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Hyperbolic Heat Conduction in a Functionally Graded Hollow Sphere

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Abstract Non-Fourier hyperbolic heat conduction in a heterogeneous sphere is investigated in this article. Except for the thermal relaxation time, which is assumed to be constant, all other material properties vary continuously within the sphere in the radial direction following a power law. Boundary conditions of the sphere are assumed to be spherically symmetric, leading to a one-dimensional heat conduction problem. The problem is solved analytically in the Laplace domain, and the final results in the time domain are obtained using numerical inversion of the Laplace transform. The transient responses of temperature and heat flux are investigated for different non-homogeneity parameters and normalized thermal relaxation constants. The current results for the specific case of a homogeneous sphere are validated by results available in the literature.

Keywords Functionally graded sphere · Hyperbolic heat conduction · Spherically symmetric · Temperature waves

1 Introduction

Functionally graded materials (FGMs) have seen increasing applications in engineering design because of their desirable properties compared with ordinary laminated composite materials. The continuous spatial variation of physical properties in such materials has eliminated some adverse effects in ordinary composites, such as stress concentration and delamination. For instance, it is possible to use thermal shock resistance of ceramics in one side of the FGM structure due to their low thermal

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conductivity, while the other side of the structure is metal, making it possible to be bolted or welded to other structures. Although they were first invented as a thermal shield to sustain very high temperature gradients in thin structures [1,2], FGMs are currently being used for many other applications such as wear-resistant linings, heat exchanger tubes, thermoelectric generators, heat-engine components.

Heat conduction analysis of FGMs is of great interest since they are frequently used as thermal barriers. Solution of the temperature field is essential to calculating thermal stresses within the structures. It is known that the Fourier heat conduction law is not accurate enough to predict the transient temperature fields in some cases. Examples include the transient temperature field caused by pulsed laser heating of thin structures, and the heat conduction in such structures in temperatures close to absolute zero. It is reported that the measured surface temperature of a slab immediately after an intense thermal shock is 300 °C higher than that obtained from the Fourier law [3]. The main drawback of the Fourier law of heat conduction is that it leads to an unbounded velocity of thermal waves. To improve the accuracy of the traditional heat conduction law, Vernotte [4] and Cattaneo [5] independently proposed the hyperbolic heat conduction theory, also called the telegraph equation for solids, which observed the relaxation time needed for temperature to adjust to the thermal disturbance. By introducing the second relaxation time showing that a non-zero time is required for both the heat flux and the temperature to be adjusted to the thermal disturbance, Tzou [6] proposed a dual-phase-lag (DPL) heat conduction equation, which smoothes the sharp wave front obtained using the Cattaneo–Vernotte (C–V) equation. Using the Boltzmann equation, other researchers attempted to enhance the heat conduction predictions [7–10]. For instance, Chen [7] proposed a ballistic-diffusive equation to better approximate heat conduction in thin film, even if the film thickness is comparable to the mean free path of the heat carrier or the characteristic time is comparable to the heat-carrier relaxation time.

To our knowledge, there are very few papers in the literature about the non-Fourier heat conduction in heterogeneous media. Fang and Hub [11] investigated the propagation of hyperbolic thermal waves caused by a spherical substrate in a semi-finite FGM medium. For simplicity, they assumed that the density and thermal relaxation are constant, while other properties vary exponentially in the direction normal to the boundary of the medium. Others dealt with the non-Fourier heat conduction analysis of homogeneous structures. For instance, Glass et al. [12] investigated the periodic heat conduction in a semi-infinite medium using the C–V equation; Zanchini and Pulvirenti [13] solved the telegraph conduction in cylindrical geometries. Ozisik and Vick [14] investigated the propagation of thermal waves in a thin strip subjected to a volumetric heat source. Al-Nimr and Naji [15] solved the problem of hyperbolic heat conduction in an anisotropic material. Tang and Araki [16–18] solved several problems of non-Fourier heat transfer using the C-V and DPL theories for different geometries and loading conditions including pulse laser heating. Jiang and Sousa [19] analyzed the hyperbolic heat conduction in homogeneous spheres. Using a combined analytical and numerical method, Tsai and Hung [20] investigated the thermal wave propagation in a bi-layered composite sphere due to a sudden temperature change on the outer surface. Kozlowska et al. [21] analyzed hyperbolic thermal waves in thin gold films induced by picosecond laser pulses. The majority of research articles about heat conduction in non-homogeneous solids investigate Fourier heat conduction in such structures. For example, Noda [22] calculated the optimum composition profile of the properties of FGM structures under severe thermal shocks to minimize the thermal stresses using the Fourier law. Eslami et al. [23] solved the thermoelasticity problem of an FGM hollow sphere analytically using the Fourier law and a power law variation of the material properties. Hosseini et al. [24] used a separation-of-variable method to solve the transient, axisymmetric problem of Fourier heat conduction within a functionally graded, cylindrical shell. A review of available analytical solutions for hyperbolic heat conduction is given by Antaki [25].

The current work is motivated by [19,26] in which hyperbolic heat transfer within a homogeneous sphere was investigated. The present study investigates non-Fourier heat conduction using the C–V equation for non-homogeneous hollow spheres. The properties of the sphere are varying in the thickness direction according to a power law with the exponent showing the non-homogeneity value of the body. The governing equation is solved analytically in the Laplace domain. The inversion of the Laplace transform is carried out numerically to obtain the results in the actual time domain. For the homogeneous case, the current results are verified by those reported in [19,26].

2 Statement of the Problem and Solution Procedure

Consider a radially graded FG hollow sphere whose inner and outer radii are, respectively, r_{i} and r_{o}, as shown in Fig. 1.

From Fig. 1, the boundary conditions of the problem can be expressed as

$$\begin{aligned} T\,(r,t)|_{r=r_{\mathrm{i}}} &= T_{\mathrm{wi}}, \\ T\,(r,t)|_{r=r_{\mathrm{o}}} &= T_{\mathrm{wo}}, \end{aligned} \tag{1}$$

in which T, r, and t are, respectively, temperature, radial coordinate, and time, and T_{wi} and T_{wo} are the imposed temperatures on the inner and outer surfaces of the hollow sphere, respectively.

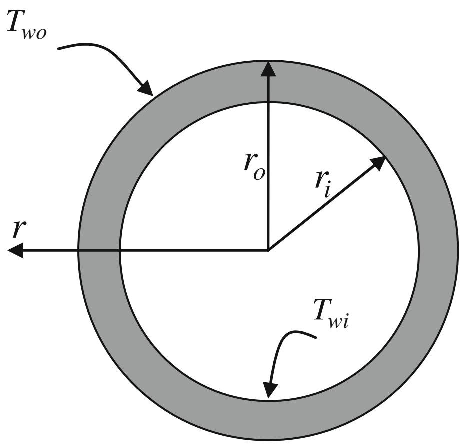

Fig. 1 A hollow FGM sphere and the imposed boundary conditions

For isotropic bodies, the hyperbolic heat conduction equation is [27]

$$\vec{q} + \tau \frac{\partial \vec{q}}{\partial t} = -K \nabla T \tag{2}$$

in which $\vec{q}$, τ, and K are, respectively, the heat flux vector, thermal relaxation time, and thermal conductivity, and ∇ is the gradient operator.

On the other hand, the energy balance is expressed as

$$\rho c_p \frac{\partial T}{\partial t} = S - \nabla \cdot \vec{q} \tag{3}$$

where ρ, c_p, and S stand for, respectively, density, specific heat, and internal heat generation.

The spherical symmetry and isotropy of the current problem imply that only the radial component of heat flux is non-vanishing. Moreover, all geometrical derivatives are zero except the derivative with respect to the radial coordinate, r. Then, in the absence of internal heat generation, Eqs. 2 and 3 become

$$-\left(1 + \tau \frac{\partial}{\partial t}\right) q_{\mathrm{r}} = K \frac{\partial T}{\partial r} \tag{4a}$$

$$-\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 q_{\mathrm{r}}\right) = \rho c_p \frac{\partial T}{\partial t} \tag{4b}$$

where q_{r} is the radial component of the heat flux. To investigate the effect of the non-homogeneity on the hyperbolic heat conduction, the properties are assumed to vary radially according to a power law, while the thermal relaxation is taken to be constant within the body, viz.,

$$K(r) = K_0 r^{n_1}, \quad \rho(r) = \rho_0 r^{n_2}, \quad c_p = c_{p0} r^{n_3} \tag{5}$$

where $n_j (j = 1, 2, 3)$ are non-homogeneity exponents. K_0, ρ_0, and c_{p0} are constants.

Substituting Eq. 5 into Eq. 4, we have

$$\begin{aligned} -\left(1 + \tau \frac{\partial}{\partial t}\right) q_{\mathrm{r}} &= K_0 r^{n_1} \frac{\partial T}{\partial r} \\ -\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 q_{\mathrm{r}}\right) &= C_0 r^{n_2+n_3} \frac{\partial T}{\partial t} \end{aligned} \tag{6}$$

It is more convenient to introduce normalized parameters as follows:

$$\begin{matrix} \eta = \dfrac{r}{r_{\mathrm{o}}} & \varepsilon_0 = \dfrac{\kappa_0 \tau}{r_{\mathrm{o}}^2} & \theta = \dfrac{T - T_0}{T_{\mathrm{wo}} - T_0} \\ \xi = \dfrac{\kappa_0 t}{r_{\mathrm{o}}^2}, & r_\gamma = \dfrac{r_{\mathrm{i}}}{r_{\mathrm{o}}}, & T_\gamma = \dfrac{T_{\mathrm{wi}} - T_0}{T_{\mathrm{wo}} - T_0}, \end{matrix} \quad Q = \frac{r_{\mathrm{o}}}{K_0 T_0} q_{\mathrm{r}} \tag{7}$$

in which $\kappa_0 = \frac{K_0}{\rho_0 c_{p0}}$ and T_0 are the thermal diffusivity of the inner surface and initial temperature of the sphere.

Normalizing Eq. 6 with Eq. 7, and then performing Laplace transform to the normalized equations and assuming a uniform initial temperature in the sphere before heat impact, i.e., $T\left(r,t\right)|_{t=0} = T_0$, we obtain the governing equations of the problem in the Laplace domain:

$$-\left(1+\varepsilon_0 s\right)\tilde{Q} = r_{\mathrm{o}}^{n_1}\frac{T_{\mathrm{wo}}-T_0}{T_0}\eta^{n_1}\frac{\partial\tilde{\theta}}{\partial\eta}, \tag{8a}$$

$$-\frac{1}{\eta^2}\frac{\partial}{\partial\eta}\left(\eta^2\tilde{Q}\right) = r_{\mathrm{o}}^{n_2+n_3}\frac{T_{\mathrm{wo}}-T_0}{T_0}\eta^{n_2+n_3}s\tilde{\theta}, \tag{8b}$$

in which $\tilde{Q}$ and $\tilde{\theta}$ are the Laplace transforms of the normalized heat flux and temperature; s is the Laplace parameter.

Eliminating $\tilde{Q}$ between Eqs. 8a and 8b, we obtain an ordinary differential equation for $\tilde{\theta}$:

$$\eta\tilde{\theta}_{,\eta\eta} + \left(2+n_1\right)\tilde{\theta}_{,\eta} - r_{\mathrm{o}}^{n_2+n_3-n_1}\eta^{n_2+n_3-n_1+1}s\left(1+\varepsilon_0 s\right)\tilde{\theta} = 0. \tag{9}$$

The solution of Eq. 8 is

$$\tilde{\theta}\left(\eta,s\right) = \eta^{-\frac{1}{2}\left(n_1+1\right)}\left(A_1 J_G\left(I\eta^H\right) + A_2 Y_G\left(I\eta^H\right)\right) \tag{10}$$

where G, H, and I are

$$\begin{aligned} G &= \frac{n_1+1}{2-n_1+n_2+n_3}, \\ H &= 1-\frac{1}{2}n_1+\tfrac{1}{2}n_2+\tfrac{1}{2}n_3, \\ I &= -\frac{2\sqrt{-sr_{\mathrm{o}}^{n_2+n_3-n_1}\left(1+\varepsilon_0 s\right)}}{2-n_1+n_2+n_3}, \end{aligned} \tag{11}$$

and $J_\alpha\left(\beta\right)$ and $Y_\alpha\left(\beta\right)$ are, respectively, Bessel functions of the first and second kind, with order α and argument β; A_1 and A_2 are integration constants to be determined by the boundary conditions.

Substituting Eq. 10 into Eq. 8a, we obtain the non-dimensional heat flux in the Laplace domain, $\tilde{Q}$, as follows:

$$\begin{aligned} \tilde{Q}\left(\eta,s\right) = P\eta^{\frac{1}{2}n_1-\frac{3}{2}}\Big(&A_1\left(MJ_G\left(I\eta^H\right)+2IH\eta^H J_{G+1}\left(I\eta^H\right)\right) \\ &+ A_2\left(MY_G\left(I\eta^H\right)+2IH\eta^H Y_{G+1}\left(I\eta^H\right)\right)\Big) \end{aligned} \tag{12}$$

where

$$
\begin{aligned}
M &= n_1 + 1 - 2HG, \\
P &= -\frac{r_{\mathrm{o}}^{n_1}\left(T_{\mathrm{wo}} - T_0\right)}{2T_0\left(1 + \varepsilon_0 s\right)}.
\end{aligned}
\tag{13}
$$

The boundary conditions of Eq. 1 can be normalized and transformed in the Laplace domain as follows:

$$
\left.\tilde{\theta}\left(\eta, s\right)\right|_{\eta=1} = \frac{1}{s},\ \left.\tilde{\theta}\left(\eta, s\right)\right|_{\eta=r_\gamma} = \frac{T_\gamma}{s}
\tag{14}
$$

Substituting Eq. 10 into 14, we can eventually obtain A_1 and A_2 as follows:

$$
\begin{aligned}
A_1 &= \frac{Z}{X}, \\
A_2 &= \frac{W}{X}.
\end{aligned}
\tag{15}
$$

in which

$$
\begin{aligned}
W &= Y_G\left(Ir_\gamma^H\right) - r_\gamma^{\frac{1}{2}(1+n_1)} Y_G\left(I\right) T_\gamma, \\
X &= s\left(J_G\left(I\right) Y_G\left(Ir_\gamma^H\right) - J_G\left(Ir_\gamma^H\right) Y_G\left(I\right)\right), \\
Z &= r_\gamma^{\frac{1}{2}(1+n_1)} J_G\left(I\right) T_\gamma - J_G\left(Ir_\gamma^H\right).
\end{aligned}
\tag{16}
$$

Substituting Eq. 15 into Eqs. 10 and 12, we can express explicitly the final solution in the Laplace domain as follows:

$$
\begin{aligned}
\tilde{\theta}\left(\eta, s\right) &= \frac{\eta^{-\frac{1}{2}(1+n_1)}}{X}\left(ZJ_G\left(I\eta^H\right) + WY_G\left(I\eta^H\right)\right) \\
\tilde{Q}\left(\eta, s\right) &= \frac{P}{X}\eta^{\frac{1}{2}n_1-\frac{3}{2}}\left(Z\left(MJ_G\left(I\eta^H\right) + 2IH\eta^H J_{G+1}\left(I\eta^H\right)\right)\right. \\
&\quad \left. + W\left(MY_G\left(I\eta^H\right) + 2IH\eta^H Y_{G+1}\left(I\eta^H\right)\right)\right)
\end{aligned}
\tag{17}
$$

To obtain the final solution in the time domain, the inversion of Eq. 17 needs to be performed. Here, we employ the fast Laplace inversion technique (FLIT) proposed by Durbin [28]. In this method, the inversion of a function $\left(\tilde{f}\left(\eta, s\right)\right)$ at time ξ_j is found numerically as follows [28]:

$$
\tilde{f}\left(\eta, \xi_j\right) = D\left(j\right)\left[-\frac{1}{2}\mathrm{Re}\left\{\tilde{f}\left(\eta, a\right)\right\} + \mathrm{Re}\left\{\sum_{k=0}^{N-1}\left(A\left(\eta, k\right) + iB\left(\eta, k\right)\right)W^{jk}\right\}\right],
$$
$$
j = 0, 1, 2, \ldots, N-1
\tag{18}
$$

in which

$$
\begin{aligned}
&A(\eta, k)=\sum_{l=0}^{L} \operatorname{Re}\left\{\tilde{f}\left(\eta, a+i(k+l N) \frac{2 \pi}{T}\right)\right\}, \\
&B(\eta, k)=\sum_{l=0}^{L} \operatorname{Im}\left\{\tilde{f}\left(\eta, a+i(k+l N) \frac{2 \pi}{T}\right)\right\}, \\
&D(j)=\frac{2}{T} \mathrm{e}^{a j \Delta t}, \quad \Delta t=\frac{T}{N} \\
&W=\mathrm{e}^{i \frac{2 \pi}{N}} .
\end{aligned} \tag{19}
$$

Furthermore, it should be noted that a, which is an arbitrary real number, should be larger than all real parts of the function's singularities; Δt is the time increment; and L and N are arbitrary parameters which influence the accuracy of the solution.

To minimize both discretization and truncation errors, it is recommended to observe the following constraints [28]:

$$
\begin{aligned}
&5 \leq aT \leq 10, \\
&50 \leq NL \leq 5,000.
\end{aligned} \tag{20}
$$

For the present work, the abovementioned parameters are taken as follows:

$$
\begin{aligned}
&aT = 7.5, \\
&L = 10, \\
&N = 500.
\end{aligned} \tag{21}
$$

3 Numerical Example

In this section, the results are given for different non-homogeneity constants and non-dimensional thermal relaxations (ε_0).

The inner and outer radii of the sphere are taken to be 0.6 and 1 $\left(r_\gamma = 0.6\right)$, respectively. The properties of the outer surface of the sphere are kept constant. The initial temperature of the sphere is assumed to be 300 K. It is worth noting that a variation of the inner-to-outer-radius ratio may affect the results, and the choice of these values is made solely to show the interesting behaviors of the thermal waves in the sphere. Other inner-to-outer-radius ratios such as 0.9 have been tested, and there does not exist significant deviations in the dynamic thermal response of the sphere.

Figure 2 shows the effect of the material heterogeneity on the temperature history of three different points within the sphere ($\eta = 0.7, 0.8, 0.9$). To validate the results with [19,26], the sudden temperature changes on the inner and outer surfaces of the sphere have been taken to be the same, i.e., $T_\gamma = 1$. In Fig. 2, all the properties are assumed to have the same non-homogeneity degree, i.e., $n_1 = n_2 = n_3 = n$.

It is seen that when n increases from -1 to 1, the maximum transient temperature increases for the point closer to the inner surface, and decreases for the point closer to

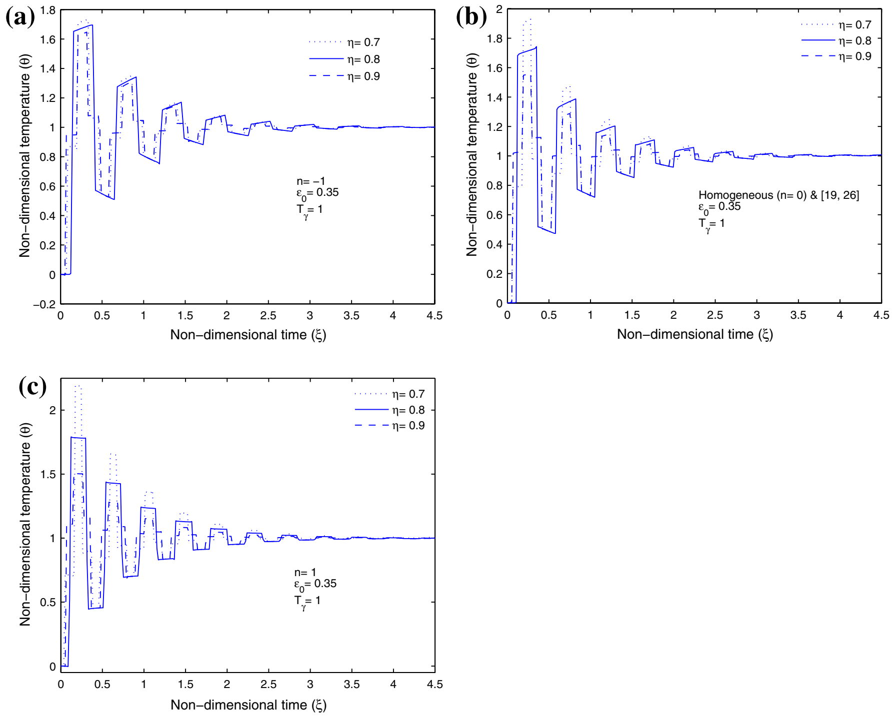

Fig. 2 Effect of the non-homogeneity exponent on the temperature history at three different locations, $\eta = 0.7, 0.8, 0.9$, within the sphere: (**a**) $n = -1$, (**b**) $n = 0$, and (**c**) $n = 1$

the outer surface. Figure 2b shows the solution for $n = 0$, which is exactly the same as that for homogeneous materials [19,26]. Further investigations show that there exists a point ($\eta = 0.825$) between the inner and outer surfaces at which the temperature history is roughly unaltered with any variation in n, as shown in Fig. 3. Before this point, an increase in n leads to an increase in the maximum transient temperature and after it, this behavior is reversed.

Figure 4 shows the temperature history of the point $\eta = 0.8$ for two non-dimensional thermal relaxation parameters, $\varepsilon_0 = 0.01$ and $\varepsilon_0 = 0.35$. When $\varepsilon_0 = 0.01$, the temperature reaches its stationary value much more quickly with fewer oscillations around it compared with when $\varepsilon_0 = 0.35$. Although for $n = 1$ the oscillation of temperature lasts slightly longer compared with that for $n = -1$ and $n = 0$, the general trends are almost the same regardless of different n values.

Figure 5 illustrates the effect of the heterogeneity parameter, n, on the velocity of the thermal wave propagation. To avoid initiation of two thermal waves when the temperatures of both the inner and outer surfaces are changed suddenly, the temperature of the inner side of the sphere is fixed to the initial temperature, T_0, while the outer surface is subjected to a sudden temperature rise of 500 K.

Figure 5a–c shows the incoming thermal wave fronts traveling from the outer surface to the inner surface at three different moments: (a) $\xi = 0.04$, (b) $\xi = 0.1$, and

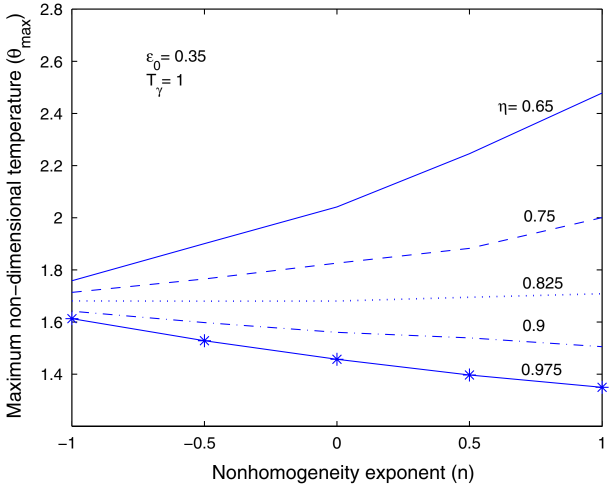

Fig. 3 Maximum temperature at different points through the thickness for different non-homogeneities

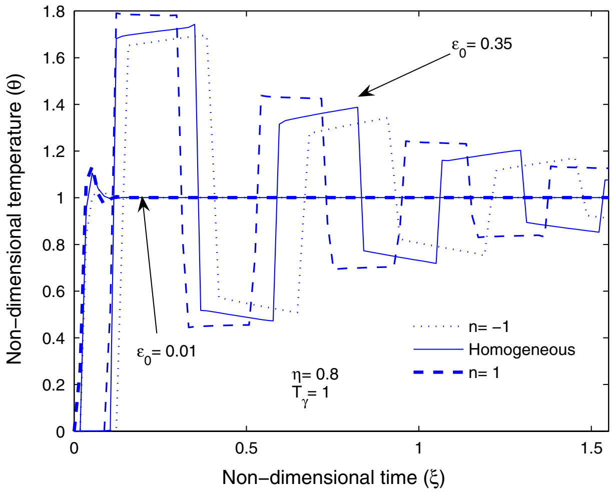

Fig. 4 Effect of the non-homogeneity exponent on temperature history for two different thermal relaxation parameters, $\varepsilon_0 = 0.01$ and $\varepsilon_0 = 0.35$

(c) $\xi = 0.18$, for different non-homogeneity parameter values. At the very beginning, although the maximum temperature is higher for larger n, all wave fronts are almost synchronous with each other without any significant delay for different n values (Fig. 5a). Shortly after, the wave front for $n = 1$ moves ahead of the others which are still traveling at roughly the same speed, as shown in Fig. 5b. The effect of the non-homogeneity parameter on the thermal wave speed is more pronounced as it approaches the inner surface, as shown in Fig. 5c. The non-homogeneity effect on the thermal wave speed becomes even more pronounced after the first travel of thermal waves from the outer surface to the inner surface, where higher n values lead to higher wave speed, as shown in Fig. 5d, e. Thermal waves eventually dissipate after several

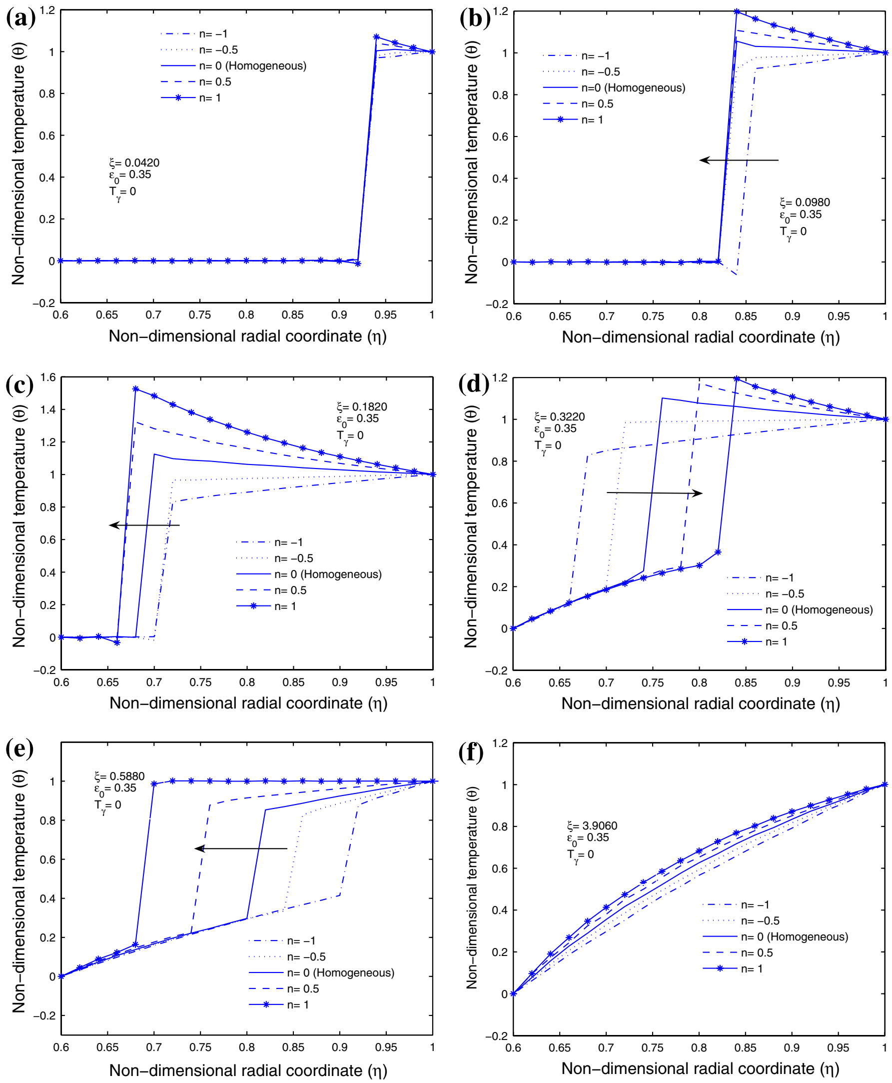

Fig. 5 Effect of non-homogeneity exponent on the speed and position of thermal waves at different moments: (**a**) $\xi = 0.04$, (**b**) $\xi = 0.1$, (**c**) $\xi = 0.18$, (**d**) $\xi = 0.32$, (**e**) $\xi = 0.59$, and (**f**) $\xi = 3.9$

reflections back and forth between the inner and outer surfaces. The final distribution of temperature along the thickness of the sphere is shown in Fig. 5f, and higher n values result in higher temperatures within the thickness.

An increase in non-homogeneity exponent leads to an increase in the heat flux wave speed, similar to the temperature wave as illustrated in Fig. 6. Thus, henceforth, we omit the results of heat flux for brevity.

To unravel the effect of different properties, among the three non-homogeneity parameters, n_1, n_2, and n_3, two are set to be zero while the remaining one is varied. Although a variation in the specific heat or density affects the transient temperature

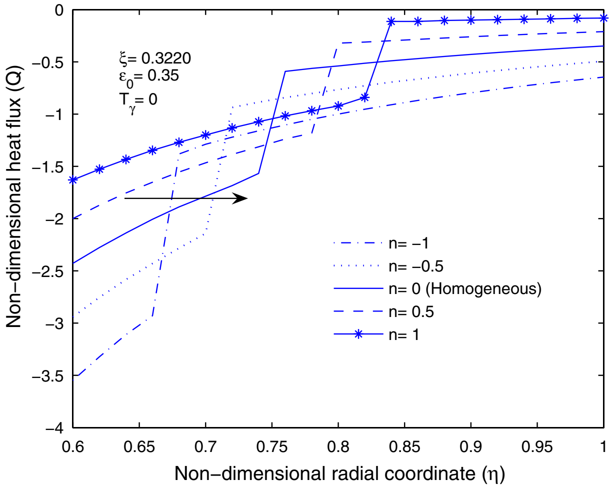

Fig. 6 Effect of non-homogeneity exponent on the speed of heat flux waves

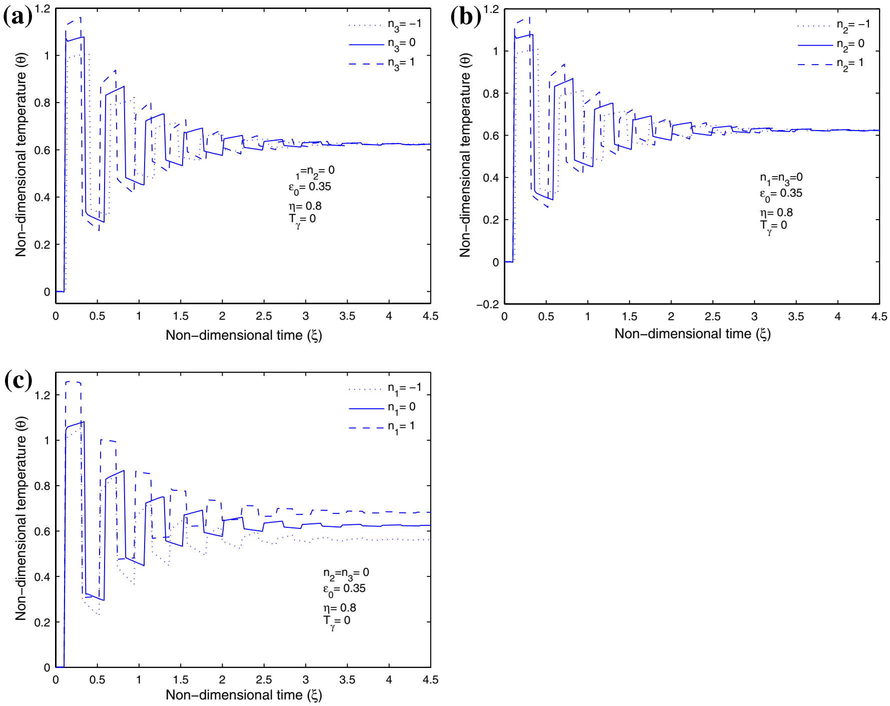

Fig. 7 Dependence of temperature history on variations of different properties: (**a**) thermal conductivity, (**b**) density, and (**c**) specific heat

within the sphere, the final temperature is almost insensitive to them, as shown in Fig. 7a and b. However, a variation in the thermal conductivity greatly affects the final temperature as well as the transient temperature distribution, as shown in Fig. 7c.

4 Conclusions

The solution of the hyperbolic heat conduction in an FGM hollow sphere is presented in this paper. The properties of the sphere are assumed to be varying in the radial direction following a power law except for the thermal relaxation parameter, which is taken to be constant.

The problem is solved analytically in the Laplace transform domain, and the numerical inversion of the Laplace transform is performed to find the results in the time domain. For the homogeneous case, the current results will reduce to the available results in the literature showing the validity of the solution. The following conclusions are drawn from the current study.

(1) An increase in the non-homogeneity parameter, n, results in a decrease in the maximum temperature at the point closer to the outer surface ($\eta = 0.9$) and an increase at the point closer to the inner surface ($\eta = 0.7$). Moreover there exists a point within the sphere at which the temperature history is unchanged by the heterogeneity of the material.
(2) When the thermal relaxation time is small, the non-homogeneities of properties do not have a significant effect on the temperature response. The temperature history is almost diffusive.
(3) The speed of thermal wave propagation, in terms of temperature or heat flux, is strongly affected by the non-homogeneity parameters. For a sudden temperature rise on the outer surface, a higher n value results in a higher reflected wave speed except for the first incoming wave from the outer surface of the sphere.
(4) The final temperature distribution is strongly affected by the thermal conductivity of the material. However, it is not noticeably affected by the specific heat and mass density.

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