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Examination of Taiwanese Mathematics Teacher Questioning

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Abstract
In this study, we aim to investigate the types of questions that Taiwanese mathematics teachers pose and in which instructional situations they do so during mathematics lessons at the secondary school level. The classroom teaching of six experienced mathematics teachers was analyzed. Quantitative analysis showed that the mathematics teachers tend to give lectures rather than ask questions. When the mathematics teachers posed questions, only about one-fifth of the questions require students to provide high-cognitive responses. We also observed that the mathematics teachers differed in the number and type of questions they asked in different instructional situations. A cross-examination of the types of questions and the lesson structures revealed that two-thirds of the mathematics teachers asked high-cognitive questions when practicing or reviewing the content with the students. The qualitative analysis further identified three instructional purposes for high-cognitive questions: connecting the meaning of mathematical concepts, stimulating multiple solutions to a problem, and exploring mathematical relationships across different problem contexts. The results imply that mathematics teaching at the secondary school level in Taiwan is more teacher-centered, and the mathematics teachers do not often ask questions during classroom teaching. However, the teachers tend to ask high-cognitive questions for assessment purposes to ensure that the students have understood the concepts and can proceed to advanced mathematics.

Keywords High-cognitive questions · Lesson structure · Taiwan · Mathematics Teaching · Teacher questioning

Examination of Taiwanese Mathematics Teacher Questioning

Questioning is one of the core instructional behaviors in classroom teaching. The National Council of Teachers of Mathematics (NCTM) (1991) claimed that mathematics teachers should manage classroom discussion by posing questions and setting up

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tasks to engage, elicit, and challenge students. Posing questions enable teachers to listen to students' ideas and ask follow-up questions so that students have the opportunity to clarify doubts and mathematical ideas.

In the past decades, teacher questioning has been widely investigated, especially the relationship between teacher questioning and student learning (e.g. Wilen, 1991). Researchers have shared a consensus that teacher questioning, especially questions that encourage students to apply high-level cognitive processes, has a significant impact on the quality of student learning (Aziza, 2018; Franke et al., 2009; Winne, 1979) and the development of mathematics knowledge (Moyer & Milewicz, 2002). In particular, students need classroom opportunities to share their mathematical thinking, discuss alternative solution strategies to problems, and use mathematical tools flexibly (Franke et al., 2007).

This study aims to investigate teacher questioning in mathematics at the secondary school level in Taiwan. Taiwan shares a similar cultural and educational background with the rest of East Asia (Clarke et al., 2006). Taiwanese students have consistently come out at the top in cross-national mathematics assessments (e.g. Mullis et al., 2020; Organisation for Economic Co-operation and Development [OECD], 2014). The exceptional performance of Taiwanese students has led to follow-up research to explore the underlying reasons, one of which is tied to effective teaching (Anthony & Walshaw, 2009; Kaiser & Vollstedt, 2007; Wang & Hsieh, 2017). Teaching in Taiwan is often described as teacher-centered (Fwu & Wang, 2006) or examination-driven (Lin et al., 2021). Researchers have attempted to articulate the cultural differences between the West and the East in terms of teaching and learning (Anthony & Walshaw, 2009; Kaiser & Vollstedt, 2007; Leung, 2006). To this end, this study contributes to understanding how Taiwanese mathematics teachers communicate with students by posing questions, especially when teacher questioning is considered as a significant predictor of effective teaching (Wilen, 1987; Wilen & Clegg, 1986).

Additionally, we attempt to examine the relationship between teacher questioning and lesson structures. Studies have shown that lesson structure can be one of the keys to student learning (Hiebert & Stigler, 2000; Hiebert et al., 2003). Kaur (2009) highlighted that the search for instructional patterns in East Asian countries enables further exploration of effective teaching. The investigation of both teacher questioning and lesson structure allows for uncovering the epistemic ecology in Taiwanese mathematics classrooms, whereby teaching affords opportunities for students to learn mathematics.

Literature Review

Teacher Questioning

A question is defined as a sentence with an interrogative form (Cotton, 1988; Wilen, 1991). Questions are instructional cues or stimuli that indicate the elements of the to-be-learned content and directions for students to understand what and how to do. Researchers claim that learning and understanding in a classroom occur through teachers' discourse and teacher-student interactions. Questioning and explaining allow teachers to diagnose students' understanding, thus ascertaining the actual learning of concepts in a particular domain (Ashlock, 2002).

The theory of social constructivism (Vygotsky & Cole, 1978) can explain the influence of teacher questioning on student learning. The social constructivism theory highlights that social context, language, and semiotic tools determine the construction of knowledge. The "zone of proximal development" concept implies that teachers' guidance during classroom discourses can scaffold students to develop their knowledge. The learning process from Vygotsky's perspective requires the acquisition of language and an understanding of the cultural context in which language is used (Wegerif, 2008). Interactions, mainly constituted and mediated by speech, allow individuals to construct meanings, internalize them, and accept them in the stream of thought.

The purposes of teacher questioning involve different aspects, such as cognition, affection, and classroom management. In the social constructivism theory, researchers have highlighted teacher questioning as an essential instructional process and tool for student learning (Chin, 2007). During classroom teaching, teachers ask questions for instructional purposes. The purposes of teacher questioning include motivating students to actively participate in learning, scaffolding the development of mathematical thinking, and clarifying mathematical ideas (Hassan et al., 2016; Zack & Graves, 2001). Teacher questioning also helps evaluate students' participation and assess their understanding of mathematics (Lemke, 1990; van Zee & Minstrell, 1997). In addition, teacher questioning may also aim to manage classroom engagement and students' behaviors (e.g. maintaining student attention) (Brown et al., 1984; Wilen, 1991).

With respect to cognition, teacher questioning has been discussed in literature since the 1950s. Influenced by the successful Soviet launch of Sputnik in 1957, the American society called for instructional strategies that could develop students' higher-order thinking and cognitive abilities, which led to the implementation of Bloom's taxonomy and the model developed by Guilford (Oliveira, 2010; Redfield & Rousseau, 1981; Wilen, 1991). Both Bloom's taxonomy and Guilford's model identify and classify components of cognitive operations that teacher questioning may aim for (Chin & Langsford, 2004). Kamii and Warrington (1999) argued that a critical characteristic of good questioning is to encourage new ideas, recall trivial facts, and construct meaning. It is expected that teachers use interrogative dialogues to motivate students to share their ideas, explore, and debate viewpoints.

Considering the complexity of instructional practices, some researchers have proposed binary categorizations to easily and systematically analyze questions posed by the teachers (Barden, 1995; Wimer et al., 2001). For example, one approach to evaluate teacher questioning categorized the questions into the open- and closed-ended types based on the number of acceptable answers (Lee & Kinzie, 2012; Sole, 2018). Another approach distinguished divergent and convergent questioning based on creativity (Tofade et al., 2013; Voss et al., 2022). In addition, considering the various cognitive behaviors and operations expected of students, teacher questioning was also categorized into higher- and lower-order questions (Ellis, 1993). In this study, we focus on cognition and attempt to explore when and how Taiwanese mathematics teachers pose cognitive questions during classroom teaching.

High-cognitive questions involve a series of interactions between teacher and students with the aim to extend or modify the student's views on mathematics and to develop higher-order thinking skills (e.g. critical thinking) (Barnes, 1979). House et al. (1990) alleged that high-level thinking demanded questions that elicited justification and application from students and expected students to invent new solutions to problems. Thus, students who answer such questions foster generative thinking and apply learned information to solve non-routine problems. High-cognitive questions motivate students to comprehend concepts, brainstorm multi-solutions, and formulate generalizations for specific problems (Ellis, 1993). Wimer et al. (2001) claimed that teachers' higher-cognitive questions encourage students to think critically, which is decisive for learning.

Low-cognitive questions, meanwhile, require students to engage in low convergent thinking with emphasis on recalling factual knowledge or information (Wilen & Clegg, 1986). Ellis (1993) claims that low-cognitive questions are concerned with "the correct answer," requiring students to transfer, identify, and organize facts and focus less on students' reasoning and critical thinking skills. Further, low-cognitive questioning taps only the memorization of facts (Wimer et al., 2001) with a teaching model that was termed by the literature as "initiation, response, and evaluation" (IRE).

The IRE model is recognized as a typical discourse pattern of teacher questioning in traditional lessons (Mehan, 1979). Teachers with this discourse pattern initiate questions to check students' knowledge and understanding (initiation), listen to students' answers (response), and assess the correctness of those answers (evaluation). The IRE discourse pattern is also similar to what Lemke (1990) termed as triadic dialogue, which is often predominant in science classroom teaching. Traditional teaching typically involves planning a series of questions before class begins. The teacher has a plan to accomplish that takes precedence over unanticipated student remarks or queries. Moyer and Milewicz (2002) indicated that teachers prefer to use leading questions to provide hints to the answers so that students can move towards the solution. Thus, low-cognitive questions often involve classroom discourse that focuses on what the teacher wants the students to learn instead of on students' thinking. Scott (1998) also indicated the authoritative function of a classroom discourse where teachers play the role of knowledge transmitters. Teachers' talks often aim to convey factual statements, and their questions are close-ended and information-seeking, requiring students to respond only in single, detached words. In this regard, the questions are only for recalling the learned factual knowledge and low-cognitive answers.

Lesson Structure

Researchers have explored lesson structures and their relation to the teaching and learning of mathematics (Clarke et al., 2006; Hugener et al., 2009; Smith, 1985; Smith & Hodgin, 1985). For example, Lemke (1990) identified a lesson as a social activity structured through specific patterns of activity organization where the teacher and the students have different roles to play. Lemke (1990) further argued that the activity structure influenced the interaction modes between the teacher and the students and the construction of meanings of a particular topic. The term "instructional situation," defined by Herbst (2006), further explains the activity structures of a lesson. According to Herbst (2006), instructional situation refers to the customary ways of framing classroom actions that would allow the teacher and the students to take responsibility and successfully exchange knowledge. Furthermore, the situation identifies the frames that participants need to know who has to do what and when to fulfill the didactical contract agreed upon by teachers and students (Brousseau, 2002). Thus, participation in instructional situations requires enacting specific norms and scripts that shape the mathematical work of the teachers and the students. In this regard, an analysis of the lesson structure allows one to realize the roles, norms, and scripts that teachers and students should follow.

The Trends in International Mathematics and Science Study (TIMSS) have collected students' data on mathematics and science achievement across countries and regions since 1995. Following this, the TIMSS video studies explored teaching similarities and differences that accounted for its effectiveness on students' learning outcomes. The lesson structure analysis followed the directions proposed by the TIMSS video studies (Hiebert et al., 2003; Stigler & Hiebert, 1999). For example, the TIMSS videotape classroom study described instructional situations such as warm-up, review, introduction, practice, and seatwork (Stigler & Hiebert, 1999). They found that lesson structures within a country shared similar recurring features of teaching, which was termed "cultural script" (Stigler & Hiebert, 1999).

The TIMSS 1999 video study (Hiebert et al., 2003) further identified three main aspects of mathematics teaching—how lessons are structured, the nature of the content implemented in a lesson, and the instructional practices. The video study applied these three aspects to examine mathematics teaching in seven countries, including Taiwan. When examining the lesson structures across countries, the TIMSS video study identified three main instructional situations in a lesson—introducing new content, practicing new content, and reviewing. The three instructional situations refer to different norms and scripts that the teachers and the students must fulfill. Introducing new content referred to the instructional situation when a new mathematics content was illustrated. Practicing new content identified the instructional segment where teachers required students to practice, apply, and summarize new knowledge and familiarize themselves with the new concepts. Practicing activities during the classroom discourse included discussing the solutions to earlier problems given in the lesson. Reviewing referred to the instructional situation where the teacher provided warm-up questions, checked students' homework, discussed examination solutions, or clarified queries about previous concepts. The review also functioned as a transition for the teacher to introduce a new concept.

Based on the literature review, we aim to examine the following research questions:

1. What types of questions, in terms of their level of cognitive demand, do Taiwanese mathematics teachers pose, and during which instructional situations do they do so?
2. What is the relationship between the questions posed by the teachers and the lesson structures they use?

3. What are the underlying instructional purposes of high-cognitive questions posed by Taiwanese mathematics teachers?

Methodology

Participants

Six experienced mathematics teachers, who teach at secondary schools in Taiwan, voluntarily participated in the study. The teaching years of the six teachers ranged from 6 to 27 years. The teachers' majors were either in mathematics or mathematics-related areas (e.g. mathematics education). All the participating teachers expressed that one of their main goals is to help students obtain high scores in high school entrance examinations.

Data Collection

The data collected and analyzed in this study concerned the teaching of a geometric unit during the second semester of 8th grade in Taiwan about the properties of parallel lines. The content included the definition of parallel lines, distances between parallel lines, angle properties related to parallel lines (e.g. corresponding angle theorem), reversed properties related to parallel lines, and geometric diagram construction. The participants' teaching of the geometry unit was videotaped and transcribed, focusing on their delivery of the content and interactions with class students. The teachers spent seven to eleven lessons teaching the unit. As a result, a total of 49 lessons taught by the six mathematics teachers were analyzed. In Taiwan, a secondary school lesson lasts for 45 min. Teachers were also interviewed during and after data collection to understand the underlying reasons for the questions posed during classroom teaching.

Data Analysis Procedure

A mix of top-down and bottom-up data analysis approaches was adopted (Carspecken & Apple, 1992; Reich, 2010). On the one hand, we followed the literature review on teacher questioning and lesson structure to examine the data collected in the study. On the other hand, we used a grounded-based approach to explore the nuances and instructional purposes of the questions asked during classroom teaching.

The data analysis process started by identifying teachers' spoken dialogues during classroom teaching. Only mathematics-related spoken dialogues were analyzed. Next, we identified the dialogues as declarative or interrogative (Hill, 2016; Tienken et al., 2009). Declarative format referred to those dialogues that conveyed mathematics content or told students what to do. Interrogative format referred to those questioning dialogues that required students to answer. The interrogative format was more likely to stimulate interactive dialogues between teacher and students. Typical interactive dialogues identified in the data followed the IRE pattern—teachers initiate questions, students respond to the questions prompted by the teachers, and teachers evaluate students' responses. Next, we counted the number of words in interrogative and declarative dialogues as well as the number of sentences in interrogative dialogues (Herbel-Eisenmann, 2007). This counting revealed the prevailing dialogues that occupied classroom teaching in Taiwan.

After identifying the questions posed by the participating teachers, we analyzed the types of questions in regard to cognitive demand. We aimed to examine the cognitive nature of the questions, thus adopting a binary classification between high-cognitive and low-cognitive questions. Based on the grounded-based approach, we found that high-cognitive questions were often close-ended and convergent-oriented. Very few questions posed by the teachers were open-ended and divergent-oriented. These high-cognitive questions had the instructional purpose of engaging students in high-order mathematical thinking. The purposes of high-cognitive questions included (1) connecting the meaning of mathematical concepts when applying them to problem solving; (2) stimulating multiple solutions to a problem; and (3) facilitating students to explore mathematical relationships across different problem contexts. Low-cognitive questions required consecutive and stepwise targeted questions to direct students from the initial acquisition to independent proficiency in mathematical concepts. The purposes of low-cognitive questions included (1) recalling previously learned factual knowledge; (2) providing answers without further clarification; and (3) calculating simple problems.

In our data, we also found that some questions posed by the teachers were related to mathematics instructions but not to cognition. Such questions had a checklist purpose as those questions aimed to ensure if students followed the instructions, instead of acknowledging students' cognitive responses (McCarthy et al., 2016; Moyer & Milewicz, 2002). Teachers asked this kind of questions using specific and repetitive verbal "checkmarks." These checkmarks found in the data were usually one- or two-word expressions, such as "okay," or simple sentences such as "is everything [mathematics] okay," which indicated that the teacher intended to move on to the next instructional activity.

We examined lesson structures based on the categories proposed by Hiebert et al., (2003), which include introducing new content, practicing new content, and reviewing. In our data, introducing new content referred to the instructional situations where teachers introduced new mathematics concepts or demonstrated examples selected from the textbooks or supplementary materials. For instance, teachers often provided examples and diagrams to help students acquire new geometric knowledge and problem-solving skills that they had not learned previously. Practicing new content identified the instructional segments where teachers asked students to practice problems that were selected from textbooks or other instructional materials (e.g. supplementary materials). The aim was to help students become familiar with the new concepts. In addition, practicing activities included student seat-work and discussions of the problem solutions with students. Reviewing referred to the instructional situations where the teacher provided warm-up questions related to previously learned mathematics concepts, checked students' homework, provided tests for students, and discussed the problems in the tests with students to ensure that they had understood the concepts. The review also functioned as a transition for the teacher to introduce new concepts. Concerning teacher interview data, it was also transcribed and analyzed to probe the underlying reasons for the questions posed at specific instructional moments.

The authors discussed the coding procedures and categories to achieve reliable coding, following which the second and the third authors coded the data individually. The authors discussed disagreements in coding until an agreement was reached. We also used Cohen's κ to check for inter-rater consistency. A result of 0.94 showed that the coding was reliable (Landis & Koch, 1977).

Findings

Analysis of Teacher Questioning and Lesson Structure

We first present the analysis of the mathematics teachers' spoken dialogues for interrogative and declarative purposes during classroom teaching. As presented in Table 1, we calculated the number of words for interrogative dialogues and declarative dialogues as well as the number of sentences for interrogative dialogues. However, during spoken declarative dialogues, it is not easy to determine when a dialogue ends. We did not calculate the number of sentences for declarative dialogues. Teachers tended to conduct the lessons with declarative dialogues rather than posing questions. On average, three-fourths of spoken dialogues were declarative, and only one-fourth were interrogative. The percentage of the spoken dialogues with an interrogative purpose ranged between 21 and 35%, showing that the mathematics teachers tended to lecture the lessons instead of posing questions for follow-up discussions with students.

The average length of words in interrogative sentences was also calculated. The purpose was to understand the characteristic of the questions posed by Taiwanese mathematics teachers. The calculation showed that the average number of words in the interrogative dialogues did not exceed ten, which indicated that the questions posed by the mathematics teachers were often concise. Examples of the interrogative dialogues posed by the teachers can be "do you find the intersecting line?" and "what relationship can you get from the diagram?".

Table 2 showed analyses of the types of questions and the lesson structure identified in classroom teaching. As evident, the analysis revealed that, on average, only 20% of the questions were high-cognitive, 43% were low-cognitive, and 37% were for checklist purposes. The analysis showed that the mathematics teachers were more likely to pose low-cognitive and checklist questions rather than high-cognitive ones. The examination of individual teachers revealed that the percentages of the questions identified as high-cognitive ranged from 9 to 29%. The percentages for low-cognitive questions ranged between 27 and 62%. Similar results were found for the percentages for checklist questions, ranging between 14 and 53%. The analyses of the types of questions revealed that the mathematics teachers had their preferences in asking the types of questions. Some teachers (e.g. Teacher Jyu) tended to ask questions to stimulate students' high-cognitive thinking, and others (e.g. Teacher Yao) more emphasized on instructional flows by posing checklist questions. A close

Table 1 The percentage of word count in sentences for each teacher

Teachers	Interrogative dialogue	Average length of interrogative dialogue	Declarative dialogue
Teacher Jyu[1]	7368[2], 1091[3] (25%)	6.75[4]	22,435[5] (75%)
Teacher Yin	12,328, 1463 (35%)	8.43	22,831 (65%)
Teacher Huang	4544, 490 (22%)	9.27	16,030 (78%)
Teacher Yao	5967, 652 (22%)	9.15	20,791 (78%)
Teacher Wen	8162, 848 (24%)	9.63	26,257 (76%)
Teacher Ying	7469, 831 (21%)	8.99	27,690 (79%)
Average	7640, 896 (25%)	8.53	22,672 (75%)

[1]All teachers' names are pseudonyms

[2]The number of total words identified in interrogative dialogues spoken by the teacher

[3]The number of interrogative sentences spoken by the teacher

4The average number of words in an interrogative sentence

[5]The number of total words identified in declarative dialogues spoken by the teacher

analysis of the low-cognitive questions and the follow-up dialogues also illustrated that the interaction models between the mathematics teachers and the students were oriented towards the IRE model.

The analysis of the questions asked during different instructional situations in a lesson (Table 2) indicated that the mathematics teachers mostly tended to ask questions when introducing new mathematics content (40%) or when reviewing the previously learned content (40%). Only 21% of the questions were posed when practicing new content. A close analysis of the individual teachers highlighted the variability in posing questions in different instructional situations. When introducing new content, the percentages of questions ranged between 27 and 71%, and when practicing new content, the percentages ranged from 6 to 38%. The percentages of the questions during reviewing of the content ranged between 10 and 55%.

Table 2 Descriptive analysis of types of questioning and lesson structure

Teachers	Types of questioning			Lesson structure			
	High-cognitive	Low-cognitive	Checklist	Introducing	Practicing	Reviewing	Sum of questions
Teacher Jyu	317[1] (29%)	292 (27%)	482 (44%)	431 (40%)	69 (6%)	591 (54%)	1091
Teacher Yin	278 (19%)	563 (38%)	622 (43%)	543 (37%)	209 (14%)	711 (49%)	1463
Teacher Huang	117 (24%)	303 (62%)	70 (14%)	346 (71%)	93 (19%)	51 (10%)	490
Teacher Yao	57 (9%)	252 (39%)	343 (53%)	222 (34%)	247 (38%)	183 (28%)	652
Teacher Wen	152 (18%)	376 (44%)	320 (38%)	225 (27%)	157 (19%)	466 (55%)	848
Teacher Ying	183 (22%)	410 (49%)	238 (29%)	247 (30%)	228 (27%)	356 (43%)	831
Average	20%	43%	37%	40%	21%	40%	896

[1]The number of questioning sentences spoken during classroom teaching

The percentage variations revealed that teachers make instructional decisions about posing questions based on the importance placed on different teaching activities. For example, Teacher Wen paid more attention to reviewing, while Teacher Huang focused more on teaching new mathematical content.

Cross-Analysis of Teacher Questioning and Lesson Structure

We examined the relationship between types of questioning and the lesson structure to realize which instructional situations and types of questions the mathematics teachers preferred. As shown in Table 3, the mathematics teachers posed different types of questions in different instructional situations. For example, two teachers posed high-cognitive questions mostly when introducing new mathematics content (Teacher Yin: 63%; Teacher Huang: 85%). The other four teachers posed high-cognitive questions when practicing new content (Teacher Yao: 68%) or when reviewing the previously learned content (Teacher Jyu: 52%; Teacher Wen: 66%; Teacher Ying: 46%).

Additionally, two teachers frequently posed low-cognitive questions when introducing new content (Teacher Jyu: 55%; Teacher Huang: 65%). One teacher asked low-cognitive questions during practicing activities (Teacher Yao: 39%), and three teachers asked low-cognitive questions during reviewing activities (Teacher Yin: 51%; Teacher Wen: 59%; Teacher Ying: 35%). Five out of the six teachers tended to ask checklist questions during reviewing activities (Teacher Jyu: 65%; Teacher Yin: 55%; Teacher Yao: 35%; Teacher Wen: 44%; Teacher Ying: 55%). Only Teacher Huang often posed checklist questions when introducing new mathematics content (70%).

The cross-analysis between the types of questions and lesson structure revealed large discrepancies among the mathematics teachers. In particular, the percentage of the questions varied, and teachers posed high-cognitive questions during different instructional situations. The analysis result also showed that the mathematics teachers tended to ask more low-cognitive and checklist questions when reviewing the learned content with the students.

Instructional Purposes Identified in High-Cognitive Questioning

The quantitative examination indicated that the Taiwanese mathematics teachers did not frequently pose high-cognitive questions. They also tended to pose high-cognitive questions during different instructional situations. The qualitative analysis further revealed that the high-cognitive questions posed by the teachers aimed to stimulate students towards higher-order thinking processes even though the questions were often close-ended or convergent-orientated. We noted that four out of the six mathematics teachers tended to pose high-cognitive questions when practicing and reviewing the content. Those questions were oriented toward the assessment of students' learning. The qualitative analysis revealed three types of instructional purposes when the mathematics teachers posed high-cognitive questions during the

Table 3 Cross-analysis between types of questioning and lesson structure

Teachers	Lesson structure	Types of questioning		
		High-cognitive	Low-cognitive	Checklist
Teacher Jyu	Introducing	134 (42%)[1]	162 (55%)	135 (28%)
	Practicing	19 (6%)	14 (5%)	36 (7%)
	Reviewing	164 (52%)	116 (40%)	311 (65%)
	Total	317 (100%)	292 (100%)	482 (100%)
Teachers Yin	Introducing	175 (63%)	188 (33%)	180 (29%)
	Practicing	21 (8%)	90 (16%)	98 (16%)
	Reviewing	82 (29%)	285 (51%)	344 (55%)
	Total	278 (100%)	563 (100%)	622 (100%)
Teacher Huang	Introducing	100 (85%)	197 (65%)	49 (70%)
	Practicing	15 (13%)	71 (23%)	7 (10%)
	Reviewing	2 (2%)	35 (12%)	14 (20%)
	Total	117 (100%)	303 (100%)	70 (100%)
Teacher Yao	Introducing	13 (23%)	95 (38%)	114 (33%)
	Practicing	39 (68%)	99 (39%)	109 (32%)
	Reviewing	5 (9%)	58 (23%)	120 (35%)
	Total	57 (100%)	252 (100%)	343 (100%)
Teacher Wen	Introducing	28 (18%)	75 (20%)	122 (38%)
	Practicing	23 (15%)	78 (21%)	56 (18%)
	Reviewing	101 (66%)	223 (59%)	142 (44%)
	Total	152 (100%)	376 (100%)	320 (100%)
Teacher Ying	Introducing	73 (40%)	139 (34%)	35 (15%)
	Practicing	26 (14%)	129 (31%)	73 (31%)
	Reviewing	84 (46%)	142 (35%)	130 (55%)
	Total	183 (100%)	410 (100%)	238 (100%)

[1]The percentage of each type of questioning under different instructional situations

instructional situations of practicing and reviewing the content. The three instructional purposes are elaborated as follows.

Connecting the Meaning of Mathematical Concepts

During the instructional situations of practicing and reviewing the content, the teachers expected the students to understand the mathematics when solving problems. In this regard, teachers posed high-cognitive questions to facilitate students to connect the meaning of the mathematical concepts to the problems.

For example, when Teacher Jyu interacted with her students during the instructional situation of reviewing the learned content, she led the students to understand a challenging problem by asking questions, enabling them to grasp the mathematical concepts embedded in the problem. The problem included a diagram where lines L and M are parallel and segments AB and AD trisect the angle EAC (Fig. 1). Given that the measure for angle 1 is 12 degrees and for angle 2 is 36 degrees, the students had to find the difference in measurements between angles ADC and ABC.

> Teacher Jyu: When you look at this problem, what do you think of? … you might think of some properties. Can you identify it [the properties] by looking at the diagram?
> Students:….The alternate interior angles and the consecutive interior angles…
> Teacher Jyu: What else?
> Students: The opposite angles
> Teacher Jyu:…. is it [the opposite angles] this one [pointing at the interaction of AD and CB]?
> Students: yes

The challenging problem shown in Fig. 1 had a cognitively demanding diagram which looked quite different from those shown in the textbooks. Teacher Jyu expected her students to know how to solve the challenging problem and understand the geometric properties required to generate a solution. She posed questions to encourage the students to reason and make connections with the properties embedded in the diagram (e.g. the property of alternate interior angles). The questions she posed, such as "what do you think of" and "can you identify it by looking at the graph?" invited students to further connect with the mathematical concepts and relationships learned in previous lessons. After the students responded to the questions, Teacher Jyu further posed the question, "what else?" that indicated that she continued to stimulate the students to identify geometric properties from the diagram that would help solve the problem. Through the interaction process of questions and answers, students gradually grasped the mathematical concepts and could find solutions to the challenging problem.

Stimulating Multiple Solutions to a Problem

The analysis also showed that teachers posed high-cognitive questions to facilitate students to generate multiple solutions to a problem.

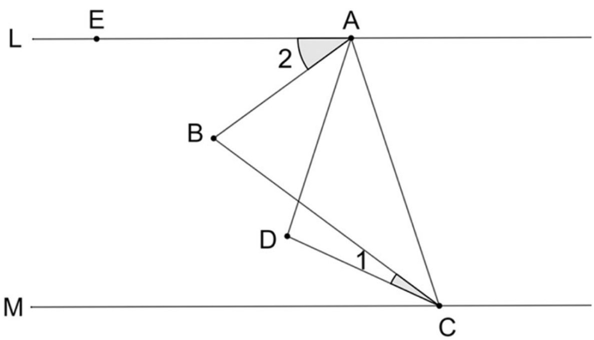

Fig. 1 The problem that required connecting mathematical concepts

Figure 2(a) was the problem that Teacher Yin discussed with her students during the instructional situation of practicing new content. The problem required them to find the sum of the degrees for the angles 1, B, C, and 2. Figure 2(b), 2(c), and 2(d) presents three solution strategies by drawing different auxiliary lines on the given diagram. Teacher Yin first discussed two methods to solve the problem by drawing lines P and Q through points B and C, respectively, that are parallel to lines L and M (see Fig. 2(b)) and drawing a line that intersects lines L and M to form a hexagon (see Fig. 2(c)). Drawing the auxiliary lines at different places required the students to visualize the diagram for identifying embedded geometric properties that can be used to formulate a solution. Teacher Yin further posed questions to facilitate students to reason alternative solution strategies (Fig. 2(d)).

Teacher Yin: Are there other methods to solve this problem?
Students: Draw a segment by connecting points A and D.
Teacher Yin: Why? What would happen if we connected AD [drawing the segment on the blackboard]? …. What can we find? [pointing at quadrilateral ABCD]
Students: The same side interior angles are supplementary [referring to angles 3 and 6].
Teacher Yin: Okay, so, can we know how to solve the problem now?

Teacher Yin encouraged the class to reason alternative solution strategies to the problem. She posed questions, such as "are there other methods to solve this problem?" and discussed students' solution ideas with the whole class. She also facilitated students to connect with mathematics concepts by asking, "what do we find?"

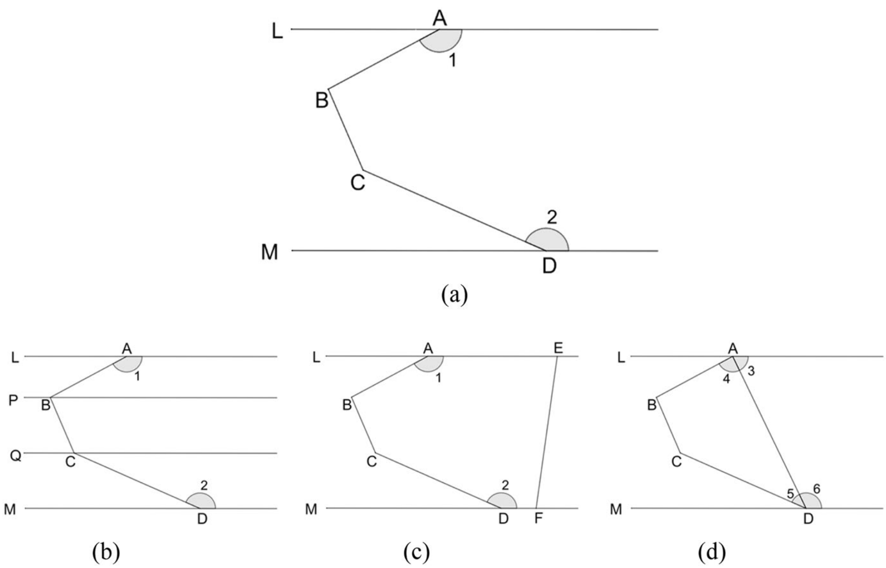

Fig. 2 The problem diagram and the strategies to solve the problem

After Teacher Yin obtained the key idea to the alternative solution strategies from the students, she discussed the solutions with the whole class.

We found that facilitating students to generate multiple solutions occurred during the instructional situations of practicing and reviewing the content as well as when teachers introduced new mathematics content. Taiwanese mathematics teachers tend to stimulate students' flexible thinking by encouraging them to find multiple solutions to a complex problem. Such questions develop students' critical thinking and evaluate their competence in applying mathematics concepts to different problem contexts.

Exploring Mathematical Relationships Across Different Problem Contexts

Facilitating students in exploring mathematical relationships embedded in a series of problem contexts is another instructional goal identified in Taiwanese mathematics classes. Exploring mathematical relationships among problems is critical to the development of students' generalization competence in problem solving. According to Mason et al. (1982), generalization refers to the competence of moving from individual examples to making conjectures about a wide class of examples.

Figure 3 presents three problems that Teacher Jyu posed during classroom teaching. She first posed a problem during the reviewing activity (see Fig. 3(a)) that aimed to facilitate students to review the properties related to parallel lines and apply them to solve relevant problems. After students identified the solutions, Teacher Jyu posed another two problems with more complex diagrams (see Figs. 3(b) and 3(c)). The instructional purpose of the two complex problems was for students to practice their problem-solving skills and explore mathematical relationships among the three problems. After discussing solution strategies for the three problems with class students, Teacher Jyu posed questions to facilitate them to explore mathematical relationships across the problems.

> Teacher: We have just done the problems … What conclusion can you get? … What general rule can you explore from those problems?
> Students: The sum of measures of the angles on the right side in the diagram is equal to that of angles on the left side.

Teacher Jyu considered this as a good opportunity to develop students the competence of generalization ability. She expected that her students can not only know how to solve the problems but also identify the general mathematical rules embedded in the problem contexts. Thus, she posed the questions "what conclusion can you get?" and "what general rule can you obtain from those problems?" to elicit students to explore mathematics. The questions triggered generalizations of solutions and facilitated the students in forming mathematical connections across problem contexts.

Discussion

In this study, we examined the types of questions that Taiwanese mathematics teachers at the secondary school level posed and in which instructional situations they do so during classroom teaching. Based on the analysis of six experienced mathematics

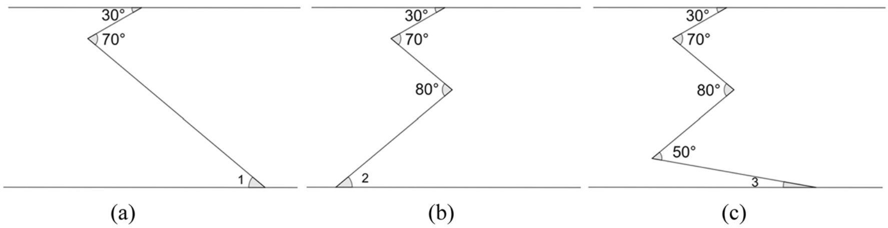

Fig. 3 The problem that required to explore the mathematics relationships

teachers, we found that only one-fourth of the mathematics teachers' spoken dialogues during classroom teaching were for interrogative purposes; others were for declarative purposes. The findings confirm that mathematics teaching in Taiwan is teacher-centered (Fwu & Wang, 2006), as the mathematics teachers tend to transmit the knowledge to the students instead of posing questions for follow-up discussions. Among questions posed by the teachers, only one-fifth of the questions required students to provide high-cognitive responses. Most questions identified in the study were either low-cognitive questions or for checklist purposes. The analysis of lesson structure showed that the teachers were more likely to pose questions when introducing new mathematics content or reviewing the learned content. A low percentage of questions were posed when the teachers and the students practiced the mathematics content.

The cross-analysis of the types of questions and lesson structures revealed that teachers preferred posing different types of questions in different instructional situations. Four out of six teachers posed high-cognitive questions when practicing or reviewing the content. The analysis indicated that Taiwanese mathematics teachers were more likely to pose high-cognitive questions for assessment purposes. The qualitative analysis further revealed three types of instructional purposes for posing high-cognitive questions during the instructional situations of practicing and reviewing the content—connecting the meaning of mathematical concepts, stimulating multiple solutions to a problem, and exploring mathematical relationships across different problem contexts.

Scaffolding students to make connections with mathematical concepts is one of the important goals of teaching and learning mathematics and the development of problem-solving competence (NCTM, 2000). These mathematical connections allow students to thread new concepts into the existing knowledge network and bridge the gap between mathematical ideas, concepts, and representations. Franke et al. (2009) highlighted the importance of teachers in initiating questions to elicit students' mathematical thinking. The study shows that Taiwanese mathematics teachers are concerned with the students' understanding of mathematics and their mathematical thinking skills. Teachers use high-cognitive questions to assess students' mathematics understanding.

A number of researchers have highlighted the value of generating multiple solutions to a problem for developing problem-solving skills and nurturing mathematical creativity (Leikin, 2009; Levav-Waynberg & Leikin, 2012; Silver et al., 2005). This study shows that Taiwanese mathematics teachers provide students the opportunities to develop mathematical creativity by posing questions to facilitate multiple solutions based on flexibly applying the learned knowledge. Lin and Tai (2015) indicated that Taiwanese students preferred adopting multiple strategies to learn mathematics. Taiwanese students tend to learn new mathematics concepts by relating them to things that they already know or by figuring out the essential parts of the newly learned material. This study shows the consistency between teachers' teaching and students' learning of mathematics in Taiwan.

Generalization has been considered an important competence in mathematics (Davydov, 1990; Dörfler, 1991; Martino & Maher, 1999). Martino and Maher (1999) argued that teachers should pose questions to facilitate students to explore and reinvent mathematical rules and relationships. Students are expected to extend their understanding of mathematics problems and grasp the problems' core ideas and mathematical relationships. The analysis indicates that Taiwanese mathematics teachers search for opportunities for students to observe mathematical relationships and develop generalization competence.

In the TIMSS video study, Hiebert et al. (1999) reported that mathematics instruction in Japan at the secondary school level aims to help students develop mathematical thinking, understand mathematical ideas, and invent new ways to solve problems. In this study, we illustrate a similar teaching pattern as Taiwanese mathematics teachers also emphasize on understanding mathematical ideas, inventing new ways to solve problems, and identifying general mathematical rules across different problem contexts. While the teaching script in East Asian countries is often described as teacher-centered (Fwu & Wang, 2006; Watkins & Biggs, 1996) and examination-driven (Ho, 2009; Leung, 2006; Lin & Tsao, 1999; Wu, 2006), teachers likely focus on demonstrating procedures through lectures. This study further specifies that, although not frequently, teachers tend to use questions to assess students' learning outcomes.

It is recognized that teacher-centered instruction may cause learning problems as students become passive knowledge receivers and may not benefit from the development of high-order competencies (e.g. critical thinking) (Yang et al., 2022). However, Kember (2016) addressed that the teacher-centered approach makes it possible for students to work through mathematics problems sequentially and systematically. The teacher-centered approach enables teachers to efficiently use class time to help students learn mathematics. Particularly, while mathematics textbooks and other instructional materials used by Taiwanese mathematics teachers often include a high portion of cognitively demanding tasks (e.g. Charalambous et al., 2010; Hsu & Silver, 2014), the teacher-centered approach allows teachers to manage the teaching with a large portion of cognitively demanding tasks. Teacher Wen described her teaching as follows:

> ….We want our students to learn mathematics actively and learn it well.... However, we are also concerned with time….If we let students freely explore mathematics, it will take much of the class time and influence students' learning progression....We expect our students to see the whole picture of mathematics and even the underlying mathematics structure....Thus, we usually do

not ask questions, especially the difficult ones, during classroom teaching…. We are likely to ask questions for assessment purposes…to check if students understand mathematics or to help students to enrich their understanding of mathematics. (Interview transcript of Teacher Wen)

The above interview transcript reveals the pedagogical dilemma encountered by Taiwanese mathematics teachers in terms of how to efficiently use class time to help their students learn mathematics. The quantitative and qualitative analyses reveal that the teachers use questions as assessment tools to balance the pedagogical dilemma and extend students' mathematics learning in a teacher-centered environment. The study's findings also relate to the findings of Hsieh et al. (2020) that indicated that Taiwanese senior high school students tend to appreciate mathematics teaching that focuses on helping them understand the mathematics concepts through a detailed illustration approach other than the instruction based on exploration. The transcript of teacher interview and the findings from Hsieh et al. (2020) provide a picture of the ecology of teaching and learning in Taiwan mathematics classrooms.

Nevertheless, we recognize the limitations of the study as it was developed based on the analysis of a small sample size of teachers. However, the quantitative and qualitative analysis reveal characteristics of Taiwanese mathematics teaching that allow one to realize how teachers manage students' mathematics learning. Therefore, this study contributes to the understanding of the ecology of the Taiwanese mathematics classrooms and provides insights into the differences in mathematics teaching between the West and the East.

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Declarations

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