



Observing Mathematical Properties in the Virtual World: An Exploratory Study of Online Independent Learning of Locus Concepts

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Received: 4 October 2023 / Accepted: 13 April 2024
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Abstract

In the post-pandemic world, metaverse technology has become a popular approach to facilitating students' online learning experiences. Drawing on Kolb's experiential learning theory, we created immersive scenes using CoSpaces Edu, a 'mirror-world' metaverse platform that enables students to observe mathematical properties virtually. We applied a quasi-experimental design and evaluated the effect of integrating these virtual experiences into our learning resources. Students in the control group used conventional resources created using GeoGebra, which allowed them to manipulate mathematical objects and observe the locus of moving points. For the experimental group, in addition to the GeoGebra resources, we incorporated scenes (e.g., scenarios featuring running dogs) via CoSpaces Edu, which enabled the students to observe mathematical properties virtually online. The post-test results demonstrated that the students in the experimental group ($N=28$) performed better than those in the control group ($N=35$). The virtual world, which offered immersive walk-through experiences and different perspectives, enhanced the students' understanding of locus concepts. However, some students encountered technical issues during their online learning, which may have hindered their mathematical exploration. This study contributes to the application of experiential learning theory using metaverse tools, thus offering educators valuable insights into creating immersive environments for online mathematics instruction.

Keywords Experiential learning theory · Mathematics education · Mirror-world metaverse · Secondary education

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Introduction

The pandemic-led suspension of campus operations served as a wake-up call for educators and highlighted the importance and potential of online instruction (Bozkurt et al., 2020). As Cao et al. (2021) emphasised, proficiency in online independent learning has become a crucial skill in the post-pandemic world. We must equip students with skills to excel in situations where traditional face-to-face teaching might not be possible (Huang et al., 2020; Stracke et al., 2022). To encourage them to become self-directed learners, one possible strategy is to task students with independently learning specific course materials before attending class (Lo, 2023).

However, providing pedagogical support for students in their online learning is not a simple task. It requires the provision of high-quality educational resources that align with curriculum standards and spark students' interest and engagement (Bugler et al., 2017). As Zourmpakis et al. (2022) cautioned, a lack of understanding of the resources could potentially lead to a decrease in performance. Therefore, in this study we showcase the development of e-learning resources that secondary school students can use to learn mathematics on a self-paced basis. Our initial attempt involved creating the resources using GeoGebra (<https://www.geogebra.org/>), a dynamic mathematics application, with the aim of making online learning more interactive by enabling students to manipulate mathematical objects. We found that this set of GeoGebra resources worked well for basic concepts but fell short when it came to helping students explore more advanced topics.

To tackle this problem, we then considered the metaverse and other virtual world platforms. These have become popular e-learning resources and educators can explore their innovative features to further develop students' knowledge and motivation. Jiang et al. (2020) noted that the virtual world can enable students to interact with digital content and build immersive environments in online flipped classroom settings. Zhai (2021) also highlighted that virtual environments offer a safe, cost-effective, and self-paced learning context and thus promote science inquiry at home.

We thus explored the use of CoSpaces Edu (<https://cospaces.io/edu/>), a web-based metaverse platform that allowed us to build an immersive virtual world in which mathematical objects and properties can be presented for student observation beyond the classroom. Unlike traditional virtual reality (VR), this platform offers accessibility, the ability to create a 3D virtual world, user-friendliness, and the integration of media elements. It enables students to experience and create virtual worlds without being confined to campus or requiring specialised VR equipment. Its intuitive interface, collaborative capabilities, and interactive elements make it a user-friendly platform that provides students with self-paced and immersive learning experiences (Ng, 2022). Due to the lack of research on the metaverse in education, however, further exploration in this area is required (Hwang & Chien, 2022; Ng et al., 2023).

To address this research gap, in this study we aimed to investigate students' mathematics learning experiences via a new metaverse-based learning approach (i.e., CoSpaces Edu) to facilitating their learning of locus concepts within the virtual environment. The following research questions (RQ1 and RQ2) guided the study.

- RQ1: To what extent does the observation of mathematical properties in CoSpaces Edu enhance students' online learning of locus concepts?
- RQ2: How do students perceive their mathematics learning experiences in CoSpaces Edu?

The subsequent sections of this paper describe the conceptual background, methods, and results, followed by a discussion and conclusion, along with limitations and recommendations for future research.

Conceptual Background

The conceptual background of this study covers three main areas. First, we establish our theoretical framework for developing educational resources by drawing on Kolb's (1984) experiential learning theory. We then describe how GeoGebra and CoSpaces Edu can be applied to mathematics learning, with the aim of operationalising this theory and facilitating support for students' online learning. We further explain how the theory and IT tools informed our instructional design.

Theoretical Framework: Experiential Learning Theory

Kolb's (1984) experiential learning theory serves as the theoretical foundation for our approach to resources development in the context of online learning. From this theoretical perspective, meaningful learning occurs when students engage in a continuous cycle of concrete experience, reflective observation, abstract conceptualisation, and active experimentation. This holistic learning process has been applied in the recent development and utilisation of e-learning resources (e.g., Ghosheh Wahbeh et al., 2023; Lo et al., 2023; Ping Por et al., 2020). The four stages of Kolb's (1984) experiential learning cycle are as follows.

- Concrete experience: Students are exposed directly to new concepts or situations. Teachers can then facilitate the acquisition of concrete experience by providing students with e-learning resources they can actively engage and interact with (Ping Por et al., 2020).
- Reflective observation: Students move on to analyse and reflect upon their experiences. Teachers can pose reflective questions to assist students in establishing connections between their observations and mathematics content (Ghosheh Wahbeh et al., 2023).
- Abstract conceptualisation: Students formulate concepts based on their observations and reflections. Teachers can use verbal instructions to explain the mathematical concepts involved, thereby facilitating students' knowledge acquisition (Lo et al., 2023).
- Active experimentation: Students apply their newly formed concepts to solve problems. By providing opportunities for active experimentation, their ability to

transfer their knowledge to practical mathematical applications is enhanced (Ghosh Wahbeh et al., 2023; Lo et al., 2023).

However, Morris (2020) identified a theoretical gap when interpreting the experiential learning cycle, centred around the ambiguity regarding “what constitutes a concrete experience, exactly” (p. 1064). He thus emphasised the importance of students physically interacting with learning objects and engaging in inquiries into real-world problems. While offering students concrete experiences through interaction with tangible learning materials in a face-to-face classroom setting is generally straightforward, delivering similar experiences in an online learning environment becomes a more challenging task. Therefore, we leveraged relevant IT tools with the aim of providing students with concrete experiences related to mathematical properties within the context of their everyday lives.

Using GeoGebra and CoSpaces Edu for Mathematical Learning

We initially used GeoGebra to develop e-learning resources. This tool allows educators to create interactive learning materials that enable students to explore mathematical concepts in online settings (Radović et al., 2020; Yohannes & Chen, 2023). Students can use GeoGebra applets to manipulate mathematical objects, visualise geometrical properties, and experiment with algebraic equations, thereby facilitating a deeper understanding of mathematics knowledge (Yohannes & Chen, 2023). For example, Radović et al. (2020) developed GeoGebra applets for elementary school students. One of their applets enabled the students to manipulate the components of a triangle and observe how the sum of the exterior angles remained constant. The researchers found that these interactive learning materials positively influenced their students’ mathematics learning. In his linear algebra course, Turgut (2022) used GeoGebra to create educational resources. He used sliders in his applet to facilitate students’ mathematical exploration of different matrices by changing the variable values. Turgut (2022) suggested that using GeoGebra applets can provide a context that supports students’ reasoning regarding mathematical observations.

However, when addressing more advanced topics, we encountered a limitation in GeoGebra’s capacity to create immersive virtual environments aimed at demonstrating mathematical properties as in the real world. Dimmel and Bock (2017) also noted this limitation, pointing out that although GeoGebra provides a top view for students to learn geometry on the screen, metaverse platforms such as CoSpaces Edu offer first- or third-person views. The students can obtain a virtual walk-through immersive experience that can help them retain knowledge (Valero-Franco & Berns, 2023). The metaverse also shows real-world phenomena in a virtual environment, enabling students to observe objects from their everyday lives (Cowin, 2020). Offering this type of immersive learning experience aligns with the current trend of using game-design technological elements as innovative educational tools (Zourmpakis et al., 2023).

As discussed by Ng (2022), educators can create meaningful immersive learning experiences to construct knowledge within the metaverse. CoSpaces Edu is a

‘mirror-world’ metaverse platform, embodying what Kye et al. (2021) described as “a metaverse where the appearance, information, and structure of the real world are transferred to virtual reality as if reflected in a mirror” (p. 4). The platform enables educators to construct virtual 3D worlds in which objects and mathematical concepts spring to life, thus “opening doorways to experiential learning for students” (Trudeau, 2020, p. 80). As Morris (2020) emphasised, the concrete experience stage in the experiential learning cycle demands engagement with real-world problems. CoSpaces Edu can facilitate this engagement by providing students with the opportunity to observe virtual real-world objects and explore mathematical properties in a fully immersive online setting. For example, Bhatia et al. (2021) and Lee and Hwang (2022) created various environments such as fields with elements from nature and decorated indoor spaces. Ng et al. (2023) further identified CoSpaces Edu projects in which students could rotate and construct geometric objects in mathematics museums. Although research in this area is currently lacking (Ng et al., 2023), this resource development tool can potentially bridge the gap between Kolb’s (1984) experiential learning theory and its practical application.

Implications for Instructional Design

Incorporating experiential learning theory (Kolb, 1984; Morris, 2020) with the use of CoSpaces and GeoGebra informed our approach to facilitating student learning in online settings. We created two scenes using CoSpaces Edu to offer students concrete experiences and opportunities for reflective observations, as in the real world. These scenes were designed to align with two teaching objectives (TO1 and TO2) of loci, a senior secondary mathematics education topic in the Hong Kong curriculum (Curriculum Development Council, 2017):

- TO1: Maintaining a fixed distance from a fixed point.
- TO2: Maintaining an equal distance from two given points.

Based on the suggestions of Bhatia et al. (2021), we designed and created both the corresponding movements and a virtual environment. For TO1, we created two dogs (representing the moving points) orbiting around their respective owners (representing the fixed points) in a field, as shown in Fig. 1(a), thereby illustrating the locus of the dogs as circles with different radii. For TO2, Fig. 1(b) shows a teacher (representing the moving point) acting as an invigilator during an examination. As the teacher moves inside the classroom, he maintains an equal distance from two students (representing the two fixed points) taking the examination, resulting in the locus of the teacher forming the perpendicular bisector of the line segment connecting the two students. According to Papadakis et al. (2023), such simulation elements have the potential to increase student engagement and learning achievement.

Taking TO2 as an example, we guided students through the four stages of Kolb’s (1984) experiential learning cycle. In the concrete experience stage, the students watched the teacher’s movement in the virtual world (Fig. 1b). We provided guidance notes to facilitate the students’ reflective observations of

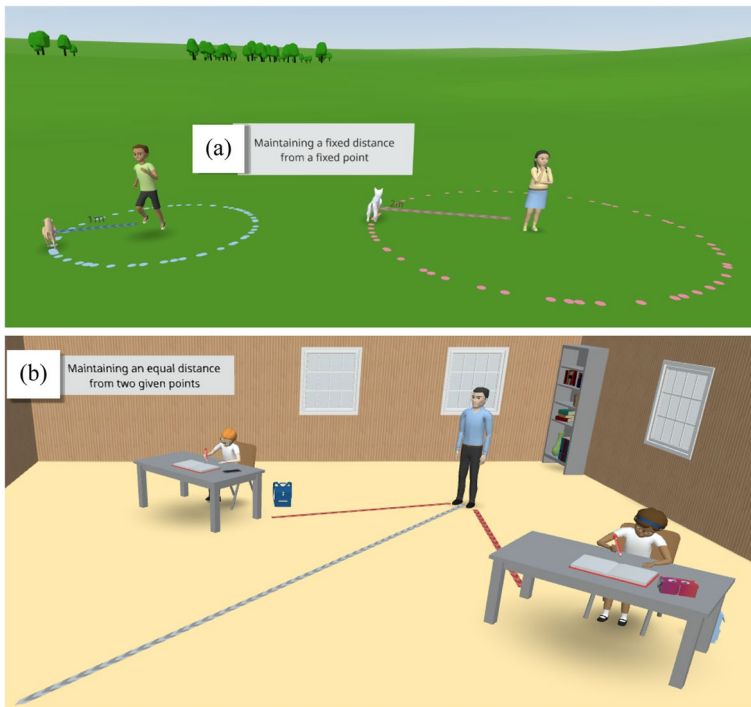


Fig. 1 Dynamic mathematical visualisation of locus concepts in CoSpaces Edu

the mathematical property (Weinhandl et al., 2020). We further supplemented the learning process with a GeoGebra applet that allowed students to manipulate mathematical objects, thus echoing their concrete experiences (Radović et al., 2020). In the abstract conceptualisation stage, we provided mathematical descriptions and work examples to help students formulate abstract concepts. In the final active experimentation stage, the students were required to apply what they had learnt and complete follow-up exercises. Figure 2 provides various snapshots of these instructional activities.

To summarise, in this study, which addresses the theoretical gap identified by Morris (2020), we explored the use of CoSpaces Edu resources in mathematics education within the framework of Kolb's (1984) experiential learning theory. The students had the opportunity to actively engage with mathematical concepts through object manipulation and exploration in a virtual environment. Our investigation provides empirical evidence of the role of concrete experience and reflective observation in enhancing students' understanding of locus concepts. This study thus contributes to experiential learning theory by demonstrating how the CoSpaces Edu resources can offer immersive experiences that align with the principles of experiential learning.

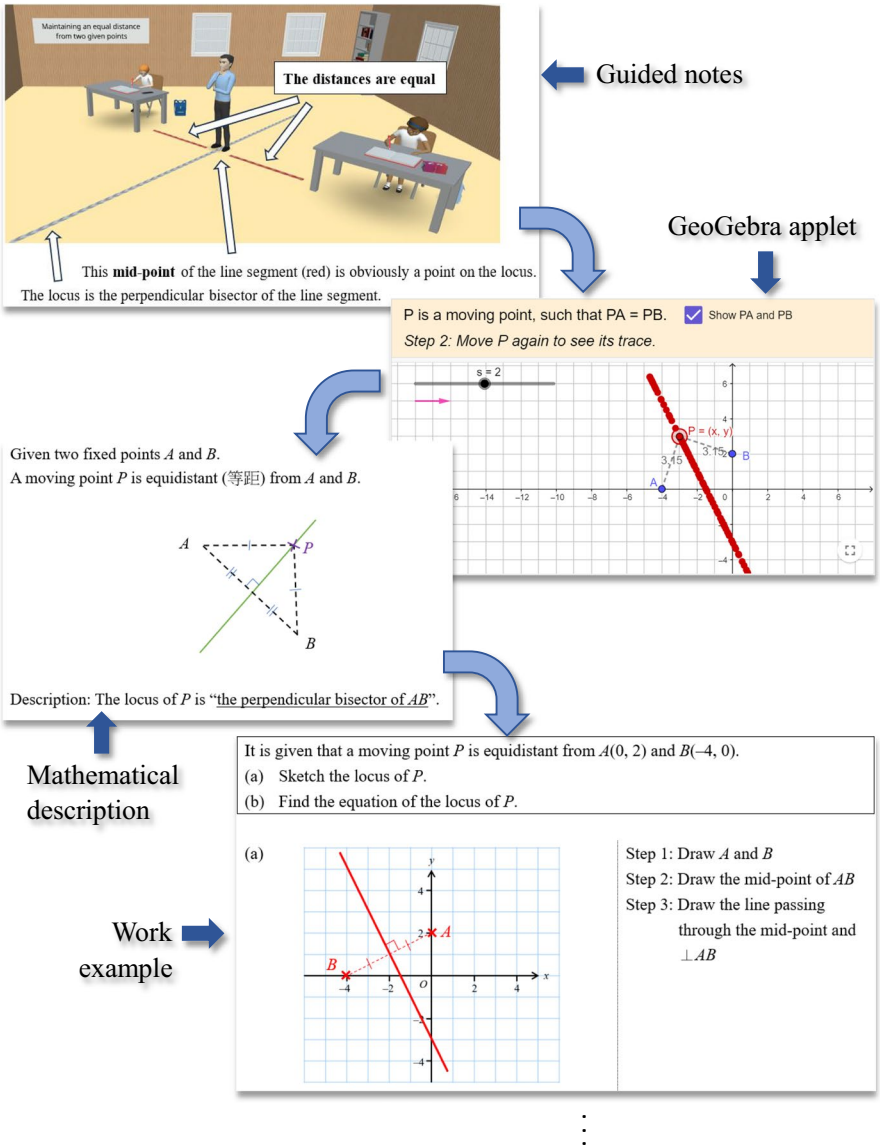


Fig. 2 Snapshots of the instructional activities associated with maintaining an equal distance from two given points

Methods

Research Design, Participants, and Procedure

We applied a quasi-experimental design that incorporated both quantitative (students' pre-test and post-test scores and survey data) and qualitative (student

interviews) methods. A total of 66 Grade 11 students from two local secondary schools in Hong Kong participated in the study. The two participating schools were similar in terms of the medium of instruction (i.e., English), number of mathematics lessons per week, students' ability, and their socio-economic background. As shown in Table 1 and in the Appendix, the students in the experimental group ($N=30$; Male: 14; Female: 16) were provided with the aforementioned resources developed using CoSpaces Edu (Fig. 1) and GeoGebra, in addition to our worksheet and instructions (Fig. 2). The holistic approach of using both GeoGebra and CoSpaces Edu resources in the experimental group was aimed at enabling the students to transfer their knowledge from the 3D virtual world to the 2D coordinate plane, thus facilitating them to complete the tasks in worksheets that represented locus problems in 2D coordinate geometry. The students in the control group ($N=36$; Male: 14; Female: 22) were given similar materials for learning, but without the CoSpaces Edu resources.

The mathematics topics addressed by our teaching objectives were TO1: maintaining a fixed distance from a fixed point; and TO2: maintaining an equal distance from two given points (Curriculum Development Council, 2017). According to their school teaching schedules, the student participants had not received any previous instruction on these topics. However, a pre-test was conducted to ascertain whether they had acquired knowledge of the topics through alternative means (e.g., private tutoring) and to establish the initial equivalence of their mathematics abilities across the experimental and control groups. To ensure the students could engage in self-paced mathematics learning using CoSpaces Edu and/or GeoGebra, the teachers demonstrated to both the experimental and control groups how to use these tools during their lessons. The students in both groups were then given a one-day window to use the e-learning resources after school. To mimic authentic online learning activities, we did not impose a time limit on their self-paced learning. They were encouraged to allocate 30 to 60 min to the learning tasks, which represents a typical daily workload for mathematics learning in Hong Kong secondary schools. Following the intervention, a post-test was administered to evaluate their achievements. We conducted semi-structured interviews with the experimental group to gain a deeper understanding of the students' learning experiences. We also used a survey to examine their perceptions of our CoSpaces Edu resources.

Table 1 The research groups and the e-learning resources provided

Research group	No. of students	CoSpaces Edu resources	GeoGebra applets	Worksheet and instruction
Experimental group	30	Yes	Yes	Yes
Control group	36	Nil	Yes	Yes

Some students were excluded from our analysis due to their test responses (as explained in the Results section)

Data Collection

The aforementioned research design involved three data sources: (1) a pre-test and post-test, (2) a survey, and (3) interviews. To address RQ1, we assessed the impact of the different instructional conditions on students' mathematics achievement through a comparison of their test scores. Pre- and post-intervention tests of 15 min each were administered, which were designed in alignment with the curriculum requirements (i.e., to stretch the loci and to find their equations; Curriculum Development Council, 2017). To ensure our assessment of the subject matter was comprehensive, the tests were co-developed with a mathematics teacher with over 10 years of experience and a pre-service teacher. The tests were reviewed by another mathematics teacher with over 5 years of experience, to further validate their quality and relevance. Although the questions in the tests differed, they were similar in terms of scope and level of difficulty. Each test was aimed at evaluating the students' mastery of TO1 and TO2. For each teaching objective, we formulated one curve sketching problem (2 marks), one basic problem (3 marks), and one advanced problem (4 marks), yielding a possible score range of 0 to 9. In combination (TO1 and TO2), the total maximum score for both the pre-test and post-test was 18.

To address RQ2, we conducted a post-intervention survey and interviews to gather data from the experimental group. The survey was administered to examine the students' perceptions of our CoSpaces Edu resources. This comprised six closed-ended questions presented on a 5-point Likert scale (ranging from strongly agree to strongly disagree), as adapted from Bugler et al. (2017). We developed this instrument for evaluating the quality of instructional materials based on a previous large-scale study involving teachers from six cities. As shown in Table 2, we focused on three dimensions relevant to the students' perspectives: accuracy and visual appeal (Items 1 and 2); ease of use and support (Items 3 and 4); and engagement and ability to meet student needs (Items 5 and 6). An open-ended question at the end of the survey prompted students to share their thoughts and comments on the e-learning resources. We also randomly selected six experimental-group students (20%) for our semi-structured interview. The interviews focused on understanding the ways in which the e-learning resources facilitated or hindered students' mastery of the mathematical concepts. This approach allowed us to explore their experiences and perceptions during their online learning journey.

Table 2 Items in the student survey by dimension (adapted from Bugler et al., 2017, p. 5)

Dimension	Survey items
Accuracy and visual appeal	1.The learning resources offer clear guidelines and instructions 2.The visual design of the learning resources is well-executed
Ease of use and support	3.The learning resources are easy to use 4.The learning resources provide sufficient information, including instructional worksheets, activities, assessments, and solutions
Engagement and ability to meet student needs	5.The learning resources stimulate interest and promote active engagement 6.The learning resources are easily understandable

Data Analysis

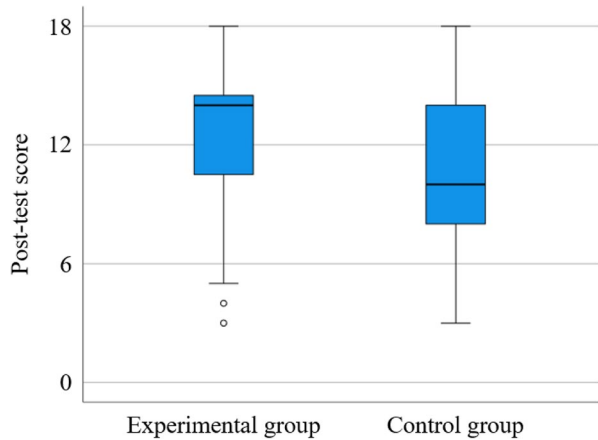
We first addressed RQ1 by assessing the normality of the data to determine an appropriate statistical test for the quantitative data analysis (Field, 2009). As the Kolmogorov–Smirnov test indicated a significant deviation from normality in the post-test data ($p=0.04$), we deemed non-parametric tests to be suitable for the analysis of our quantitative data (Field, 2009). We thus used the Mann–Whitney test to compare the post-test scores between the experimental and control groups. The average pre-test scores for both groups were close to zero, as the students had not generally been exposed to the locus concepts before the intervention. We did not use within-group effect sizes to interpret the students' learning gains, as they could not meet the basic benchmarks (Stern & Piper, 2019). We instead compared their post-test scores to reveal the extent to which the proposed tool (i.e., CoSpaces Edu) enhanced their mathematics learning. The effect size (r) was calculated using the formula suggested by Field (2009) with the benchmarks of 0.1 (small), 0.3 (medium), and 0.5 (large).

To address RQ2, we analysed the students' quantitative results and written responses in the post-intervention survey in conjunction with the interview data. The quantitative responses were summarised using descriptive statistics. We conducted a thematic analysis to explore the qualitative data, utilising an initial framework adapted from Bugler et al. (2017). This framework encompassed three major themes: (1) accuracy and visual appeal, (2) ease of use and support, and (3) engagement and ability to meet student needs. Although the work of Bugler et al. (2017) provided the basis for the thematic analysis of students' perceptions of our CoSpaces Edu resources, we remained flexible in terms of refining or augmenting the framework when new themes emerged. Frequency analysis was used to process the identified narratives. To establish the reliability of the coding, all qualitative data were double-coded by the first two authors. A Cohen's Kappa coefficient of 0.90 indicated excellent inter-coder reliability (Nili et al., 2020). Any disparities in coding were resolved through discussion to achieve consensus. We also included direct quotations from the student participants when reporting our interview findings (Eldh et al., 2020). Therefore, some of the data were translated from Chinese to English for reporting purposes.

Results

RQ1: To What Extent Does the Observation of Mathematical Properties in CoSpaces Edu Enhance Students' Online Learning of Locus Concepts?

This RQ examined students' knowledge acquisition of locus concepts in CoSpaces Edu via knowledge tests. The pre-test results indicated that except for one control-group student, they all lacked knowledge of the locus concepts, as most of them either left the pre-test blank or answered the questions incorrectly. The student

Fig. 3 Boxplot of the post-test scores by research group**Table 3** The post-test results of the experimental ($N=28$) and control ($N=35$) groups by teaching objective

Teaching objective	Mean (<i>SD</i>)	<i>Mdn</i>	<i>Z</i>	<i>p</i>	<i>r</i>
TO1: Maintaining a fixed distance from a fixed point (9 marks)					
• Experimental group	8.07 (0.34)	9.00	2.78	0.01	0.35
• Control group	7.03 (0.31)	8.00			
TO2: Maintaining an equal distance from two given points (9 marks)					
• Experimental group	4.64 (0.56)	5.00	1.50	0.13	n/a
• Control group	3.51 (0.55)	4.00			
Overall score (18 marks)					
• Experimental group	12.71 (0.78)	14.00	2.15	0.03	0.27
• Control group	10.54 (0.69)	10.00			

who achieved an almost perfect score in the pre-test and thus exhibited previous understanding was excluded from our analysis. Two experimental-group students were also excluded as they had not completed either the pre-test or the post-test. We therefore analysed the interventional effect using post-test data from 28 students in the experimental group and 35 in the control group. We observed no significant difference in the pre-test scores of the two research groups.

Figure 3 presents a boxplot of the post-test scores of both the experimental and control groups. The results of the Mann–Whitney test in Table 3 indicate that students in the experimental group ($Mdn = 14.00$) significantly outperformed those in the control group ($Mdn = 10.00$), $U = 335.50$, $Z = 2.15$, $p = 0.03$, with a small effect size ($r = 0.27$). We then examined students' learning outcomes for each teaching objective. Unfortunately, we only found an increase in student learning with regards to TO1 after the addition of our CoSpaces Edu resources. Those in the experimental group ($Mdn = 9.00$) scored significantly higher than those in the control group ($Mdn = 8.00$), $U = 229.50$, $Z = 2.78$, $p = 0.01$, with a medium effect size $r = 0.35$. In

contrast, the difference between the experimental group ($Mdn=5.00$) and control group ($Mdn=4.0$) was not statistically significant in terms of their learning of TO2, with $p=0.13$.

RQ2: How Do Students Perceive Their Mathematics Learning Experiences in CoSpaces Edu?

This RQ was aimed at investigating students' perceptions of mathematics learning experiences in CoSpaces Edu through a post-intervention survey and their qualitative feedback (i.e., written responses and interviews). First, we analysed the results of the student survey (Table 4) to assess their perceptions of their online learning experiences using the CoSpaces Edu resources. In terms of the accuracy and visual appeal, a significant proportion of students agreed or strongly agreed that the learning resources offered clear guidelines and instructions (Item 1; $N=19$, 63.3%) and they appreciated the well-executed visual design (Item 2; $N=19$, 63.3%). Regarding the ease of use and support, the majority of students found the learning resources easy to navigate (Item 3; $N=20$, 66.7%). In addition, more than half of the students said that the resources provided sufficient information through the instructional worksheets, activities, assessments, and solutions (Item 4; $N=18$, 60.0%). Regarding the dimension of engagement and meeting student needs, the majority of students agreed or strongly agreed that the resources were easily understandable (Item 6; $N=21$, 70.0%). However, less than half of them were of the opinion that the learning resources stimulated their interest and promoted active engagement (Item 5; $N=14$, 46.7%).

In addition to the closed-ended survey items, we considered the students' written responses and interview data to further examine their perceptions of the CoSpaces Edu resources. Table 5 presents the findings of our thematic analysis with representative student quotes. First, we found that some students expressed positive views regarding the accuracy and visual appeal of the platform, as they appreciated the clarity of the 360-degree views and the zoom function, which enabled a closer examination of the virtual elements. In terms of ease of use and support, the students found the resources to be user-friendly, enabling them to immerse themselves in a first-person perspective when exploring loci. Regarding engagement and meeting their needs, the students generally found that the resources offered useful guidelines and animations that helped them understand challenging concepts. One student suggested that adding background music and instructional narratives would offer a more engaging experience.

However, a few students voiced concerns about technical issues and usability challenges (Table 5). Some reported occasional technical problems, particularly with the platform link. One student expressed reservations concerning the task of maintaining an equal distance from two given points, feeling that this was beyond their current ability. Another pointed out that although CoSpaces provides the ability to view loci from the perspectives of objects or avatars, he found

Table 4 Survey results of the experimental group (*N* = 30)

Dimension and survey items	SA	A	N	D	SD
Accuracy and visual appeal					
1. The learning resources offer clear guidelines and instructions	5 (16.7%)	14 (46.7%)	10 (33.3%)	1 (3.3%)	
2. The visual design of the learning resources is well-executed	3 (10.0%)	16 (53.3%)	10 (33.3%)	1 (3.3%)	
Ease of use and support					
3. The learning resources are easy to use	5 (16.7%)	15 (50.0%)	9 (30.0%)	1 (3.3%)	
4. The learning resources provide sufficient information, including instructional worksheets, activities, assessments, and solutions	4 (13.3%)	14 (46.7%)	11 (36.7%)	1 (3.3%)	
Engagement and ability to meet student needs					
5. The learning resources stimulate interest and promote active engagement	3 (10.0%)	11 (36.7%)	15 (50.0%)	1 (3.3%)	
6. The learning resources are easily understandable	5 (16.7%)	16 (53.3%)	7 (23.3%)	2 (6.7%)	

SA = strongly agree, A = agree, N = neutral, D = disagree, and SD = strongly disagree

Table 5 Main themes identified in the students' perceptions of the CoSpaces Edu resources

Theme and sub-categories (count)	Representative student quotes
Accuracy and visual appeal	
<ul style="list-style-type: none"> • Positive ($N=3$) 	<p>“When using it, I can see the details clearly in 360 degrees. The zoom function enables both close and distant views, and I can see the details of the dog’s appearance very clearly.” (Student A)</p> <p>“There are many interesting elements, e.g., animated cartoons, human characters and virtual objects.” (Written response 2)</p>
Ease of use and support	
<ul style="list-style-type: none"> • Positive ($N=2$) 	<p>“I can view the locus from a first-person perspective. Normally, when solving problems, I observe from the side, but now I can zoom in and view the objects by walking closer and head-turning.” (Student C)</p> <p>“Overall, it’s quite user-friendly. I can rotate the objects and move the view.” (Student B)</p>
<ul style="list-style-type: none"> • Negative ($N=2$) 	<p>“Sometimes the link has issues, there are occasional technical problems... I can’t locate the virtual objects when moving my head.” (Student F)</p> <p>“I can’t change the location and shapes of the objects.” (Written response 1)</p>
Engagement and ability to meet student needs	
<ul style="list-style-type: none"> • Positive ($N=4$) 	<p>“I found understanding the concept of loci challenging. But after watching this AR animation, I grasped the concept.” (Student D)</p> <p>“I haven’t learnt about the equation for the circle yet. But with the guidelines and animation, I can get the idea.” (Student E)</p>
<ul style="list-style-type: none"> • Suggestion ($N=1$) 	<p>“Having background music would make it more enjoyable.” (Student B)</p>
<ul style="list-style-type: none"> • Negative ($N=2$) 	<p>“I don’t want to do this exercise about maintaining an equal distance from two given points. It seems to be a challenge beyond my current level.” (Student F)</p> <p>“There are too many objects in the virtual world. Although I can observe loci from various perspective, I don’t know where I should start.” (Student G)</p>

it challenging to decide where to start. These comments aligned with the few negative responses in the survey (refer to Table 4).

Discussion

In this section, we first assess the improved student performance resulting from the integration of our CoSpaces Edu resources and its theoretical implications, based on our research results. We then address the challenges identified when using these resources and propose several suggestions for educators.

Opportunities for Using CoSpaces Edu to Complement Mathematical Learning

In this study, we developed GeoGebra applets and supplementary materials to facilitate students' mastery of locus concepts in secondary school mathematics education. The efficacy of GeoGebra in terms of supporting students' mathematical exploration has been substantiated in previous research (Radović et al., 2020; Turgut, 2022; Yohannes & Chen, 2023). In our control group, over half of the students scored 10 out of 18 in their post-test (Table 3). We further sought to enhance the efficacy of our instructional activities by integrating CoSpaces Edu, a mirror-world metaverse platform. Based on the theoretical perspective of experiential learning (Kolb, 1984; Morris, 2020), we proposed that our CoSpaces Edu resources could provide students with concrete experiences involving virtual mathematical objects, enabling their reflective observation without the need to be physically present in a classroom.

The results of our study provide evidence of this positive impact on student learning, thus supporting the perspective of Hwang and Chien (2022) regarding the metaverse in education. In terms of the post-test results (Fig. 3), the students in the experimental group, who had access to our CoSpaces Edu resources alongside the traditional GeoGebra materials, significantly outperformed those in the control group. Our qualitative findings (Table 5) from the student interviews and written responses reveal the mechanisms underlying this improvement. Some students said that the visual and immersive nature of the CoSpaces Edu resources enabled them to comprehend complex mathematical ideas more effectively. This is consistent with the research suggesting that students can benefit from scaffolding mathematical concepts through interacting with objects using body movements, such as dragging, walking, and head-turning (Bock & Dimmel, 2021). This can enable them to visualise mathematical concepts and derive arguments about how to generate formulae from loci.

In addition to knowledge-building, the students expressed enthusiasm for the immersive nature of CoSpaces Edu, emphasising its visual appeal and dynamic features. They found the ability to observe loci from various angles and zoom in to be useful, as it enabled them to examine loci in more detail. This finding supports the findings of Papadakis et al. (2023), whose students expressed a positive view regarding the simulation elements and emphasised their value in learning. Echoing the study conducted by Bhatia et al. (2021), our students also found the virtual environment to be engaging and they enjoyed interacting with the objects within this metaverse platform.

Theoretical Contributions

Our research results have important theoretical implications. The design of our CoSpaces Edu resources allowed students to observe virtual mathematical properties within the scenarios offered, thereby facilitating the transition from abstract

mathematical concepts to tangible experiences. This aligns with the essence of Kolb's (1984) experiential learning theory. Students' concrete experiences in the virtual world enabled their subsequent engagement in reflective observation, abstract conceptualisation, and active experimentation. This resonates with the findings of Lee and Hwang (2022), who found in their metaverse-supported course that concrete experiences could stimulate students' reflection, which then became a catalyst for improvement. The design of our CoSpaces Edu resources thus addresses the theoretical gap concerning what constitutes a concrete experience (Morris, 2020). Our use of virtual mathematical objects also helps to bridge the gap between Kolb's (1984) theory and its practical application in fully online mathematics education.

In the context of virtual worlds, our study on the application of CoSpaces Edu in mathematics education contributes to the current research into the benefits of VR for learning. Although VR has unsurprisingly been found to improve students' learning (Fromm et al., 2021; Schott & Marshall, 2018), our study provides further insights into the metaverse learning experience within the framework of Kolb's (1984) experiential learning theory. The findings regarding students' perceptions of their online mathematics learning experiences when using CoSpaces Edu reveal the roles of concrete experience and reflective observation in facilitating their learning. Our findings regarding students' perceptions in terms of accuracy and visual appeal, ease of use and support, engagement, and the ability to meet their needs (Bugler et al., 2017) also contribute to our understanding of how virtual environments can be optimally designed and implemented to enhance learning outcomes.

In addition, previous studies have not addressed the potential advantages of CoSpaces Edu over conventional methods such as using GeoGebra in teaching mathematics. This study reveals how students may perceive this new approach of encouraging mathematical observations in a virtual world. Our results suggest that CoSpaces Edu can offer students accurate and visual clues that enable them to transfer their mathematical understanding about locus concepts from a 3D environment to a 2D-coordinate plane in worksheets or in GeoGebra. This tool provided our students with alternative perspectives that enabled them to observe mathematical properties in the virtual world, rather than simply viewing them on the screen. They could virtually move and walk through the virtual environment, manipulate scenarios, change the parameters of objects (e.g., size and position), and observe real-time changes. Thus, they were able to make connections between the concepts of distances and shapes and relate them to real-world objects to gain a better understanding of locus concepts.

Challenges When Using CoSpaces Edu to Learn Locus Concepts

Although the inclusion of the CoSpaces Edu resources demonstrated a positive impact on student performance, a closer assessment of the two teaching objectives revealed that for TO2, the resources fell short of increasing student performance (Table 3). This teaching objective entailed maintaining an equal distance from two given points, which is a more difficult concept to grasp than that presented in TO1.

The qualitative results (Table 5) of the student interviews and written responses provide insights into this limitation. The students generally found the TO2 concept to be more challenging to acquire, even with the aid of the CoSpaces Edu resources. Although this was in line with the reported general performance of students in local public examinations (see HKEAA, 2021 as an example), as they struggled to master knowledge through online learning, further improvement is required. Teachers should provide additional guidance on how to tackle more advanced mathematics problems in an independent online learning environment (Lo & Hew et al., 2021; Vidergor & Ben-Amram, 2020). This is consistent with Szeto's (2015) emphasis on the crucial role of teacher presence in online settings, which can be manifested through proactive strategies such as providing spontaneous feedback to students.

In addition to the mathematical challenges, some of the students encountered technical issues when utilising the CoSpaces Edu platform (Table 5). For example, one student reported difficulties in locating virtual objects when moving their heads, indicating a potential problem with the tracking or display of objects in the virtual environment. Another expressed frustration regarding the inability to change the location and shape of objects in CoSpaces Edu and found it difficult to determine where to start observing the loci. These technical challenges warrant attention as they can impair the overall user experience, increase the cognitive load, and potentially hinder students' full engagement in learning activities (Albus et al., 2021; Jost et al., 2020). Educators should therefore address these issues and ensure a seamless learning journey for students in the virtual world.

Recommendations for Educators

Based on our research findings and the literature, we offer the following three recommendations to enhance future metaverse-supported instructional activities. First, a majority of students found our learning resources comprehensive and user-friendly (Table 4). However, we did not integrate recent technology such as artificial intelligence (AI)-based chatbots to address students' learning challenges. Wollny et al. (2021) categorised AI-based chatbots in the education sector into the three main functional types of assisting, mentoring, and learning chatbots, which are specifically designed to support students in comprehending course materials. For example, a chatbot that serves as a mentor can be developed to encourage students who may feel discouraged while undertaking learning tasks (e.g., Student F). As highlighted by Hew et al. (2021), chatbots trained to address questions that students frequently ask can provide prompt feedback to help resolve students' learning issues.

Second, we primarily focused on students' independent online learning. However, the role of peer support in the learning process should be recognised. In addition to Kolb's (1984) experiential learning theory, Ghosheh Wahbeh et al. (2023) emphasised the application of Vygotsky's (1978) sociocultural theory and the importance of collaborative learning. Integrating peer interaction into metaverse-supported instructional activities can create a stimulating environment for students, in which they can share insights and co-construct knowledge online (Hwang & Chien, 2022). Such interactions can enable students to view course materials from various

perspectives, which can enhance their understanding of difficult concepts (Roy & Verma, 2020). Facilitating opportunities for peer interactions within the virtual environment can potentially increase the educational benefits of metaverse-supported learning experiences.

Third, the technical aspect of future interventions should be considered. While most students reported a smooth experience with our instructional activities (Table 4), some encountered technical issues (e.g., Student F). However, we could not replicate the reported problems. This finding suggests that first, more extensive trials of the resources are required as involving more student participants would increase the likelihood of identifying any potential technical issues before widespread implementation, thus helping to ensure a seamless user experience. Second, some students will require training on how to use the resources, and proactively offering such support can address any technical issues encountered during online learning (Lo, 2023).

Conclusion

By drawing on experiential learning theory and IT tools, in this study we created immersive learning resources aimed at further developing online mathematics education. The results regarding RQ1 highlighted a significant improvement in student performance after the integration of our CoSpaces Edu resources. RQ2 was aimed at exploring students' perceptions of the CoSpaces Resources and further confirmed the positive impact of immersive virtual experiences on students' mathematics achievement. The study further informs the practical application of Kolb's (1984) experiential learning theory in the context of metaverse-supported mathematics learning. However, guidance is still necessary when teaching challenging topics. Enhancing instructional effectiveness, for example, by encouraging peer interactions on the metaverse platform, should also be a priority. We recommend that future studies investigate the design and implementation of virtual mathematical exploration across diverse topics and educational settings. Any technical concerns should also be addressed through extensive trials, and student training is essential to ensure a seamless user experience.

Limitations and Recommendations for Future Research

We acknowledge several limitations of our study. First, we primarily focused on two types of loci included in the curriculum requirements. Future studies could create other scenarios in the virtual world that include other types of loci, such as those involving maintaining an equal distance from two parallel lines (Curriculum Development Council, 2017), along with other mathematics topics. Second, the study was conducted in local senior secondary mathematics classes, and thus the generalisability of our results to other educational settings is limited. Future research

should explore the effectiveness of similar approaches across educational settings (e.g., primary and junior secondary school mathematics) to establish a broader scope of applicability and compare their intervention effects. Third, this study offers approaches for extending the evaluation of learning outcomes beyond students' mathematics achievement. Future research can compare experimental and control groups in terms of students' interests, motivation, and engagement. This can offer a more comprehensive perspective on metaverse-supported mathematics learning. Finally, we suggest that the learning resources could be further enhanced by exploring other pedagogical strategies. For example, the incorporation of a gamification approach, as recommended in various studies (Papadakis et al., 2023; Zourmpakis et al., 2022, 2023), could enhance the learning process.

Appendix

Table 6 Comparison of the learning activities in CoSpaces Edu and GeoGebra

Learning activities in the experimental group (CoSpaces Edu + GeoGebra resources)	Learning activities in the control group (GeoGebra resources only)
<ul style="list-style-type: none"> • Students were presented with two scenes created using CoSpaces Edu, each aligning with one of the two teaching objectives (TO1 and TO2) of loci • For TO1, a scene was created in which two dogs (representing the moving point) orbited around their respective owners (representing the fixed point) in a field • For TO2, a scene was created in which a teacher (representing the moving point) acted as an invigilator during an examination, maintaining an equal distance from two students (representing the two fixed points) • In the stage of concrete experience, the students watched the movements of the dogs (TO1) and the teacher (TO2) in the virtual world. The learning process was supplemented with GeoGebra applets, allowing students to manipulate mathematical objects • They were provided with guidance notes to facilitate their reflective observations of the mathematical property • In the stage of abstract conceptualisation, they were given mathematical descriptions and work examples • In the stage of active experimentation, they applied what they had learnt to complete follow-up exercises 	<ul style="list-style-type: none"> • Students were introduced to the same teaching objectives (TO1 and TO2) of loci • In the stage of concrete experience, they utilised GeoGebra applets designed to support the learning process, allowing them to manipulate mathematical objects • They were provided with guided notes to facilitate their reflective observation of the mathematical property • In the stage of abstract conceptualisation, they were given mathematical descriptions and work examples • In the stage of active experimentation, they applied what they had learnt to complete follow-up exercises

Acknowledgements We would like to express our gratitude to Hiu Laam Naomi Lee and her team for creating the CoSpaces Edu resources.

Author Contributions All authors contributed to the study conception and design, material preparation, data collection and analysis. The first draft of the manuscript was written by Chung Kwan Lo and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

Funding The work described in this paper was partially supported by a grant from the Research Grants Council of the Hong Kong Special Administrative Region, China (Project No. EdUHK 28604623) and by Department of Mathematics and Information Technology (Departmental Research Grant; MIT/DRG01/23–24), The Education University of Hong Kong.

Data Availability Data are not publically available for this article.

Declarations

Ethical Approval and Consent to Participate. The participants have been well informed about the research purpose and methods. The study has obtained approval of the university's ethics application committee. The approval has been obtained from school and students via consent form.

Conflict of Interest The authors declared there is no conflict of interest.

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