International Journal of Science and Mathematics Education (2022) 20:1677–1698
https://doi.org/10.1007/s10763-021-10225-7

Mathematical Assemblages Around Dynamic Aspects of Angle in Digital and Physical Space

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Received: 3 October 2020 / Accepted: 26 September 2021 / Published online: 28 October 2021

Abstract

Drawing upon constructionism and neo-materialist approaches on the role of physical activity and the human body in teaching and learning mathematics, this paper investigates the meanings constructed by students while collaboratively carrying out tasks focusing on the dynamic aspects of the concept of angle in the educational context of a 6th grade class of a public primary school in Greece. Paying particular attention to the specific material configurations at play, we try to analyse the construction processes through which angle was used both as a measure represented by a number and as a directed turn in the simulated 3D space of a digital tool that combines 3D Turtle Geometry and dynamic manipulation. Following a design-based research method, mathematical meanings are analysed as contingent assemblages involving gestures, embodied metaphors, navigation in virtual spaces, viewpoints, figural representations of 2D and 3D geometrical objects and ways of verbal and symbolic expression.

Keywords Angle · Embodiment · Gestures · Turtle metaphor · Virtual space · Assemblage · Constructionism

Introduction

Spatial reasoning importantly relies on the use of representations that provide access to spatial information such as spatial locations, adjacency, symmetry and transformations. It is linked to skills such as locating, orienting, shifting

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dimensions, mentally rotating and visualising. Recently, spatial reasoning has gained renewed prominence in the attention of mathematics education researchers (Davis, 2015; Eilam & Alon, 2019), as it underpins and supports mathematical understanding and problem solving even in abstract and advanced levels. A more spatial and embodied approach to the teaching and learning of mathematics has been advocated (Shapiro, 2014) while spatial skills are considered as essential beyond mathematics classrooms, in fields such as science, technology and engineering (Abrahamson & Lindgren, 2014). In this framework, the teaching and learning of Geometry—as a topic of mathematics concerned with the space around us—has been given fresh attention through the focus on its unique potential to connect children's experience with space to mathematical meaning making.

The interest in Geometry has also been renewed with respect to what digital media can bring to meaning making through affordances such as interactivity, multiple interlinked representations, dynamic manipulations and dynamic visualisations (Ball et al., 2017; Laborde et al., 2006). However, the new possibilities of encounters with geometrical objects engendered through 3D digital artefacts and the embodied and material dimensions of that encounters have only recently been a focus of research (de Freitas et al., 2017; Ng, 2019; Sinclair et al., 2017). Three-dimensional digital media could play a significant role in redefining the way we can associate geometrical concepts with human sensorimotor activity while it seems that they open up new directions of research in mathematics education concerning corporeality and material physicality of mathematical concepts (de Freitas, 2016; de Freitas & Sinclair, 2014).

In this paper, we aim to contribute to our understanding of meaning construction processes followed by students at the end of primary school concerning the notion of angle in 3D space as represented on a computer screen. For many years, there has been research showing that carefully designed computational environments on a two-dimensional (2D) plane—such as Turtle Geometry environments—are an effective medium in offering rich mathematical experiences and in encouraging the construction of meanings in relation to the notion of dynamic angle-as-turn (Clements & Sarama, 1997; Kynigos, 1997). It is unclear, however, how similar computational environments representing 3D space can be used to this end. In particular, it is unclear how the moving entity in Turtle Geometry environments may be put to use and how deeply rooted intuitions about experiencing space and locomotion can be exploited so as to make sense of angle in space (Kynigos et al., 2009; Latsi & Kynigos, 2010; Papadopoulos et al., 2016).

In the design research we employed, we have tried to combine and coordinate two rather complimentary theoretical frameworks (Ng & Ferrara, 2020), that of constructionism and that of neo-materialism, in order to deepen our understanding of students' meaning-making processes. Our pedagogical aim was to engage students in navigating a moving entity to construct graphical digital objects through Logo programming with a digital tool, called MaLT2. MaLT2 is a programmable tool embedding 3D Turtle Geometry but also uniquely providing an affordance for the dynamic manipulation of a graphical output—resembling a Dynamic Geometry Environment (DGE) behaviour—by sliders changing variable procedure values (Kynigos & Grizioti, 2018). Our research aim was to investigate the meanings about dynamic aspects of angle constructed by our 6th grade students while paying particular attention to the material and embodied aspects of their mathematical activity. Thus, meanings were investigated not as immaterial mental objects but as contingent assemblages (Thompson, 2020), involving digital artefacts, physical activity, embodied metaphors, 2D and 3D geometrical figures as well as ways of verbal and symbolic expression that came together in productive relations and formed a whole not reducible to its parts.

Related Work and Theoretical Considerations

Even though angle is one of the most important mathematical tools for describing and analysing physical space (along with length and distance), it is present in a wide variety of physical situations that are not easily correlated or connected by children at the end of primary (Kontorovich & Zazkis, 2016; Mitchelmore & White, 2000). In typical school education, angle is basically approached as a static geometric figure, while the notion of angle as turn is usually underrepresented, although it is considered the most natural, the most instinctive aspect of angle (Freudenthal, 1983). Even in cases where angle is approached as turn or as a relationship between two directions on the plane or in space, this is done only through static 2D representations, which, no matter how cleverly designed, may delay the development of dynamic aspects of the concept and their integration with the static ones (Clements et al., 1996).

Aiming to understand the way in which students' intuitions and ideas concerning dynamic aspects of angle are challenged with 3D digital media, we developed a set of computational environments and a set of activities adopting a constructionist theoretical perspective (Kafai & Resnick, 1996). Constructionism puts emphasis on the role of the material culture considering individual and social meaning-making activity in the context of discursive bricolage of artefacts (Kynigos, 2015). It has also emphasised the importance of external constructions as well as the importance of the activity of the body since the 1970s.

Papert (1980) put notable emphasis on embodiment through what he called 'body-syntonicity' in the particular case of students engaged in activities with Turtle Geometry (see also Kynigos, 1992). According to Papert (1988), turtle—the moving entity used in Turtle Geometry—is vastly more important than Logo programming language, as, through its anthropomorphism, it can bridge mathematics and users' bodily action schemas and sensorimotor experiences. However, this aspect of constructionist activity was not hence given a lot of attention, despite a large interest in embodied metaphors (Gibbs, 2008) and the role of the body and human activities in the process of mathematical meaning making (Lakoff & Núñez, 2000).

Navigating an avatar such as Papert's turtle on the computer screen requires the formation of essentially novel methods of spatial orientation, where the reference point is not the position of the user's body but the turtle's body, relative to which the entire system of orientation may change. Body-syntonicity is thus a critical concept in doing mathematics with 2D Turtle Geometry (Papert, 1980) and refers to: (a) navigating the turtle by coordinating one's body posture, physically or imaginarily, with the turtle-vehicle of motion and (b) solving geometrical problems drawing upon personal embodied motional experiences. A critical feature of the turtle, which is closely associated to the idea of body-syntonic representation of mathematical concepts, is the 'play the turtle' technique which refers to the possibility of putting yourself in the place of turtle and recast turtle's trip on the floor (Papert, 2002).

Moreover, when working with a programmable medium such as Turtle Geometry, students have to reconceptualise geometrical figures in the engineering terms of sequences of specific programming commands (Abelson & diSessa, 1981; Kynigos, 1993). By creating geometrical figures through Logo programming, users have to develop and communicate mathematical meanings in spatially dynamic ways and thus to extend the notion of embodiment beyond the biological body demonstrating 'embodied' mathematics.

The intimate relationship between the functioning of the brain and body experience (with or without the use of tools) even when the most abstract mathematical notions are considered is now commonly recognised (Lakoff & Núñez, 2000). In parallel, new ways of associating mathematical knowledge with the sensorimotor system may arise (de Freitas, 2016) while using digital tools, for instance through the kinaesthetic engagement with the technology or through the 'temporalisation' of the behaviour of mathematical objects through dragging facilities, as in Dynamic Geometry Environments (Laborde et al., 2006). Recently, there is a clear research interest on the perceptions students have in 3D virtual environments, on the spatial dimensions of interactions through 3D avatars and on the way these technological advances occasion new ways of moving and thinking and thus of doing mathematics (Hollebrands et al., 2008).

In order to shed new light on the relationship between mathematical meanings and the material activity with the turtle in the virtual 3D space of Turtle Geometry, we have tried to combine and co-ordinate constructionism with a rather complementary theory, that of neo-materialism (Ng & Ferrara, 2020). Nowadays, embodied theories of learning compel renewed attention to the primacy of the experiencing body (Abrahamson & Lindgren, 2014; Nemirovsky, 2003), while neo-materialist approaches move beyond how individual actions inform emerging understandings and argue that there are no sharp boundaries between bodies of knowers and bodies of knowledge (de Freitas & Sinclair, 2014; Ferrara & Ferrari, 2017; Fox & Alldred, 2019). These approaches foreground the activity of the body and the physical aspects of mathematical meaning making, assembling learners, concepts and tools. An effort is made so as to rethink the relationship between the sensing subject and the sensed object as mutually constitutive. Mathematical objects partake of their material realisations, and concurrently the understanding of a mathematical concept comprises a material arrangement of things and relations.

Located in the physical world—through playing the turtle—or in the virtual environment of the 3D Turtle Geometry, the embodied students' acts can potentially evoke mathematical meanings within body-material assemblages. In this framework, gestures are thought as a special form of action or as a 'disciplined distribution of mobility', to use Châtelet's (2000) words, that offer students unique ways to engage in spatial reasoning. In mathematics education, gestures are conceived as a non-propositional and body-centric mode of thinking and as an interface between abstract and symbolic mathematics, and mathematical metaphors that are on their part grounded on human sensorimotor experience (Lakoff & Núñez, 2000). Research interest has also arisen in relation to the role of gestures while students are constructing geometrical figures in 2D and 3D Turtle Geometry computational environments (Morgan & Alshwaikh, 2010; Psycharis & Morgan, 2012). These studies provided empirical support for the embodied means used by students in their effort to carry out particular geometric tasks highlighting the connections of certain gestures with the mathematical knowledge integrated in such tools while raising questions about the interpretation and use of the same gestures by different groups of interlocutors.

However, the role of gestures in mathematics education has been analysed through various theoretical perspectives (McNeill, 2000; Radford, 2009; Maffia & Sabena, 2016; Alibali et al., 2014). In the present research, we are less interested in any sort of classification or thorough description of gestures but more in their implications on meaning making. The embodied aspects of students' construction processes are analysed not as mediational means between human mobility and abstract mathematical concepts but as mutually entailed and entangled with mathematical concepts.

Neo-materialist approaches might allow us to re-examine meaning-making processes in the framework of 3D Turtle Geometry environments not so much as learning trajectories towards a fixed abstract mathematical concept—here the concept of angle—but as human-technology assemblages that may comprise new forms of movement in virtual and real spaces and new forms of expression verbally or symbolically through mathematical notation and Logo code. In our research, the notion of assemblage (Deleuze & Parnet, 2007), which has been used extensively in various new materialist approaches, is emphasised so as to surpass dichotomies (such as concrete/abstract, body/mind, subject/object of activity, human/non-human actants) while foregrounding learners, concepts and tools not as mere components of a meaning-making process but in a constant productive interplay. In particular, we use the term mathematical assemblage to describe provisional dynamic physical arrangements that bring together corporeal bodies and incorporeal things (e.g. words, figures, ideas) on purpose and in order to construct mathematical meanings. Mathematical assemblages are conceived as contingent entities that hold characteristics and meanings not reducible to its parts while these characteristics and meanings are activated differently according to an assemblage's arrangement.

The meanings constructed by our 6th grade students are conceived and analysed as assemblages involving gestures, embodied metaphors, viewpoints of the 3D simulated space as well as dynamic manipulation of 2d and 3d geometrical figures. In particular, in this research, we focus on: (a) how the material mobility of human body, as it is realised by gestures and Logo turn commands, came to produce meanings about the dynamic aspects of angle and (b) how the moving entity metaphor, the dynamic manipulation of variables' values and the various viewpoints of the simulated 3D space were assembled during meaning construction processes concerning angle and 3D figures' geometrical properties.

The Computational Environment

MachineLab Turtlesphere (MaLT2, http://etl.ppp.uoa.gr/malt2) is a freely available on-line Logo Programming environment based on Brian Harvey's Berkeley Logo (Harvey, 1997; Kynigos & Grizioti, 2018). It embodies Turtle Geometry in 3D space by employing two new primitives, 'roll' and 'pitch', which turn the turtle on two planes perpendicular to the traditional one on the 2D version (Reggini, 1985). These are executed by the commands 'uppitch/downpitch n degrees', which pitches the turtle's nose up and down, and 'leftroll/rightroll n degrees', which moves the turtle around its trunk/vertical axis. However, the distinct feature of MaLT2 is that Logo-based Turtle Geometry is integrated with variation tools affording the dynamic manipulation of graphical representations—resembling a DGE-like dynamic behaviour—by means of equivalent manipulation of variable procedure values (Kynigos et al., 1997). In particular, the dynamic manipulation tools available can be divided in two categories:

- Dynamic manipulation of graphical figures by means of sequentially changing the variable values of the programs that create them through the use of specially designed variation tools (Fig. 1a).
- Dynamic manipulation of the viewpoint of the 3D space without interfering with the code and the respective figure: (a) by using toggle buttons where the user can pick among 3 default views (front, side, top-down) (Fig. 2), (b) by manipulating through mouse a specially designed vector tool, called the active vector, where the user can define camera's direction or position (Fig. 1b).

Methodology

As our research questions warranted an interpretive approach in educational research, we adopted a design-based research method. Design-based research entails the 'engineering' of tools and task, as well as the systematic study of both

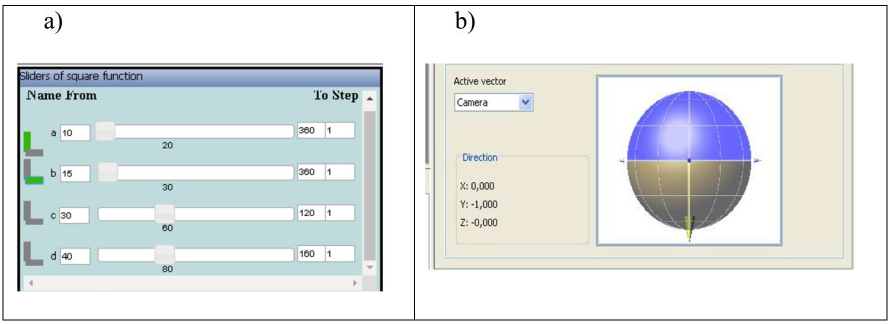


Fig. 1 **a** The one-dimension variation tool on the left and **b** the active vector tool on the right

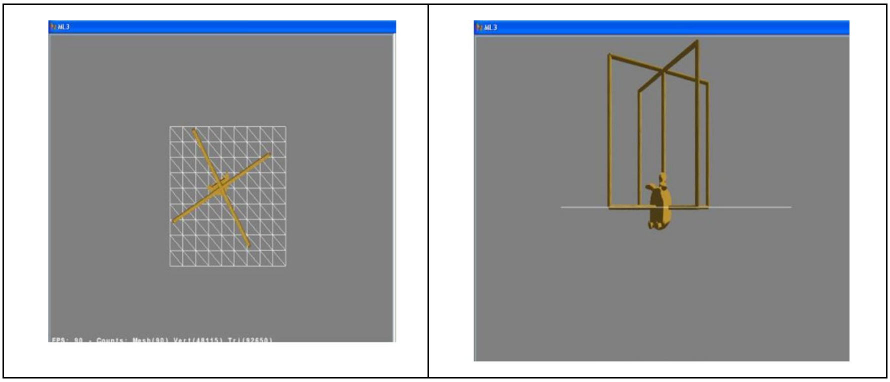


Fig. 2 Top-down and side view

the process of learning and the means of supporting it (Gravemeijer & Cobb, 2006) in order to create new theoretically expressed understandings about areas for which little is known. In this paper, we present the design of tools and tasks, and we focus on the analysis of some aspects of their implementation in real classroom context according to the purpose of our research. It should be mentioned that the design of tools and tasks presented here came as a result of a prior pilot design phase, teaching experiment and initial retrospective analysis.

The research took place in the 6th grade of a public primary school in Greece. The class consisted of 23 pupils, who had totally sixteen 45-min teaching sessions. The pupils did not have any previous experience with 3D Turtle Geometry environments, but they were accustomed to programming with 2D Turtle Geometry. They worked collaboratively in mixed-gender groups of two or three in the school's computer laboratory in the framework of 'Flexible Zone'. 'Flexible Zone' is an innovative educational program that gives teachers in Greece the means (officially allocated time, resources, etc.) in order to study subjects that are up to date and stem from teachers' and students' personal interests. The first author acted both as a researcher and as an educational practitioner.

The activity sequence comprised four tasks with accompanying activity sheets desiged in order to engage students in activities foregrouding angle both as a directed turn in the simulated 3D space and as a measure represented by a number. In task 1, pupils had to navigate the turtle in such a way so as to simulate the take-off and the landing of an aircraft. In task 2, pupils had to construct rectangles in at least two different planes of the graphical space of MaLT2 simulating the adjacent walls of a virtual room. In task 3 and 4, pupils experimented with two 'half-baked' microworlds (Fig. 3), i.e. didactically engineered incomplete digital artefacts that students had to investigate how they work and to change and fix them (Kynigos, 2007).

In particular, pupils had to use the one-dimension variation tool to control and experiment with the variables of the two half-baked microworlds—'movedoor' (task 3, Fig. 3a) and 'revolving door' (task 4, Fig. 3b)—that corresponded to different turtle turns, so as to create the simulation of a door opening and closing, and the simulation of a revolving door respectively. Both procedures were deliberately designed to include more variables than are necessary, and pupils had to amend them so as to redevelop the procedures with the least possible variables. Finally, pupils had to extend the procedure of the revolving door in order to create a simulation of the fan of a watermill.

For data collection, we adopted a participant observation method while the main corpus of data included video-recorded observational data (two focus groups were videotaped, while a camera was videotaping the whole class), the experimenting teacher's observational notes as well as the sorting and archiving of the corpus of the students' work on and off computer. As far as students' work on the computer is concerned, we used a specially designed screen capture software—called Hypercam—which allowed us to record students' voices and at the same time to capture all their actions on the screen.

Trying to attend to the full range of the materialities (everyday language, gestures, visual images, instances of students' symbolic work on and off computer, etc.) used by students in the meaning-making process, we followed a multimodal data transcription (Kress et al., 2001). In parallel to speech transcription, in an extra column, we recorded: (a) video-copied instances of students' actions and interactions (e.g. Figures 5 and 6), (b) instances of students' work on and off computer (e.g. Figures 7 and 8) and (c) extracts of researcher's notes. The selection of these instances was inevitably a product of an interpretative process between us and the various modes of data, apart from speech. The 'multimodal episode' was then used as the unit of analysis. The multimodal data were divided in episodes that constituted 'easily discernible parts of children's actions and interactions with a clear focus point' (Noss & Hoyles, 1996, p. 148). We proceeded data analysis through open and axial

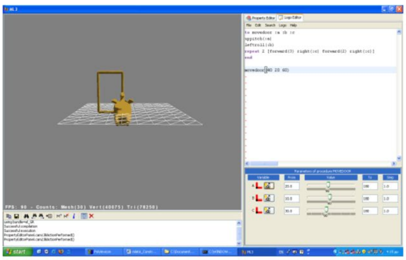
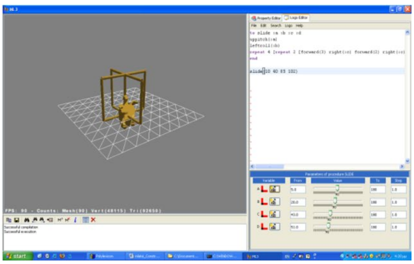
a)	b)
to movedoor :a :b :c uppitch(:a) leftroll(:b) repeat 2 [forward(7) right(:c) forward(4) right(:c)] end 	to revolving door :a :b :c :d uppitch(:a) leftroll(:b) repeat 4 [repeat 2 [forward(7) right(:c) forward(4) rt(:c)] leftroll(:d)] end 

Fig. 3 **a** The Logo code of the 'Movedoor' and **b** the 'Revolving door' half-baked microworlds

coding (Strauss and Corbin, 1994), trying to categorise multimodal episodes and to interconnect the themes that had arisen having as a central theme the construction of angles in the simulated 3D space.

Data Analysis and Discussion

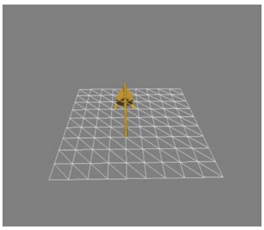
The analysis of our results is organised around three main categories that reflect critical themes in students' construction processes of angles in the simulated 3D space. Initially, students were rather concerned with turning the turtle-agent in the simulated 3D space while body-syntonicity with the turtle was the main point of focus. Then, they progressively focused more on the graphical results of specific turtle's turns, while finally angle was rather used as a dynamic property of 3D shapes.

Turning the Turtle in the Simulated 3D Space: Focusing on Body-Syntonicity

Students seemed to have initially tried to get 'immersed' in the virtual 3D space, explore it and navigate it though the turtle/agent, a perspective that could be characterised as intrinsic (Tversky, 2005). They focused on angle as a directed turn trying to syntonise their embodied motional experiences with turtle's motion as it was evident: (a) from the view of the 3D space preferred and the kind of turn commands used and (b) from students' gestures.

During task 1, students tried to imitate the take-off and landing of an aircraft using turn and forward commands. They executed each Logo command, and they watched its graphical representation on the computer screen before typing the next one. In particular, they used *the uppitch/downpitch* set of turn commands which had the graphical result they wanted and they opted for the default front view (Fig. 4).

The preferred viewpoint, the set of turn commands used and the way they were executed (line by line) could be explained by students' efforts to coordinate their body posture physically and imaginatively with the turtle-vehicle of motion, drawing upon their earthly embodied motional experience. Flying the turtle along the Z axis students' body posture and orientation in the lived-in 3D space coincided with the orientation of the moving entity in the simulated 3D space in the computer



Uppitch(45)
Forward(2)
Downpitch(45)
Forward(2)
Downpitch(45)
Forward(2)
Uppitch(45)

Fig. 4 Simulating the take-off and landing of an aircraft along the Z axis and the respective Logo code screen. In the preferred front view, the use of certain turn commands, such as *right* and *left*, coincided also with the standard way of referring to things presented in the computer screen, where the right/left and up/down directions are fixed. Moreover, this way, the *uppitch/downpitch* commands could be more easily coordinated with students' earthly experience were the 'up' and 'down' direction are fixed, as a result of the gravitational effect (see also Latsi & Kynigos, 2012).

However, other relative research have shown that students working in a 3D simulated space usually prefer to navigate the turtle in a 2D plane parallel to the computer screen, so as to draw upon their experiences with 2D figures in school textbooks or in 2D Turtle Geometry environments (Kynigos & Latsi, 2007). This contradiction in findings could rather be explained by the kind of task and the metaphor used: students' aim was not to construct just a slanted line but to simulate the take-off and the landing of the turtle—aircraft, and this had influence both on the conception of the simulated space and on the kind of turn commands used. Using the *uppitch/downpitch* set of turn commands and flying the turtle along the Z axis, that gave the impression of depth, were rather more easily coordinated with representations of flying aircrafts.

However, it seems that the 'play the turtle' metaphor (imitating turtle's motion with human body in real space) cannot be realised physically using the full body as far as the 3D Turtle Geometry environments are concerned. In these environments, the turtle moves in all 3 dimensions without any restriction, while in real 3D space the human body can move only in a 2D horizontal plane. As a result, students did not use full-body motion but used gestures as a spatial form of action to link lived experiences in real space with the embodied turtle metaphor and the symbolic expression in Logo code. In the following episode (Fig. 5), students are trying to decide how to carry on the turtle's journey (task 1) using gestures. They are using their hands so as to represent the 3D entity and its orientation (that is why the gestures are characterised as representational) as well as its motion (that is why the gestures are characterised as dynamic). The use of the hand seems to contribute to the

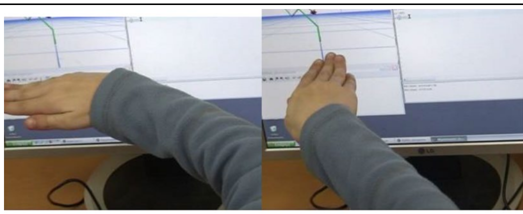
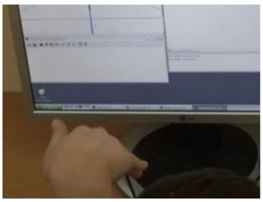
S1: Now, do you know what we should do? As it is like that, to turn it this way and to move it forward.	
S2: Not to move it down a bit?	

Fig. 5 Dynamic representational gestures—episode 1

enactment of a series of successive spatial representations before these representations are systematically articulated either verbally in everyday language or symbolically through Logo code. Although characterised as representational, these gestures should not be conceived as fixed signs existing prior to, or independent of, their realisation by students in the context of specific activities. Rather, these representational gestures constitute the material traces of a spatio-temporal re-imaging of the intangible visual turtle's trip.

In this research, hand was not used only representationally but also deictically to indicate turtle's direction in the 3D space. In the following episode (Fig. 6), students are trying to decide how many degrees the turtle should turn so as to take the intended position and direction. As the focus point is not the direction of the turn but its degrees, the hand is rather used as an indication of the various positions of the turtle for certain angular turns in the 3D space and in particular for the left turn of 45°, of 90° and of 135°. It should be stressed that the students' hands are not used deictically as far as certain concrete objects or attributes of the context of the activity are concerned. The gestures used in this episode integrate abstract deictic characteristics that call for mathematical interpretation and imply a metaphoric use of the real space, where certain angular measures have acquired spatial properties.

In the episodes presented above (Figs. 5 and 6), it seems that students' constructionist activity, in the framework of certain tasks in the 3D Turtle Geometry environment, foregrounded the notion of angle as a directed turn in the simulated 3D space and as a measure represented by a number. The analysis shows that the meanings constructed by students could be approached as mathematical assemblages comprising: Logo turn commands, the turtle metaphor, the graphical representation of turtle's trip on the computer screen and embodied enactments of angle as turn through gestures.

However, the following two sections show how students progressively focused more on the graphical results of specific turtle's turns and less on body-syntonicity and how this change of focus was related to changes (a) in the preferred viewpoint

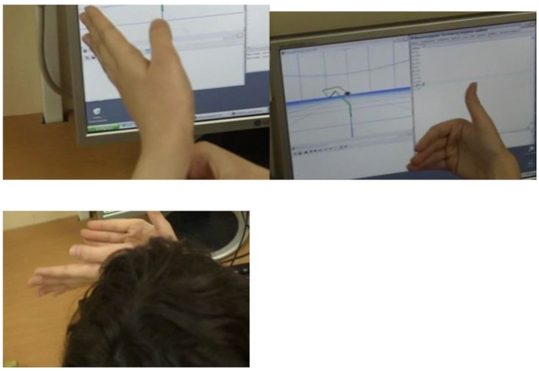
S1: Fine. We will turn it. Wait, it is like that. So half of it, approximately 45, 90, approximately at 135 …	

Fig. 6 Abstract deictic gestures—episode 2

of the simulated space and (b) in the construction strategies followed alongside changes in the kind of gestures used.

Constructing Dihedral Angles: Using Multiple Viewpoints and Focusing on the Intrinsic Characteristics of Turtle Geometry

While trying to construct dihedral angles in the simulated 3D space, students used extensively the various viewpoint manipulation tools. Drawing mainly upon their embodied navigational experience and based upon visual cues, rather than upon objects geometrical properties, they tried to construct 3d objects by navigating the turtle through a step-by-step procedure. They did this by (a) inserting and immediately executing each Logo command, (b) observing the graphical results on the screen through various viewpoints and (c) deciding about the Logo command to be given next. The following episode (Fig. 7) is indicative:

In the above episode (Fig. 7), the two students are trying to construct the 'wall' of a room (task 2) following a trial and error method. Based only on the graphical results of each command's execution, they proceed tentatively command by command. It is indicative that they have entered sixteen lines of commands (8 of which are turn commands) in order to construct the 3 sides of a parallelogram. When it is not visually clear if a closed figure is constructed, the students resort to the use of the various viewpoint manipulation tools and not to the figure's geometrical

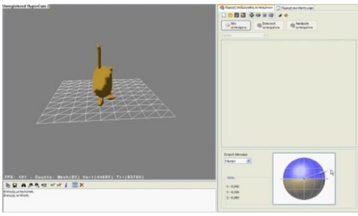
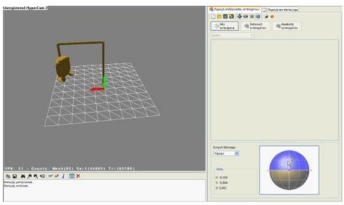
S1: Fd, more Fd, more…we should forward the turtle 0.5. Does it have 0.5? S2: Wait, wait the turtle is here. S1: Yes, but it hasn't touch the ground. Has it? Wait. I have to see it.	 	Up(180) Dp(45) Dp(45) Fd(4) Rt(90) Rt(90) Rt(90) Rt(90) Rt(90) Fd(4) Bk(1) Bk(1) Bk(1) Fd(1) Fd(1) Fd(1)

Fig. 7 Episode 3—First column: Students' dialogue. Second column: Changing viewpoints. Third column: The respective Logo code

properties (e.g. that the opposite sides of the rectangular figure should have equal lengths). These bricolage construction strategies followed by pupils could not be attributed only to personal styles in programming (Turkle & Papert, 1990) or to their Logo inexperience (Kafai, 1995), but rather to an emphasis on the 'embodied' spatial activity of the 3D moving entity. It seems that programming through the turtle metaphor in the 3D simulated space promoted initially a step-by-step construction that is rather strongly related to the intrinsic characteristics of Turtle Geometry (Abelson & diSessa, 1981), where a given geometrical state of the turtle is fully defined by its relation only to the turtle's immediately previous state (see also Latsi & Kynigos, 2012).

The multiple views were not only used when students' focus was on the turtle's navigation but also when they were focusing on the graphical results of specific turn commands. For instance, during task 4 students had extra difficulties in finding out the role of variable 'd'. In the following episode (Fig. 8), students do not find the default front view convenient, they test all of the available default views and they choose to continue working with the top-down view active, where the number of doors/rectangles created by the turtle is more clearly visible.

Through the default views, students came into contact with simplified 2D views of the simulated 3D space, which possibly helped them: (a) focus on specific aspects of their construction as a result of turning commands; and (b) more easily discern dihedral angles, as they looked more like the 2d geometrical figures students were accustomed to.

In parallel when the students' focus point was on the represented 3D graphical objects, the gestures used were initially static representational ones. They are characterised as static representational as they constituted static instances and as they had a degree of resemblance (although idiosyncratic and not yet systematised) with the graphical objects (Fig. 9).

When trying to match real to corresponding virtual 3D objects during task 2 (Fig. 9), students are using firstly the one hand to represent the position and direction of the one wall/plane. Then, the intended figure and the spatial relationship between the two walls/planes are represented through the use of both hands. These representational gestures are not used in order to represent existing mathematical knowledge

S1: Let's see how many doors there are if the value is only one? This perspective is not convenient, I will change it.

S2: Yes, exactly like in the case of 360.

S1: Yes, it collects all of them in one. When we move it, the doors are changing position. They are sticking together or they are unsticking.

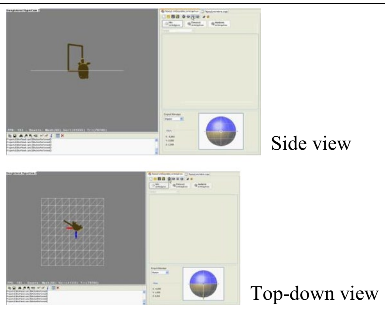


Fig. 8 Episode 4

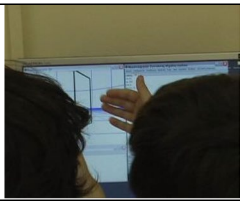
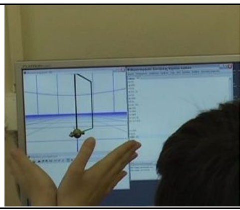
S1: Now as it looks this way, let's make the one wall like that.	
S2: Yes, that's better.	
S1: Otherwise we can do it this way.	

Fig. 9 Static representational gestures—episode 5

but rather to 'structure' real space and enact both the visible and the intended geometrical figures changing the relationship between the virtual and the actual.

Constructing dihedral angles in the 3D Turtle Geometry environment, students generated meanings about angle as a geometric shape in a process of assembling Logo turn commands, turtle's virtual trip and multiple views of the constructed graphical objects as well as embodied enactments of these graphical representations in real space through gestures. The next section provides the analysis of how students progressively reconceptualised 3D space in terms of Logo turn commands and opted for an extrinsic perspective of the simulated space so as to manipulate and construct 3D objects according to their geometrical properties.

Angle as a Dynamic Property of 3D Shapes and the 'Holistic' 3D View of an External Observer

During tasks 3 and 4, the simulated 3D object's position and orientation could be dynamically arranged through the combined use of Logo programming and of the variation tool. The sequential change of the values of variables 'a' and 'b', in both respective procedures, created a film-like succession of the different instances of the 3D model and gave the impression of rotation. In the following episode (Fig. 10), students are trying to change the orientation of the revolving door model in relation to the ground plane so as to create the fan of a watermill. They can easily discern that it is the same construction with a different orientation in 3D space (*we should lie it down*) and they start experimenting with variable 'a' through the variation tool so as to achieve this change of orientation. It should be also noticed that throughout their experimentation students are using a fixed 3D view.

Changing the values of variable 'a' sequentially, they observe the turtle and the whole 3D construction to roll around x-axis and to have the desired orientation in

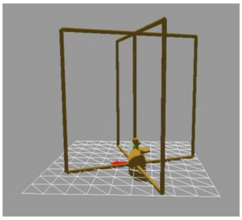
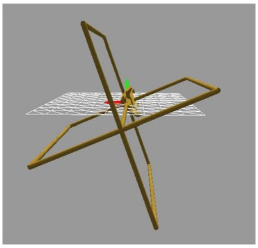
S2: Now we should lie it down. *(First print screen on the right)* S1: It's the same as before. S2: Yes, it's like viewing it from the top. *(Second print screen on the right)* S1: Move variable 'a'…yes 360, it makes a whole circle and it lies down. S2: or 720, 1080 etc.		

Fig. 10 Episode 6

3D space for various multiples of 360°. It seems that dragging the variation tool fostered experimentation and rendered the various turn commands descriptors of the evolving geometrical objects' place and orientation. Students observe these changes in geometrical objects' place and orientation through the more 'holistic' view of an external observer who views the whole 3D construction from a stable point while not trying to syntonise his position with that of the turtle's.

Adopting the 'extrinsic' perspective of an external observer (Tversky, 2005) was not only evidenced across tasks but also within tasks. At the end of Task 2, a group of students spontaneously decided to try to construct a closed figure building upon their previous experimentation (Fig. 11). The students decided to reuse the 'flight' commands (see code in Fig. 4). Each take-off and landing of the turtle was used as the building block of a 'peculiar' figure that came as result of four repeats of the initial turtle's journey, turning the turtle 90° before each re-execution. Students described it as a kind of square where each take-off/landing of the turtle corresponded to each one of its sides. It is also interesting that the students adopted a more analytic programming strategy, visualising the whole turtle's journey and explaining it to each other before entering commands in the Logo editor. Moreover, they adjusted the view of the 3D space through the active variation tool so as to have a clear 3D view of the simulated space and they kept it fixed throughout their construction.

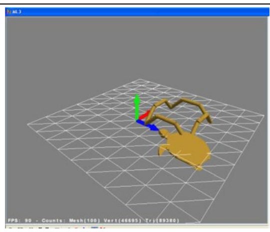
S1: Let's make a square. So not right 45 but right 90, so as to go this way and then again 90. S2: Yes, and again 90 and we will come back.		Uppitch(45) Forward(2) Downpitch(45) Forward(2) Downpitch(45) Forward(2) Uppitch(45) Right(90)

Fig. 11 Episode 7

Working with a fixed view during this kind of construction rather required a relatively high degree of abstraction with respect to spatial visualisation and orientation. However, constructing the simulation of a 3D object, while viewing the simulated space in perspective, allowed students to reconceptualise 3D objects in terms of Logo commands, while paying attention not only to turtle's immediately previous state but also to the whole 3D simulated space and to the geometrical properties of the figure. In the above episode (Fig. 11), students executed the command right (90) four times so as to *come back*, to use students' words. Thus, having a fixed 3D view, students seem to have intuitively articulated the total turn trip theorem (Papert, 1980) in the simulated 3D space: walking all the way round a polygon and returning to its initial position with the same orientation, the turtle makes a full turn.

It seems that as students got progressively more accustomed to the 3D turtle's motion and the software's representational infrastructure, they were not so much concerned about body-syntonicity. It was more important to them to have a clear sense of the three dimensionality both of the simulated space and of the simulated objects. This way they would rather more easily co-ordinate the two prevalent but different viewpoints: the viewpoint of the turtle which must be moved in an appropriate way so as to draw a figure and the viewpoint of an external observer who looks at the figural results of turtle's movement.

As children were gradually changing focus from static 3D representations to constructing geometrical figures through Logo programming, they again used dynamic representational gestures. However, this kind of gestures was not used only to re-image the turtle's journey but also to enact the 3D object as a result of turtle's motion. In the following episode (Fig. 12), the two students are discussing how they should construct a staircase in the 3D simulated space of MaLT2. It was a construction that was carried out spontaneously by a group of students at the end of the Task 2.

Initially, student 2 suggests turning the turtle *uppitch* 90° while representing this motion with his hand. The other student, having focused not on the turtle's navigation but on the staircase's inclination in relation to the horizontal level, corrects the former showing with his hand the inclination that the staircase should have, which in any case should not be vertical to the horizontal plane. Student 2 reacts by asking if they should do the staircase straight, understanding the other student's gesture as a straight inclined line. Then, he represents both the turtle's motion and the staircase moving his both hands. The hands side by side enact the horizontal and vertical planes of the staircase, while the hands' motion seems to enact the turtle's motion in the simulated 3D space. In parallel student 2 tries to translate verbally in Logo code the turtle's motion trying to find out the right turn command.

These gestures captured mobility and created new dimensions and structures in the 3D space around the computer screen. As students brought into being shapes of the screen in full sensuous inventiveness, a virtual gesture space was created in front of them, where the various represented mathematical objects were placed, processed and interconnected. This virtual gesture space rather fostered imagery and helped students focus on the intended images' structure (e.g. actualising with the hands the various horizontal and vertical planes and their angular relationship) rather than on accuracy (e.g. the exact degree of turtle's turn) while coordinating turtle's motion with the gradual depiction of the resulting geometrical object.

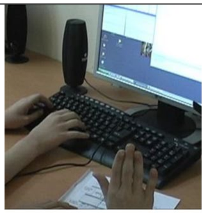
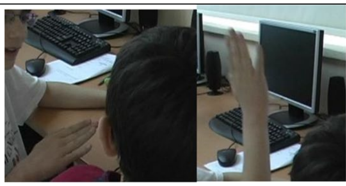
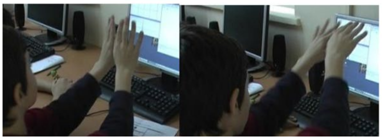
S1: Up (90). From this to go this way.	
S2: The staircase should rather be this way and not that way.	
S1: What? Will we make it straight?	
S2: Of course.	
S1: It should move this way. Then it should turn. … rt, no. No, there is something else, how is it called?	
This way and then this way.	

Fig. 12 Dynamic representational gestures—episode 8

The episodes presented and analysed in this section show how the meanings constructed about angle as a dynamic property of 3d geometrical objects and as a spatial visualisation concept came out as assemblages of: (a) Logo turn commands, (b) specific views of the simulated space and of the geometrical objects represented on the computer screen, (c) dynamic manipulation of geometrical objects and (d) material enactments of the construction process of the geometrical objects through gestures (see also Fig. 13).

Conclusions

In this research, we have tried to shed more light on students' constructionist activity in a 3D Turtle Geometry Environment analysing its material and embodied dimensions (an aspect that is rather neglected by constructionism) through a

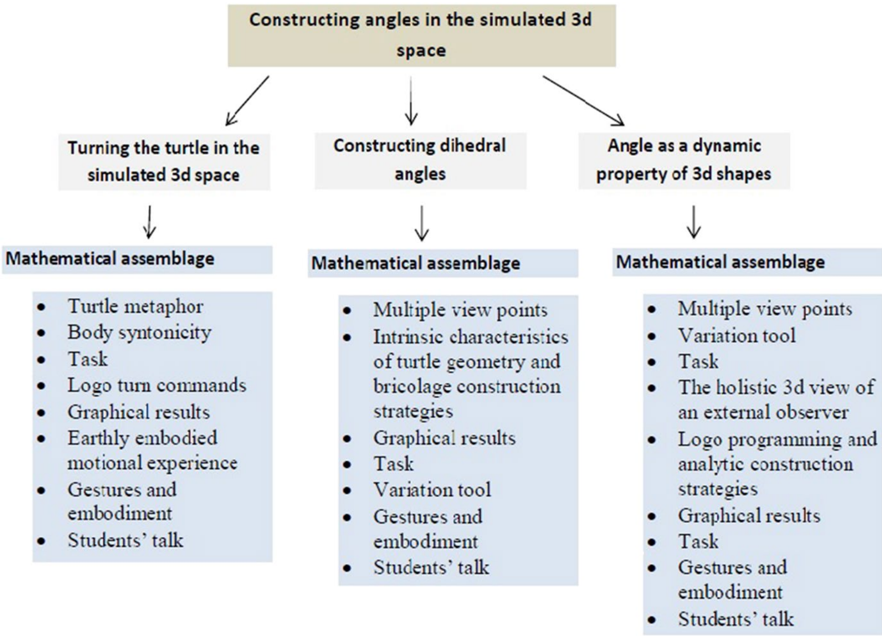


Fig. 13 Main categories and main parts of each mathematical assemblage

neo-materialist lens (Ng & Ferrara, 2020; Thompson, 2020; Fox & Alldred, 2019; de Freitas & Sinclair, 2014). The processes of students' meaning construction about the concept of angle were analysed as part of a growing indeterminate assemblage where students' (mathematical) activity entailed a constant intra- and interplay between gestures, embodied metaphors and enactments, navigation and viewpoints, 2D and 3D geometrical figures as well as ways of verbal and symbolic expression.

The gestural mobility in the lived-in space and the virtual mobility on the computer screen (either of the turtle/agent or of 2D and 3D geometrical figures) were pivotal sources of mathematical meanings, while at the same time these meanings could not be separated from their material realisations. The didactical notion of building upon intuitive embodied metaphors of locomotion (Clements & Sarama, 1997; Papert, 2002) changes in 3D virtual spaces, taking into account that our earthly experiences are confined in a two-dimensional (ground) plane and rather evolve from the interaction between body and culture.

The role of embodiment, mainly through the embodied form of gestures, was crucial in turning the turtle and experiencing the simulated 3D space both in an intrinsic and in an extrinsic perspective (Tversky, 2005). In an intrinsic perspective, body-syntonicity with the turtle-vehicle of motion was the central focus of interest. The viewpoints chosen were those that helped students get immersed in the 3D space and syntonise their body with the turtle, while the gestures used were mainly those enacting the turtle's trip in real space (dynamic representational gestures). Drawing mainly upon their embodied navigational experience and based upon visual cues, students tried initially to construct 3D objects navigating the turtle through a step-by-step procedure while using multiple viewpoints. This intrinsic perspective of navigating the turtle in the simulated 3D space was rather associated with an emphasis on the intrinsic characteristics of Turtle Geometry (Abelson & diSessa, 1981).

As students gradually changed focus from managing turtle's spatial movements through body-syntonicity to constructing 3D figures according to their geometrical properties, within and across tasks, they experienced virtual space through an extrinsic perspective, as external observers who looked at the figural results of turtle's motion from a fixed 3D point of view. Such a stable 3D view rather supported a holistic perception of geometrical objects while offering space and shape constancy (Wickens et al., 2005). Moreover, a holistic/external view of the 3D space was in accordance with analytic construction strategies where pupils were trying to visualise the whole turtle's journey taking into account the graphical objects' geometrical properties before executing sets of commands. In this case the gestures used instantiated not the turtle's trip but the intended figure. Initially, the intended figure was instantiated pictorially through static representational gestures, while gradually the intended figure was denoted both through actions, and as a result of them.

When navigating the turtle in the simulated 3D space, students infused spatial meaning (Davis, 2015) to a geometrical concept—that of angle as a directed turn. By foregrounding the relationship of geometry to the experienced and virtual spaces, angle was rather made more meaningful (Freudenthal, 1983). As students got more accustomed to 3D turtle's motion and the software's representational infrastructure, they used angle both as a directed turn and as a measure represented by a number. They also used angle not only in order to navigate the turtle and explore the simulated 3D space, but also as, a dynamic property of 3D shapes and as a descriptor of geometrical objects' place and orientation in 3D space.

Human-technology assemblages, such as the one presented in this research, may give rise to new ways of 'moving' in real and virtual spaces and thus of constructing meaning about geometrical concepts, such as angle. This would have consequences on the way we design mathematical activities and curriculum sequences. However, much of further research is needed in order to investigate the way in which mathematical concepts are entangled with spatial thinking and reasoning along with embodied metaphors of locomotion (Abrahamson & Lindgren, 2014; Shapiro, 2014; De Freitas & Sinclair, 2014; Lakoff & Núñez, 2000; Gibbs, 2008) in virtual environments.

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