International Journal of Science and Mathematics Education (2022) 20:1141–1160
https://doi.org/10.1007/s10763-021-10204-y

Predicting Pre-service Mathematics Teachers' Teaching and Learning Conceptions: The Role of Mathematical Beliefs, Mathematics Self-efficacy, and Mathematics Teaching Efficacy

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Received: 8 August 2020 / Accepted: 17 July 2021 / Published online: 13 August 2021

Abstract

Pre-service mathematics teachers (PSTs) enter university with diverse beliefs and understandings of teaching and learning; yet, they may not be aware of how these conceptions are related to their epistemological and efficacy beliefs. This study explored how the mathematical beliefs, mathematics self-efficacy, and mathematics teaching efficacy of PSTs predict their teaching and learning conceptions. Participants were 80 PSTs (59.7% men and 40.3% women) who were studying in a 5-year full-time undergraduate degree program at a comprehensive research university in Hong Kong. Multiple regression analyses were performed to investigate the relationships between the PSTs' teaching and learning conceptions and various predictors. The results indicated that mathematical beliefs and personal mathematics teaching efficacy, a component of mathematics teaching efficacy, predicted traditional conceptions. Mathematics self-efficacy and mathematics teaching outcome expectancy, another component of mathematics teaching efficacy, predicted constructivist conceptions. This study yields insights into the different roles of mathematical beliefs, mathematics self-efficacy, and mathematics teaching efficacy in explaining and predicting the teaching and learning conceptions among PSTs, which demonstrates the potential to inform and improve PSTs' professional learning and development.

Keywords Pre-service mathematics teachers · Teaching and learning conceptions · Mathematical beliefs · Mathematics self-efficacy · Mathematics teaching efficacy

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Introduction

Teaching is an enormously complex lifetime undertaking, and during the process of becoming a teacher, teaching and learning conceptions play crucial roles in the development of teachers' professional competence (Blömeke et al., 2008). Empirical studies have provided evidence that teaching and learning conceptions are associated with teachers' curriculum fidelity (Baş & Şentürk, 2019) and instructional practices (Yang et al., 2020). Teaching and learning conceptions often refer to teachers' beliefs about their favored teaching and learning methods, and two such conceptions are common in the literature: traditional and constructivist (Chan & Elliott, 2004). In traditional conceptions, teaching is viewed as the transfer of knowledge from teachers to students, and learning is viewed as the mastery of knowledge acquired through the teaching process. In constructivist conceptions, teaching is regarded as a process in which teachers act as the learning facilitator, and learning is regarded as the construction of knowledge by students through inquiry and investigation.

Teachers are influential agents in the educational system. In schools, teachers drive classroom and school improvement, which collectively lead to student engagement and learning as well as leadership and mobilization (Fullan et al., 1990). Teachers are generally educated in three stages: initial teacher education, induction for beginning teachers, and in-service teacher education. Teacher education aims to improve teachers' beliefs, knowledge, and practices to develop students' affective and cognitive skills (Krainer & Llinares, 2010). Pre-service teachers have existing beliefs regarding different subject areas and develop beliefs on teaching and learning as they receive formal professional training at universities. These beliefs may be incompatible with those advocated in the curriculum guides. Although misaligned beliefs may be changed as a consequence of learning, some pre-service teachers still hold stable beliefs upon graduation from their teacher education programs. These future teachers will make teaching decisions based on beliefs formed before entering university (Vesga-Bravo et al., 2021). For instance, a pre-service teacher may learn by rote and excel in examinations. If this traditional learning belief remains unchallenged, it is likely to be passed on to the next generation of students and thus impede their conceptual understanding of subject matter and cognitive development.

Research has established that the epistemological and efficacy beliefs of pre-service teachers considerably influence their teaching and learning conceptions (Chan & Elliott, 2004); however, much less is known about such an influence in different disciplines. In mathematics, although studies have investigated the relationships among the mathematics teaching efficacy, mathematics self-efficacy, and mathematical beliefs of pre-service mathematics teachers (PSTs) (Briley, 2012), little research has been conducted on the combined effects of these variables on the teaching and learning conceptions of PSTs. Because of the close relationships between teachers' beliefs about mathematics and its teaching and learning and their instructional practices, changing these mathematics-related beliefs to inquiry-oriented beliefs is crucial for improving mathematics teaching quality (Potari, 2020). This study aimed to reveal the complex relationships between the teaching and learning conceptions of PSTs and various belief factors and was guided by the following research question: what is the role of mathematical beliefs, mathematics self-efficacy, and mathematics teaching efficacy in predicting the teaching and learning conceptions of PSTs?

Effects of Mathematical Beliefs on Teaching and Learning Conceptions

Mathematical beliefs, as domain-specific epistemological beliefs, are concerned with the nature of mathematics. Extensive research has been conducted on these beliefs since the 1990s. Different conceptualizations have been proposed over the years. Collier (1972) suggested two categories of mathematical beliefs: formal mathematics based on fixed and established forms and informal mathematics with original and creative elements. Ernest (1989) proposed three categories of mathematical beliefs: mathematics as a collection of facts, rules, and skills used to achieve certain goals (instrumentalist view); mathematics as a static but unified body of knowledge with interrelated structures and truths for discovery (Platonist view); and mathematics as a constantly evolving field of human invention subject to revision (problem-solving view). Stipek et al. (2001) introduced two categories similar to those of Collier (1972): traditional mathematics and inquiry-oriented mathematics. Individuals with traditional mathematics view tend to regard mathematics as a static body of knowledge that involves using a set of rules and procedures to obtain the correct answer to a problem. Those with inquiry-oriented mathematics view conceptualize mathematics as a continually changing discipline that provides a tool for problem solving and cultural understanding. To reconcile the various conceptualizations of mathematical beliefs, Roscoe and Sriraman (2011) concluded that the formal mathematics view, instrumentalist view, and traditional mathematics view tend to be dualistic. By contrast, the informal mathematics view, problem-solving view, and inquiry-oriented mathematics view tend to be relativistic.

Beswick (2005) described the conceptual relationships among beliefs regarding the nature of mathematics, mathematics teaching, and mathematics learning. For example, teachers with the instrumentalist view focus on content and emphasize students' academic performance. Teachers with the Platonist view focus on content and emphasize students' understanding of concepts and processes. Teachers with the problem-solving view adopt the constructivist approach of mathematics teaching and learning, which involves learners' construction of meaning from experience by doing mathematics. Tang and Hsieh (2014) surveyed future lower-secondary mathematics teachers from 15 countries and found that they had an extremely strong belief in the open and creative aspect of mathematics, as well as student initiative in studying mathematics.

Effects of Mathematics Self-efficacy on Teaching and Learning Conceptions

Self-efficacy is an important motivational construct that affects human behavior and performance. According to Bandura (1977), it encompasses two components: efficacy expectations and outcome expectations. Efficacy expectations refer to an individual's conviction of his or her own competence to behave successfully, whereas outcome expectations refer to an individual's prediction that a behavior will produce certain outcomes. These two types of expectations are distinct because some actions are likely to produce specific outcomes; yet, some individuals lack the ability to complete the actions. Bandura (1986) further defined self-efficacy as one's assessment of his or her own ability to carry out plans of action to achieve the expected performance; thus, self-efficacy concerns the evaluation of one's ability to complete a task more than the skills one possesses. Therefore, mathematics self-efficacy can be understood as one's beliefs in one's capacity to do mathematics and is generally measured by one's own evaluation of one's capability to answer certain mathematics problems, complete mathematics activities, and succeed in mathematics courses (Bates et al., 2011). PSTs utilize multiple sources such as past performance, vicarious experiences, verbal persuasions, and career goals to develop their mathematics self-efficacy and learning goals (Phelps, 2010).

Teachers who display high self-efficacy tend to work longer with struggling students, identify students' errors more easily, and explore new teaching methods more willingly to help students in need (Swackhamer et al., 2009). In teaching, teachers with high self-efficacy are more prone to use student-centered and constructivist instruction, whereas those with low self-efficacy are more prone to use teacher-centered and traditional instruction (Zee & Koomen, 2016). This is because teachers with high self-efficacy beliefs are more effective in managing classroom situations that require more spontaneous pedagogical actions than those that simply follow a strict and predictable learning trajectory.

Effects of Mathematics Teaching Efficacy on Teaching and Learning Conceptions

Teaching efficacy originates from self-efficacy theory. Similar to self-efficacy, there are two components in teaching efficacy: personal teaching efficacy and teaching outcome expectancy (Briley, 2012). Personal teaching efficacy is understood as a teacher's belief in his or her ability as an effective teacher, whereas teaching outcome expectancy is defined as a teacher's belief that effective teaching results in students' academic success, irrespective of the external factors. The belief in one's capacity to teach mathematics successfully is referred to as mathematics teaching efficacy (Giles et al., 2016), and accordingly it comprises personal mathematics teaching efficacy and mathematics teaching outcome expectancy. PSTs primarily develop their mathematics teaching efficacy through enactive mastery experiences, vicarious experiences, social persuasion, and physiological and emotional arousal (Thomson et al., 2020).

Generally, teachers who report high teaching efficacy would be more willing to employ constructivist instructional strategies rather than didactic instructional strategies (Nie et al., 2013). Swars et al. (2006) found that PSTs with high teaching efficacy valued learning experience incorporating problem solving, reasoning, and communication; this approach is considered relatively constructivist. Those with low teaching efficacy emphasized the memorization of mathematical knowledge and procedures; this approach is considered more traditional. Efficacious PSTs were confident that they could understand and deliver mathematics content much better, and less efficacious PSTs believed that they shared the same struggle as that of at-risk students. When mathematics teachers teach the topics that they are most confident in, they are more inclined to focus on understanding concepts and use constructivist teaching methods. For topics that they are less confident with, they prefer to focus on procedures and use traditional teaching methods (Kahle, 2008).

Theoretical Framework of the Study

This study employed Perry's theory of intellectual development (Perry, 1970) and Bandura's social cognitive theory (Bandura, 1986) to determine the effects of the mathematical beliefs, mathematics self-efficacy, and mathematics teaching efficacy of PSTs on their teaching and learning conceptions. Perry (1970) found that university students typically follow a trajectory through which they are transformed from dualist to relativist thinkers as they recognize uncertainty in knowledge. The author identified nine positions that can be grouped into four positions along the trajectory: dualism, multiplicity, relativism, and commitment within relativism. Students in the position of dualism believe that a clearly right or wrong answer exists for a question, and that they should learn to find the right answer. Students in the position of multiplicity believe that the right answer may not exist, and that all the ideas are equally important. Those in the position of relativism realize that not all the ideas are equally valid and their validity should be determined based on logic and context. Finally, those in the position of commitment within relativism integrate ideas from various sources with their experience, and evaluate the consequences and implications of their commitments. Although Perry's work was developed in the 1970s, subsequent research that has utilized his theory in various higher education settings has supported its validity (Crooks, 2017).

Ernest (1991) aligned the four aforementioned positions with mathematical beliefs at three levels. As per dualistic views, mathematics is regarded as fixed and exact structures that contain facts, rules, procedures, and truths determined by absolute authority. Doing mathematics entails following the established rules. Multiplistic views of mathematics allow multiple approaches to a problem, and all approaches are equally valued. A certain degree of creativity is possible when doing mathematics. Relativistic views of mathematics accept varieties in doing mathematics and the evaluation of these varieties based on the mathematical system or context under concern. Entwistle et al. (2000) theorized the relationships between epistemological levels of knowledge and conceptions of teaching, in which dualistic thinking and relativistic thinking are linked to the teacher-centered and student-centered conceptions of teaching, respectively. Dualistic views of mathematics are thus hypothesized to be positively associated with traditional conceptions but negatively associated with constructivist conceptions, and relativistic views of mathematics are hypothesized to be positively associated with constructivist conceptions but negatively associated with traditional conceptions (Beswick, 2005; Tang & Hsieh, 2014).

Social cognitive theory postulates that people learn in a social context of dynamic and reciprocal interactions among people, environment, and behavior (Bandura, 1986). An important goal of this theory is to explain how people manage their behaviors through control and reinforcement to achieve their goal-directed behaviors and maintain them over time. Self-efficacy, as a defining feature of the theory, influences how personal, environmental, and behavioral factors mutually interact. For example, through cognitive, motivational, affective, and selection processes (Bandura, 1977), self-efficacy affects the way of goal setting, the effort and perseverance required to fulfill the goals, and the ability required to cope with adversity. All these decisions have a considerable effect on subsequent beliefs and behaviors. According to the theory, mathematics self-efficacy is hypothesized to be positively associated with constructivist conceptions but negatively associated with traditional conceptions among PSTs (Zee & Koomen, 2016). This is also true for the relationships between mathematics teaching efficacy and teaching and learning conceptions (Kahle, 2008; Nie et al., 2013; Swars et al., 2006).

Method

Participants

The participants in this study were 80 PSTs (59.7% men and 40.3% women) who were studying in a 5-year full-time undergraduate degree program at a comprehensive research university in Hong Kong. They were 18 to 24 years old. PSTs comprised 16.7% Year 1 students, 12.5% Year 2 students, 22.2% Year 3 students, 18.1% Year 4 students, and 30.6% Year 5 students. On average, they completed 7.9 mathematics courses (standard deviation = 2.6), 3.3 mathematics pedagogy courses (standard deviation = 2.6), and 3.8 education courses (standard deviation = 1.2). Over half of them (55.6%) did not commence teaching practicum in schools, whereas 25.0% completed teaching practicum in primary schools. The remaining 19.4% of PSTs completed teaching practicum in both primary and secondary schools. PSTs who graduate from this double-major program in mathematics and mathematics education are professionally qualified to teach mathematics in schools in Hong Kong. Currently, only two universities in Hong Kong offer such a program to train PSTs, and considering the diverse demographic background of participants, the sample in this study is believed to be representative of all PSTs in the territory.

Procedure

The participants were recruited to participate in the study voluntarily via mass emails. Their informed consent was obtained prior to data collection. To encourage participation, every PST was offered a HKD50 cash coupon for providing data for analysis. An online survey was used to collect anonymized data from PSTs. They began by providing demographic information, including gender, age, and year of study. Then, they were asked questions about their teaching and learning conceptions, mathematical beliefs, mathematics self-efficacy, and mathematics teaching efficacy. The wording of some questions was changed slightly to enhance clarity while retaining their original meaning.

Instruments

This study used the Teaching and Learning Conceptions Questionnaire (Chan & Elliott, 2004) to examine the PSTs' teaching and learning conceptions. The questionnaire consists of two subscales: the traditional conceptions subscale and the constructivist conceptions subscale. It was developed and validated previously with teacher education students in Hong Kong (Chan & Elliott, 2004). The items are scored on a 5-point Likert scale (1 = strongly disagree, 5 = strongly agree). For brevity, in this study, the teaching and learning contexts in the items always pertained to mathematics. As argued by Lovelace and Brickman (2013), individuals can indicate their preference by showing their degree of agreement with statements presented in dichotomous agree/disagree, semantic-differential, and Likert formats. According to Chan and Elliott (2004), Cronbach's alphas for the traditional and constructivist conceptions subscales were both 0.840.

The Mathematical Beliefs Scale developed by Seaman et al. (2005) and later revised by Ly and Brew (2010) was used to measure the PSTs' mathematical beliefs in this study. The revised scale has two subscales: a formal approach to mathematics subscale and a constructivist approach to mathematics subscale; these subscales are rated on a 6-point Likert scale (1 = strongly disagree, 6 = strongly agree), as revealed by the studies of Seaman et al. (2005) and Ly and Brew (2010). The formal and constructivist approaches to mathematics align with dualistic and relativistic mathematical beliefs (Perry, 1970; Roscoe & Sriraman, 2011). Unlike the other instruments in this study that used 5-point Likert scales, a 6-point Likert scale was used to measure PSTs' mathematical beliefs. However, Preston and Colman (2000) revealed that when additional response categories (up to roughly 7) are offered, the indices of reliability, validity, and discriminating power are considerably higher than those of 2-point, 3-point, and 4-point scales; moreover, not much difference exists in these performance indicators between 5-point and 6-point scales. Ly and Brew (2010) found acceptable Cronbach's alphas for the subscales of the formal approach to mathematics (0.600) and the constructivist approach to mathematics (0.750).

The Mathematics Self-Efficacy Scale for Future Teachers (Zimmermann et al., 2011) was used to assess the mathematics self-efficacy of PSTs in this study. The scale was developed and validated previously with university education students in Germany. PSTs rate their confidence in successfully solving three types of mathematics problems, namely mathematics problems without context, real-world mathematics problems, and reasoning problems, on a 5-point Likert scale (1 = I am not at all confident, 5 = I am totally confident). Because the structure of efficacy scales depends on the form of competence considered in a given domain of functioning (Bandura, 1997), the scale consists of three dimensions corresponding to the three problem types. Cronbach's alpha was 0.770 for mathematics problems without context, 0.700 for real-world mathematics problems, and 0.740 for reasoning problems, respectively (Zimmermann et al., 2011).

The Mathematics Teaching Efficacy Beliefs Instrument, which was developed and validated for PSTs in the United States by Enochs et al. (2000), was used to assess mathematics teaching efficacy among the PSTs in this study. The instrument is divided into two subscales: the personal mathematics teaching efficacy subscale and the mathematics teaching outcome expectancy subscale, which correspond to the two dimensions of teaching efficacy (Bandura, 1977). The personal mathematics teaching efficacy subscale measures a PST's belief of his or her ability to teach mathematics. Some items in the subscale are reverse coded, meaning that low scores indicate high efficacy and vice versa. The mathematics teaching outcome expectancy subscale measures a PST's belief that effective mathematics instruction benefits student learning. The items are scored on a 5-point Likert scale (1 = strongly disagree, 5 = strongly agree). Cronbach's alphas for these two subscales were 0.880 and 0.770, respectively (Enochs et al., 2000).

Data Analysis

Because the factor structure of a scale may change when applied to different samples, exploratory factor analysis with geomin rotation was used to examine whether the original factor model is present in the current study. Among the various methods available to determine the number of factors extracted, this study selected parallel analysis because it has been demonstrated to be the most accurate and appropriate method (Schmitt, 2011). Parallel analysis compares the eigenvalues obtained from the correlation matrix of the original dataset with those generated from a random dataset with the same number of observations and variables. A factor is retained if its corresponding eigenvalue is greater than the 95th percentile of the generated values. As shown in the Results section, parallel analysis revealed the number of factors purported in all the instruments used in this study, except for the mathematics self-efficacy scale. In this case, a single-factor solution instead of a three-factor solution was supported. PSTs may have limited knowledge and experience to differentiate between the different aspects of mathematics self-efficacy measured by the scale (Duffin et al., 2012). Other results concerning exploratory factor analysis, such as sampling adequacy for the analysis, total variance explained by the factors, and factor loadings, are also provided in that section.

Multiple regression analyses were performed to examine the relationships between the PSTs' teaching and learning conceptions and various predictors. Dependent variables included traditional and constructivist conceptions, and predictors included mathematical beliefs, mathematics self-efficacy, and mathematics teaching efficacy. The tenability of the assumptions underlying the analyses was examined (Tabachnick & Fidell, 2014). There were no outliers found because no case had a standardized residual that differed by more than three standard deviations from the predicted value. All of the predictors had tolerance values higher than 0.7, indicating that the variance explained by one predictor was not adequately explained by the others. Multicollinearity was not evident. The linearity and homoscedasticity assumptions were tested using residual plots showing how standardized residuals and standardized predicted values were related. Because the standardized residuals were randomly distributed around a horizontal line indicating zero standardized residuals, these assumptions were justified. The normal probability plots of the standardized residuals revealed that the points largely lay on straight lines, confirming the normality assumption. The Durbin–Watson statistic evaluates serial correlation between residuals and thus verifies the independence assumption. The values of this statistic were 1.739 and 2.084 for the two regression models. The residuals were considered to be uncorrelated, and the assumption was supported.

Results

Teaching and Learning Conceptions

The Kaiser–Meyer–Olkin measure of sampling adequacy was 0.689, and Bartlett's test of sphericity was significant ($\chi^2(406) = 980.500$, $p < 0.001$). The data were suitable for factor analysis. Parallel analysis revealed two factors: one for the traditional conceptions subscale (17 items) and the other for the constructivist conceptions subscale (12 items) (Table 1). The total variance explained by the two factors was 37.047%. Cronbach's alphas for the traditional and constructivist conceptions subscales were 0.880 and 0.825, respectively.

Table 1 Results of exploratory factor analysis with geomin rotation for the Teaching and Learning Conceptions Questionnaire items

		Factor loadings
Traditional conceptions (TRAC, $\alpha=0.880$)		
TRAC1	The primary goal of learning is to absorb as much information as possible.	0.819
TRAC2	During a lesson, student learning should be confined to textbooks and the classroom.	0.740
TRAC3	Teaching is merely the act of telling, presenting, or explaining the subject matter.	0.646
TRAC4	A teacher's main responsibility is to provide students with knowledge/information, allocate them drill and practice tasks, and assess their memory.	0.628
TRAC5	Learning is all about remembering what the teacher has taught.	0.620
TRAC6	Students cannot learn unless they are managed.	0.604
TRAC7	The traditional/lecture way of teaching is the most satisfactory because it covers more information/knowledge.	0.596
TRAC8	Learning to teach entails merely repeating the ideas of lecturers without challenging them.	0.581
TRAC9	Teachers should always be able to regulate what their students do.	0.556
TRAC10	It is ideal if teachers have as much power in the classroom as possible.	0.556
TRAC11	When the teacher speaks most of the time in the classroom, this is a sign of good teaching.	0.510
TRAC12	Learning occurs primarily from drilling and practice.	0.496
TRAC13	Students must be summoned at all times to maintain control.	0.471
TRAC14	The primary function of a teacher is to transfer knowledge to students.	0.409
TRAC15	Teaching entails giving students correct and comprehensive knowledge rather than encouraging them to find it on their own.	0.394
TRAC16	A teacher's duty is to rectify students' learning misconceptions as soon as possible rather than allowing them to check their own.	0.376
TRAC17	In class, good students stay quiet and obey the teacher's instructions.	0.356
Constructivist conceptions (CONC, $\alpha=0.825$)		
CONC1	Learning involves giving students a variety of opportunities to investigate, debate, and express their views.	0.747
CONC2	Every child is unique or exceptional, and they all need an education that is suited to his or her own requirements.	0.675
CONC3	Effective teaching encourages students to engage in more dialog and hands-on activities.	0.658
CONC4	A good teacher constantly makes his or her students feel valued.	0.580

Table 1 (continued)

		Factor loadings
CONC5	Individual differences among students should be accommodated through flexible instruction.	0.567
CONC6	A democratic and open atmosphere occurs in effective classrooms, which encourages students to think and engage.	0.518
CONC7	Students' ideas are valuable and should be carefully examined.	0.497
CONC8	A teacher should be aware of his or her students' feelings.	0.494
CONC9	Students should have numerous opportunities to express themselves.	0.468
CONC10	Good teachers constantly encourage their students to think and arrive at the answers themselves.	0.460
CONC11	Individual learning objectives and expectations should be set for different students.	0.455
CONC12	Instead of transferring knowledge, the emphasis of teaching is on assisting students in creating knowledge from their learning experience.	0.379

Mathematical Beliefs

The Kaiser–Meyer–Olkin measure of sampling adequacy was 0.740, and Bartlett's test of sphericity was significant ($\chi^2(28) = 136.677$, $p < 0.001$), indicating the suitability of the data for factor analysis. Parallel analysis suggested two factors for the formal approach to mathematics subscale (three items) and the constructivist approach to mathematics subscale (five items) (Table 2). The total variance explained by the two factors was 54.982%. Cronbach's alphas similar to those obtained in Ly and Brew's (2010) study were found for the formal approach to mathematics subscale (0.617) and the constructivist approach to mathematics subscale (0.774).

Mathematics Self-efficacy

The Kaiser–Meyer–Olkin measure of sampling adequacy was 0.899, and Bartlett's test of sphericity was significant ($\chi^2(105) = 665.429$, $p < 0.001$), indicating the appropriateness of the data for factor analysis. Parallel analysis revealed only one factor, namely teachers' mathematics self-efficacy, from the 15-item Mathematics Self-Efficacy Scale for Future Teachers. The factor accounted for 50.113% of the total variance of the data (Table 3). Cronbach's alpha for this factor was 0.922.

Mathematics Teaching Efficacy

The Kaiser–Meyer–Olkin measure of sampling adequacy was 0.760, and Bartlett's test of sphericity was significant ($\chi^2(120) = 364.537$, $p < .001$), supporting the use of the data for factor analysis. Parallel analysis indicated two factors for the personal mathematics teaching efficacy subscale and the mathematics teaching outcome expectancy subscale. Each subscale contains eight items (Table 4). The two factors accounted for 43.706% of the total variance of the data and had Cronbach's alphas of 0.755 and 0.828, respectively.

Table 5 shows descriptive statistics and zero-order correlations between the major variables of the study. Except for the pairs between constructivist conceptions and the constructivist approach to mathematics, constructivist conceptions and teachers' mathematics self-efficacy, the constructivist approach to mathematics and teachers' mathematics self-efficacy, and personal mathematics teaching efficacy and mathematics teaching outcome expectancy, significant differences were found in the means of the other 17 pairs from the results of paired-samples *t*-tests ($p < 0.05$). Moreover, traditional conceptions were significantly positively correlated with the formal approach to mathematics but significantly negatively correlated with the constructivist approach to mathematics and personal mathematics teaching efficacy. Constructivist conceptions were significantly positively correlated with all other variables, except for the formal approach to mathematics. Other significant positive correlations were found between the formal approach to mathematics and the constructivist approach to mathematics, the constructivist approach to mathematics and mathematics teaching outcome expectancy, and teachers' mathematics self-efficacy and personal mathematics teaching efficacy.

Although both correlation and regression can quantify the strength and direction of the relationship between two numeric variables, correlation measures the extent to which the two variables are linearly related, whereas regression assesses how two or

Table 2 Results of exploratory factor analysis with geomin rotation for the Mathematical Beliefs Scale items

		Factor loadings
Formal approach to mathematics (FATM, α=0.617)		
FATM1	Mathematicians are mostly engaged by scientists to perform exact measurements and computations.	0.691
FATM2	Mathematics is a discipline that stresses problem solving through the application of formulas.	0.640
FATM3	Mathematics is a rigorous discipline that follows unbreakable rules.	0.461
Constructivist approach to mathematics (CATM, α=0.774)		
CATM1	Studying mathematics helps to enhance creativity.	0.755
CATM2	Many of the most beautiful and graceful inventions of the human intellect can be found in the field of mathematics.	0.686
CATM3	An inquisitive nature is a necessary element of mathematics achievement.	0.673
CATM4	Mathematics necessitates independent and creative thought.	0.649
CATM5	Originality and ingenuity can be found in mathematics possibly more than in other fields.	0.414

Table 3 Results of exploratory factor analysis with geomin rotation for the Mathematics Self-Efficacy Scale for Future Teachers items

		Factor loadings
Teachers' mathematics self-efficacy (TMSE, α=0.922)		
TMSE1	There are same number of men and women sitting in a room. When eight women leave the room, the number of men in the room doubles. Provide the number of people sitting in the room.	0.830
TMSE2	Solve equations such as $\frac{x}{x+6}+\frac{x}{x+1}+\frac{x}{x+1}=1$ and $\frac{t}{\frac{2}{3}+6}+\frac{t}{\frac{2}{3}+1}=1$ for x and t.	0.824
TMSE3	Solve equations such as $k \cdot p^x = c$ for x (k, p, and c are constant real numbers).	0.796
TMSE4	A room is to be painted. Together, Adam, Betty, and Cedric can paint the room in 6 h less than Adam can paint it alone. They can paint it in 1 h less than Betty can paint it, and they need only half as much time as Cedric needs to paint it alone. How long do Adam and Betty need to paint the room together?	0.726
TMSE5	Why is three always a factor of the sum of three successive natural numbers?	0.704
TMSE6	Determine the solution of fraction equations such as $\frac{b-k}{b}=c$ (c and k are constants and natural numbers).	0.697
TMSE7	To divide 313 sweets equally among a group of people, how many people can be in the group so that every person gets more than 2 sweets?	0.696
TMSE8	Identify the extremum of the function $f(x)=-x^2+bx$.	0.683
TMSE9	Explain why a natural number is divisible by 4 if and only if the number formed by its final two digits is divisible by 4.	0.664
TMSE10	Determine if a 2.5 m long board can be carried in a van.	0.628
TMSE11	Explain how five numbers between 0.1 and 9.9 can be chosen to get an average of 6.3.	0.625
TMSE12	Prove that $\sqrt{2}$ cannot be represented by a fraction.	0.609
TMSE13	Solve the following system of equations: $x+y=-7$ and $x \cdot y=-30$.	0.608
TMSE14	Why is 491 a prime number?	0.573
TMSE15	Calculate the capacity of a water barrel.	0.506

Table 4 Results of exploratory factor analysis with geomin rotation for the Mathematics Teaching Efficacy Beliefs Instrument items

		Factor loadings
Personal mathematics teaching efficacy (PMTE, α=0.755)		
PMTE1	In general, I am ineffective when it comes to teaching mathematics.	0.732
PMTE2	I am not very good at keeping track of mathematics activities.	0.625
PMTE3	I have no idea to promote students' interest in mathematics.	0.605
PMTE4	I am not sure if I am qualified to teach mathematics.	0.560
PMTE5	I am generally stumped when a student is having trouble grasping a mathematics concept.	0.546
PMTE6	Even if I work really hard, I am unable to teach mathematics as well as I can in many other subjects.	0.451
PMTE7	Using manipulatives to demonstrate how mathematics works to students is tough for me.	0.444
PMTE8	I would not invite the principal to evaluate my mathematics instruction if I had the option.	0.428
Mathematics teaching outcome expectancy (MTOE, α=0.828)		
MTOE1	In most cases, the mathematics accomplishment of students is the responsibility of the teacher.	0.739
MTOE2	If parents indicate that their child is more engaged in mathematics at schools, it is most likely because of the teacher's performance.	0.716
MTOE3	The efficacy of a teacher's mathematics instruction directly impacts on students' mathematics success.	0.681
MTOE4	When students' mathematics grades increase, it is usually because their teacher teaches more effectively.	0.663
MTOE5	When a low achiever in mathematics improves, it is usually because of the teacher's extra attention.	0.623
MTOE6	If students are performing poorly in mathematics, it is almost always because of poor mathematics instruction.	0.581
MTOE7	Effective instruction can compensate for a student's lack of mathematical knowledge.	0.493
MTOE8	When a student performs better than expected in mathematics, it is usually because of the teacher's additional effort.	0.424

Note. Items in the PMTE subscale are reverse coded

Table 5 Descriptive statistics and zero-order correlations between the major variables

	1	2	3	4	5	6	7
1. TRAC	-						
2. CONC	−0.144	-					
3. FATM	0.349**	0.198	-				
4. CATM	−0.286*	0.377**	0.220*	-			
5. TMSE	−0.054	0.266*	−0.014	0.218	-		
6. PMTE	−0.567**	0.287*	−0.136	0.223	0.255*	-	
7. MTOE	−0.068	0.295*	0.211	0.394**	−0.178	−0.013	-
Mean	2.783	4.261	3.917	4.325	4.188	3.308	3.377
Standard deviation	0.581	0.417	0.824	0.793	0.684	0.574	0.584

Note. TRAC traditional conceptions, *CONC* constructivist conceptions, *FATM* formal approach to mathematics, *CATM* constructivist approach to mathematics, *TMSE* teachers' mathematics self-efficacy, *PMTE* personal mathematics teaching efficacy, *MTOE* mathematics teaching outcome expectancy

*$p < 0.05$. **$p < 0.01$

more independent variables affect the dependent variable. Moreover, the standardized regression coefficient is, in general, not equal to the correlation between the independent and dependent variables unless the predictors are orthogonal, that is, all the predictors are uncorrelated (Reddon & Ho, 2007). Therefore, in this study, multiple regression analyses were performed, in which traditional and constructivist conceptions were regressed on the formal approach to mathematics, the constructivist approach to mathematics, teachers' mathematics self-efficacy, personal mathematics teaching efficacy, and mathematics teaching outcome expectancy. Table 6 clearly indicates that the formal approach to mathematics, the constructivist approach to mathematics, teachers' mathematics self-efficacy, personal mathematics teaching efficacy, and mathematics teaching outcome expectancy significantly predicted traditional conceptions ($F(5, 69) = 12.347$, $p < 0.001$) and constructivist conceptions ($F(5, 69) = 5.487$, $p < 0.001$). They explained 47.2% and 28.4% of their total variances, respectively. Significant predictors of traditional conceptions included the formal approach to mathematics ($\beta = 0.358$, $p < 0.001$), the constructivist approach to mathematics ($\beta = -0.282$, $p < 0.01$), and personal mathematics teaching efficacy ($\beta = -0.478$, $p < 0.001$). Significant predictors of constructivist conceptions included teachers' mathematics self-efficacy ($\beta = 0.233$, $p < 0.05$) and mathematics teaching outcome expectancy ($\beta = 0.249$, $p < 0.05$).

Discussion

Despite our conviction that theoretical relationships are present among mathematical beliefs, mathematics self-efficacy, mathematics teaching efficacy, and teaching and learning conceptions in PSTs (Bandura, 1986; Perry, 1970), empirical findings of this study offer a nuanced perspective of the effects of these epistemological and efficacy beliefs on their teaching and learning conceptions. Specifically, traditional conceptions were positively predicted by the formal approach to mathematics but negatively

Table 6 Multiple regression analyses predicting teaching and learning conceptions by mathematical beliefs, mathematics self-efficacy, and mathematics teaching efficacy

	TRAC	CONC
Predictors	*β*	*β*
FATM	0.358***	0.136
CATM	−0.282**	0.159
TMSE	0.104	0.233*
PMTE	−0.478***	0.217
MTOE	−0.021	0.249*
R^2	0.472	0.284
F	12.347***	5.487***

Note. *TRAC* traditional conceptions, *CONC* constructivist conceptions, *FATM* formal approach to mathematics, *CATM* constructivist approach to mathematics, *TMSE* teachers' mathematics self-efficacy, *PMTE* personal mathematics teaching efficacy, *MTOE* mathematics teaching outcome expectancy

*$p < 0.05$. **$p < 0.01$. ***$p < 0.001$

predicted by the constructivist approach to mathematics and personal mathematics teaching efficacy. Constructivist conceptions were positively predicted by teachers' mathematics self-efficacy and mathematics teaching outcome expectancy. These findings align with those of a prior study showing that for PSTs, the formal approach to mathematics is positively related to their traditional conceptions, whereas the constructivist approach to mathematics is negatively related to their traditional conceptions (Lau, 2021). As the mathematics self-efficacy of PSTs increased, their constructivist conceptions also increased (Phelps, 2010). Moreover, personal mathematics teaching efficacy and mathematics teaching outcome expectancy were negatively and positively linked to traditional and constructivist conceptions, respectively (Swars et al., 2009). This pattern of relationships recognizes the unique role of each of these belief-related variables in explaining and predicting the teaching and learning conceptions of PSTs. As mentioned previously, because of the close relationships between teachers' beliefs regarding mathematics and mathematics teaching and learning and their instructional practices, changing these mathematics-related beliefs to inquiry-oriented beliefs would improve mathematics teaching quality (Potari, 2020).

Although the findings are largely consistent with previous results (Kahle, 2008; Tang & Hsieh, 2014), some of the expected results were not derived from multiple regression analyses. For example, constructivist conceptions were not predicted by the formal or the constructivist approach to mathematics. Inconsistency between mathematical beliefs and teaching and learning conceptions has also been discussed in other studies. Howard (2013) observed various beliefs regarding the nature of mathematics and mathematics teaching and learning that were inconsistent with the theoretical descriptions of Beswick (2005). In his study, one PST endorsed the three views on the nature of mathematics put forward by Ernest (1989), namely the instrumentalist, Platonist, and problem-solving views, to varying degrees. The PST showed a dominant instrumentalist view on the nature of mathematics but a problem-solving view on mathematics teaching and learning. Penn (2012) found that PSTs held absolutist beliefs regarding the nature of mathematics (mathematics is infallible) but constructivist beliefs regarding mathematics teaching and learning. A subsequent interview with eight PSTs who had misaligned beliefs indicated that constructivist instructional methods are in fact used to teach didactically. The author explained that misalignment occurred because PSTs changed their teaching approaches superficially without a genuine understanding of the rationale underlying the change. These findings suggest that PSTs have multiple and contrasting beliefs regarding mathematics and its teaching and learning that are not well organized or coherent, although they attempt to seek methods of accommodating different approaches and ideologies. PSTs use these kaleidoscopic beliefs as resources to address their professional needs contingent upon their experience and the context. Therefore, the presence of various conflicting beliefs supports the notion that teachers' beliefs are contextualized, indicating that teachers are acculturated into real teaching settings considering the specific learning context that constantly influences their beliefs and decisions (Levin & Nevo, 2009).

In particular, although PSTs in this study were trained to teach using the constructivist approach, they were also exposed to other teaching environments where constructivist-based teaching may not be promoted. For example, when they were required to teach in practicum schools or had private tutorial sessions with students where drilling and practice were more prevalent due to external examination pressure, PSTs often endorsed constructivist conceptions but tended to espouse traditional conceptions (Yang & Li, 2009). Furthermore, because the instruments used in this study were developed in educational contexts different from those in Hong Kong, this may affect how PSTs respond to those items. As contended by Murphy and Alexander (2016), task, context, and even individual differences are likely to alter one's judgments on the validity of beliefs; thus, contextual and situational effects must be examined when evaluating the standards of evidence, even within the same domain. Therefore, researchers should regularly identify critical factors shaping the formation of teachers' belief systems and assess the complex relationships of their components.

In summary, this study offers empirical evidence for PSTs to critically reflect on their professional beliefs and make informed teaching and learning decisions, which constitute a part of professional competence for future teachers (Blömeke et al., 2008). The study findings enable teacher educators to understand how the teaching and learning conceptions of PSTs are influenced by various belief factors. This can help them design learning experience for PSTs to strengthen or weaken conceptions that agree or contradict with existing beliefs. As shown by Eren (2013), dissonance and consonance between prospective teachers' values and practices can provide important initial information to bridge the belief–practice gap in the future. This study also contributes to the literature on learning to teach from the perspective of PSTs; this topic is often overlooked in relevant studies. As argued by Cady and Rearden (2007), pre-service teachers' beliefs influence what and how they learn to teach and students' belief systems. The present study identified the relationships between epistemological and efficacy beliefs among PSTs and examined the extent to which these beliefs predict their teaching and learning conceptions, which are ultimately linked to their instructional practices.

Conclusions

Studying the teaching and learning conceptions of PSTs is crucial. PSTs enter university with diverse beliefs and understandings of teaching and learning; yet, they may not be aware of how these conceptions are related to their epistemological and efficacy beliefs. During initial teacher education, the efficacy beliefs of PSTs are still malleable and can be enhanced (Swars et al., 2009). As PSTs become more efficacious, their instructional quality is likely to rise by providing a supportive classroom atmosphere, successful classroom management, and cognitive activation (Künsting et al., 2016). This study explored PSTs' teaching and learning conceptions and identified their predictors. The study findings advance our understanding of the unique role of belief-related variables, such as mathematical beliefs, mathematics self-efficacy, and mathematics teaching efficacy, in their relationships with teaching and learning conceptions. Overall, this study demonstrates the potential to inform and improve PSTs' professional learning and development.

Future studies should address some limitations of the present research. First, although this study offers valuable insights into the relationships between PSTs' epistemological and efficacy beliefs and their teaching and learning conceptions in a specific sociocultural context, more efforts are required to replicate the current investigation in a large random sample of PSTs to increase the generalizability of the findings. Second, qualitative data should be collected from PSTs and analyzed to interpret the quantitative results of the multiple regression analyses. Third, understanding how PSTs' teaching and learning conceptions develop with respect to their epistemological and efficacy beliefs over time is paramount; such longitudinal relationships are crucial when testing causality among variables is needed. Finally, some other mathematical affect, such as emotions and attitudes, may be included as predictors of PSTs' teaching and learning conceptions in future studies.

Acknowledgements The author would like to thank PSTs in this study for providing data for analysis.

Funding This study was supported by the Direct Grant for Research (2017-18) (Grant Number 4058053) of the Chinese University of Hong Kong awarded to the author of the paper. The quantitative findings on PSTs' teaching and learning conceptions were presented at the Psychology of Mathematics Education (PME) and Yandex Russian Conference: Technology and Psychology for Mathematics Education, Moscow, Russia in 2019.

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