabular data. These representations of the results are utilized to discuss the influence of pertinent parameters.

2 Mathematical Formulation

Consider the steady, incompressible and two-dimensional flow of a viscous fluid past on a curved surface coiled in a circle of radius R (Fig. 1). The boundary layer equations describing the flow phenomena are [43]:

$$\frac{\partial}{\partial r}\left\{(r+R)\,v\right\}+R\frac{\partial u}{\partial s}=0, \tag{1}$$

$$\frac{u^2}{r+R}=\frac{1}{\rho}\frac{\partial p}{\partial r}, \tag{2}$$

$$v\frac{\partial u}{\partial r}+\frac{Ru}{r+R}\frac{\partial u}{\partial s}+\frac{uv}{r+R}=-\frac{1}{\rho}\frac{R}{r+R}\frac{\partial p}{\partial s}+\nu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r+R}\frac{\partial u}{\partial r}-\frac{u}{(r+R)^2}\right), \tag{3}$$

The boundary conditions for the velocity field are

$$\begin{aligned} u &= 0=v \quad \text{at } r=0,\\ u &\to U_\infty \quad \text{as } r\to\infty, \end{aligned} \tag{4}$$

for the Blasius flow and

$$\begin{aligned} u &= U_w, v = 0 \quad \text{at} \;\; r = 0, \\ u &\to 0 \quad \text{as} \;\; r \to \infty, \end{aligned} \tag{5}$$

for the Sakiadis flow.

Here u and v are components of the velocity in s- and r-directions, respectively, ρ is the density of the fluid, ν the kinematics viscosity, p the pressure, U_∞ a constant free stream velocity and U_w the surface velocity.

Introducing the similarity variables

$$\begin{aligned} &u = Uf'(\eta), \quad v = \frac{1}{2}\sqrt{\frac{U\nu}{s}}\frac{R}{r+R}\left[\eta f'(\eta)\right] - f(\eta), \\ &\eta = \sqrt{\frac{U}{\nu s}}r = \frac{r}{s}\sqrt{Re_s}, \quad p = \rho U^2 P(\eta), \end{aligned} \tag{6}$$

where η and f are the similarity variable and the nondimensional stream function, respectively, and $Re_s = Us/\nu$ is the local Reynolds number. In Eq. 6 we set $U = U_\infty$ for Blasius flow and $U = U_w$ for Sakiadis flow.

With the help of Eq. 6, Eq. 1 is satisfied and Eqs. 2–5 give

$$\frac{\partial P}{\partial \eta} = \frac{f'^2}{\eta + K}, \tag{7}$$

$$\begin{aligned} &f''' + \frac{K}{2(\eta+K)}ff'' - \frac{K}{2(\eta+K)^2}\left(\eta f'^2 - ff'\right) \\ &\quad + \frac{\eta K}{2(\eta+K)}\frac{\partial P}{\partial \eta} + \frac{f''}{\eta+K} - \frac{f'}{(\eta+K)^2} = 0, \end{aligned} \tag{8}$$

$$f(0) = 0, \quad f'(0) = 0, \quad f'(\infty) = 1, \qquad \text{(Blasius flow)} \tag{9}$$

$$f(0) = 0, \quad f'(0) = 1, \quad f'(\infty) = 0, \qquad \text{(Sakiadis flow)} \tag{10}$$

In order to eliminate pressure gradient using Eq. 7 in (8) we get

$$f''' + \frac{K}{2(\eta+K)}ff'' + \frac{f''}{\eta+K} - \frac{f'}{(\eta+K)^2} + \frac{K}{2(\eta+K)^2}ff' = 0, \tag{11}$$

where $K = RU/\nu$ is the dimensionless radius of curvature. It is worth mentioning here that by taking $K \to \infty$, Eq. 11 reduces to classical momentum equation.

3 Transport Equation

The equation that governs the heat transport subject to nonlinear thermal radiation and neglecting the viscous dissipation effect is

$$v\frac{\partial T}{\partial r}+\frac{uR}{r+R}\frac{\partial T}{\partial s}=\alpha\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r+R}\frac{\partial T}{\partial r}\right]-\frac{1}{\rho c_p\left(r+R\right)}\frac{\partial}{\partial r}\left(r+R\right)q_r, \quad (12)$$

where $\alpha = k_1/\rho c_p$ is the thermal diffusivity, k_1 the thermal conductivity, c_p the specific heat at constant pressure, q_r the radiative heat flux and $T = T_w$ the surface temperature, where $T_w > T_\infty$ with T_∞ being the uniform temperature of the ambient fluid.

The thermal boundary conditions for the energy Eq. 12 are

$$\begin{aligned} T &= T_w \quad \text{at } r=0, \\ T &\to T_\infty \quad \text{as } r\to\infty. \end{aligned} \quad (13)$$

Incorporating the Rosseland approximation for thermal radiation [48], the radiative heat flux is given by

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial r}=-\frac{16\sigma^*}{3k^*}T^3\frac{\partial T}{\partial r}, \quad (14)$$

where σ^* is the Stefan–Boltzmann constant and k^* the mean absorption coefficient.

The dimensionless temperature is defined as

$$\theta\left(\eta\right)=\frac{T-T_\infty}{T_f-T_\infty}, \quad (15)$$

with

$$T=T_\infty\left[1+\left(\theta_w-1\right)\theta\right], \quad (16)$$

and

$$\theta_w=T_w/T_\infty \qquad \text{(temperature parameter).}$$

The right-hand side energy Eq. 12 can be expressed as

$$\frac{\alpha}{(r+R)}\left(\partial/\partial r\right)\left[\left\{1+Rd\left(1+\left(\theta_w-1\right)\theta\right)^3\right\}(r+R)\partial T/\partial r\right], \quad (17)$$

here $Rd = 16\sigma^* T_\infty^3/3k_1k^*$ denotes the radiation parameter.

Substituting Eqs. 6 and 14–17, Eq. 12 takes the following form

$$\frac{1}{\Pr\left(\eta+K\right)}\left[\left(1+Rd\left(1+\left(\theta_w-1\right)\theta\right)^3\right)\left(\eta+K\right)\theta'\right]'+\frac{K}{2\left(\eta+K\right)}f\theta'=0, \quad (18)$$

with boundary conditions

$$\theta\left(0\right)=1,\quad \theta\left(\infty\right)=0. \quad (19)$$

where $Pr = \mu c_p/k_1$ is the Prandtl number.

The energy Eq. 18 can be transformed to classical energy equation in the absence of thermal radiation by taking $Rd = 0$. The physical quantities of interest are the skin-friction coefficient and rate of heat transfer along the s-directions, which are given as

$$C_f = \frac{\tau_{rs}}{\rho U^2}, \quad Nu_s = \frac{s q_w}{k_1 (T_w - T_\infty)}, \tag{20}$$

where τ_{rs} is the shear stress at the surface and q_w the heat flux at the surface in the s-direction, which are given by

$$\tau_{rs} = \mu \left(\frac{\partial u}{\partial r} - \frac{u}{r+R} \right) \Bigg|_{r=0}, \quad q_w = -k_1 \frac{\partial T}{\partial r} \Bigg|_{r=0} + (q_r)_w, \tag{21}$$

Using Eqs. 6 and 21, Eq. 20 becomes

$$Re_s^{\frac{1}{2}} C_f = f''(0) - \frac{f'(0)}{K}$$
$$Re_s^{-1/2} Nu_s = -\left[1 + Rd\theta_w^3 \right] \theta'(0),$$

where $Re_s = Us/\nu$ is the local Reynolds number.

4 Numerical Solution

Numerical solution for the momentum Eq. 11 subject to either boundary conditions (9) (i.e., for the Blasius flow) or the boundary conditions (10) (i.e., for the Sakiadis flow) and energy Eq. 18 along with the boundary conditions (19) is obtained by using shooting method along with the Runge–Kutta algorithm. For application of shooting method, first we reduce the boundary value problems into initial value problems with missing condition of $f''(0)$ and $\theta'(0)$ as follows

$$\begin{aligned} &f' = z, z' = q, \\ &q' = \frac{-Kfq}{2(\eta + K)} - \frac{q}{\eta + K} + \frac{z}{(\eta + K)^2} - \frac{Kfz}{2(\eta + K)^2}, \end{aligned} \tag{22}$$

$$\begin{aligned} \theta' = g, g' = &-\frac{g}{(\eta + K)} - \frac{3Rd(\theta_w - 1)(1 + (\theta_w - 1)\theta)^2 g^2}{1 + Rd[1 + (\theta_w - 1)\theta]^3} \\ &- \frac{-PrKfg}{2\left(1 + Rd(1 + (\theta_w - 1)\theta)^3\right)(\eta + K)}, \end{aligned} \tag{23}$$

with boundary conditions.

$$f(0) = 0, \; z(0) = 0, \quad \theta(0) = 1, \qquad \text{(Blasius flow)} \tag{24}$$

$$f(0) = 0, \quad z(0) = 1, \quad \theta(0) = 1, \qquad \text{(Sakiadis flow)} \tag{25}$$

For integration of Eqs. 22 and 23 as an initial value problem subject to any boundary conditions (24) or (25), we need the value of $q(0)$, i.e., $f''(0)$ and $\theta'(0)$, i.e., $g(0)$,

Table 1 Comparison of the Sakiadis momentum transfer solution with [22], ()* represents for the flat surface by taking $K \to \infty$, i.e., $\left(K = 10^5\right)$, and ()** represents for the curved surface by taking $K = 100$

η	f	f'	$-f''$
0.0	0.00 000, (0.00 000)* (0.00 000)**	1.0000, (1.0000)* (1.0000)**	0.44 374, (0.44 374)* (0.44 201)**
0.1	0.09 778, (0.09 778)* (0.09 779)**	0.95 566, 0.95 566)* (0.95 586)**	0.44 265, (0.44 265)* (0.44 051)**
0.2	0.19 113, (0.19 113)* (0.19 118)**	0.91 153, 0.91 153)* (0.91 197)**	0.43 946, (0.43 946)* (0.43 695)**
0.3	0.28 010, (0.28 010)* (0.28 020)**	0.86 783, 0.86 783)* (0.86 853)**	0.43 430, (0.43 430)* (0.43 150)**
0.4	0.36 472, (0.36 472)* (0.36 491)**	0.82 473, 0.82 473)* (0.82 572)**	0.42 735, (0.42 735)* (0.42 432)**
0.5	0.44 507, (0.44 507)* (0.44 537)**	0.78 241, 0.78 241)* (0.78 372)**	0.41 878, (0.41 878)* (0.41 558)**
0.6	0.52 124, (0.52 124)* (0.52 168)**	0.74 102, 0.74 102)* (0.74 265)**	0.40 877, (0.40 877)* (0.40 547)**
0.7	0.59 331, (0.59 331)* (0.59 394)**	0.70 070, 0.70 070)* (0.70 266)**	0.39 753, (0.39 753)* (0.39 418)**
0.8	0.66 142, (0.66 142)* (0.66 225)**	0.66 155, 0.66 155)* (0.66 385)**	0.38 525, (0.38 525)* (0.38 190)**
0.9	0.72 567, (0.72 567)* (0.72 675)**	0.62 367, 0.62 367)* (0.62 631)**	0.37 211, (0.37 211)* (0.36 881)**
1.0	0.78 620, (0.78 620)* (0.78 756)**	0.58 715, 0.58 715)* (0.59 011)**	0.35 831, (0.35 831)* (0.35 509)**
3.0	1.43 273, (1.4 3273)* (1.4438)**	0.14 401, 0.14 401)* (0.14 937)**	0.10 983, (0.10 983)* (0.11 050)**
5.0	1.57 884, (1.5 7884)* (1.5985)**	0.02 994, 0.02 994)* (0.03 297)**	0.02 392, (0.02 392)* (0.02 512)**
10	1.61 611, (1.61 611)* (1.6409)**	0.00 053, 0.00 053)* (0.00 076)**	0.00 043, (0.00 043)* (0.00 056)**
20	1.6154, (1.6154)* (1.6420)**	0.00 000, 0.00 000)* (0.00 000)**	0.00 000, (0.00 000)* (0.00 000)**

but no such values are given. Since momentum and energy equations are uncoupled, $f''(0)$ is not influenced by the Prandtl number, radiation and temperature parameters. The value of $\theta'(0)$ is computed for different combinations of Prandtl number Pr, temperature parameter θ_w and radiation parameter Rd. In order to validate the method used in the present study and to analyze the accuracy of the present results, the missed wall shear stress $f''(0)$ is calculated for $K \to \infty$ (for the flat surface case) which is 0.3320 and -0.4437 for the Blasius and the Sakiadis flows, respectively, as reported by Bataller [22]. For the flow of Newtonian fluid over a flat surface, the shear stress is more than that in Blasius flow. The same trend exists for the curved surface, i.e., for $K = 100$, the missed wall shear stress $f''(0)$ is 0.2269 (for Blasius flow) and -0.5400 (for Sakiadis flow). Tables 1 and 2 are made for the second validation test in

Table 2 Comparison of the Blasius momentum transfer solution with [22], ()* represents for the flat surface by taking $K \to \infty$, i.e., $\left(K = 10^5\right)$, and ()** represents for the curved surface by taking $K = 100$

η	f	f'	f''
0.0	0.00 000, (0.00 000)* (0.00 000)**	0.0000, (0.0000)* (0.0000)**	0.33 206, (0.33 206)* (0.361 27)**
0.1	0.00 166, (0.00 166)* (0.00 180)**	0.03 320, (0.03 320)*, (0.03 610)**	0.33 205, (0.33 205)* (0.36 090)**
0.2	0.00 664, (0.00 664)* (0.00 722)**	0.06 641, (0.06 641)* (0.07 217)**	0.33 199, (0.33 199)* (0.36 044)**
0.3	0.01 494, (0.01 494)* (0.01 624)**	0.09 960, (0.09 960)* (0.10 820)**	0.33 182, (0.33 182)* (0.35 990)**
0.4	0.02 656, (0.02 656)* (0.02 885)**	0.13 276, (0.13 276)* (0.14 415)**	0.33 148, (0.33 148)* (0.35 915)**
0.5	0.04 149, (0.04 149)* (0.04 506)**	0.16 589, (0.16 589)* (0.18 002)**	0.33 092, (0.33 092)* (0.35 813)**
0.6	0.05 973, (0.05 973)* (0.06 485)**	0.19 894, (0.19 894)* (0.21 577)**	0.33 008, (0.33 008)* (0.35 680)**
0.7	0.08 127, (0.08 127)* (0.08 821)**	0.23 189, (0.23 189)* (0.25 137)**	0.32 893, (0.32 893)* (0.35 509)**
0.8	0.10 611, (0.10 611)* (0.11 512)**	0.26 471, (0.26 471)* (0.28 677)**	0.32 739, (0.32 739)* (0.35 295)**
0.9	0.13 421, (0.13 421)* (0.14 556)**	0.29 736, (0.29 736)* (0.32 194)**	0.32 544, (0.32 544)* (0.35 032)**
1.0	0.16 557, (0.16 557)* (0.17 950)**	0.32 979, (0.32 979)* (0.35 682)**	0.32 301, (0.32 301)* (0.34 715)**
3.0	1.39 684, (1.39 684)* (1.4982)**	0.84 605, (0.84 605)* (0.89 757)**	0.16 135, (0.16 135)* (0.16 015)**
5.0	3.28 332, (3.28 332)* (3.4752)**	0.99 154, (0.99 154)* (1.0279)**	0.01 590, (0.01 590)* (0.00 602)**
7.0	5.27 933, (5.27 933)* (5.5245)**	0.99 992, (0.99 992)* (1.0169)**	0.00 022, (0.00 022)* (0.00 927)**
8.82	7.09 920, (7.09 920)* (7.3598)**	1.00 000, (1.00 000)* (1.00 000)**	0.00 000, (0.00 000)* (0.00 918)**

which a numerical comparison (for $K \to \infty$) and the present study (for $K = 100$) of the solution for f and its derivatives are given for both Sakiadis and Blasius flows, respectively, with the results reported by Bataller [22]. It is evident from Table 2 that the missed wall condition $f''(0)$ is the same for the Blasius flow over flat surface case (i.e., $K \to \infty$) but for the curved surface (i.e., $K = 100$) it is 0.36 127. The calculated value of $f'(\infty)$ is compared with the given boundary condition $f'(\infty) = 1$, and the value of $f''(0)$ is adjusted by a root finding algorithm. For better approximation, here we used Newton–Raphson method. The step size is taken as $\Delta\eta = 0.005$. Also, it is noticed from these tables that the integrating domain for the Sakiadis flow is larger as compared to the Blasius flow.

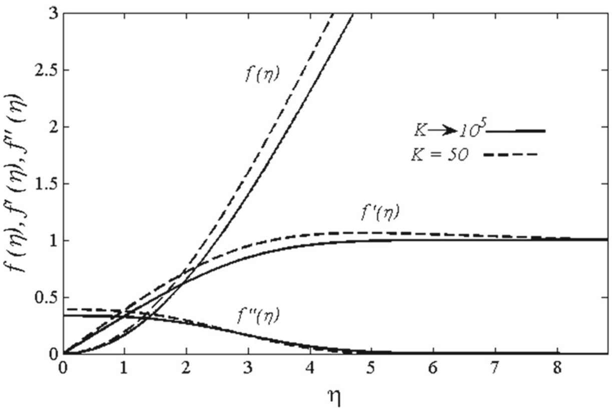

Fig. 2 Variation of nondimensional radius of curvature K on functions f, f', f'' for Blasius flow

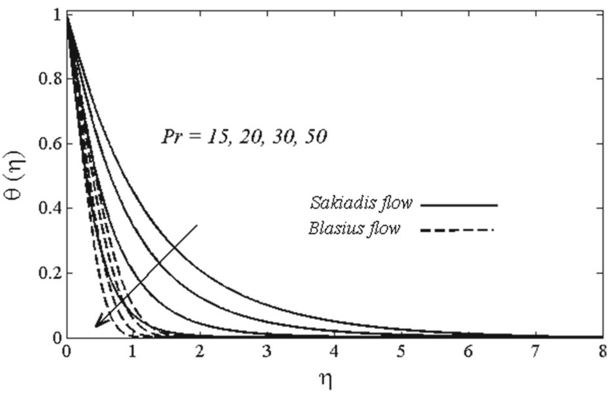

Fig. 3 Variation of Prandtl number Pr on temperature profile θ by keeping $Rd = 0.2, \theta_w = 1.3$ and $K = 7$ fixed

5 Results and Discussion

Numerical solution for the momentum Eq. 11 is subject to either boundary conditions (9) (i.e., for the Blasius flow) or the boundary conditions (10) (i.e., for the Sakiadis flow), and energy Eq. 18 subject to boundary conditions (19) is obtained by shooting method along with the Runge–Kutta algorithm. The effect of nondimensional radius of curvature K on f and its derivatives is shown in Fig. 2. Figure 3 depicts effects of Prandtl number Pr for both Sakiadis and Blasius flows. From this figure it is observed that the temperature of the fluid is decreased in both the cases. It is due to the fact that Prandtl number has an inverse relation with the thermal diffusivity of the fluid. Hence, an increase in Prandtl number results in a decrease in thermal diffusivity. Also, the thermal boundary layer thickness is thicker for the Sakiadis flow in comparison with the Blasius flow. Figure 4 demonstrates the influence of nonlinearized radiation parameter Rd on temperature distribution for both Sakiadis and Blasius flows. The temperature of the fluid is increased by increasing the value of Rd. Also, the thermal

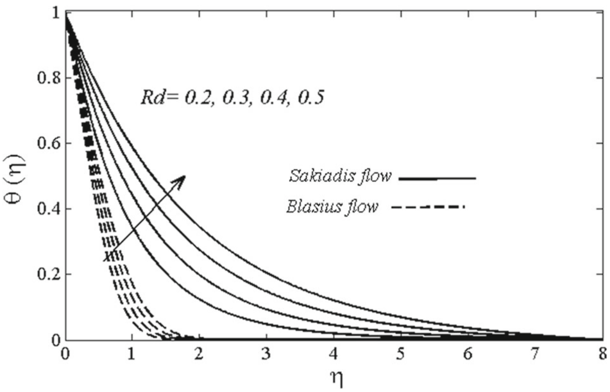

Fig. 4 Variation of nonlinearized radiation parameter Rd on temperature distribution θ by keeping $Pr = 20$, $\theta_w = 1.3$ and $K = 7$ fixed

Table 3 Comparison of the results of Nusselt number $-\theta'(0)$, for the Sakiadis flow for several values of the temperature and radiation parameter θ_w and Rd, respectively, with previously published data [37] with $Pr = 5.0$ and $K \to \infty$ (i.e., $K = 1000$) fixed, and $()^{**}$ represents for the curved surface by taking $K = 10$

$Rd = 10$		$Rd = 100$	$Rd = 1000$
Battaler [36] Linear Rosseland approximation			
	1.1349	1.2072	1.2160
Pantokratoras and Fang [37] present results of linear Rosseland approximation			
	1.1342, (1.1342)* (1.1830)**	1.2071, (1.2071)* (1.2559)**	1.2165, (1.2165)* (1.2654)**
θ_w	Nonlinear linear Rosseland approximation		
1.5	0.9516, (0.9516)* (1.0001)**	1.1811, (1.1811)* (1.2299)**	1.2124, (1.2124)* (1.2613)**
3	0.4196, (0.4196)* (0.4668)**	0.9671, (0.9671)* (1.0157)**	1.1837, (1.1837)* (1.2326)**
5	0.1671, (0.1671)* (0.2120)**	0.5907, (0.5907)* (0.6389)**	1.0792, (1.0792)* (1.1279)**

Table 4 Comparison of the results of reduced Nusselt number $-\theta'(0)$, for the Blasius flow for different values of the temperature and radiation parameter θ_w and Rd, respectively, with previously published data [38] with $Pr = 5.0$ and $K \to \infty$ (i.e., $K = 1000$) fixed, and $()^{**}$ represents for the curved surface by taking $K = 10$

$Rd = 10$		$Rd = 100$	$Rd = 1000$
Battaler [35] Linear Rosseland approximation			
	0.5528	0.5741	0.5764
Pantokratoras and Fang [38] present results of linear Rosseland approximation			
0.5533, (0.5533)* (0.6014)**		0.5745, (0.5745)* (0.6227)**	0.5767, (0.5767)* (0.6248)**
θ_w	Nonlinear linear Rosseland approximation		
1.5	0.4755, (0.4755)* (0.5243)**	0.5640, (0.5640)* (0.6122)**	0.5756, (0.5756)* (0.6238)**
3	0.2422, (0.2422)* (0.2883)**	0.4714, (0.4714)* (0.5193)**	0.5635, (0.5635)* (0.6117)**
5	0.1266, (0.1266)* (0.1706)**	0.3068, (0.3068)* (0.3540)**	0.5171, (0.5171)* (0.5651)**

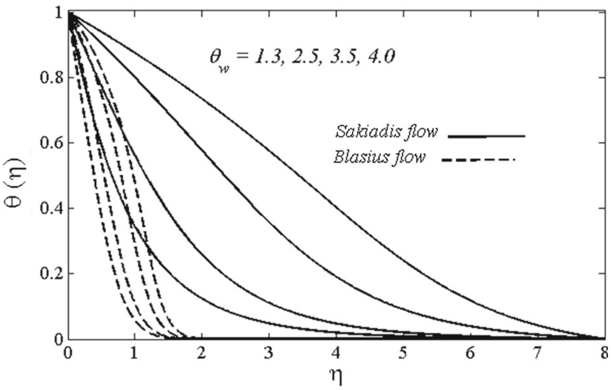

Fig. 5 Variation of temperature parameter θ_w on temperature distribution θ by keeping $Pr = 20$, $Rd = 0.2$ and $K = 7$ fixed

boundary layer thickness is higher for the Sakiadis flow as compared to the Blasius flow because the Nusselt number for Sakiadis flow is more than Blasius flow which can be seen in Tables 3 and 4. Figure 5 is made to see the variation of temperature parameter θ_w on the temperature distribution for both Sakiadis and Blasius flows. From this figure it is found that temperature is increased for higher values of θ_w. Also, one can observe from this figure that the temperature distribution becomes S shaped (as discussed by Pantokratoras and Fang [37,38]) showing the existence of adiabatic case for large values of θ_w. It is also worth mentioning here that when θ_w is near 1, the nonlinear Rosseland approximation tends to linear Rosseland approximation, which can be seen in Tables 3 and 4.

6 Concluding Remarks

In the present study the Blasius and Sakiadis flows are considered in the presence of nonlinear radiation over a curved surface. The following conclusions have been drawn from this study:

- Both the temperature and thermal boundary layer thickness of the fluid are decreased for higher values of Pr.
- An increase in nonlinear radiation parameter Rd raises the temperature and the thermal boundary layer thickness.
- Temperature of the fluid increases and becomes S shape for large value of θ_w.
- By taking $K \to \infty$, the results for flat surface for both Sakiadis and Blasius flows are recovered.

Acknowledgements We are thankful to the honorable reviewers for their constructive suggestions.

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