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Mathematical Reading: Investigating the Reading Comprehension Strategies Implemented by Middle School Students

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Abstract
Mathematical literacy is a keystone of contemporary mathematics education research. We collectively, thoroughly explore this set of literacy practices from the perspectives of mathematical writing and mathematical discussion. Mathematical literacy practices, of course, include a third component—reading—which takes a number of forms. This document explores the mathematical reading processes of 22 middle school students, identifying the strategies most and least used by these students, and the ways in which strategy implementation aids their reading process. From this study, we can begin to identify how this knowledge can be used by teachers, curriculum designers, and educational researchers in an effort to aid their students.

Keywords Mathematical literacy · Mathematical reading · Middle grades · Skills and strategies

This study found students employ known reading strategies to help them decipher mathematical text. It also explored how these efforts aid middle school students' understanding of the task at hand. In the USA, our public education system has evolved to a place where two disciplines, mathematics and reading, are seemingly so valued above all others that states often mandate students pass specific standardized exams before being promoted to their next grade (Common Core State Standards Initiative [CCSSI], 2020; Florida State Legislature, 2016; Michigan Education Agency, 2017; Texas Education Agency, 2016; United States Department of Education [USDE], n.d.a/n.d.b). Furthermore, the Common Core State Standards (2020) for education place focus instructional foci on two disciplines—English-language arts/literacy and mathematics. Through the development of these Common Core

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State Standards, the National Governors Association and the Council of Chief State School Officers have clearly stated a desire that American students be proficient at both literacy skills and mathematics skills (CCSSI, 2020).

Lost in this legislative effort is the inevitable overlap between the two disciplines, moments when mathematical ideas show up within the study of literacy (e.g. the logic found Through the Looking Glass) or a need to employ literacy skills to help decipher the mathematics at hand. We often explore ideas of reading and literacy through our study of English-language arts (ELA; e.g. Pearson & Cervetti, 2015) and mathematical concepts throughout mathematics education (e.g. Byrd et al., 2015; Powell & Fuchs, 2014). In more recent years, mathematical literacy has become an increasingly important focus for mathematics education researchers (e.g. Brozo & Crain, 2018; Herbel-Eisenmann et al., 2015). While mathematical reading strategies and skills would be found within the study of mathematical literacy, few studies explicitly focus on the identification and effective implementation of mathematical reading practices for students (Beaudine, 2018).

The goal for this document is to explore this intersection of mathematics and literacy, as encountered by middle school students. More precisely, the text below seeks to highlight the ways middle school students employ reading strategies answering the following question:

- What reading comprehension strategies were students utilizing to make sense of the provided passage?

Literature Review

From Literacy to Mathematical Literacy

Any discussion of mathematical literacy (ML) ought to begin with an underlying understanding of literacy practices established by our ELA peers and the roles played by both content-area literacy and disciplinary literacy. Literacy, itself, is an "interplay of meaning-making systems (alphabetic, oral, visual, etc.) that teachers and students should strive to study and produce" (National Council of Teachers of English [NCTE], 2005) This NCTE definition builds upon the historic understanding of literacy—reading, writing, and arithmetic as "tools that equip people for intelligent participation in daily life" (Hildreth, 1947, p. 1). One may argue Hildreth's definition of literacy to be archaic, but the argument for the "three R's" is at least colloquially understood. Gibbons (2009) offers another yet another, and more modern definition of "literacy," writing "literacy involves the integration of listening, speaking, reading, writing, and critical thinking and includes the cultural knowledge that enables a speaker, writer, or reader to recognize and use language appropriate to different social situations" (p. 7). Note that reading is found in each of the definitions above.

These same literacy practices are embedded in the study of mathematics. The Organisation for Economic Co-operation and Development (OECD, 2013) describes a need for mathematically literate students in similar terms to Gibbon's definition.

They suggest students "formulate, employ and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena" (OECD, 2013, p. 14–15), allowing students to effectively interpret a variety of mathematical texts. The similarity of the OECD description of mathematical literate beings and the idea of literacy stated by Gibbons matches up well—listen and read (interpret), speak and write (explain, describe), and think critically (formulate, employ).

Approaches to ML have included the use of content-area literacy (CAL) and disciplinary literacy (DL) practices. CAL begins with the premise that "every teacher is a teacher of reading" (Brozo et al., 2013, p. 353) and promotes the use of the same broad and generic reading strategies, many of which come directly from ELA research, when reading text form all disciplines (Armstrong et al., 2018). DL explores "the idea that we should teach the specialized ways of reading, understanding, and thinking used in each academic discipline, such as science, history, or literature" (Shanahan & Shanahan, 2014, p. 636), as each discipline has varying literacy needs.

Mathematical literacy has been described as the ability to create, utilize, and interpret a range of mathematical representations and "analyze and communicate ideas as they pose and interpret solutions to mathematical problems" (Chen & Chui, 2016, p. 265) and Firdaus et al. (2017) suggested "mathematical literacy is about usability or mathematical functions that have been learned by the students in the school to everyday life in order to compete in a globalized world" (p. 213). Given all the ideas and definitions presented thus far, it can be said that ML pertains to one's ability to listen, speak, read, write, and think critically about mathematics, allowing the literate individual to understand and use language appropriate for the mathematics setting (Beaudine, 2020), thus reading can be stated as a vital component of mathematics.

Many projects have been conducted in the realm of ML. Some studies focus on the teacher and how they approach instruction in their classroom (e.g. Herbel-Eisenmann et al., 2015; Temple & Doerr, 2018), while others look at the student discourse within the classroom (e.g. Moschkovich & Zahner, 2018; Sigley & Wilkinson, 2015). Few, in comparison, explore the role reading plays in math, how mathematical reading is instructed in classrooms, and the reading skills and strategies used by students as they engage with mathematical text. The latter is explored in this present study.

Herbel-Eisenmann et al. (2015), through the development and implementation of professional development materials, focused on the classroom discourse inside secondary mathematics spaces. They suggest "that engagement with readings and analyzing student work and textbooks have promise toward developing nuance in teachers' understandings and talk about the mathematics register" (Herbel-Eisenmann et al., 2015, p. 40).

Similarly, Sigley and Wilkinson (2015) and Moschkovich and Zahner (2018) explored student discourse in mathematics spaces, but from a student perspective. Sigley and Wilkinson (2015), conducted a case study in which they worked with a single bilingual middle school student, exploring his interaction with both oral and written forms of mathematical communication. Moschkovich and Zahner's (2018) project also attended to the communicative efforts of bilingual students as they worked in groups to solve the given mathematics problems. In each of these studies—Herbel-Eisenmann et al. (2015), Sigley and Wilkenson (2015), and Moschkovich and Zahner (2018)—they focused on some form of discourse, whether verbal or written. Studies like these seek to improve student mathematical literacies through written or verbal discourse, but typically do not explore mathematical reading (Beaudine, 2018).

Mathematical Reading

In a mathematical reading focused study, Beaudine (2019) asked each of the ten participants how they felt about reading and about mathematics, and how the two disciplines overlapped. Those ten middle school students suggested that solving word problems (9 students), understanding Eqs. (2 students), and reading directions and definitions (1 student each) were components of mathematical reading. While two participants identified equations, the students characterized reading in mathematics as attending to text written in word, sentence, and paragraph formats. This provides some insight into the perspective of these students, but these views do not encapsulate all that is mathematical reading.

Adams' (2003) described mathematical reading as an effort to "acquire comprehension and mathematical understanding with fluency and proficiency through the reading of numerals and [all non-numeric] symbols, in addition to words" (p. 786). Years later, Molina (2012) published work focusing on the possible confusion between English words and their mathematical counterparts, suggesting that words with different meanings in both the ELA and mathematics spaces may get conflated (e.g. plane, range). Hillman (2014) added the idea that students must also interpret or "read" images such as diagrams, graphs, and tables. So mathematical reading requires one to decipher words, symbols, and a wide range of visual images to understand the provided text.

Adams et al. (2015) suggest that the study of mathematics seeks to simplify and condense information found in the text, allowing the writer and reader to communicate as efficiently as possible. Conversely, process of reading in ELA spaces tends to expand the text on the page into a representation packed with meaning (Adams et al., 2015). This paradox is certainly not insurmountable but must be considered as we help students with their study of mathematics. One approach is to identify those mathematical reading practices employed by expert readers of mathematics, and help students learn strategies to help develop those same mathematical reading skills. We have evidence of the ways experts approach mathematical reading, whether they are classroom teachers, graduate students, or university professors (e.g. Doerr & Temple, 2016; Shanahan et al., 2011; Shepherd & van de Sande, 2014).

Doerr and Temple (2016) worked with two 6th grades over a 5-year period. They found that, initially, their cooperating teachers thought little of the reading-heavy curriculum chosen for that school. They felt the students' efforts were already taxed by the mathematics they were asked to learn, and that was further complicated by the requirement to read a text-heavy, story-driven curriculum. In the fifth year of the study, Doerr and Temple (2016) reported a large shift in the teachers' perspectives, having finally embraced the wordy curriculum as opposed to pushing against the reading requirements.

The studies from both Shanahan et al. (2011) and Shepherd and van de Sande (2014) utilized those with or those pursuing a graduate degree in mathematics. For Shanahan et al. (2011) they explored the ways a mathematician, a historian, and a chemist read their respective documents, drawing the strongest contrast between the mathematician who seemed not to care who wrote the document being read, as they were more interested in the argument being made, and the historian who could not separate the text on the page from the individual who wrote the document or the period within which it was written. Shepherd and van de Sande (2014) also interviewed academics, though they were all graduate students or professors of mathematics.

The mathematicians in the studies above make those expert readers of mathematics often "read the meaning" of the work on the page, as opposed to reciting each and every notation (Shepherd & van de Sande, 2014). They also try to treat prose and equation equally and are willing to pause and re-read passages that were not initially clear (Shanahan et al., 2011). While more expert readers may skim over things understood or already known (Shepherd & van de Sande, 2014), more novice readers tend to slow their reading down and read every word or symbol (Doerr & Temple, 2016) as they read the entire passage (Adams et al., 2015).

Research exists related to the ways teachers, graduate students, professors, and other mathematical experts read mathematical text (Doerr & Temple, 2016; Shepherd & van de Sande, 2014). Additionally, there is work to identify and improve upon some literacy practices in mathematics spaces (Herbel-Eisenmann et al., 2015; Moschkovich & Zahner, 2018; Sigley & Wilkinson, 2015). Missing from this conversation are the strategies and skills implemented by students as they read, and ways those efforts aid student understanding while reading mathematics texts. This study seeks to clarify this last point.

Study Methodology

The decision to work with middle school students was targeted. Before continuing, three things must be noted. First, research suggests that students as young as second grade can successfully report their thoughts while reading (Hilden & Pressley, 2011). Second, "the perspective of [elementary and secondary] students are recognized as valuable, but not often queried or considered" (Zheng et al., 2014, p. 279), often leaving researchers and administrators to unilaterally make decisions about instruction that directly affects our students' academic pursuits. Through past studies, I have found that middle school students have very clear thoughts related to mathematical reading that need to be more deeply explored than the structure of my previous studies allowed (Beaudine, 2018, 2019), To this extent, the selection of middle school participants falls well within supported research, and the work and perspectives shared by said students ought to be recognized as valuable.

Third, Afflerbach et al. (2008) have identified a clear difference between reading strategy—“deliberate, goal-directed attempts to control and modify the reader’s efforts”—and reading skill—an “automatic action that result in decoding and comprehension with speed, efficiency, and fluency” (p. 368). Through this work, I make no effort to differentiate between a reading skill or a reading strategy, as the primary intent is to identify which strategies are utilized, when they are used, and the purpose the strategy implementation may serve. While focusing on identification, I did not attend to the level of automaticity or seek to identify the intention of students, evidence that is necessary to distinguish between a strategy and a skill. As such, each implementation will be identified as a “reading comprehension strategy,” as students are reading in an attempt to comprehend both the text and the mathematics that follows.

Student Interviews

This project was designed in two parts—student interviews and the analysis of those interviews. For the student interviews, two related views were considered. Ginsberg (1997) described two ways to identify the knowledge of students—testing and clinical interviews. They go on to suggest that a semi-structured clinical interview not only allows students to demonstrate their knowledge, but also allows the interviewer an opportunity to follow up on the various ideas presented during the interview. Hilden and Pressley (2011) have suggested that “People are actually quite good at reporting the contents of their working memory” (p. 427) and before that, Pressley and Afflerbach (1995) established that a verbal reading protocol (VRP) is an effective way to solicit information about how an interview participant is thinking. Because of the work of Ginsberg (1997), Hilden and Pressley (2011), and Pressley and Afflerbach (1995) a semi-structured VRP was designed for this study. This design allowed each interview to follow the same path from start to end but provided the leeway necessary for me to follow unique lines of thought with each student participant.

Study Population

This study explored the reading processes of the 7th grade students enrolled in two different mid-west middle schools. To recruit the students for the study, two cooperating teachers, Ms. H at site 1, and Ms. C at site 2, were asked to assist. Site 1, as described by cooperating teacher Ms. H, is a high-performing, traditional middle school that is “a suburban school, a little bit rural too.” The school is not ethnically diverse, but “we have a pretty big diversity in terms of socioeconomic status” (Ms. H, personal communication). Site 2, described by Ms. C as a STEM-focused private school within the public-school system. The school serves students in the 3rd through 8th grade, and Ms. C characterizes the students as being “on the higher socioeconomic scale, for sure,” and “very racially diverse.”

In all, 22 7th grade students—12 from site 1 and ten from site 2—participated in this study, each working though the same semi-structured interview protocol, outlined below. In doing so, they read and answered questions about the chosen textbook passage (Appendix 1). Students were asked to select a pseudonym for this project, but no other personal identifiers were requested.

Study Task

The Rand Reading Study Group (RRSG) clearly stated the importance of assessing the textual features present in any reading passage (Snow, 2002). They identified three pieces—"the surface code (the exact wording of the text), the text base (idea units representing the meaning of the text), and the mental models (the way in which information is processed for meaning)" (Kirby, 2003, p. 2). Each of these three pieces must be considered and change with the genre and purpose of the text. For this project, a broadly used textbook with a variety of textual features—words, symbols, and images (Adams, 2003; Hillman, 2014)—covering a mathematical concept that is revisited and built upon throughout one's mathematical learning was critical. As such, several texts were considered, and a three-page passage from a nationally distributed American mathematics text, covering unit rates and ratios, was selected for this project.

Study Interview Protocol

As stated above, students are quite capable of sharing thoughts they had while reading. In preparation for the interviews, the three-page reading passage was divided into six sections at logical breaks in the text, with each of the six stopping points labelled with a "stop" marker. At each of these points, the students were asked "what are you thinking about?," allowing them to self-report any thoughts going through their mind as they read. The students were, at times, asked to "say more about that" or clarify "what do you mean by _____?" The purpose of this VRP was to allow these students the opportunity to thoroughly describe the thoughts they were having about the reading and allow me to explore the strategies and decisions used by these middle school students without projecting unused strategies onto the study participants.

Each student was to work through the three-page passage, completing six tasks and a debrief conversation during their interview. One student was unable to complete task 6 due to time constraints but was able to complete all the work up to that point and the debrief conversation. In all, 21 of the 22 participants completing all seven sections—131 tasks and 22 debriefing conversations—for a total of 153 completed interview segments. One student missed the sixth section due to time constraints. Each interview was audio and video recorded, translated to text, and analyzed first for reading comprehension strategy implementation, then for evidence of how the chosen reading comprehension strategy aided the participants' pursuit of understanding. The 22 interviews spanned an average of 37 min, with an average of 22 min dedicated to the reading of the passage and approximately 15 min for the debrief conversation. The figures of specific passages will be shared, below, as they relate to the discussion.

Interview Analysis

To identify the reading skills and strategies used by these students while reading a mathematical passage, the research question from above, a list of reading comprehension strategies and skills (RCS) must be identified to which student efforts can be prepared. To address the second question—an effort to identify how these used strategies aid student mathematical reading comprehension, the RAND (a corporation) Reading Comprehension Analysis model (Kirby, 2003; Snow, 2002) was adopted and utilized. This model is described in more detail, below.

Reading Comprehension Strategy Identification

The RCS identified at the beginning of this study were informed by a great many researchers. The development of this of RCS and skills began with the Characteristics of Responsive Readers (CRR; Hilden & Pressley, 2014; Pressley & Afflerbach, 1995), which outlines 40 distinct characteristics observed across a collection of expert readers, as defined by Pressley and Afflerbach (1995). Through the experience of a previous study (Beaudine, 2019) which relied heavily on the work of Pressley and Afflerbach (1995), as well as a review of relevant CAL, DL, and ML literature, a list of 21 RCS was constructed. During the effort to find reliability one strategy, "pause to reflect and self-check," was separated into two separate codes, and two new codes were introduced—"Question or critique the text" and "Make connections across the text." Finally, through analysis of student interviews, one final strategy was identified and added to the list—"Create a mental image." In all, 25 reading skills, strategies, and characteristics were considered while analyzing these students' completed work. The strategy number, name, and source can be found below (Table 1).

With the use of the list above, the strategies implemented by individual students could be tracked, counted, and analyzed. This effort, though, yields very little information beyond the recognition of RCS observed while the student was reading. In addition to the identification of strategies used by middle school students as they read mathematics, this study sought to better understand what purpose these strategies served. The adoption and implantation of the RAND Reading Comprehension Analysis helps identify how the strategies were effectively utilized for the students in this study.

RAND Reading Comprehension Analysis

"*Reading comprehension* as the process of simultaneously extracting and constructing meaning through interaction and involvement with written language" (Kirby, 2003, p. 1, emphasis in original). To better understand the comprehension of these 7th grade students, I implemented the RAND Reading Comprehension Analysis model (Fig. 1; Kirby, 2003; Snow, 2002), which explores the interplay

Table 1 Reading comprehension strategies

Reading comprehension strategy	Definition
Use no strategy	Participant is not observed to using a reading comprehension strategy of any kind
Preview text	Participant takes a moment to consider the text prior to the first read of each section (Doerr & Temple, 2016; Pressley & Afflerbach, 1995)
Apply prior knowledge	Participant links content read to something already known (Adams et al., 2015)
Read aloud	Participant choses to read the passage out loud (Pressley & Afflerbach, 1995)
Plan a solution or predict result	Participant offers a hypothesis as to what might be coming later in the text or how they might find a solution (Brozo & Crain, 2018)
Modify a plan or prediction	Participant changes the way they view the text or recognizes new information, that changes their understanding of what is being asked (Bergeson & Rosheim, 2018)
Make notes while reading	Participant makes notes, either on the text itself or a on a space separate from the text (e.g. whiteboard, notebook) as they read (Armstrong et al., 2018)
Paraphrase text	Participant revoices some component of the text, may relate effort to help with remembering (Bergeson & Rosheim, 2018)
Read text closely	Participant slows pace, attends carefully to each image, word, symbol, or step in a passage (Armstrong et al., 2018; Shanahan et al., 2011)
Read entire passage	Participant suggests that words or symbols mean something different as a whole than of considered each individually (Adams et al., 2015; Pimm 1987)
Read symbols as words	Participant reads symbolic phrase as opposed to the individual symbols themselves (Doerr & Temple, 2016)
Decode the text	Participant uses decoding knowledge to decipher unknown text passages (Armstrong et al., 2018)
Attends to prose and equations equally	Participant makes no distinction between importance of prose passages and the symbolic ones (Shanahan et al., 2011)
Seek clarification or external assistance	Participant seeks or explores text outside of the passage being read to improve understanding (Brozo & Crain, 2018)
Pause to reflect	Participant pauses while reading the text (Brozo & Crain, 2018)
Reread the text	Participant revisits a passage already read (Harkness & Brass, 2017; Pressley & Afflerbach, 1995)
Seeks important information	Participant looks to identify specific points or key bits of information within the text to aid with understanding (Bergeson & Rosheim, 2018)
Selective reading	The participant skips or omits a portion of the text (Harkness & Brass, 2017)
Skims the text	Participant quickly progresses through portions of the passage, with limited engagement (Shepherd & van de Sande, 2014)
Uses text clues	Participant makes note of page lay-out or structure of text during initial reading or solution to help guide later effort (Doerr & Temple, 2016)

Table 1 (continued)

Reading comprehension strategy	Definition
Create analogy or metaphor	Participant uses an analogy or metaphor, whether extra-mathematical or structural (Pimm, 1987, p. 95), to aid in their understanding (Pressley & Afflerbach, 1995)
Question or critique the text	Participant openly questions or critiques the text to generate a more personal connection with the document (displayed by participants)
Make connections across the text	Participant discusses connections between one passage and another (displayed by participants)
Self-check	Participants demonstrated an explicit effort to gauge their own understanding as they read or worked through the passage (Doerr & Temple, 2016)
Create a mental image	Participant suggests an effort to imagine or picture something related to the problem at hand (displayed by participants)

The above list of codes and definitions were based on either current literature or observations of students as they worked. Each code was established and defined with the assistance of a colleague with expertise in literacy practices

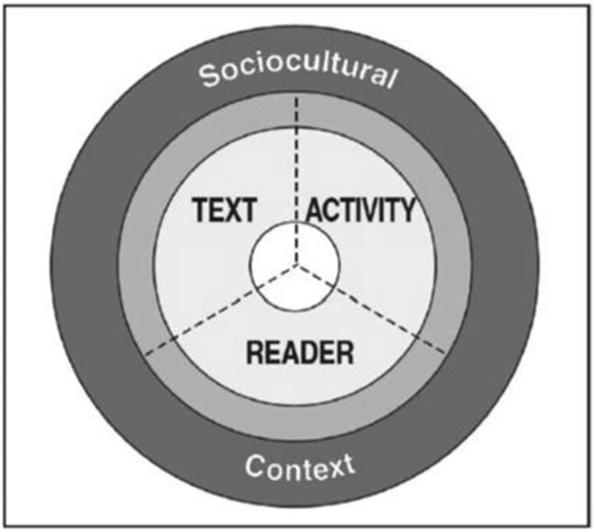


Fig. 1 RAND model for reading comprehension

of three components—text, activity, and reader—all while considering the sociocultural context within which the reading activity is taking place.

Snow (2002) explained that fluency, knowledge, and reading capability all change as the reader gains experience. With more fluency and knowledge, the readers' skill improves. In this sense, fluency is the translation of text with speed and accuracy (Fuchs et al., 2001), and knowledge is a student's understanding of the contents' "vocabulary and topic knowledge, linguistic and discourse knowledge, and knowledge of comprehension strategies" (Kirby, 2003, p. 2).

When considering the text being read, the RRCA clearly stated the importance of assessing the textual features present in any reading passage (Snow, 2002). They identify three levels of text code—the surface code, the text base, and the mental model (Kirby, 2003). Purcell-Gates et al. (2007) adds to this textual analysis the need to consider the texts' written genre—texts with "differentiated and identifiable written text types" (p. 11). Gibbons (2009) outlines four roles played by the reader—a codebreaker, a participant in the text, a user of the text, and a text analyst. Each role is dictated by the sociocultural context surrounding the activity.

The final component for the RRSA is the activity of reading. The activity of reading is where the reader and the text begin their interaction. During this process, the reader is expected to "expand and unstuff meaning" (Fuentes, 1998, p. 81) found within the condensed and largely symbolic text structure found in mathematics. The RRSG suggested that "the reading activity includes one or more purposes or tasks, some operations to process the text, and the outcomes of performing the activity, all of which occur within some specific context" (Kirby, 2003, p. 2).

As discussed above, reading comprehension hinges on the reader, the text, and the activity or interaction between the two. Each of these components "occurs within a larger sociocultural context that shapes and is shaped by the reader and that interacts with each of the three elements" (Snow, 2002, p. 11). Snow explained the need to consider the origins of the reader's motivation and instruction that may influence strategy selection and implementation, as well as both the long- and short-term implications for their effort, as each of these components can affect the approach taken by the student-reader.

For these interviews, students were asked to read a pre-selected passage while sitting at a table with a stranger asking them questions throughout. The setting and activity were new to each student, a fact that must be considered as their work was recorded and analyzed. The interview held no academic implications for the volunteering participants. Based on the RAND model, each of the three components, the reader, the text, and the activity, are affected by the setting in which the interaction takes place. For this study, both I, as the interviewer, and the setting of the interview were unfamiliar, which may have led the participants to perform differently than they would within their class or during homework. The setting, though, presents a space where they and I can speak about their RCS without the added pressure or distraction of a classroom setting.

Findings

The remainder of this manuscript will outline the findings from the study, that is the strategies used and how those strategies aid understanding, a discussion of what these findings mean in the context of mathematics education, and a proposal of implications and next steps to further this line of inquiry.

Reading Comprehension Strategies

When centering attention on the strategies explored in this project, I found all but two of the 25 strategies—*Use no strategy* and *Create analogy or metaphor*—were observed in use by at least one of the 22 study participants. Conversely, the strategies *Read symbols as words* (15 students), *Use text clues* (16), *Read aloud* (18), *Seek important information* (19), *Modify a plan or prediction* (20), *Pause to reflect* (21), *Question or critique the text* (21), *Self-check* (21), *Plan a solution or predict result* (22), and *Paraphrase text* (22) were all utilized at least once by at least two-thirds of the study participants.

A picture of the RCS most and least used by the study participants becomes clearer, when viewed by task. These students used 851 unique strategy implementations across the seven interview segments. The five RCS most used by the students in this study, across all tasks, were *Read aloud* (104 implementations), *Plan a solution or predict result* (97), *Pause to reflect* (89), *Paraphrase text* (86), and *Self-check* (77), each employed in over half of the completed interview tasks. Six strategies were used in fewer than ten tasks across the 153 completed tasks: *Use no strategy* (0), *Create analogy or metaphor* (0), *Decode the text* (2), *Create a mental image* (4), *Skims the text* (7), and *Make notes while reading* (7).

Overall, the study participants used an average of 38.68 strategies during their interview and 5.53 strategies per task. Students used more strategies during the earlier tasks (an average of 7.18 strategies used in task 1) than during the later tasks (fewer than five strategies used during tasks 4, 5, or 6). Task 7 was the debrief conversation held between me and each student. In each of the first six tasks, the reading comprehension strategy identification was based on my observation of their reading process. In task 7, the students and I openly discussed the strategies reported, and the coding was based on the ideas that surfaced during those conversations.

Exploration of Reading Comprehension Strategies Trends

Fewer Reading Comprehension Strategies Used

As mentioned above, the average number of student-implemented RCS used to complete each task fell as the interview progressed. The participants used an average of 7.2 RCS to complete their work for task 1. A decrease is seen on the second page where the same students used an average of 6.4 and 5.4 RCS to complete task 2 and task 3, respectively. The rate of decrease slowed for task 4 and task 5, where students used an average of 4.6 and 4.2 RCS per task, respectively. The only uptick in RCS was between task 5 and task 6, where students used an average of 4.8 RCS to complete their work related to task 6. The question, then, is why? I propose three explanations for the shrinking number of RCS implemented—comfort, demand, and space. By this, I am suggesting that readers' strategic approach may be affected by their level of comfort with a text, the demands made by the text, and the space provided for them, in the textbook, to work.

The participants in this study knew the chosen passage came from a mathematics textbook but were unfamiliar with Forseman's (2017) *enVision Mathematics* text. Some asked what parts of the text they were expected to read. I, to avoid leading them into a read-aloud, asked them to read the text as they normally would, in whatever manner they chose. The first page brought with it a lot of white space, a single introductory problem, and some guiding questions. The second page held task 2 and task 3, the page had less whitespace and more structure than the first page. On the third page, the students found task 4, task 5, and task 6 and comparatively little whitespace. As each student became more familiar with the stylistic layout, with the reading protocol, and with the audio and video recordings, they seemed to become less reliant on strategies to solve the problems.

Most Used Strategies

Of all the strategies observed, four appear at the top of the list of strategies when considering the both the number of students who used the strategy and the number of separate implementations. A discussion of common ways students implemented these four strategies—*paraphrase the text*, *plan or predict a solution*, *pause to reflect*, and *self-check*—will follow in the text below. Two strategies were used by a lot of students (*read aloud*) or was implemented often (*questioning or critiquing the text*), but in the interests of space and time, only those four strategies found at the top of both lists will be explored below. This creates a line of research exploring the questions students pose at their mathematical texts as they read.

Least Used Reading Comprehension Strategies

Of the selection of strategies that began this project, two—*using no strategy* and *create an analogy or metaphor*—went unobserved throughout the 22 interviews. The former may simply be a consequence of the study setting. Four other strategies—*decoding the text*, *creating a mental image*, *making notes while reading*, and *skim the text*—were used by fewer than ten students and were implemented fewer than ten times. For this discussion, we will set aside decoding and creating mental images, as they are difficult to identify while observing a reader. The focus of this discussion will be placed on the latter two strategies—making notes while reading and skimming the test—as they tend to be quite observable and were surprisingly sparsely used.

Both skimming the text and taking notes are presented as effective mathematics-standardized test-taking strategies (e.g. Penn State Learning, 2020; Todd, 2020). Just like a textbook, the text found within any standardized assessment must be read by the test taker. When one considers that both strategies are often found within the Content Area Literacy literature, encouraging students to skim, take notes, and define a purpose for their reading process (Harkness & Brass, 2017), their application in test-taking spaces is a logical extension of mathematical reading practices.

These participants tended to avoid both. Six students made notes while reading, for a total of seven implementations. Seven students skimmed the text as they worked, each identifying the strategy once. Six of the seven times *Skim the text* was assigned can be found in the debriefs. Charlotte, for example, stated, "Well it, [pause] I just thought like you can make it a little like, it's confusing cause like, um, like I could figure it out if I just like look at it, but like, um, just by looking at it really quickly." During our debrief conversation, Charlotte suggested that if she spent more time looking at example 2, she might have been able to more clearly understand the information presented. Instead, she just skimmed example 2. This demonstrates that her effort to pause and to self-check was ineffective, and might suggest that her desire to keep progressing forward was stronger than her desire to understand the very cluttered example (Fig. 6, above).

As for making notes, examples might include "highlighting, underlining, circling, making notes, outlining, or somehow flagging important points in text, including important examples" (Pressley & Afflerbach, 1995, p. 44). In her work, Charlotte identified a need to take notes while reading on two occasions. On one such occasion, during the first task she reported, "I'm gonna like, I feel like I should, like, record, like, I should, like, record it on paper—Like, write it down, like, the number of free throws." The information she needed is represented on the page, and yet the act of taking notes was employed by Charlotte to better understand what was being asked of her or more clearly identify the numbers with which she needed to work. The extra step to write down important information helped Charlotte solidify her understanding of her interview's first task, allowing her to build upon that understanding toward a solution.

When interviewing Ms. H about her class, we both shared experiences where we showed students how they might write on the problem, underline information, cross out unneeded pieces, or taking notes might help their problem-solving process. Ms. H uses a mnemonic device, one posted on the board for all students to see, to encourage note taking as student encounter new problem. Even with the instruction to do so, the practice within class, and the discussion of the strategy's usefulness when taking a test, only six of these 22 students made notes while reading.

Charlotte's Work

This next section focuses on the strategies Charlotte (all names are student chosen pseudonyms) implemented as she works through the text. This effort highlights ways participants utilized their chosen RCS to drive understanding. Immediately after each of these strategy explorations, a discussion of what this says about her reading understanding will follow. Charlotte's work was selected because she demonstrates and discusses each of the six strategies discussed below which allows consistency throughout the remainder of this text. Charlotte was a Site 1 student, attending a traditional middle school.

Paraphrase the Text

Pressley and Afflerbach (1995) identified this strategy as "Repeating/restating text just read to hold in working memory … [or] repeating/restating a thought that occurred during reading" (p. 35). All 22 students took time to paraphrase some component of their reading and did so on a total of 86 tasks. Eighty-three of the 86 observed usages came during the students' reading and working segment, with three happening as students were explaining their work.

This strategy was employed in two distinct ways. The students paraphrased the text as a reminder of what was done before moving forward with their reading or work or they paraphrased to simplify a more complicated representation. Charlotte did both in her interview. While finalizing a solution to the Explain it! problem (task 1), she looked back to the text to find out what the solution wanted, to which she announced "Out of the times-, like, okay. Who made more out of how many attempts they had?" In this moment, Charlotte seemed to be repeating the text to remind herself about the goal of the task at hand.

In task 5 (Fig. 2), Charlotte paraphrased the table presented in the text. She reported, "Oh, okay. So, [pause] okay, so the ratios, [pause] okay. So, the rabbit jumps three jumps in eight meters. The kangaroo is five jumps in 12 m. So, you keep, you keep like adding on onto the table until you find, um, [pause] the distance." In this effort, Charlotte translates the information found in the table into meaning that helps her better understand the solution presented in example 3.

By the time Charlotte and her peers arrived at task 5 (Fig. 2), they were well into their reading of this text allowing them to feel more comfortable with the text I chose, the interview setting, and the process of reading and reporting. Collectively they used the fewest number of RCS on this fifth task. Recognizing that Charlotte has little control over the text and the sociocultural setting within which she worked, we look toward the activity and reader. In the passage above, we can see Charlotte paraphrasing the meaning of the images attached to the task, identifying the jumping abilities for both the rabbit and kangaroo rat. She takes the information from the diagram and applies the repeated addition to the table as she describes the solution outlined by the text. The paraphrasing allowed Charlotte to build her knowledge, made sure she understood, and then moved forward.

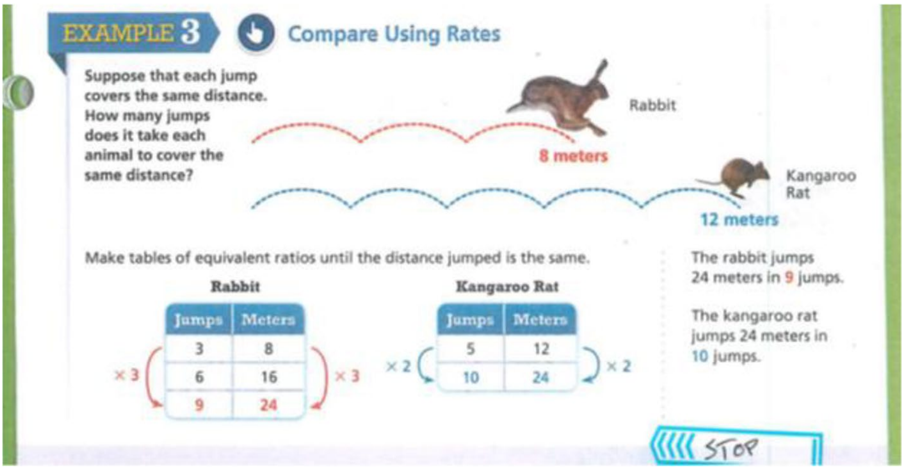


Fig. 2 Example 3: the fifth task in the interview

Plan or Predict

"Generating an initial hypothesis about what the text is about, one that can be revised or refined" (Pressley & Afflerbach, 1995, p. 33). Much like the paraphrasing, all 22 students planned out a solution or predicted the outcome before doing any work. The reading comprehension strategy of planning and predicting was utilized in 97 of the completed tasks.

In an ELA study, one might see a student make a guess about the presented reading or sharing some ideas about "what comes next" in the text. The study participants were rarely seeking to state a hypothesis at any point in the readings, and almost never guessed at the end result unless otherwise directed by the text. In this study of mathematical reading, however, students chose to talk through the steps they believe they needed to take to find a solution.

As an example, Charlotte could be observed planning out a solution in task 3. After reading the details of the example and being asked what she's thinking about, she shares "So I'm mostly thinking that, um, [whispering] okay, we did this before [/whispering]. Um, I need to find like how many, like I need to divide this by nine to find how many per hour?" In the sentence following her whispering she is formulating a plan to find the pay for one of the two lifeguards.

During example 1 (Fig. 3) and the Check for Understanding exercise that follows (Fig. 4), Charlotte and her peers read through an example of how one might find the hourly wage of a lifeguard who works x number of hours and received \$$y$ pay for their work. They are then asked to provide a solution to check for understanding, at the bottom of the page. This effort to plan a solution as she is reading the problem is aided by the work on the page, above, so Charlotte is utilizing her limited experience with the text to design solutions that come later in the text. In doing so, Charlotte constructs a solution for task 3 that falls in line with those from above the example text above, suggesting that she effectively read the example, created enough meaning to act upon it, then replicated the process as she completed the page.

Pause to Reflect and Self-Check

At the beginning of this project, these two strategies were one. Shanahan et al. (2011) stated, "When explaining how he thought about the ideas in a text, one mathematician said that he asks himself questions. 'Did I see this fact before? Did I see a special instance of this fact before? Do I know if the statement is correct? Can I prove it?'" (p. 418) and based on this definition the code began as "pause and self-check." Through the reliability coding effort, my colleague and I discovered that, while the two are closely related, they are not universally entwined. One may very well pause and not make any noticeable effort to correct or assess their own work. Similarly, one may realize a mistake as they are working, make the correction, and not pause while doing so.

The two were still very strongly associated with one-another, as evidenced by the same 21 students using both strategies during their interviews. One student, Gracie, chose not to use either of the two strategies. The two strategies were clearly not always used in tandem, as the different counts demonstrate. In several cases,

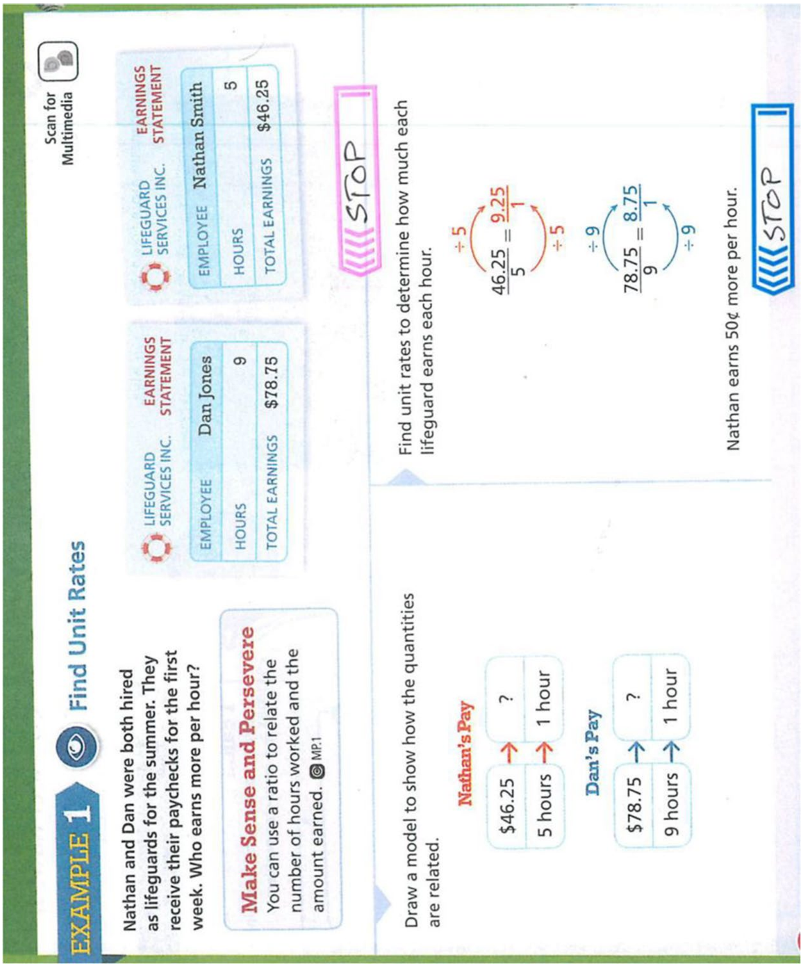


Fig. 3 Example 1: the second task in the interview

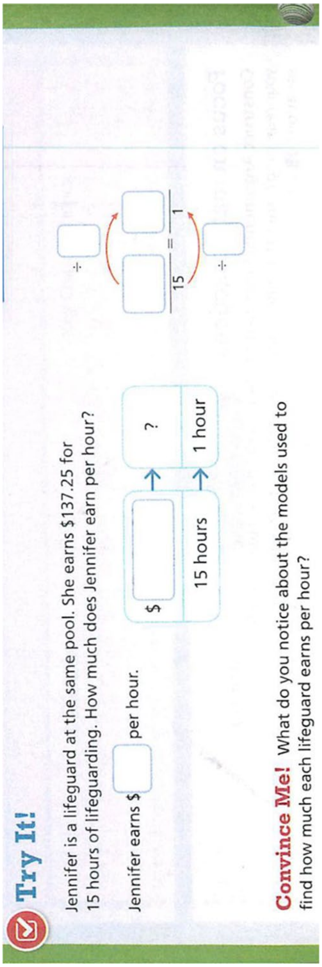


Fig. 4 Try it!: the third task in the interview

however, either the pause was followed by an assessment or "check" of the participant's work, or the pause occurred during a self-check.

Charlotte provided an example of each. In one moment, while solving the sixth task (Fig. 5), she stated, "All right, so zero. Okay. Three gallons. We have to label that. [pause] no. [pause] all right, now I got it," where the pause and the self-check were closely related to each other, with the audible pause coming before the audible self-check. In this moment, she pauses, mentally checks her thoughts, decides a route is incorrect and adjusts, then moves forward. In this case, the pause and self-check work together to help Charlotte verify her understanding of the problem based on her reading, and then construct a logical solution for her final task.

In other moments, like when Charlotte was reading through example 2 (Fig. 6), her pause could be found after the self-check had begun. As she worked through this example, Charlotte shared the following:

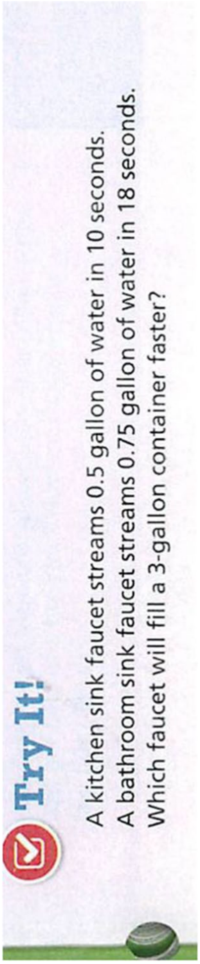
> *Um, okay. So, seven days. Okay. So, and this one you need to do. Okay. So, I think we needed to do 128 times seven. Wait. [pause] Okay. And then you need to find out how much each dog, like, eats per day.*

In the passage above, Charlotte can be observed making sense and self-checking her understanding as she reads example 2. The repeated use of the word "okay" suggests an affirmation that she understands that which is being explained and is ready to move on. At one point, she pauses to double check her assertion that she needs to multiply 128 by 7. She then changes tack to focus on the dogs' daily food consumption as the route to the correct solution.

Students do read mathematical texts and in doing so employ a wide range of established RCS. We can also observe how these strategy implementations can aid these students' understanding of the mathematics text. If we, in mathematics education, embrace the facts that mathematical text is read—students read their texts and use known RCS as they engage with the text—then we can build an understanding of mathematical reading that benefits three groups—curriculum designers, teachers, and teacher educators.

Conclusion and Implications

This document serves to introduce the strategies used by 22 students as they explored an unfamiliar passage of mathematical text. These students seem to take RCS acquired outside of the mathematics space and apply them as they engaged with an unfamiliar passage of mathematical text. In American schools, we often suggest students skim text and make notes as they work, an act that most of these students avoided. These students did, almost universally, use six strategies—read aloud, pause to reflect, plan a solution or predict results, paraphrase text, question or critique the text, and self-check—while they worked. Charlotte's work was added to illustrate how students may employ these strategies as they work, how we as educators can think about student mathematical reading processes, and an introduction to the RAND reading comprehension analysis model for identifying what students gain from their reading.


Try It!

A kitchen sink faucet streams 0.5 gallon of water in 10 seconds.
A bathroom sink faucet streams 0.75 gallon of water in 18 seconds.
Which faucet will fill a 3-gallon container faster?

Fig. 5 Try it!: the sixth task in the interview

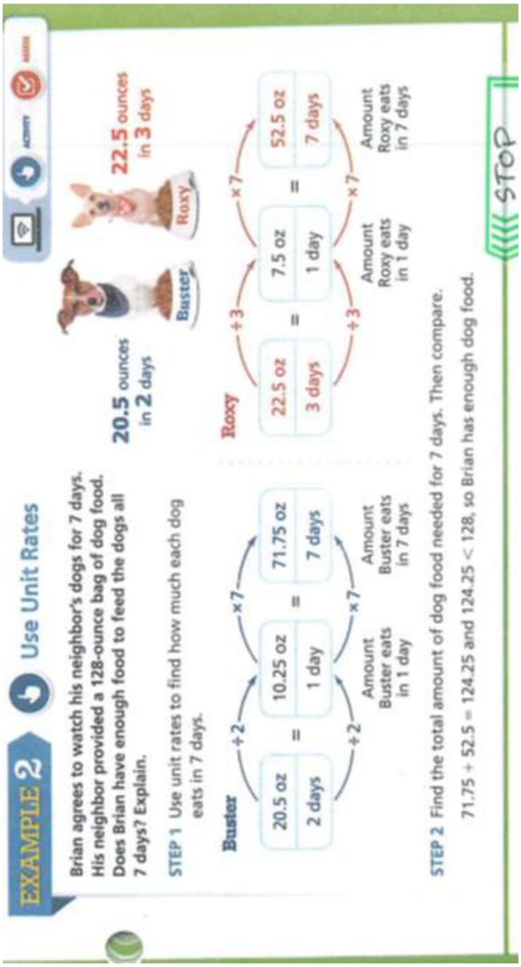


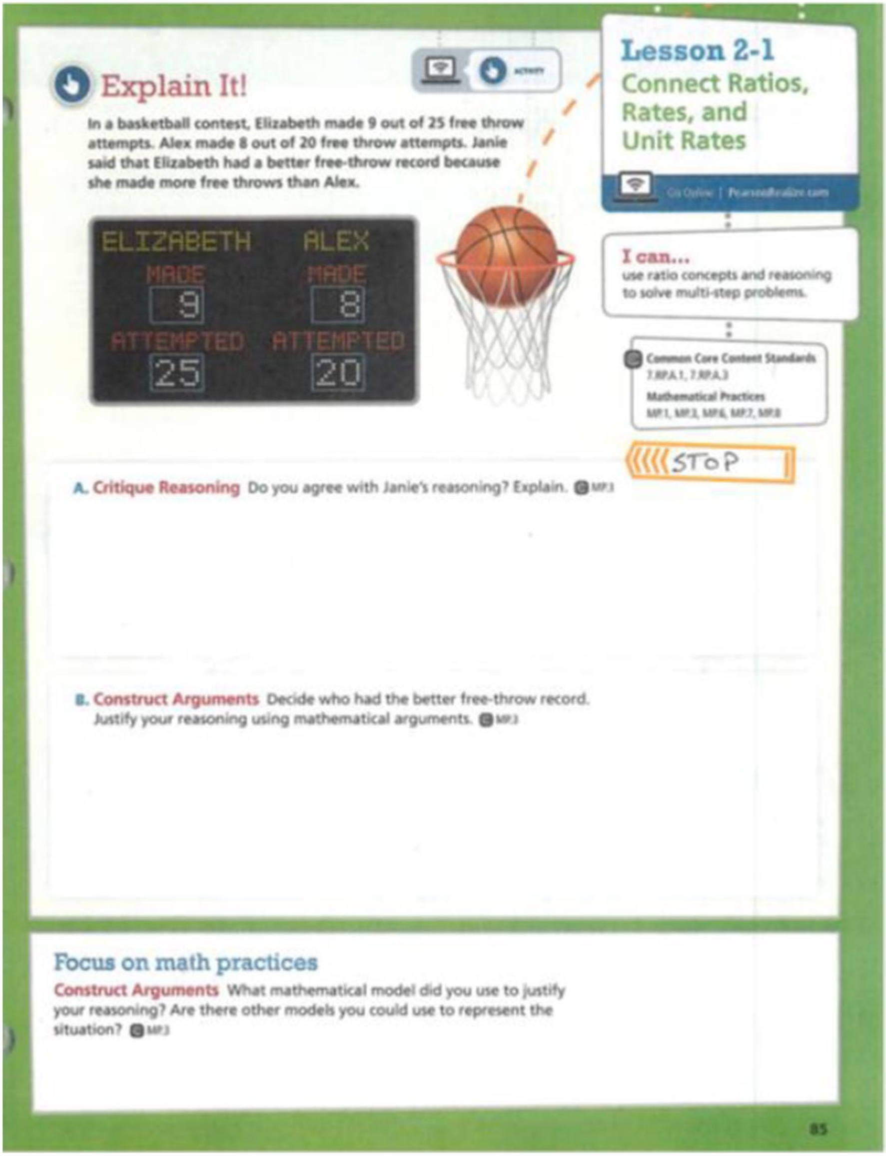
Fig. 6 Example 2: the fourth task in the interview

Ideas for future research in this arena include the ways reading is viewed by teachers leading mathematics classes, seeking effective ways to teach RCS within a mathematics space, and student reading practices in other mathematical spaces (e.g. arithmetic, geometry, trigonometry). The more knowledge about students' natural mathematical reading efforts and better understanding of how students most effectively read mathematics—two different ideas—would allow curriculum designers an opportunity to craft lessons that lead students toward more efficient processes. The textbooks, or the student consumable version of the textbook increasingly found in US classrooms, could provide more whitespace for notes, continue the self-check process, and embrace multiple representations—equations, words, and diagrams/tables—of the same information. Through the teacher manual, curriculum designers can help teachers better understand how reading can accentuate the text and their students' understanding of mathematics.

Teachers should be aware of common RCS that aid students' mathematical reading comprehension and be ready to introduce those strategies to their students, as needed. If, for example, we know students in higher levels of mathematical work effectively critique the text as they work, we can encourage younger, less experienced students to start doing the same. Through this process, teachers should adopt strategies they see within the classroom that are effective, introduce new strategies known to be effective for students, and model each strategy clearly and consistently. If we take this project as an example, a class may not utilize all 25 strategies presented, but a core set of strategies will surface, and improving students' efficiency with strategy implementation may help mathematical studies of more complex texts.

Preservice teachers will be making curricular decisions related to mathematics as they progress through their program and begin their teaching career. They will choose or design units. They will highlight what is important for their students. They will model how one might approach the discipline. As teacher educators, we have a responsibility to open our thought process up to future teachers, and this should include the ways we read mathematical texts. We should explore new ideas that aid student understanding (e.g. mathematical reading practices), model our approach to curriculum design and selection (e.g. find texts that help students improve their mathematical reading skill), embracing influence from other disciplines (e.g. look to other disciplines for reading strategy ideas), and deliberately choosing what is and is not included in our lessons (e.g. spending less time on rote memorization efforts in favor of deliberate mathematical reading efforts while problem solving). Our decisions affect these preservice teachers as much as their decisions will affect their own students. We would be wise to prepare them to make those decisions and our use of meta-conversations surrounding the action of teaching or reading mathematics will help guide their practice.

Appendix 1 – Three-page reading passage from enVision Mathematics Gr. 7.



Lesson 2-1
Connect Ratios, Rates, and Unit Rates

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Explain It!

In a basketball contest, Elizabeth made 9 out of 25 free throw attempts. Alex made 8 out of 20 free throw attempts. Janie said that Elizabeth had a better free-throw record because she made more free throws than Alex.

ELIZABETH	ALEX
MADE 9	MADE 8
ATTEMPTED 25	ATTEMPTED 20

I can… use ratio concepts and reasoning to solve multi-step problems.

Common Core Content Standards 7.RP.A.1, 7.RP.A.3
Mathematical Practices MP.1, MP.3, MP.6, MP.7, MP.8

STOP

A. Critique Reasoning Do you agree with Janie's reasoning? Explain. MP.3

B. Construct Arguments Decide who had the better free-throw record. Justify your reasoning using mathematical arguments. MP.3

Focus on math practices

Construct Arguments What mathematical model did you use to justify your reasoning? Are there other models you could use to represent the situation? MP.3

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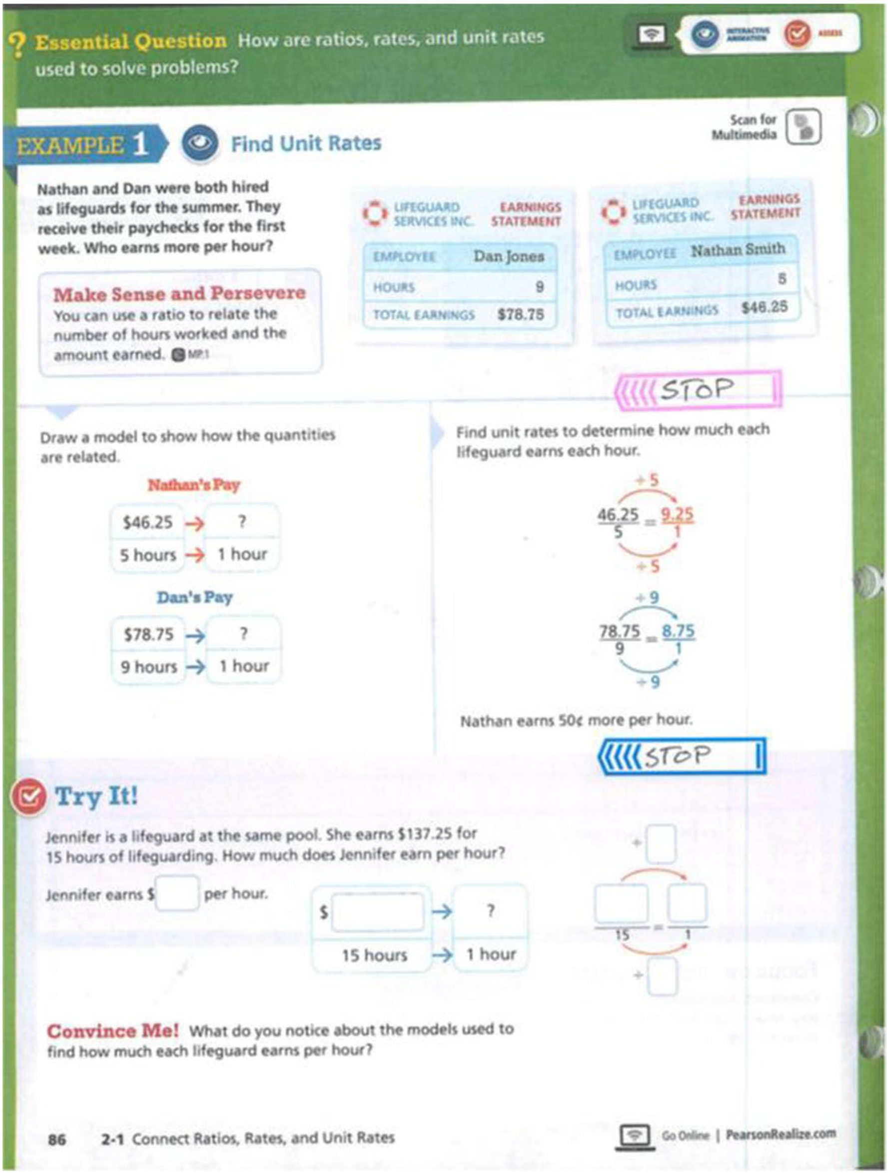

? Essential Question How are ratios, rates, and unit rates used to solve problems?

EXAMPLE 1 Find Unit Rates

Scan for Multimedia

Nathan and Dan were both hired as lifeguards for the summer. They receive their paychecks for the first week. Who earns more per hour?

Make Sense and Persevere
You can use a ratio to relate the number of hours worked and the amount earned. MP.1

LIFEGUARD SERVICES INC. EARNINGS STATEMENT

EMPLOYEE	Dan Jones
HOURS	9
TOTAL EARNINGS	$78.75

LIFEGUARD SERVICES INC. EARNINGS STATEMENT

EMPLOYEE	Nathan Smith
HOURS	5
TOTAL EARNINGS	$46.25

STOP

Draw a model to show how the quantities are related.

Nathan's Pay

$46.25	→	?
5 hours	→	1 hour

Dan's Pay

$78.75	→	?
9 hours	→	1 hour

Find unit rates to determine how much each lifeguard earns each hour.

$$\frac{46.25}{5} = \frac{9.25}{1} \quad (\div 5)$$

$$\frac{78.75}{9} = \frac{8.75}{1} \quad (\div 9)$$

Nathan earns 50¢ more per hour.

STOP

Try It!

Jennifer is a lifeguard at the same pool. She earns $137.25 for 15 hours of lifeguarding. How much does Jennifer earn per hour?

Jennifer earns $ ___ per hour.

$ ___	→	?
15 hours	→	1 hour

$$\frac{\square}{15} = \frac{\square}{1} \quad (\div \square)$$

Convince Me! What do you notice about the models used to find how much each lifeguard earns per hour?

86 2-1 Connect Ratios, Rates, and Unit Rates

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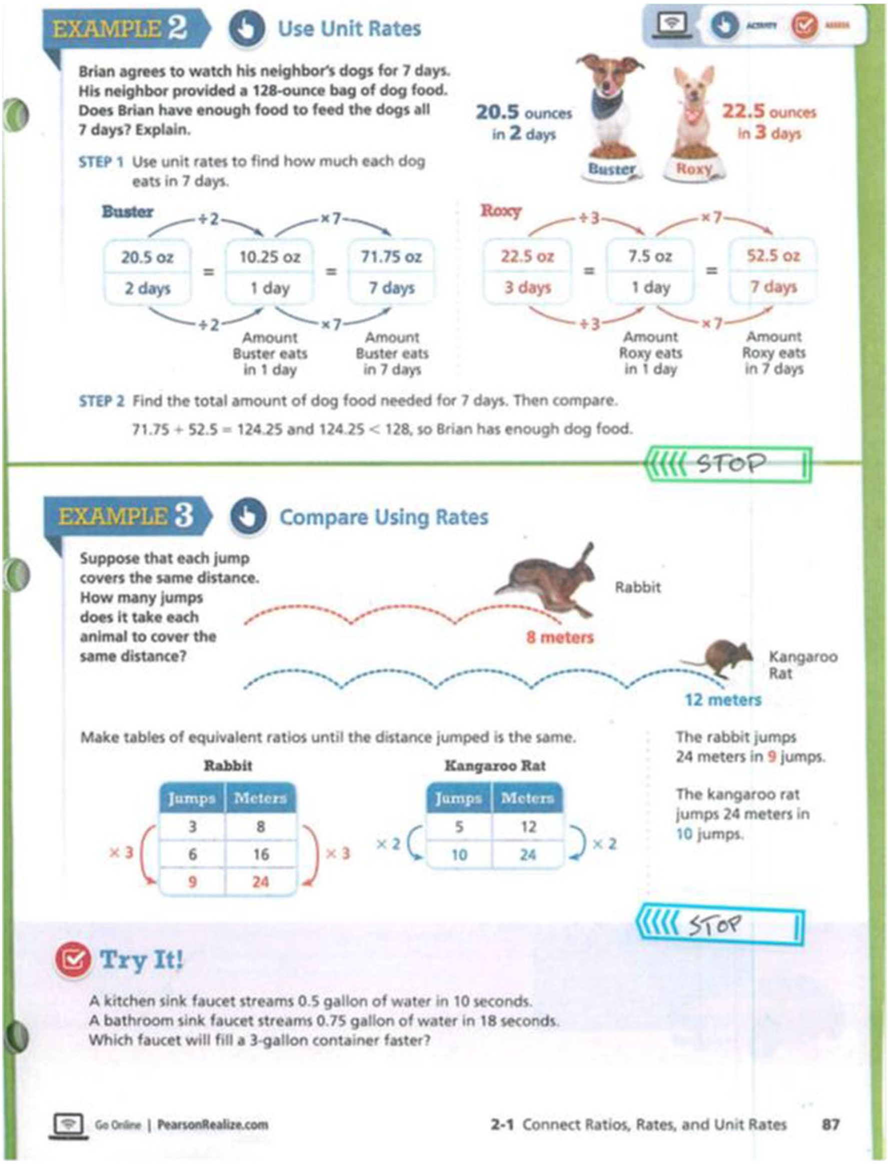

EXAMPLE 2 Use Unit Rates

Brian agrees to watch his neighbor's dogs for 7 days. His neighbor provided a 128-ounce bag of dog food. Does Brian have enough food to feed the dogs all 7 days? Explain.

20.5 ounces in 2 days — Buster

22.5 ounces in 3 days — Roxy

STEP 1 Use unit rates to find how much each dog eats in 7 days.

Buster

20.5 oz	=	10.25 oz	=	71.75 oz
2 days		1 day		7 days

÷2, ×7 — Amount Buster eats in 1 day; Amount Buster eats in 7 days

Roxy

22.5 oz	=	7.5 oz	=	52.5 oz
3 days		1 day		7 days

÷3, ×7 — Amount Roxy eats in 1 day; Amount Roxy eats in 7 days

STEP 2 Find the total amount of dog food needed for 7 days. Then compare.

71.75 + 52.5 = 124.25 and 124.25 < 128, so Brian has enough dog food.

STOP

EXAMPLE 3 Compare Using Rates

Suppose that each jump covers the same distance. How many jumps does it take each animal to cover the same distance?

Rabbit — 8 meters

Kangaroo Rat — 12 meters

Make tables of equivalent ratios until the distance jumped is the same.

Rabbit

Jumps	Meters
3	8
6	16
9	24

×3, ×3

Kangaroo Rat

Jumps	Meters
5	12
10	24

×2, ×2

The rabbit jumps 24 meters in 9 jumps.

The kangaroo rat jumps 24 meters in 10 jumps.

STOP

Try It!

A kitchen sink faucet streams 0.5 gallon of water in 10 seconds. A bathroom sink faucet streams 0.75 gallon of water in 18 seconds. Which faucet will fill a 3-gallon container faster?

Go Online | PearsonRealize.com 2-1 Connect Ratios, Rates, and Unit Rates 87

References

Adams, A. E., Pegg, J. & Case, M. (2015). Anticipation guides: reading for mathematics understanding. *Mathematics Teacher, 108*(7), 498–504. https://doi.org/10.5951/mathteacher.108.7.0498

Adams, T. L. (2003). Reading mathematics: More than words can say. *The Reading Teacher, 56*(8), 786–795.
Afflerbach, P., Pearson, P. D. & Paris, S. G. (2008). Clarifying differences between reading skills and reading strategies. *The Reading Teacher, 61*(5), 364–373. https://doi.org/10.1598/RT.61.5.1
Armstrong, A., Ming, K. & Helf, S. (2018). Content area literacy in the mathematics classroom. *The Clearing House: a Journal of Educational Strategies, Issues and Ideas, 91*(2), 85–95. https://doi.org/10.1080/00098655.2017.1411131
Beaudine, G. (2018). From mathematical reading to mathematical literacy. *Mathematics Teaching in the Middle School, 23*(6), 318–323. https://doi.org/10.5951/mathteacmiddscho.23.6.0318
Beaudine, G. (2019). Reading and math go side-by-side: Students explore the complexities between two core disciplines. In S. Otten, A. G. Candela, Z. de Araujo, C. Haines & C. Munter. (Eds). *Proceedings of the forty-first annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. University of Missouri.
Beaudine, G. (2020). *Reading to learn mathematics: Exploring reading strategy implementations of middle school students through their reading of mathematical texts* [Doctoral dissertation]. Michigan State University. ProQuest Dissertations Publishing.
Bergeson, K. T. & Rosheim, K. (2018). Literacy, equity, and the employment of iPads in the classroom: A comparison of secure and developing readers. *International Journal of Education in Mathematics, Science and Technology, 6*(2), 173–181.
Brozo, W. G. & Crain, S. (2018). Writing in math: A disciplinary literacy approach. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas, 91*(1), 7–13. https://doi.org/10.1080/00098655.2017.1342435
Brozo, W. G., Moorman, G., Meyer, C. & Stewart, T. (2013). Content area reading and disciplinary literacy: A case for the radical center. *Journal of Adolescent & Adult Literacy, 56*(5), 353–357. https://doi.org/10.1002/JAAL.153
Byrd, C. E., McNeil, N. M., Chesney, D. L. & Matthews, P. G. (2015). A specific misconception of the equal sign acts as a barrier to children's learning of early algebra. *Learning and Individual Differences, 38,* 61–67. https://doi.org/10.1016/j.lindif.2015.01.001
Chen, C. & Chiu, C. (2016). Collaboration scripts for enhancing metacognitive self-regulation and mathematics literacy. *International Journal of Science and Mathematics Education, 14*(2), 263–280. https://doi.org/10.1007/s10763-015-9681-y
Common Core State Standards Initiative [CCSSI] (2020). Common core state standards initiative: Preparing America's students for success. corestandards.org.
Doerr, H. M. & Temple, C. (2016). It's a different kind of reading: Two middle-grade teachers' evolving perspectives on reading in mathematics. *Journal of Literacy Research, 48*(1), 5–38. https://doi.org/10.1177/1086296X16637180
Firdaus, F. M., Wahyudin & Herman, T. (2017). Improving primary students' mathematical literacy through problem based learning and direct instruction. *Educational Research and Reviews, 12*(4), 212–219.
Florida State Legislature (2016). The 2016 Florida statutes (1008.25). Tallahassee, FL. http://www.leg.state.fl.us/Statutes/index.cfm?App_mode=Display_Statute&URL=1000-1099/1008/Sections/1008.25.html
Forseman, S. (2017). *enVision math 2.0, student edition, grade 7* (Vol. 1). Pearson.
Fuchs, L. S., Fuchs, D., Hosp, M. K. & Jenkins, J. R. (2001). Oral reading fluency as an indicator of reading competence: a theoretical, empirical, and historical analysis. *Scientific Studies of Reading, 5*(3), 239–256. https://doi.org/10.1207/S1532799XSSR0503_3
Fuentes, P. (1998). Reading comprehension in mathematics. *The Clearing House: a Journal of Educational Strategies, Issues and Ideas, 72*(2), 81–88. https://doi.org/10.1080/00098659809599602
Gibbons, P. (2009). *English learners, academic literacy, and thinking: Learning in the challenge zone*. Heinemann.
Ginsburg, H. (1997). Entering the child's mind: The clinical interview in psychological research and practice. *Cambridge University Press*. https://doi.org/10.1017/CBO9780511527777
Harkness, S. S. & Brass, A. (2017). How preservice teachers make meaning of mathematics methods texts. *International Journal for the Scholarship of Teaching and Learning, 11*(2), Article 17. https://doi.org/10.20429/ijsotl.2017.110217
Herbel-Eisenmann, B., Johnson, K. R., Otten, S., Cirillo, M. & Steele, M. D. (2015). Mapping talk about the mathematics register in a secondary mathematics teacher study group. *Journal of Mathematical Behavior, 40,* 29–42. https://doi.org/10.1016/j.jmathb.2014.09.003
Hilden, K. & Pressley, M. (2011). Verbal protocols of reading. In N. K. Duke & M. H. Mallette (Eds.), *Literacy research methodologies* (2nd ed., pp. 427–440). Guilford.

Hildreth, G. H. (1947). *Learning the three R's* (2nd ed.). Educational Publishers.
Hillman, A. M. (2014). A literature review on disciplinary literacy: How do secondary teachers apprentice students into mathematical literacy? *Journal of Adolescent & Adult Literacy, 57*(5), 397–406. https://doi.org/10.1002/jaal.256
Kirby, S. N. (2003). *Developing an R&D program to improve reading comprehension*. RAND Corporation. https://www.rand.org/pubs/research_briefs/RB8024.html
Michigan Education Agency (2017). Learn what's in the new third grade reading law. https://mea.org/learn-whats-in-the-new-third-grade-reading-law/
Molina, C (2012). *The problem with math is English: A language-focused approach to helping all students develop a deeper understanding of mathematics*. John Wiley & Sons.
Moschkovich, J. & Zahner, W. (2018). Using the academic literacy in mathematics framework to uncover multiple aspects of activity during peer mathematical discussions. *ZDM – Mathematics Education, 50*(6), 999–1011. http://doi.org/10.1007/s11858-018-0982-9
National Council of Teachers of English [NCTE] (2005). Position statements: Multimodal literacies. Author. https://ncte.org/statement/multimodalliteracies/
Organisation for Economic Co-operation and Development [OECD] (2013). *PISA 2012 assessment and analytical framework: Mathematics, reading, science, problem solving and financial literacy*. Author.
Pearson, P. D. & Cervetti, G. N. (2015). Fifty years of reading comprehension theory and practice. In P. D. Pearson & E. H. Hiebert (Eds.), *Research-based practices for teaching Common Core literacy* (pp. 1–24). Teachers College Press.
Penn State Learning (2020). *Test taking tips*. PennState Undergraduate Education. https://pennstatelearning.psu.edu/test-taking-tips
Powell, S. R. & Fuchs, L. S. (2014). Does early algebraic reasoning differ as a function of students' difficulty with calculations versus word problems? *Learning Disabilities Research & Practice, 29*(3), 106–116. https://doi.org/10.1111/ldrp.12037
Pressley, M. & Afflerbach, P. (1995). *Verbal protocols of reading: The nature of constructively responsive reading*. Lawrence Erlbaum Associates.
Purcell-Gates, V., Duke, N. K. & Martineau, J. A. (2007). Learning to read and write genre-specific text: roles of authentic experience and explicit teaching. *Reading Research Quarterly, 42*(1), 8–45. https://doi.org/10.1598/RRQ.42.1.1
Shanahan, C. & Shanahan, T. (2014). Does disciplinary literacy have a place in elementary school? *The Reading Teacher, 67*(8), 636–639. https://doi.org/10.1002/trtr.1257
Shanahan, C., Shanahan, T. & Misischia, C. (2011). Analysis of expert readers in three disciplines: History, mathematics, and chemistry. *Journal of Literacy Research, 43*(4), 393–429. https://doi.org/10.1177/1086296X11424071
Shepherd, M. D. & van de Sande, C. C. (2014). Reading mathematics for understanding—From novice to expert. *The Journal of Mathematical Behavior, 35,* 74–86. https://doi.org/10.1016/j.jmathb.2014.06.003
Sigley, R. & Wilkinson, L. C. (2015). Ariel's cycles of problem solving: an adolescent acquires the mathematics register. *Journal of Mathematical Behavior, 40,* 75–87. https://doi.org/10.1016/j.jmathb.2015.03.001
Snow, C. (2002). *Reading for understanding: toward an R&D program in reading comprehension*. RAND Corporation. https://www.rand.org/pubs/monograph_reports/MR1465.html
Temple, C. & Doerr, H. M. (2018). How do teachers develop and enact a disciplinary view of literacy in mathematics? *Journal of Adolescent & Adult Literacy, 61*(5), 483–488. https://doi.org/10.1002/jaal.664
Texas Education Agency (2016). *Grade-level retention and student performance in Texas public schools, 2014–15* (Document No. GE17 601 09). http://tea.texas.gov/acctres/retention_student_performance_2014-15.pdf
Todd, J. (2020, February 20). *Sadlier's math blog: math test-taking strategies for elementary students*. Sadlier School. https://www.sadlier.com/school/sadlier-math-blog/math-test-taking-strategies-for-elementary-students-math-multiple-choice-test-taking-strategies
United States Department of Education [USDE] (n.d.a). No child left behind: Elementary and Secondary Education Act (ESEA). https://www2.ed.gov/nclb/landing.jhtml
United States Department of Education [USDE] (n.d.b). Every Student Succeeds Act (ESSA). https://www.ed.gov/essa?src=rn
Zheng, B., Arada, K., Niiya, M. & Warschauer, M. (2014). One-to-one laptops in K-12 classrooms: Voices of students. *Pedagogies: An International Journal, 9*(4), 279–299. https://doi.org/10.1080/1554480X.2014.955499