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Learning to Lead: an Approach to Mathematics Teacher Leader Development

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Abstract

This paper describes a partnership between a university and an urban school district, formed with a goal of preparing mathematics teacher leaders to conduct professional development (PD) at their schools. The university and district partners worked together to achieve the district's mission of providing every student with high-quality instruction and equitable learning opportunities in mathematics by building the district's capacity to conduct school-based PD for mathematics teachers. Given the power of school-based subject-specific PD for improving instructional quality, we worked with Teacher Leaders from participating schools to prepare and support them to lead PD workshops at their schools. In this paper, we examine how Teacher Leaders learn and adapt key elements of a PD model over three school years through the lenses of Prediger et al.'s Three-Tetrahedron-Model (2019) and the university's Learning to Lead model. Over 3 years, we see that Teacher Leaders use the key structures of the PD model; make adaptations in response to school goals, interests, and priorities; and gain confidence in their work with colleagues. By viewing the adaptations through the lens of pedagogies of practice as well as the relationships illustrated by the 3-T model, this work offers insights into the complexities of teacher leadership development.

Keywords Facilitator preparation · Mathematics education · Professional development · Teacher leader

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In her 2008 chapter, "Facing the challenge of educating educators to work with practicing mathematics teachers," Even pointed out that "There is general agreement today that the continued professional development of teachers is key to improving students' opportunities to learn mathematics" (p. 57). She also observed that there is a growing international interest in the education of teacher educators who work with practicing teachers to improve their teaching. The preparation of these teacher educators is essential to providing all teachers with the opportunity for continued professional development (Borko, Koellner & Jacobs, 2011). Yet, as the call for this special issue noted, "there continues to be little research available on how to support the professional growth of mathematics and science teacher educators in their work with teachers." Furthermore, we "lack research-based principles to guide the design of leader preparation" (Lesseig et al., 2017, p. 592). This paper adds to the literature by examining the impact of a teacher leader preparation program on the PD provided by teacher leaders to the mathematics teachers at their schools.[1] It also addresses Potari's (2019) recommendation for additional research to see whether models for designing and analyzing mathematics PD such as Prediger, Roesken-Winter and Leuders (2019)'s "can be transformed in other settings and what forms of adaptations they need" (p. 329).

This paper focuses on the efforts of a Research-Practice Partnership (Coburn & Penuel, 2016) between a university research center and an urban school district to build capacity within the district for mathematics teachers to become Teacher Leaders (TLs) who lead mathematics PD in their schools. At the heart of this work was the district's stated mission to provide every student with high-quality instruction and equitable learning opportunities. Because both university and district leaders saw school-based PD as a powerful way to improve instructional quality, we formed the Research Practice Partnership to support this and other district goals. Our partnership's approach to building district PD capacity is multi-level. It entails preparing TLs who, in turn, conduct mathematics PD for the other mathematics teachers in their schools.

To guide the implementation of their mission, the district math department developed a task-based mathematics curriculum to align their mathematics courses to the Common Core State Standards (2010) (the standards for literacy and mathematics learning adopted by most states in the USA; http://www.corestandards.org/) and secured funding for a comprehensive PD program to support teachers as they learned to use the new curriculum. Simultaneously, a cross-subject team at the district level developed a guiding document that identified three Dimensions of Teaching and Learning. They created this document to support a shift in teaching and learning across the district so all students, especially those who have been historically underserved, have high-quality learning experiences. The three dimensions include developing students' agency, authority, and identity (AAI), providing access to content, and using formative assessment.

In support of these larger initiatives, the district mathematics leaders worked in close partnership with a team from a research center in the (blinded for review) Graduate School of Education (referred to as the research team) to adapt, implement, analyze,

[1] This large, multi-year project entailed a large research team working for several years of collecting and analyzing multiple sources of data, which resulted in a large group of authors for the paper. All authors participated in designing the data collection instruments and developing the analysis plans, and then drafted the manuscript and worked on manuscript revisions.

and revise two established models of professional development. One model focused on leadership preparation for Teacher Leaders and the other offered a model for school-based mathematics PD for teachers. These models, described in the Literature Review and Conceptual Framework, came from our previous work that had been successful in other school districts (Borko, Jacobs, Koellner & Swackhamer, 2015). In the initial phase of the project, we adapted them to fit the unique circumstances of this district and highlight the district's mission (Borko et al., 2017).

Literature Review and Conceptual Framework

As indicated in the call for proposals for this special issue and reiterated above, the literature base regarding the nature of programs for preparing leaders of PD for practicing teachers or the effects of such programs is sparse. This section begins with a brief review of that literature, focusing on programs that informed the design of our leadership preparation model. We then present the conceptual framework for the project, which integrates the Three-Tetrahedron (3-T) Model of Professional Development developed by Prediger and colleagues (cf. Prediger et al., 2019) and a four-phase Learning to Lead cycle that focuses on the elements of the Facilitator PD level in the 3-T model.

The Preparation of Professional Development Leaders

The MANOR program, established in 1993 by Even and colleagues at the Weizmann Institute in Israel and conducted through 2003, was one of the first preparation programs for educators of practicing mathematics teachers. Case studies of two teacher educators who participated in the program revealed similarities in their PD practices, such as modeling lessons in which the teachers participated as students, despite very different working conditions (Even, Robinson & Carmeli, 2003). Over a decade after the conclusion of MANOR, Even noted, in her commentary for a special issue of *ZDM* on the joint activity of teachers and teacher educators (didacticians), that although PD for both practicing mathematics teachers and didacticians had received growing international attention, "research on the latter is missing and has been identified as an area that could greatly profit from stronger and more systematic research" (2014, p. 331).

The following year *ZDM* published a theme issue on the related topic, "Evidence-based CPD [Continuous Professional Development]: Scaling up sustainable interventions." One section of that issue focuses on "conceptualizing scaling up CPD." The three empirical articles and one commentary in that section address the role of "multipliers" (another term for PD facilitators) and their development (Jackson et al., 2015; Krainer, 2015; Roesken-Winter, Hoyles & Blomeke, 2015; Roesken-Winter, Schüler, Stahnke & Blömeke, 2015). The article by Jackson and colleagues is most relevant to our work, however, because it specifically addresses both their approach to preparing multipliers and the multipliers' facilitation of PD.

In Jackson et al.' (2015) project, the research team worked with a school district's director of secondary mathematics (the Director) to prepare and support three math leaders (their term for educators of mathematics teachers) to design and lead mathematics PD for middle-school mathematics teachers in the district. This project, like

ours, consisted of two levels of professional development, one for the district's math leaders (math leader PD) and one for classroom mathematics teachers (teacher PD). The research team and Director engaged in four cycles of support for the math leaders over the course of one academic year. Each cycle began with a math leader PD session co-planned and conducted by the research team and the Director, which focused on planning the upcoming teacher PD workshop. Extrapolating from the literature on teacher PD, these sessions included pedagogies of investigation and enactment (Grossman et al., 2009) such as modeling PD activities and planning upcoming workshops. During the next phase of the cycle, the math leaders led and video-recorded the teacher PD workshop they had planned. In the third phase, the research team co-planned the next math leader PD session with the Director. The authors reported mixed findings. The math leaders shifted the focus of the teacher PD to target core aspects of instruction with less time devoted to peripheral issues. However, they sometimes modeled instructional activities such as classroom discussions without pressing the teachers to analyze key features of those activities. Based on these findings, the researchers recommended designing mathematics leader PD to focus on a small number of teacher PD activities and to scaffold the facilitation of these activities, for example, by modeling them and then explicitly examining the facilitator's decisions specific to the activity.

Two articles in a special issue of the *International Journal of STEM Education* focused on video-based professional development programs for teachers also addressed the preparation of PD facilitators. The study by Jacobs, Seago and Koellner (2017) examined the fidelity with which one facilitator, Hannah, implemented Learning and Teaching Geometry, a highly specified video-based PD program that is designed to be implemented with fidelity, rather than adapted to specific contexts. The program provides an extensive set of resources that include a detailed agenda for each session, PowerPoint slides, videocases with classroom video clips, mathematics tasks, and other materials designed to support facilitators to implement the program as intended. Hannah prepared to implement the PD program by studying the PD materials, discussing them with the PD developers, and then conducting a set of rehearsal sessions that were observed by the developers in order to document fidelity. Given this extensive preparation Hannah was able to implement the program with fidelity, and the adaptations she made were in line with the intentions and goals of the materials.

Lesseig et al. (2017) reported on a project that used videocases of facilitation to support PD leaders in learning to facilitate mathematics PD. They compared two phases of the work, highlighting revisions they made in Phase 2, reasons for those revisions, and their impact on the leaders' discussions of facilitation practices. Similarly, to Jackson et al. (2015), the design of the leader PD in Phase 1 was guided by ideas from classroom research such as practices for orchestrating mathematics discussions. Sessions began with the leaders working on the mathematics task used in the PD videocase. They then watched the videocase and unpacked the teachers' mathematical thinking and the facilitator's actions. In Phase 2, the researchers revised the mathematics task that the leaders worked on to include analyzing possible solution methods, evaluating affordances and limitations of representations, recognizing common student errors, and considering the reasoning that might underlie correct and incorrect solutions. They also revised the prompts for viewing and discussing the videocases, for example by highlighting justifications the teachers offered for their solution methods.

These changes led to PD leader discussions that centered more on facilitation practices that kept teacher conversations focused on unpacking mathematical representations or solution methods.

The elements of our four-phase Learning to Lead cycle incorporate several ideas from prior research, such as activities centered on mathematics tasks (Even et al., 2003; Jacobs et al., 2017; Lesseig et al., 2017) and video (Jacobs et al., 2017; Lesseig et al., 2017), and the use of modeling (Even et al., 2003; Jackson et al., 2015) and rehearsals (Jacobs et al., 2017). Our project also overlapped with key ideas from both the MANOR project and Leiseig and colleagues' work. Specifically the MANOR project (Even et al., 2003) used tasks from a new curriculum rather than tasks specific to the project, a change we also made to address district priorities. Leiseig and colleagues' revised Phase 2 tasks informed our design to incorporate what we refer to as "teacher analysis tasks." With respect to video, participants in Lesseig et al.' (2017) project analyzed videocases of PD, whereas in our project classroom videos were used in modeling and rehearsal activities. Hannah's use of classroom video in her rehearsals for Leading the Learning and Teaching Geometry PD (Jacobs et al., 2017) was more similar to ours. Our project is the only one, however, that scaffolded leader learning by first modeling PD activities where the TLs participated as learners, then explicitly discussing the facilitators' decisions and practices.

The Three-Tetrahedron Model of Professional Learning Design and Research

We use the Three-Tetrahedron Model of PD (3T-Model—see Fig. 1) that Prediger et al. (2019) developed to account for the multifaceted and multi-level structure of PD design and research to situate the specific aspects of our project that are the focus of this paper in the broader terrain of professional learning for teachers. The elements of the three tetrahedrons, and their relationships both within and across the tetrahedrons, capture the complexity of learning and teaching at the classroom, teacher professional development (TPD), and facilitator professional development (FPD) levels. The model begins with the didactic triangle of content, students, and teachers, and extends it to a classroom level didactic tetrahedron by adding the element "classroom resources" (see Rezat & Sträßer, 2012). Drawing upon the fact that these four elements are also crucial to the professional development of teachers, Prediger and colleagues created a TPD tetrahedron in which teachers are the learners, facilitators act in teaching roles, the PD content is the aspects of teaching and learning addressed in the PD program, and the resources are the materials and activities in that PD program. They extended it to the third level, FPD, using the same general structure. Here facilitators are the learners, facilitator educators act in teaching roles, the content is the substantive focus of the FPD, and the resources are the materials and activities in that program.

Using the 3T-Model as a starting point, Prediger and colleagues identified three general strategies for designing programs and conducting research at the TPD and FPD levels: lifting, nesting, and unpacking. As they explained, a lifting strategy can be used to lift design and research approaches from the classroom level to the TPD level or from the TPD level to the FPD level. For example, research approaches for investigating classroom practices can be lifted to the TPD level and used to examine facilitation practices in analogical ways. However, while the lifting strategy exploits the structural analogies of the tetrahedrons, there are also significant differences due to the higher

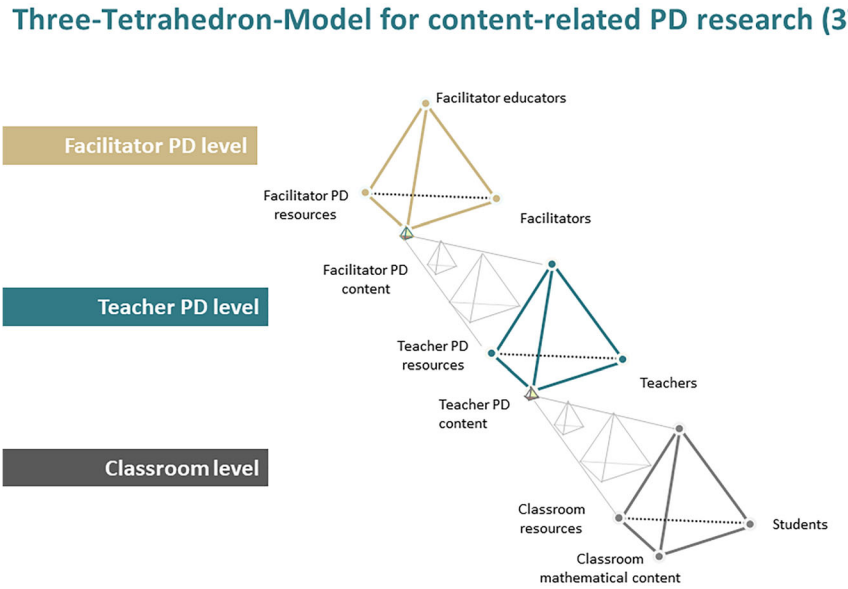

Fig. 1 Three-tetrahedron model of professional development (Prediger et al., 2019)

complexity of the TPD content. Nesting refers to a design strategy whereby the classroom tetrahedron is nested in the TPD content, and the TPD tetrahedron is nested in the FPD content (as indicated in Fig. 1 by the gray lines). Finally, unpacking is a design-based research strategy in which the content at one level is unpacked with respect to elements of the tetrahedrons of the levels below. We return to these strategies when discussing our TPD/FPD program and our research, where we also use unpacking as an activity in the leadership preparation program for TLs.

Prediger and colleagues do not suggest that research programs should account for all levels of the 3T-Model or all elements in each level simultaneously. Rather, the model can be used to understand existing PD research studies and their location in the 3-D professional development space, and to consider questions that these studies leave unanswered. Our program of research is no exception; the analyses we present in this article represent only one component of that program. Specifically, in keeping with the special issue, we focus on the preparation of Teacher Leaders who will facilitate school-based PD.

The Learning to Lead Cycle and Pedagogies of Practice

The four-phase Learning to Lead Cycle (Fig. 2) provides PD leaders with an opportunity to experience, analyze, practice, and reflect on components of the PD programs they will lead. Our enactment of the Learning to Lead Cycle is described in the section that follows.

We drew from the literature on effective PD in designing learning opportunities for the Teacher Leaders (TLs) in our project, extrapolating (Jackson et al., 2015) or lifting (Prediger et al., 2019) from that literature to create a sequence of professional learning activities focused on practices of leading PD for teachers. Specifically, the pedagogies of investigation and enactment that Grossman et al. (2009) identified in their study of

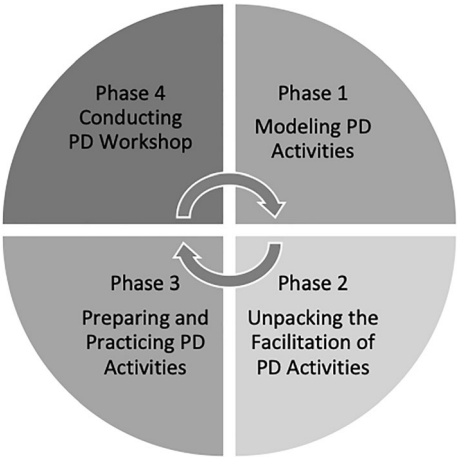

Fig. 2 The learning to lead cycle for designing FPD (adapted from McDonald, Kazemi & Kavanagh, 2013)

professional learning played a central role in the design of the phases of the Learning to Lead Cycle. In our work, pedagogies of investigation included modeling, using classroom video, and analyzing student work to deepen TLs' understanding of mathematics and how students understand mathematics. Pedagogies of enactment included opportunities to plan and rehearse components of facilitation in a low-stakes environment with peers and to receive targeted feedback after their rehearsals (Kazemi, Ghousseini, Cunard & Turrou, 2016).

Although these pedagogies have been used extensively in PD for teachers, their use is much less common in facilitator preparation. As described in the previous section, several of the pedagogies were also used in other programs for the preparation of PD leaders, sometimes in similar ways. What is unique to our project is the enactment of this combination of pedagogies in the Learning to Lead Cycle over multiple years with the same Teacher Leaders. We instantiate Prediger et al.' (2019) Three-Tetrahedron model by incorporating the Learning to Lead Cycle and our model of PD for mathematics teachers and providing concrete examples of lifting, nesting, and unpacking. Finally, we add to the literature by describing the enacted PD program for mathematics TLs in detail, as well as providing accounts of the leaders' facilitation.

The TLP and PSC Models of Professional Development and Teacher Leader Preparation

The Learning to Lead Cycle is an overarching framework that supports Teacher Leader development. The work TLs do in phases 1, 2, and 3 occurs during the Teacher Leader Preparation (TLP) model, while phase 4 encompasses all the work done during the Problem- Solving Cycle (PSC) workshops. To illustrate how these two models work within the Learning to Lead Cycle, we describe each model in detail below.

Embedding the Problem-Solving Cycle in the professional learning of mathematics teachers at their schools is at the heart of our work so we describe the PSC model first and then look at how the TLP model supports this goal. The PSC consists of three interconnected workshops, all organized around a mathematics task (see Fig. 3).

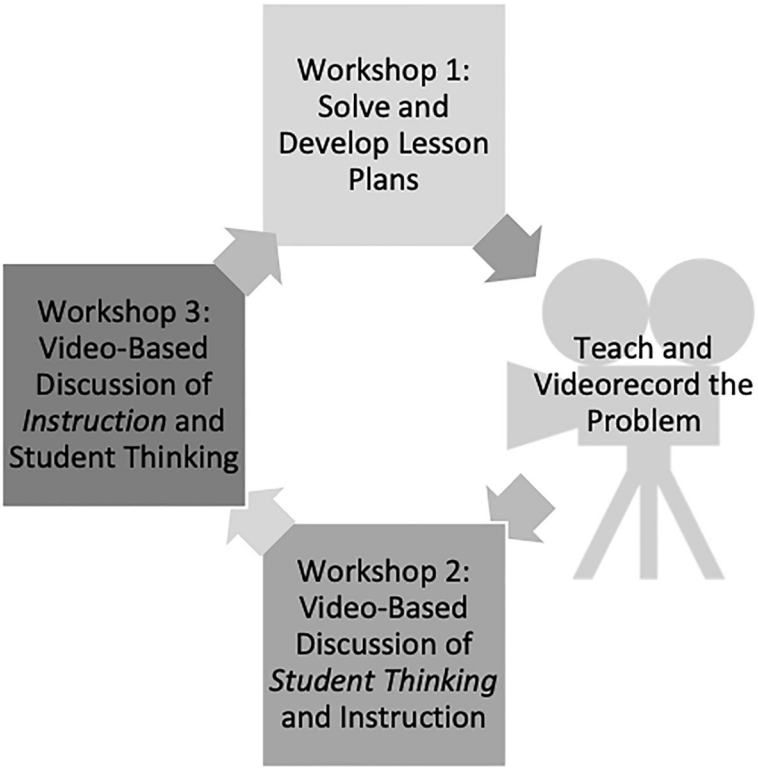

Fig. 3 The problem-solving cycle model of professional development

In our previous work, we found that the most effective tasks for the PSC model have the following characteristics: they (1) address multiple mathematical concepts and skills with high cognitive demand, (2) are accessible to learners with different levels of mathematical knowledge and allow for divergent thinking, (3) have multiple entry points that allow for initial success, (4) are relevant and engaging, and (5) provide a foundation for productive mathematical communication (Koellner et al., 2007). See Fig. 1-s in the supplemental materials for an example of a mathematics task from the UUSD curriculum used in PSC workshops.

During Workshop 1, teachers solve a mathematics task and prepare to teach it with their students. We refer to this activity as *Do the Math*. These workshops often include a "teacher analysis task" in which teachers analyze the task by considering such features as multiple solution strategies, the affordances and limitations of different representations, and the reasoning that might underlie correct and incorrect student solutions (Lesseig et al., 2017). At this point in the PSC, TLs are "lifting" a classroom resource (the math task) into the PD for their colleagues.

Participants teach the lesson on a day that falls between Workshops 1 and 2, recording video of both their instruction and students working on the task in small groups. In workshops 2 and 3, teachers work collaboratively to discuss video clips from these lessons (a video-based discussion), analyzing student reasoning and instructional practices respectively. In these two workshops, TLs are supporting their colleagues through "nesting" a video clip of classroom practice in the PD for the teachers. The video-based

discussions often use a focus question to direct teacher attention to something specific such as the role of student agency or authority while working in a small group. Participants engage in this cycle of three workshops twice during the school year. Each cycle focuses on a different mathematics task and highlights specific issues related to teaching and learning.

The Teacher Leadership Preparation model is designed to prepare Teacher Leaders to plan and lead PSC workshops with mathematics teachers in their schools (see Fig. 4).

The model consists of two components: a summer institute in which the TLs learn about the PSC and key facilitation practices and six TLP sessions that provide structured guidance throughout the academic year as TLs prepare to lead the PSC workshops at their schools. A key component of the interplay between the TLP model and the PSC model is how the central activities of each model alternate with one another, as illustrated in Fig. 4. After each TLP summer academy or session, the TLs facilitate an introductory workshop at their schools (Borko, Koellner & Jacobs, 2014).

We structured the learning experiences in each TLP session using the Learning to Lead Cycle. During each session, the TLs have an opportunity to engage in the pedagogies of investigation (Phases 1 and 2) and enactment (Phases 3 and 4) (see Fig. 2). In Phase 1, they participate as teacher learners in an activity led by the Facilitator Educators. For example, they collaboratively complete a teacher analysis task or participate in a video-based discussion (VBD) in which they view a short video clip of students working on a math task and analyze the student reasoning or instructional practices that are visible. They return to the role of TL in Phase 2 to experience "unpacking" the facilitation as the Facilitator Educators explicitly name and discuss their goals for the activity, the reasons for their pedagogical choices, and facilitation moves. In Phase 3, the TLs plan a PSC workshop based on the learning experience from Phase 1, but tailored to the goals and priorities of their school. They then rehearse portions of the workshop with TLs from the other schools and receive feedback from other TLs and the Facilitator Educators. In Phase 4, they conduct the workshop with the mathematics teachers at their school.

Research Questions, Methods, and Analytic Framework

With respect to the 3T-Model, this paper focuses on three of the four vertices of the tetrahedrons: resources, content, and the person leading the PD—either the Facilitator

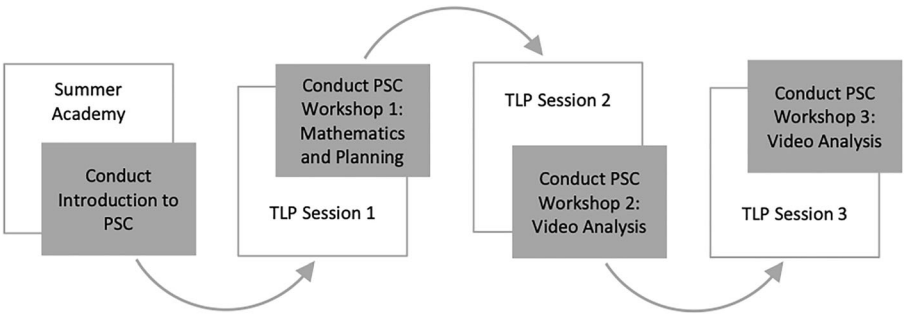

Fig. 4 The teacher leader preparation (TLP) model

Educator (the role of the research team or district math leaders during the TLP sessions) or the Facilitator (the role the TLs have at their schools when they lead PSC workshops). We consider the content and resources of the TLP sessions and the associated PSC workshops—as well as what seemed to influence the relationships between those elements over time. We examine these relationships with the expectation that they will change over time because the TLP and PSC models are intended to be adaptive to the goals and priorities of the users. See Fig. 5 for a visual mapping between the TLP and PSC models and the 3T-Model. Our guiding research questions are as follows: (1) How did the TLP sessions evolve over time? (2) What key PSC ideas did the TLs adopt and how did they adapt them over 3 years of planning and facilitating workshops?

The School District Context

Urban Unified School District (UUSD) is a large urban district with high proportions of students from non-dominant cultural and linguistic communities (approximately 80% students of color, 28% English learners, and 55% students who qualify for government-subsidized lunch) (California Department of Education, 2019). As such, this district provides an important setting for studying mathematics instructional practices that meet the needs of diverse learners and the complexity of an urban district with a commitment to high-quality learning.

As mentioned above, to align its middle school mathematics courses to the Common Core State Standards, UUSD developed a task-based curriculum for use in grades 6 through 8. Each curriculum unit is designed around four tasks. These tasks offer all students the opportunity to engage in meaningful and rigorous mathematics and to demonstrate their understanding of the essential concepts and standards in the unit. The first task helps students access their prior knowledge. The second and third tasks require students to work collaboratively, and to justify their results to one another as they apply the new concepts and skills they are learning. Teachers use the fourth task to assess students' understandings of the concepts and skills in the unit.

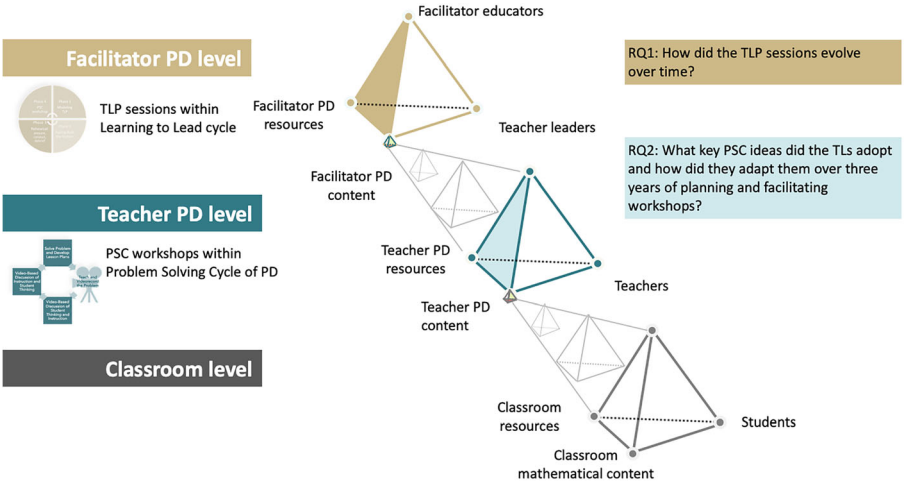

Fig. 5 Mapping the TLP and PSC models onto the 3-T model

Participants

Beginning in the 2014–2015 academic year, the district expected all middle school mathematics teachers to use the new curriculum units. This project was leveraged to assist in introducing the curriculum to the teachers. UUSD identified two to three TLs from half their middle schools to participate in the PSC/TLP PD program. TLs were asked to deepen their understanding of, and broaden their skills for implementing the instructional approach taken by the curriculum, and then bring that knowledge and skills back to their school sites.

Teacher Leaders

District staff and school administrators recruited middle school (grades 6–8) mathematics teachers to participate as TLs in the PSC/TLP program. The commitment TLs made was to attend the 3-day summer institute and six TLP sessions throughout the school year and to conduct six 1-h PSC workshops each year with the mathematics teachers at their schools.

Timeline

We conducted a pilot year with two schools during the 2015–2016 school year (Year 0). In Fall 2016, we expanded the project to seven schools and 16 TLs and continued working with these schools as well as others during the following three school years: 2016–17 (Year 1), 2017–18 (Year 2), and 2018–19 (Year 3). One school joined only for Year 2 and another only for Year 3. There was regular turnover in TLs for a variety of reasons including position changes, sabbaticals, retirements, and parental leave. (See Table 1 in supplemental materials for details.) The data we use in this paper comes from Years 1–3.

The Research-Practice Partnership

The overall plan for the partnership has been to build PD capacity at the school level by preparing TLs to conduct PSC workshops. To accomplish this, we also needed to build capacity at the district level so that the district could assume a larger role in designing and facilitating the TLP sessions over the course of the project. This aspect of the partnership evolved as the project unfolded, with the research and district teams working as Facilitator Educators to co-design and co-facilitate TLP sessions during Year 1, UUSD taking a larger role in planning and leading during Year 2, and the research team moving solely to research activities during Year 3 while district mathematics leaders became the primary Facilitator Educators for TLP sessions.

Data Sources

The large data corpus for this project includes video recordings, artifacts, interviews, and field notes collected over the course of the project, and across a range of settings. We drew upon the following data sources in the analyses for this paper:

PSC Workshop Videos. The TLs facilitated approximately six PSC workshops each year, which were video-recorded. One hundred fourteen workshop videos were included in this analysis.

Artifacts for TLP Sessions and PSC Workshops. For TLP sessions, we collected the agendas and presentation slides that were posted online, copies of mathematics tasks, worksheets, and other handouts. In addition, we photographed posters, mathematics work, and other artifacts that were created during the sessions. For the PSC workshops, we collected slide presentations, mathematics tasks, and other handouts.

Field Notes for TLP Sessions and PSC Workshops. Our team took notes during all of the TLP sessions and PSC workshops. The TLP field notes included an attendance record, the agenda, and detailed notes about each activity. The PSC notes included attendance and more general notes about the activities and participation structures.

PSC Workshop Memos. After recording a PSC workshop, the researchers wrote memos with their reflections about the session. These reflections included, for example, any interactions, comments, or dynamics that stood out to them.

Activity Logs for PSC Workshops and TLP Sessions. For each PSC workshop and TLP session, a member of the research team watched the videos and created a time stamped activity log that summarized what was done in each of the activities.

Teacher Leader Interviews. We interviewed TLs twice a year, each year that they participated in the project. For this paper, we drew on responses to questions about TLP activities they found useful and their perception of their growth as Teacher Leaders.

Analysis

Our analyses occurred in three stages. For the first stage of analysis, we worked across data sources to create a spreadsheet with information for each TLP session and PSC workshop. The spreadsheet included information such as who participated, who facilitated, activities they engaged in, and whether or not activity structures central to the PSC model occurred.

Three pairs of researchers—one pair for the PSC 1 workshops (the first workshop in each PSC cycle), one pair for PSC 2, and one pair for PSC 3—reviewed the materials for each PSC workshop conducted over three school years and completed the spreadsheet section for all workshops. These materials included the workshop video, field notes, artifacts, activity logs, and memos. This process was completed for all 114 workshops. Each pair then reviewed the spreadsheet entries for their workshops and wrote an analytical memo describing patterns and other observations related to the research questions.

The purpose of the Stage 2 analysis was to make initial comparisons between the TLP sessions and corresponding PSC workshops, to identify central elements of the TLP sessions that the TLs had adapted, modified, or omitted in their PSC workshops.

We developed another spreadsheet that included these elements for the TLP sessions and PSC workshops. Working in the same pairs as Stage 1, researchers began with the TLP sessions corresponding to their set of workshops and reviewed activity logs, field notes, and artifacts to fill in key aspects of the sessions. Next, they used the Stage 1 spreadsheet to add information for the PSC workshops. They then reviewed their spreadsheet entries together and added insights about the modifications or adaptations between TLP sessions and PSC workshops that they noticed. After completing the spreadsheets, the full project team met to discuss trends that each pair noticed across workshops and TLP sessions, with a focus on changes that occurred over time.

To complement the analysis of the PSC workshops and TLP sessions, a team of three researchers independently went through the Cycle 1 and Cycle 2 interviews for all TLs. Each researcher focused on 1 year, identifying comments in the interviews about the activities from the TLPs that TLs described as being helpful and how they described having grown as a teacher leader. Then, the researchers shared findings with each other and identified patterns and commonalities across the years.

In this stage, we leveraged Prediger et al.'s (2019) model to examine the patterns in the PSC workshops for the content and resources introduced in the TLP sessions. We examined patterns of nesting, lifting, and unpacking, and looked for anomalies that indicated adaptations to the models reflecting the needs of individual schools and responses to district priorities.

Findings

In considering the 3T-Model to understand TL development in this project, we focus on two of the three tetrahedrons—the Facilitator PD (FPD) level and the Teacher PD (TPD) level. The FPD level is equivalent to the TLP sessions and aligns with Research Question 1, while the TPD level is equivalent to the school-based PSC workshops that the TLs led and aligns with Research Question 2. Both levels are encompassed in the Learning to Lead cycle (see Fig. 2).

Changes in the Enactment of the TLP Model

With Research Question 1 we ask, "How did the TLP sessions evolve over time?" The TLP and PSC models were introduced into this project as models that can, and should, be adapted to meet the needs of the district and specific school contexts so we anticipated changes to both models as the district Facilitator Educators and the TLs increased their ownership of the work. The Facilitator Educators changed often in the first 2 years and this interrupted the ownership process. We also observed that the key structural elements of the TLP model stayed relatively constant: there were six sessions each school year; "Do the Math" was a dominant activity; the use of video-based discussions (VBDs) remained prominent in sessions 2, 3, 5, and 6; and TLs typically had time to plan and rehearse their PSC workshops. On the other hand, the content emphasized and resources used during TLP sessions changed annually to reflect the priorities of the district overall and the mathematics department in particular.

To understand the stability of the structural elements of the TLP model, we reviewed the Learning to Lead Cycle and how the TLP sessions changed over time. We also

looked at the choices of content and resources for TLP sessions each year. (A summary table of the pedagogies, content, and resources is included in the supplemental materials.)

Pedagogies of Investigation: Do the Math and Video-Based Discussions

In Phase 1, *we lifted* "Do the Math" tasks from the district curriculum with an emphasis on multiple solution strategies and multiple representations. In interviews, a number of TLs noted how helpful it was to see multiple perspectives on problem solving, to learn new strategies from others, and to do mathematics enthusiastically with colleagues. Specifically, TLs found it helpful to see others' perspectives, as this allowed them to see new strategies for solving a problem and to get feedback on their own strategies. Six TLs found it helpful to have a focus question during the Do the Math activity. TLs also mentioned that they found joy in doing mathematics together and were inspired by their peers' enthusiasm for doing so. To support the deepening of content knowledge during Do the Math, the Facilitator Educators frequently incorporated resources such as protocols for looking at solutions and sample student work for those tasks.

Pedagogies of Investigation: Unpacking the Facilitation

In Phase 2 of the Learning to Lead Cycle, the TLs changed roles from "teacher learner" to "facilitator learner" so they could consider the activity they had just completed as a teacher learner from the perspective of a facilitator. The Facilitator Educators (members of the Research-Practice Partnership team) described how they had planned and facilitated the activity so TLs could understand the goals for the activity and the reasoning for particular facilitation moves. As one TL commented in Year 1, "We saw the humanism in the whole thing, the transparency around the thinking." This pedagogy demystified the background thinking in professional learning and made effective facilitation seem more achievable. The following comment elaborates on the usefulness of this approach for TLs' development:

> What stands out to me is that I really liked when one of you … present[s] something where we do some type of activity. Then after that activity I think [author] would ask whoever was leading the activity, "What was your thought process?" or "How did you prepare for this?" I think it was cool to just keep switching our hats and our roles to see, what does it look like to be presented to? What does it look like to prepare for this presentation? And just having a lot of time to reflect. It just changes my perspective on things. (TL, Year 1, Cycle 1 Interview)

Pedagogies of Enactment: Planning and Practice

Midway through Year 1, we realized that the TLs were struggling to design PSC workshops that emphasized the nature of the task-based curriculum and its associated instructional strategies. At that point, we incorporated "rehearsals" as specific pedagogy of enactment in Phase 3 of the Learning to Lead Cycle. Once they were

introduced, rehearsals continued to be a dominant structure within TLP sessions. In interviews, we learned that TLs considered rehearsals the most helpful structure of the sessions:

> The rehearsal was great. That one especially 'cause [Facilitator Educator] pulled me aside and was like, "hey, the people you're going to be rehearsing with, they've never done this before so feel free, you're going to go first and just know that you're going to lead the way here because this is all new." That gave me a weird confidence that made me feel I could go wherever with it and it also made me want to do it pretty darn well. It felt a little bit more pressure than just a first draft run through, but that was healthy pressure, if you will. (TL, Year 3, Cycle 1 Interview)

TLs appreciated getting feedback during rehearsals because it helped them think through their plans and anticipate colleagues' reactions. They valued the structure of the feedback protocol that was used, which asked others to offer points of praise, ask clarifying questions, and suggest points of polish, as this helped TLs anticipate and understand the needs of their teacher colleagues. In interviews, when asked about the valuable components of their experience, at least half of the TLs said they appreciated time to co-plan for their workshops. In addition, they found it useful to see the facilitation strategies of other TLs.

Pedagogies of Enactment: Conducting PSC Workshops

Conducting PSC workshops is one of the most stable components of the TLP model and is the primary pedagogy of enactment in Phase 4. During Year 1, if a school stayed active in the program, they typically conducted all six workshops (two schools only completed five workshops). In Year 2, the majority of participating schools conducted two of three workshops in the second cycle and most schools did not conduct a third workshop. In Year 3, the majority of schools conducted five of six workshops, although 14 of 47 workshops did not include the signature structures of Do the Math or VBD. When TLs varied the structure of their workshops, it was often a reflection of school priorities or a shortage of time.

Content and Resources

Our TPD content and FPD content extended beyond the mathematics in the curriculum to the classroom level tetrahedron. This content included concepts related to agency, authority and identity, formative assessment, educational equity, and TLs as influencers and advocates. The TPD and FPD resources comprised the tools, strategies, and activities designed to support the concepts and skills the TLs or teachers engaged in during their PD experiences. Some common resources were mathematics tasks, video clips, sample focus questions, and district documents such as the Dimensions of Teaching and Learning. The TLs developed their understanding of these resources to strengthen their work as Teacher Leaders in a variety of ways beyond participating in Do the Math and VBD structures. They also developed specific strategies for leading PSC workshops. These specific strategies include setting norms for viewing videos,

developing focal and back-up questions, learning, and discussing facilitation techniques.

Over the 3-year period of the project, the greatest area of evolution in TLP sessions occurred in the arena of content and resources. Each year had key areas of focus, and the content and resources used were modified over time to reflect the district's priorities, the mathematics team's goals, and TLs' interests and school goals. In Year 1, the mathematics curriculum and the three-part Dimensions of Teaching and Learning (especially agency, authority, and identity) were central, so the district priorities were unpacked in FPD content and then used as the TPD content. FPD content in Year 2 again focused on the TPD content as the Dimensions document stayed central but shifted in emphasis from agency, authority, and identity to access to content. Formative assessment and attention to English Learners also began to emerge as important concepts. By Year 3, the district Facilitator Educators organized key activities around developing teacher and student voice as well as understanding how TLs can be influencers and advocates, so some FTP content focused on elements of the TPD tetrahedron other than TPD content.

Teacher Leader Development as Evidenced in PSC Workshops

Research Question 2 explores different aspects of TL growth and development by examining how the TLs adopted and adapted specific ideas from the PSC model over time based on what we observed in the school-based workshops. During TLP sessions, TLs experienced pedagogies of practice and were introduced to content and resources to develop their PSC workshops. In examining PSC workshop videos, we traced the use of those pedagogies, content, and resources for 3 years to describe the ways in which TLs adopted and adapted their experiences from the TLP sessions.

To study the TLs' adoption and adaptation of aspects of the PSC model, we first consider what would be expected in a typical set of PSC workshops. If all schools were implementing the PSC model as originally designed, we would expect to see the Do the Math structure in the first workshop of each PSC cycle and the combination of Do the Math and VBD structures in the second and third workshops of each cycle. This expected pattern is evident in Year 1 and the first half of Year 2. The pattern falls off quite a bit in the second half of Year 2, in part due to teachers in the school mathematics departments preparing for a state test and in part due to a shift in the schedule for the third TLP session in each cycle. The third TLP sessions were scheduled as short, after-school sessions. Attendance was low during these sessions, and there was no time allotted to prepare or rehearse a school-based workshop. By Year 3, the content focus and use of key structures during the TLP sessions had shifted even more and VBDs were not used. However, the majority of PSC workshops, particularly in the first cycle, continued to reflect the pattern of starting a cycle with the Do the Math structure followed by workshops that include a VBD.

Use and Adaptations of Do the Math

A typical first workshop in each PSC cycle involves teachers solving a mathematics task and then considering the mathematics concepts and the range of possible solution

strategies for the task (a Do the Math activity). Student work may be incorporated as a resource for examining the concepts and strategies central to the task.

We found that TLs typically included a Do the Math activity in the first workshop of each cycle in Years 1 and 2, but this activity occurred somewhat less frequently in Year 3. When schools did not include a Do the Math activity, it was more often during the second PSC cycle of the year. For example, in Year 3, six of eight schools did a Do the Math activity during their first cycle of PSC workshops, but only three schools included a Do the Math activity in their second cycle. Some common alternatives included conducting a VBD or discussing a topic of interest such as amplifying student voice or considering adaptations for students in special education.

Whether or not a Do the Math activity was done during the first PSC workshop in a cycle appeared to be more contingent on decisions made by individual TLs than what was modeled in the TLP sessions. For example, School 8 had a site goal of improving instruction for English Learners and made multiple adaptations to the PSC workshops, both in structure and content, to reflect that emphasis. When TLs included a Do the Math activity in the first workshop of a cycle, the emphasis often mirrored the emphasis of the same activity in the TLP session. Over the course of the 3 years, the TLs modified their goals for the Do the Math structure. Initial goals often centered on a critique of student work. These goals shifted to focus on implications of the task design for teaching, which was more consistent with the learning goals of the TLP sessions.

Video-Based Discussions and TL Development

As designed, the PSC model includes two workshops in each cycle that use video-based discussions (VBDs). VBDs are crafted to help teachers focus on a specific aspect of learning or teaching mathematics and are dependent on understanding the mathematics task that is part of the video clip. When designing a PSC workshop that includes a VBD, we would expect TLs to use *nesting.* That is, TLs use video clips to account for the multifaceted complexities of a classroom—even a very short clip can show student talk, student work, and teacher instruction. By nesting a piece of a classroom experience within a PD activity, participating teachers can consider mathematical thinking more deeply.

Variability is the key word in understanding how VBDs were implemented at schools in this study. Some of this variability is seen in how structures were used over the 3 years of data analyzed for this paper and some of the variability is evident in the content and resources used to set up VBDs. For example, the mathematics tasks that were the focus of VBDs varied across schools and workshops. During Year 1, we found that most TLs started the VBDs in their workshops with no focal question followed by a second viewing, which was framed by a focal question. When TLs used focal questions, they were very similar (sometimes identical) to what had been modeled in the TLP session. For example, we emphasized the concepts of agency and authority, from the district's Dimensions, in the last TLP session of the cycle. We then saw extensive use of the focal question "What does authority look like in the math classroom?" during the PSC workshops—a question that TLs grappled with during their TLP session.

In Year 2, there was greater variability in the format of the schools' workshops, the types of VBD focal questions, and how the mathematics tasks were used with the

VBDs. During this year TLs designed their PSC workshops using a wide range of focal questions that did not mirror what had been done in the TLP session and often focused on student learning, e.g., "How do students use shared learning to challenge and support each other?" or "What evidence do we see of productive struggle?" These questions related loosely to the content of the TLP sessions but tended to emphasize an issue or goal that was specific to the school.

In Year 3, TLs designed workshops that were more similar in content to each other, as evidenced in the second PSC workshop in the second cycle, where the focal questions across all sites had to do with either "amplifying student voice" or "pushing students' ideas." These ideas reflected the content of the TLP session. On the other hand, the third workshops in the second cycle were most divergent among schools this year with format, content, and focus questions all varying widely. Based on comments in interviews, the TLs acknowledged the importance of stronger planning and, in particular, making each activity build toward a stated goal through backwards planning. In addition, they noted that they became more organized and efficient at planning over time and in general their workshops became more valuable. This degree of goal-oriented and intentional planning resulted in VBD workshops that were more reflective of priorities that mattered at the school site.

Overall, it appears that schools were more tightly aligned in their use of VBDs toward the beginning of the project (2016–17). Over time, we saw TLs making more site-specific decisions about the focus of their VBDs (content adaptations) and whether to conduct VBDs at all (structural adaptations). These adaptations occurred as the design and facilitation of the TLP sessions shifted from the research team to the district team, more ideas were introduced during the TLP sessions, and TLs grew in their confidence with planning and conducting VBDs.

VBDs provided a valuable discussion structure and resulted in a greater interest for TLs to understand facilitation moves that support adult learning. For example, identifying key questions in advance helped TLs steer conversations in ways that fostered professional community. During their interviews, the TLs described the need to pay attention to the participation patterns of their colleagues in order to build community. Many TLs worked to design workshops in which more teachers contributed. This goal became important as the TLs recognized their PSC workshops as a site of adult learning where increased, or higher quality, participation from colleagues was very desirable. In addition, as TLs gained more confidence as facilitators, they recognized that they did not need to be experts about everything. Rather they could make the space available for colleagues to grapple with instructional challenges.

Discussion and Conclusion

Drawing upon our conceptual framework, we analyzed 3 years of data to understand how to support the professional growth of school-based teacher leaders in their roles as mathematics teacher educators with teachers in their schools. By examining the role of the pedagogies of investigation and enactment (Grossman et al., 2009) as used within the Learning to Lead Cycle, as well as using Prediger et al.'s (2019) 3T-Model to guide this analysis, we were able to see how pedagogy, content, and resources contributed to teacher leader development as these features of TPD were lifted, nested, and unpacked

in designing and enacting the FPD model—for example by focusing on the content of PSC workshops in the TLP sessions (nesting) and modeling (lifting) and debriefing (unpacking) Do the Math and VBDs. By looking across 3 years of workshops conducted by TLs at eight different schools, these analyses are necessarily at a relatively large grain size.

Because both the TLP and PSC models are designed to be adapted by users, we expected to see modifications made over time (Koellner & Jacobs, 2015). As the TLs increased their focus on adapting the design of their PSC workshops in response to the goals of their schools and needs of their colleagues, they became more comfortable seeing the PSC model as a malleable PD model through which they could address these priorities. Our analysis reveals an interesting blend of aspects of the models that did not change over time as well as changes that were driven by either intention or disruption. For example, we saw stability in how the Facilitator Educators (the district mathematics leaders) used modeling and debriefing (pedagogies of investigation), and planning and rehearsals (pedagogies of enactment) to support TLs' preparation for conducting activities central to the PSC model, such as Do the Math and VBDs.

In their interviews, TLs explicitly named rehearsals and Do the Math as strategies that were key to their growth. The role of rehearsals grew stronger over time by focusing on challenging aspects of workshop and incorporating feedback protocols. Do the Math was used from the beginning and remained central throughout the project. TLs took ownership of the way they led Do the Math by adapting the substance to goals of the school. TLs also continued to lead VBDs, and they made them their own by using video clips and launching questions that reflected their school goals. As Johnson, Severance, Penuel and Leary (2016) point out with their "design tensions framework", it is critical within RPP work that there is space for evolution of key activities so that district and school priorities are met and adjustments are made as priorities evolve. The choices TLs made, such as how to adapt Do the Math activities and which video clips and questions to use in VBDs to address their school goals, all reflected this type of evolution.

In addition to changes in the content that were emphasized in PSC workshops, we noticed changes in how the teachers learned from each other that reflected Nelson, Slavit, Perkins and Hathorn's research (2008) on the development of professional learning communities where they found that developing an inquiry stance was challenged by difficulties related to communication that could be mediated by the introduction of the explicit use of norms and protocols. Prior to their engagement in this project, many TLs considered department meetings a time for updates and announcements rather than collaborative learning. By periodically using the PSC model as the structure for their meetings, TLs recognized that they could create places of adult learning. To make this shift, many TLs had to learn skills to facilitate equity of voice in an adult learning environment.

School-based characteristics also affected leadership development. Like other researchers who have engaged in RPPs for mathematics professional learning, we saw evidence of how school and department culture influenced the ways in which TLs were able to facilitate PSC workshops (Givvin & Santagata, 2011; Santagata, Kersting, Givvin & Stigler, 2011). A typical challenge came when principals launched a school-wide initiative, then TLs were challenged to find time to take up key features of the PSC, such as Do the Math or VBDs.

Finally, over the course of 3 years, the project experienced disruptive changes that caused inconsistencies in how key elements of the PSC model were introduced in the TLP sessions and taken up in PSC workshops as the core district Facilitator Educator team formed and reformed. The district team stabilized in the year after the data reported here, and the TLP sessions are now more focused and have a more coherent arc of content. This type of district turnover and its disruptive impact on adoption of innovations is a common challenge for RPPs (Coburn, Penuel & Geil, 2013).

Broader Implications for the Field

Prediger et al. (2019) intended the 3-T model to capture the complexity of learning and teaching at multiple levels to inform the design of and research into PD, and to uncover gaps in the literature. By viewing patterns in the evolution of TLP sessions and PSC workshops through the lens of the relationships illustrated by the 3-T model and the Learning to Lead cycle, we have a window into the specific complexities of teacher leadership development. This work adds to the literature by suggesting that these two frameworks, when used in combination, provide powerful insights about effective strategies for supporting PD facilitators to incorporate TPD content and skills introduced in FPD sessions into their own practice. When TPD activities and resources were *nested* in FPD sessions, modeling was used to introduce the activities and resources (Phase 1 of the Learning to Lead Cycle), debriefing was used to *unpack* those activities and resources (Phase 2), and rehearsing was used by PD facilitators to practice those activities and resources (Phase 3), we found that the TLs (the PD facilitators in our project) consistently adopted or adapted those elements into their own practice to align with their school goals and needs. In this way, we could successfully apply the lifting, nesting, and unpacking strategies for the FPD design (and partially for researching its effects). This application of integrating the Learning to Lead Cycle with the 3-T model adds to the knowledge base about developing the leadership capacity of experienced teachers.

Our results also illustrate the value of rehearsals for developing experienced teachers' leadership capacity. In recent years, rehearsals have become more prominent in teacher education (c.f. Kazemi et al., 2016). Most of this work focuses on preservice and novice teachers' rehearsals of classroom lessons. By contrast in this study, experienced teachers rehearsed potentially challenging aspects of the workshops they planned to lead. The rehearsals had an added level of authenticity compared to preservice teachers' rehearsals of classroom lessons because the TLs practiced with other TLs who were similar to the colleagues at their school sites.

In addition, this project contributes to the field's understanding of Research Practice Partnerships in several ways. RPPs are a relatively new approach to research in education. Given their potential for fostering lasting educational change, Coburn and Penuel (2016) argue for the importance of additional research on the dynamics of RPPs and how their designs and strategies can address the challenges they face. Our project offers several insights in this regard. As one example, it provides an illustration of working across multiple levels of an educational system, which is important for building capacity for sustainable improvements in classroom learning and teaching (Penuel, 2018). Specifically, the patterns of consistency and change over time suggest

that to create and maintain professional learning communities at school sites across a district over multiple years, it is useful to have intertwined models that focus on both teacher leader development at the district level (FPD) and professional development at the schools (TPD).

The project also provides an example of mutualism—sustained interaction that benefits both researchers and practitioners—which Coburn et al. (2013) identify as a key feature of successful RPPs. Mutualism came into play as we responded to changes in personnel and renegotiated decisions about who would be responsible for designing and leading the TLP sessions. It was also evident in the support the research team provided to district TLP leader and TLs as they adapted the TLP and PSC models to address changing district and school priorities. Henrick, Cobb, Penuel, Jackson and Clark (2017, p. 12) emphasize that "high turnover in practice organizations can influence an RPP's impact." For this project, the impact of turnover at the district level was most noticeable in the inconsistency about the design and emphasis in the TLP sessions. Our experiences highlight the importance of both anticipating and attending to the inevitable changes in key personnel, especially during a multi-year project. These changes create challenges for developing the strong relationships with school district personnel that are necessary for sustained change. Our work indicates that if researchers are going to influence the links from professional development to instruction to improved mathematics learning opportunities for all students, it is critical to address these challenges, and to focus on understanding and creating sustained, systemic approaches for building capacity at both the district and school site levels.

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