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Using Community Artifacts to Support Novice Math Teacher Educators in Teaching Prospective Teachers

Alison Castro Superfine[1] · Kathleen Pitvorec[1]

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Abstract

Despite the centrality of math teacher educators (MTEs) in teacher education, we know little about the nature of professional learning opportunities for MTEs to develop and enhance the knowledge needed to teach prospective teachers. Existing models for supporting MTEs in developing their knowledge and practice do not address how to prepare novice MTEs in initially learning to teach prospective teachers. We present a professional learning model we have been pursuing for supporting novice MTEs and the generation of and role for community artifacts, namely lesson plans, in that model. We outline the process by which we implement, analyze, and collectively revise lesson plans so that they are continually improved over time to serve as artifacts that better instantiate what members of the local community are learning about how to support novice MTEs through identification of their problems of practice. Finally, we problematize the model we are investigating and propose implications of this model and questions raised by our work with the goal of inviting further discussion about supporting novice MTEs.

Keywords Elementary prospective teachers · Lesson plans · Math teacher educator · Novices

The importance of preparing prospective teachers to teach math effectively has long been recognized. Indeed, the issue of math teacher preparation has been the subject of considerable debate for at least two decades (see Adler, Ball, Krainer, Lin, & Novotna, 2005; Brown & Borko, 1992). Over only the past decade, however, has research on math teacher educators (MTEs), individuals who are primarily responsible for the

✉ Alison Castro Superfine
amcastro@uic.edu

Kathleen Pitvorec
kaptivor@uic.edu

[1] Learning Sciences Research Institute, University of Illinois at Chicago, 1240 W. Harrison Street, Suite 1535, Chicago, IL 60607, USA

mathematical preparation of prospective teachers (PTs), begun to emerge (see Jaworski & Huang, 2014; Jaworski & Wood, 2008). Despite the centrality of MTEs in teacher education, we know little about the nature of professional learning opportunities for MTEs to develop and enhance the knowledge needed to teach PTs. This is particularly important as MTEs who teach PTs include mathematicians, graduate students, math educators, and classroom teachers who not only have different professional backgrounds, but who are often not professionally prepared for the work of teaching teachers.

There are few, if any, formal structures for MTEs, particularly novice MTEs, to learn to do the work of teaching teachers. Some exceptions might be conducting research in math teacher education as a form of professional development (e.g. Chen, Lin & Yang, 2018; Rowland, Turner & Thwaites, 2014), or engaging in self-study as a means by which to further develop one's knowledge (e.g. Garcia, Sanchez, & Escudero, 2007; Taylan & da Ponte, 2016). Other examples might include ones in which experienced MTEs provide formal professional learning opportunities for their colleagues (Castro Superfine & Li, 2014; Even, 1999), or establish communities of MTEs that promote collective reflection on MTE programs and practices (e.g. Jaworski, 2003; Zaslavsky & Leikin, 2004). Models like these hold promise for supporting experienced MTEs, but they do not particularly address how to prepare novice MTEs in initially learning to teach PTs. Indeed, novices do not have the same wealth of knowledge and experience that more expert practitioners have to draw on when teaching (e.g. Borko & Livingston, 1989).

Professional learning models for addressing the particular needs of novice MTEs also need to be explored and developed. For example, one option might be to enlist experienced MTEs to develop graduate-level programs for preparing novice MTEs to teach PTs. However, currently, in the USA for example, there are few, if any, formal graduate programs devoted exclusively to the professional preparation of teacher educators (Goodwin, Smith, Souto-Manning, Cheruvu, Tan, Reed, & Taveras, 2014). Indeed, Goodwin et al. (2014) found that the majority of the teacher educators they surveyed learned to teach PTs "on the job" with no formal preparation for the work. Another option might include formal professional development programs that integrate academic knowledge learned in graduate programs with knowledge learned in practice (e.g. conducting research) as in Even (1999). Such programs, however, do not include opportunities for novices to integrate their knowledge with learning to teach teachers alongside more experienced MTEs.

The professional learning model we have been investigating for the preparation of novice MTEs aims to work within existing graduate-level programs in math education to create apprenticeship-type opportunities within our local community of MTEs (see Castro Superfine & Wagreich, 2010). We have identified several advantages to this model. First, working within existing graduate programs means the model does not require much in the way of additional resources nor programmatic changes. Second, including explicit work with novices potentially strengthens the local MTE community, transforming it into an inquiry community (Jaworski, 2003) by providing opportunities for community members to collectively examine their own practice as well as work towards the common goal of apprenticing novices. Finally, generating and archiving collaboratively designed artifacts (lesson plans and related handouts and power point presentations) that improve as a result of iterations of an implementation–analysis–

revision cycle promotes the establishment of a knowledge-building system—a system present outside of education that Hiebert and Morris (2009) describe as a potentially powerful context for improving the work of MTEs over time.

In this paper, we present a study embedded in the context of a professional learning model the local MTE community has been pursuing for supporting novice MTEs. The model draws on features from the aforementioned examples in its focus on collective reflection (Jaworski, 2003), and the integration of academic knowledge with knowledge learned in practice (Even, 1999). This study particularly focuses on the creation and revision of, and role for lesson plans within the professional learning model. We specifically attend to how a novice MTE's problems of practice might inform the development and revision of lesson plans which then reflexively support MTEs in managing future problems of practice in their early efforts to teach a PT math content course. We focus on the following research question: How can the analysis of one novice MTE's problems of practice inform the creation of lesson plans that might support novice MTEs as they learn the practice of teaching PTs? In answering this question, we first outline the general process by which the local MTE community has engaged in implementing, analyzing, and revising lesson plans. We then specifically delineate the process that is the focus of this study and was developed by a subset of our local MTE community. We describe what we believe might be promising outcomes of this work, and outline some of the limitations. Finally, we propose implications of and questions raised by our work with the goal of inviting further discussion about improving professional learning for novice MTEs.

Framework

Learning to Teach Within a Community of Practice

Consistent with apprenticeship models (e.g. Lave & Wenger, 1991), we assume that learning is a social phenomenon situated within contexts and communities (Wenger, 1998). For a community, coming to know in that community means developing competence at enacting practices valued by the community, where practices refer to those things that are done constantly and habitually in the service of the goals of the community. In our work, we are concerned with two levels of community—a broader global community of MTEs and a local community—each of which shares related goals, practices, and tools and artifacts. The broader MTE community includes professionals who work with teachers and/or PTs to develop and improve the teaching of math (Jaworski & Wood, 2008).

Individuals working together in a particular institutional context differentiates our local MTE community from the broader MTE community. Generally speaking, the goals of the local community align with those of the broader community in that they include a focus on improving the preparation of PTs through the lens of various recommendations for what PTs need to know (e.g. AMTE, 2017). In our local community, the goals of the institution inform and influence the local MTE community goals. For example, the broader MTE community includes the goal of preparing teachers to meet the needs of diverse learners. The institution providing the context for our local MTE community strives to prepare teachers for the unique challenges of

teaching in urban contexts—which includes not only preparation for meeting the needs of diverse learners but also an emphasis on seeing *all* students as capable and encouraging a growth mindset for both teachers and students (Boaler, 2015; Dweck, 2006). The broader MTE community goal gets refined in the local community to better focus on preparing PTs specifically for urban contexts.

Similar to the group surveyed by Masingila, Olanoff, and Kwaka (2012), our local MTE community includes math educators, learning scientists, mathematicians, and graduate students. We agree with Seaman and Szydlik (2007) who explain that it is crucial for MTEs who prepare teachers to teach math have the mathematical sophistication of mathematicians as well as the specialized content knowledge for teaching. Traditionally, an individual MTE rarely possesses both in equal measure (Seaman & Szydlik, 2007). We have pursued enhancing the preparedness of MTEs by establishing a local community of MTEs who bring expertise and experience from all three categories described by Bergsten and Grevholm (2008)—that is, mathematicians with mathematical sophistication, math educators with pedagogical expertise, and MTEs who have classroom teaching experience at the elementary, middle, and high school levels. The variety of expertise and experience in our local community then enriches our joint activities of planning, teaching, and reflecting (Jaworski, 2003).

This local MTE community constitutes a community of practice in several ways. First, the local MTE community has the shared goal of preparing preservice teachers to teach in urban environments. In order to achieve this goal, one set of shared practices in the community is related to the iterative design and implementation of a math content course for PTs. These practices include articulating learning goals for PTs; developing and revising lesson plans, slides, and handouts; and implementing lesson plans so as to address the intended learning goals. Finally, the local MTE community has a shared history, having been brought into existence 3 years prior to the current study. Consistent with communities of practice, new members of the community, in particular, novice MTEs are apprenticed into the practices, working alongside current members as they increasingly engage in the practices of the community.

Building on Lave and Wenger's (1991) notion of legitimate peripheral participation in a community of practice, learning is visible in the changing practices of novices as they become fuller participants of a community, in this case, the local community of MTEs. However, the language, goals, tools, and activities that are central to the domain of math teacher education need to be made visible to novices—for example, made visible in the context of lesson plans—in order for novices to gain access to and participate in the practices of the community. As novices move to fuller participation, they take on more responsibility and thereby, eventually become a fully participating member of the community. Novices not only need access to the domain knowledge, but they need experiences with the practices for teaching PTs that are valued and useful in the community. Evidence of novice learning in this context takes the form of novices engaging in and contributing to the valued practices of the community (Wenger, 1998) and increasingly experiencing their participation as meaningful.

In the model our local community has been developing, the novice MTEs first serve as apprentices working with more expert MTEs in the local community. The novice MTEs may begin their engagement in the local community by reviewing lesson plans, observing lessons, and attending planning and debriefing meetings associated with the lessons. As the novice MTEs move towards fuller participation, they engage in the

local community's collaborative inquiry process by contributing to the planning before and reflection after teaching lessons (Ball, Sleep, Boerst & Bass, 2009). They then gradually take over all of the responsibilities for teaching PTs. As novice MTEs transition from peripheral observer to taking full responsibility for teaching the PT courses, they encounter some novel and some predictable challenges or problems of practice.

Problems of Practice

In the current study, two members of the local community (referred to hereafter as the *research team*, and also as the authors) pursued a deeper understanding of the structure and content of the lesson plans by investigating one MTE's problems of practice. The research team focused on problems of practice because as the novice MTE (for the year in which this study took place) began to assume full teaching responsibilities, her contributions to the weekly planning meetings tended to take the form of articulating problems she had experienced.

Following Lampert (2001), we conceive of problems of practice as a variety of decisions and challenges that influence the course of instruction, and ultimately, students' opportunities to learn. Consistent with Lampert (2001), we focus on the challenges of making these decisions and conceptualize these challenges—anticipated and unanticipated—as the "problems" in the practice of teaching math. Parallels in these problems exist across levels of teaching. For example, just as in their future practice, PTs might face the problem of anticipating their students' errors and misconceptions, an MTE might face a similar problem in anticipating PT errors and misconceptions. Relatedly, the MTE might face the problem of supporting PTs in identifying their future students' misconceptions. For teachers, addressing problems of practice is further complicated because they often occur simultaneously and not sequentially. Problems of practice were relevant in our community discussions as the group considered how to address the problems in revisions to community artifacts, including lesson plans, slides, and handouts.

Lesson Plans as Artifacts Reifying Shared Knowledge Within a Community

As described by Lave and Wenger (1991) and Wenger (1998), knowledge within the local MTE community is reified in the practices of the community, as well as in the tools and artifacts (e.g. lesson plans) that are used to engage in these practices. In our local community, reification of the knowledge MTEs require to achieve the local community's goals takes several forms, including course syllabi, lesson plans, slides, and handouts, that are all important as part of the (re)design of PT courses. MTEs both create and use such tools and artifacts as they engage in the practices of the community. In our local community, these tools and artifacts have evolved, being shaped and revised in an iterative cycle of implementation, analysis, and revision.

Following Hiebert and Morris (2009), we emphasize revising our tools and artifacts (i.e. lesson plans) and continually testing our revisions as an essential feature of our knowledge-building system—a system that makes the local community knowledge accessible to novice MTEs. Similar to how Hiebert and Morris (2009) and Zaslavsky and Leikin (2004) describe how local communities of MTEs meet to discuss and

generate lesson plans, a subset of MTEs from our local community teach from lesson plans they have jointly designed and observe the teaching. The community then discusses and debriefs the observations, and revises lesson plans and other artifacts accordingly. These iteratively revised lesson plans serve to reify the knowledge and valued practices of the community. In the next section, we present an additional process for revising lessons that the research team developed based on information derived from the problems of practice one novice MTE encountered during her first semester of teaching.

Methods

Study Context

As described earlier, this study is situated in a local community of MTEs. The course that is the focus of this study took place at a large urban US university and included twenty-nine 110-min class periods. Each section of the course included 15–30, mostly female, PTs. The course was structured around learning math and connecting the math to the work of teaching.

During this study, the local community included two math educators, a learning scientist, and a mathematician, all of whom had previously served as primary instructors for the required content courses for PTs. In addition, there was one novice MTE. The two math educators (also with backgrounds in the learning sciences and math) comprise the research team (and authors) for this study. The novice MTE who participated in the process, Kristina, was new to the community.[1] She was enrolled in a graduate program focused on math education. She had completed coursework equivalent to that of a graduate student in pure math, and had experience teaching K-12 math and large undergraduate math courses such as pre-calculus and calculus.

Process for Reifying Local MTE Community Knowledge

Within the local MTE community, the process for lesson plan revisions includes comparing written lesson plans with descriptions of the implementation of those plans and then using this comparison as a tool for meeting one of their primary shared goals—better understanding the ways in which course activities can and do support PTs' learning about math. This iterative process of course design is accomplished through regular weekly meetings of the local community to review, reflect on, and revise lesson plans created in the previous revision cycle. Based on discussions of what occurred in the previous week's classes and of the overall course goals, each meeting results in the development of prepared lesson plans and lesson artifacts for the following week. Several iterations of lesson plans have been produced over the course of 3 years since the inception of the local community.

Because lessons plans function as an artifact of the collective decision-making of the local MTE community, lesson plans also provide a resource for making sense of the valued community practices. For example, lesson plans articulate specific learning

[1] Pseudonyms are used to protect the identities of all MTEs and PTs.

goals and then provide prompts designed to leverage content-rich discussions related to those goals. Lesson plans include activities that elicit possible PT misconceptions and then provide suggestions for addressing those misconceptions. Lesson plans emerge as a key resource for supporting the engagement of MTEs new to the community in the practices valued by the community.

As a new community member in her first semester, Kristina observed more experienced MTEs teaching the lessons and participated peripherally in planning meetings. In addition, Kristina had opportunities to co-teach class sessions, apprenticing into the valued practices of the community. Kristina gradually moved to a more central role in the planning and implementation of the course. At the start of her second semester as a community member, Kristina transitioned to fully participating in the practices of the community as she assumed primary responsibility for teaching the same PT content course in which she had apprenticed. During the second semester, the experienced MTEs observed her lessons to provide multiple perspectives for discussion. One such lesson discussion revolved around the previously taught Candy Box Problem[2]:

Candy Box Problem

> There was a box of candy on the table. Thuy was hungry because she hadn't had breakfast, so she ate half the candy. Then Hana came along and noticed the candy. She thought it looked good and had not packed a lunch so she took two-thirds of what was left in the box. Alicia came by and decided to take three-fourths of the remaining candies with her to her next class. Then Anayeli came dashing up and took one piece of candy to munch on. When Mia looked at the candy box, she saw that there was just one piece of candy left. "How many pieces of candy were there in the box to begin with?" she asked Thuy suspiciously.

Table 1 shows an example of the lesson plan instructions for implementing the Candy Box Problem (minus the information about how to group students and the length of the lesson, which was included in the lesson plan) that guided Kristina's implementation of the lesson.

In the weekly meeting with the local MTE community, Kristina described one of the challenges she encountered in the lesson—that she could see in the lesson plan *why* she was supposed to engage PTs around particular ideas, she could not identify information about *how* to engage them. The lesson plan did not suggest specific questions for engaging PTs. In response to this challenge, the local MTE community added prompts and questions like those below, hoping the revised lesson plan might better support a future novice MTE[3]

[2] This task comes from materials developed by a different community of MTEs of which the first author was a member prior to joining the local community in the current study.

[3] Note that we provide a more complete excerpt of the revised lesson plan instructions for the Candy Box Problem in the results section of this paper.

Table 1 Excerpt from The Candy Box problem lesson plan that Kristina implemented

Details	Cautious points
Candy Box Problem Read it over. Discuss what it is asking. Write in notebook first impressions of the problem, but not yet work on it. Any instincts, reactions, feelings? Ideas about what might be involved? Some sharing of these comments? *Goals:* To develop representations for the problem To build correspondences among representations To begin to develop criteria for what counts as a mathematical explanation Try to come up with at least two different ways to represent the problem and solve it, and be able to show how the two relate. Focus on clear explanations of representations and how those lead to solutions, asking questions about others' solutions, justifying mathematical reasonableness of solutions Explain why work on mapping matters and the level of detail with which we will try to build such correspondences For algebraic representation: Consider what sort of representation this language affords: What does it make visible? What do the other representations make visible? Compare. Try to "demystify" algebraic notation. Think of it as language with certain kinds of power.	Try to make sure that students do not treat this as a problem to use with children, or as what we are doing as directly about that. Awareness of how people may react to an algebraic representation of the problem.

- Describe what you think you have to do in the problem and how you might begin to solve the problem. *Sample response: You have to figure out how many candies there were before any were taken. You might work backwards from the 1 that is left.*
- What else might we want or need to know about this solution strategy based on what we see or do not see in the representation?
- Explain which steps of the solution strategy are visible (explicit) in the representation being used?
- Describe some mathematical idea related to the solution strategy that is NOT visible (or is implicit rather than explicit) in the representation? (explain)

Thus, the lesson plans served as an evolving reification of the knowledge, experiences, and valued practices of the community, more fully representing the public and shareable accumulated knowledge (Hiebert & Morris, 2009). Moreover, lessons plans functioned as artifacts that might ultimately support novice MTEs in perceiving the practices of the community as meaningful.

As Kristina participated more fully in the community, both as the primary instructor for the course (during the second semester) and by sharing her insights and experiences at the weekly meetings, the research team noted that Kristina often contributed by sharing what we would later come to describe as her problems of practice—activities and/or related instructions in the lesson plan that posed challenges for her. Listening to

Kristina describe her problems of practice led to the authors (research team) to hypothesize that these problems potentially elucidate components of a knowledge base related to the content of lesson plans. We pursued a more systematic investigation of Kristina's problems of practice, speculating that such an investigation might result in productive lessons plan revisions. In the next sections, we describe the data we collected around Kristina's problems of practice and our analysis of the data.

Data Collection

As the weekly planning meetings progressed, the research team speculated that, if we could identify types of information as well as specific information that might better prepare a novice MTE for developing practices related to teaching PTs, we could include this information in future lesson plans. We hypothesized that analyzing Kristina's reflections on her problems of practice might inform this type of lesson redesign. Thus, we asked her to more systematically reflect before and after teaching lessons so that we might develop some insight into her perceptions of the problems of practice she faced. Kristina audio recorded reflections before and after lessons, and participated in several semi-structured interviews based on her reflections. These reflective interviews took place outside of the weekly planning meetings.

Kristina made 17 brief (2–5-min) reflective audio recordings immediately before and/or after teaching the lessons. These were distributed over about 60% of her lessons. Kristina generally had specific issues she was focused on before teaching the lesson and issues that came up for her during her teaching. For example, after one lesson, Kristina reflected on one confusing aspect of her lesson.

> So, one of the things that I think I said I was going to tend to, was like really listening to students and what they said, um, and responding to them. So I think I was doing that but, then I was, having a hard time like, um, like I would second guess like what I was going to say, like so I would just basically not say anything cause I'd be standing there and like worrying like about a million things and like how to respond to this student.

Kristina also participated in six semi-structured interviews during the semester. Their duration ranged from 15 min to just over 1 h. For each interview, several initial questions were generated based primarily on her audio-recorded pre- and post-lesson reflections. The preliminary interview questions, however, were used to invite and promote discussion rather than to serve as the organizing structure for the interview. Such questions included: *What knowledge do you think they [the students] had about the concept of addition before you introduced the content today? What would make students want to work together?* Kristina briefly addressed these questions, but then moved on in the conversation to issues that were more pressing for her at that moment. For example, the interview focus might shift from small group work to Kristina's concerns about her own use of questions to facilitate discussions.

In addition to Kristina's reflections and interviews, as part of the local MTE community, we observed and videotaped Kristina during each class session and recorded field notes attending to how the initial, lesson plan compared to the lesson implementation. The field notes paid particular attention to the ways in which

Kristina's enactment aligned with and differed from the initial written lesson plan that had been produced in a planning meeting. These field notes constitute an enacted lesson plan as do the transcribed videotapes from each class session.

Data Analysis

The aforementioned interviews provided opportunities to discuss the unanticipated problems of practice—that is the decisions in the moment—that occurred during lesson enactments and that were highlighted in Kristina's reflections. Her perceptions of her work and of her progress in more fully participating in the practices of the local community provided opportunities to identify and understand problems of practice that she experienced. Our analysis of Kristina's reflections became central to our decision-making process for the lesson plan revisions described in detail in the next section.

Using a grounded theory approach (Glaser & Strauss, 1967), one author iteratively segmented and categorized the transcripts from Kristina's 23 interviews and reflections. This resulted in 57 segments according to the topics she discussed. For each segment, both authors discussed and then characterized the main ideas of what was discussed in the form of concise statements, resolving any discrepancies through further discussion. The segment of Kristina's comments below and the statements we assigned to this segment illustrate the process.

> I can just feel their eyes on me when somebody else is presenting, waiting for me to ask a question… So I need some more prompts. Um, I need to start using some more prompts to get, um, students, um, more engaged. Maybe like, um, besides, "Does anybody have any questions?" or "What do people think about the way that's represented?" or "Does anybody have any questions?" I, I kind of said one of those questions like does anybody have questions about, you know, the way the numbers are represented. But then I like, I don't know. I don't know if that was really a good question, um, because nobody said anything.

We characterized this segment with these two statements: Considers if a question is good because students did not respond; and Desires to develop prompts beyond, "Do you have any questions?" We organized the statements into broad categories related to the kinds of decisions Kristina was making in the moment. Building on these categories, and following other analyses of problems of practice (Horn & Little, 2010), we identified problems through linguistic cues that signaled instructional interactions experienced as challenging, confusing, recurrent, unexpectedly interesting, or otherwise worthy of comment. Such cues included references to trouble, for example, or expressions of emotional distress, many of them marked by changes in intonation. Each of the aforementioned categories suggested a broader problem of practice Kristina was attending to in the moment. We identified the preceding segment as relating to the problem we called "Promoting Discussions with Questions."

After identifying the range of broadly defined problems of practice, both authors together subdivided these "problems" into more specific components according to groups of associated statements, resolving any discrepancies through further discussion. For example, the reflections for the broadly defined problem of Promoting Discussions with Questions include references to the components of (a) knowing the purpose of questions, (b) anticipating likely PT responses, (c) identifying and using

productive PT responses, and (d) crafting productive questions. For the preceding sample segment, the first statement was categorized as (a) knowing the purpose of questions. The second statement was categorized as (d) crafting productive questions. Comparisons among relevant segments from videotaped lessons, field notes about lesson implementation, and lesson plans were used to triangulate information in Kristina's reflections and interviews—that is compared her descriptions of classroom interactions and events with information in lesson plans and field notes, and the videotaped lesson enactments—to further define and/or validate the problems of practice that emerged. We conducted this process for each segment we identified as related to a problem of practice.

Results

One Novice MTE's Problems of Practice

In this study, the authors (research team) focused on the following research question: How can the analysis of one novice MTE's problems of practice inform the creation of lesson plans that might support novice MTEs as they learn the practice of teaching preservice teachers? To address this question, we analyzed Kristina's reflections and interviews to identify broad problems of practice that Kristina repeatedly referenced. In our analysis of her statements, we identified six broader problems of practice with which she was primarily concerned (Table 2).

For purposes of this paper, we focused on the three most prevalent problems of practice for further investigation. For these three problems of practice, we pursued a deeper understanding of the issues arising from the comparison of Kristina's reflections

Table 2 Kristina's problems of practice

Problem of practice	Categorized as Kristina's comments or questions about	Number of segments	Number of recordings
Promoting discussions with questions	Facilitating whole-class discussions with PTs, in particular using questions as prompts for further thinking	30	14
Setting and addressing goals for common and specialized content	Setting goals for PTs during planning or focusing on identified goals for PTs while teaching	13	6
Developing mathematical language	Fostering, supporting the development of mathematical language used by PTs	7	4
Managing content for PTs	Scaffolding, supporting, and extending content for PTs	4	2
Clarifying board work	Communicating clearly to PTs when writing/recording on the board	2	2
Assessing	Collecting formative and summative assessment information about PTs	1	1

with videotaped lesson implementations and field notes—that is, we used the lesson video and field notes to better understand how problems of practice that Kristina described in her reflections had unfolded in lessons in real time. For each of these three problems of practice, we refined our descriptions of the problems as we identified the components from Kristina's statements that more completely decomposed and defined each problem (Table 3).

When comparing the components of the problems of practice with the relevant lesson plans, we found that first, the problems aligned with some of the information in the existing lesson plans and second, that the lesson plans consistently lacked adequate information to fully address the components of the problems. We do not mean to suggest that a lesson plan could or should anticipate or solve all problems of practice. Rather, we argue that the identification of where and how problems of practice arise during the implementation of lessons may inform the improvement of the structure and content of lesson plans so as to better support novice MTEs as they transition to more fully participating in the practices of the community.

Revising Lesson Plans

Our in-depth analysis of Kristina's problems of practice exposed underdeveloped and potentially confusing information in our lesson plans. What we learned about the challenges and decision-making of one novice MTE guided our lesson plan redesign, both in structure and content. For example, Kristina had expressed confusion around goals. We recognized that our plans included content, specialized content, and practice goals intertwined and not specific to activities. The revised plans list the goals at the beginning, explicitly connect them to activities, and provide suggestions for what observations or artifacts might be used to evaluate PT progress.

The template in Fig. 1 resulted from our discussion of how to refine and add categories and content in order to better address the components of the problems as described in Table 2. Our revised template has four sections. Section I describes what to prepare. Section II highlights a *manageable* list of goals (explicitly connected to specific activities,

Table 3 Components of Kristina's three most prevalent problems of practice

Identified problems of practice	Specific components highlighted in MTE reflections
1. Promoting discussions with questions	a. Knowing the purpose of questions b. Anticipating likely PT responses c. Identifying and using productive PT responses d. Crafting productive questions
2. Developing mathematical language	a. Managing confusing explanations b. Revising and enhancing PT language c. Making connections from PT-generated language to concepts d. Modeling appropriate language
3. Setting and addressing goals for common and specialized content	a. Managing connections to multiple big ideas b. Sequencing content according to goals c. Identifying goals beyond concepts d. Following lesson plan goals in tension with PT-generated ideas

Section I
What to prepare for class *A list of materials, any advance preparation instructions, and a list of relevant resources that elaborate lesson content*

Section II
Lesson Goals *List of lesson goals for prospective teachers categorized as content, practice, and specialized content knowledge (SCK) goals*

Section III
Lesson Overview *A 1-page chart that includes the information listed below for each activity of the lesson*

Activity	Time	Description	Content, Practice, and SCK Foci	Rationale	Materials
1. Title of activity	Duration of activity	Brief description of activity (1-2 sentences)	List of specific goals from Section II above that are related to this activity	Description of how this activity contributes to preparing prospective teachers—that is, how this activity relates to them developing necessary knowledge and skills for teaching math	Materials list for this activity

Section IV

Lesson Details *Lists separate steps of activity and ties specific questions, prompts, and anticipated student thinking/contributions to the individual steps. Provides detailed outcomes and expectations related to activities and ties these back to goals in Section II.*

Time/Format	Activity/ Task	Questions / Prompts / Extensions	Anticipating Prospective Teachers' Questions, Misconceptions, Etc.	Outcomes/ Expectations
1. Title of activity; duration; grouping recommendation (small group, whole group, partner, individual)	a. A list of instructions describing the individual tasks, steps, or actions of the activity	**Q1:** The questions and prompts that will be used with the activity tied to the individual tasks, steps, or actions from the activity Rationale for the prompt (when appropriate) *Sample student answers, responses, and representations illustrating their anticipated contributions and reasoning (when appropriate)* Sample responses to student contributions listed in this section used to illustrate how the discussion might flow and to highlight key discussion points (when appropriate)	Notes about anticipated possible student strategies, misconceptions, challenges, errors, and incomplete understandings related to content, practices, or teaching and learning of content Notes about how to address above and how to incorporate them into the lesson as sites for learning	Descriptions of what prospective teachers will be able to do within and as a result of this activity Notes about how to assess these outcomes and expectations Each outcome and expectation for the activity is tied back to goals from the list in Section II

Fig. 1 Lesson plan template

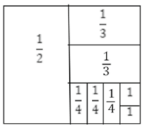
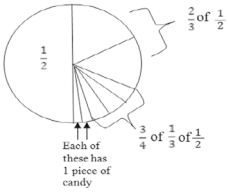
outcomes, and assessments throughout Section IV). Section III provides a lesson overview and a rationale for activities—that is, a description of how the local MTE community envisions each activity as contributing to the preparation of PTs. Section IV explains the implementation of activities. In addition to questions and prompts, lesson plans incorporate sample PT responses and strategies. Section IV includes suggestions for assessing outcomes. See Table 4 for an excerpt demonstrating these revisions for the Candy Box Problem lesson plan.

We argue that the lesson plan revisions may provide significant support for Kristina's problems of practice. For example, the plans address problems related to Promoting Discussions with Questions in Section IV in both *Questions/Prompts/Extensions* and *Anticipating Prospective Teachers' Question, Misconceptions,* etc. These columns include prompts for the MTE and sample PT solutions and representations. Samples include references to important mathematical ideas and misconceptions aiming to better prepare a novice MTE for flexibly responding to and capitalizing on PT contributions. Following Ball, Sleep, Boerst, and Bass (2009), we do not view these sections as scripts, but rather as ways to provide images of how a discussion might unfold.

To provide support for the problem of Developing Mathematical Language, the revised plan includes informal language juxtaposed with domain language and connects these in sample PT responses. This connection may assist a novice MTE in modeling "teacher" language. The lesson plan highlights language issues pertinent to teaching to provide guidance for focusing the discourse. For example, a place value plan includes a reference to "adding a zero" accompanied by a description of why this language could result in place-value confusion.

Finally, in support of the problem of Setting and Addressing Goals for Content and Specialized Content, the lesson plan outlines a set of goals connected to specific activities. Because MTEs need to make sense of the goals, but also need to evaluate PTs' progress

Table 4 Revised lesson plan excerpt for Candy Box Problem implementation instructions

Time/ Format	Activity/ Task	Questions/Prompts / Extensions	Anticipating PT's Mathematical Thinking	Outcomes/ Expectations
3. Solve Candy Box Problem [30 min]	**a) PPT #4:** PTs read the problem and think about what the problem means. **Candy Box Problem** There was a box of candy on the table. Thuy was hungry because she hadn't had breakfast, so she ate half the candy. Then Hosa came along and noticed the candy. She thought it looked good and had not packed a lunch so she took two-thirds of what was left in the box. Alicia came by and decided to take three-fourths of the remaining candies with her to her next class. Then Aniyeli came dashing up and took one piece of candy to munch on. When Mia looked at the candy box, she was sure that there was just one piece of candy left. "How many pieces of candy were there in the box to begin with?" she asked Thuy suspiciously.	**Q1 (with a):** Describe what you think you have to do in the problem and how you might begin to solve the problem. *Sample response: You have to figure out how many candies there were before any were taken. You might be able to work backwards from the 1 that is left.*		
	b) Invite PTs to work individually to solve the Candy Box Problem. After 5-10 minutes, they can share their thinking in their groups. **Invite** PTs to create at least two different representations of the problem and solution in their groups.	**Q1 (with b):** As PTs work, prompt them to explain and clarify the math, but invite them to respond to and build on each others' ideas. *Sample representations:* *Representation 1*  *This representation shows the literal description of each step of the problem without portraying the changing whole. Without explanation, this representation does not make sense because the fractional parts shown are all fractions relative to a different whole.* *Representation 2*  *This representation shows the fraction that was taken in each step of the problem until the last step. Note that the parts are labeled so that the fractional part is described in relation to the whole box. The last step is not recorded as a fraction, but rather the last piece taken and the last piece remaining are both labeled as 1 piece from all the candy in the box.* *Representation 3* $x - \frac{1}{2}x - \frac{2}{3}(\frac{1}{2}x) - \frac{3}{4}[\frac{1}{3}(\frac{1}{2}x)] - 1 = 1$ *This equation represents subtracting the fractional part for each step of the problem and then taking away one of the last two pieces to leave the last piece. Solving for* x *yields the original total number of pieces.* *Representation 4* $\frac{1}{4}[\frac{1}{3}(\frac{1}{2}x)] - 1 = 1$ *This equation represents the fractional part of the remaining two pieces and then shows subtracting one of those pieces to get the final piece in the box. Solving for* N *yields the original total number of pieces.*	PTs may take away only 1/3 instead of 2/3 in the 2nd step. This is especially common when PTs are trying to use algebra to solve the problem.	*PTs will be able to solve a fraction problem that involves analyzing units. (C2 & C3) *PTs will be able to represent and solve the problem in multiple ways and are able to make connections between and explain how each part of the problem is shown in different representations. (P1) [*Recorded assessment*: Collect a scan of their Candy Box solutions to assess their understanding of the fraction problem] *PSTs will be able to share their ideas with each other working towards understanding and building on each other's ideas. (P2) [Informal assessment based on observations of students as they discuss ideas in their groups.]

towards meeting those goals, observable outcomes for potentially evaluating progress towards the goals are described. The combined revisions are intended to provide a foundation from which an MTE can make informed decisions: for example, deciding what to emphasize; which discussion threads to follow and which to abandon; as well as deciding which outcomes PTs are making sense of and where they may still need enhanced opportunities in order to reach the lesson goals.

Implications and Conclusions

In this paper, we present our professional learning model for novice MTEs and describe our process for identifying and using a novice MTEs problems of practice as an analytic lens for determining how to enhance and improve lesson plans such that they might better support novice MTEs in developing their teaching practices as they learn to teach PTs. Although we have not systematically investigated the question of how lesson plans might support the professional learning of novice MTEs, a more recent implementation of the revised lesson plans suggests that the new structure and additional content resulting from our analysis of one novice MTE's problems of practice do seem to provide enhanced support for a novice MTE in developing teaching practices valued by the local community—that is, in three informal interviews, the novice MTE reported feeling she had adequate information for promoting discussion with questions, developing mathematical language, and making sense of the lesson goals. Unsurprisingly, the novice MTE implementing the revised lesson plans still faced problems of practice. Her persistent questions over the semester stemmed from how to assess whether students were meeting the specified goals. Even informally, this implies that our addition of specific activity-related pointers for assessing PTs' progress could be revised and improved. We speculate that the formative nature of many of the suggestions might be unwieldy for the novice MTE. Continued investigations related to novice MTEs' problems of practice may suggest further revisions for supporting MTEs as they learn to teach PTs.

All of this is predicated on a collaborative local MTE community. It is unlikely that the lesson plans alone as static documents would impact a new MTE without the supporting discussions in local MTE planning meetings. In fact, in the process presented in this paper, the planning meetings were the beginning of and an integral part of the entire process. The MTEs in the local community collaboratively revised existing versions of lesson plans for the initial lesson implementation, and it is the community's subsequent discussions of the problems of practice occurring after lesson implementations that resulted in the revisions to the community product of lesson plans. Writing the lesson plan for individual lessons should be a community-based activity. Individual MTEs within a local community might initiate the efforts, but we envision lesson plan documents as living artifacts that are responsive to the growing and changing knowledge of the broader MTE community.

Our work builds on existing research related to structures or opportunities for MTE professional learning in the following ways. First, our model established a local community of inquiry similar to Jaworski (2003), a central practice of which was collective reflection on artifacts of teaching practice. Indeed, research on MTEs' reflective practices suggest that reflection plays an important role in connecting theory to teaching practice, and consequently enhances teacher educators' teaching practice (Zasklavsky, 2009). Second, as our model is situated within an existing graduate program, it provides opportunities for novices in the program to integrate what they are learning from coursework into community discussions of enacting and revising lesson plans. This dual integration of types of knowledge is similar to features of other MTE professional learning programs (Even, 1999). Finally, while not a feature described in our model, we can envision opportunities within the model for individuals to engage in self-study or conduct research on PT learning, activities others have

identified as contributing to MTE professional learning (e.g. Garcia et al., 2007; Rowland et al., 2014).

We recognize that the process presented here of using problems of practice as an analytic lens to determine lesson plan revisions has a number of limitations. For example, lesson plans can only present so much information before becoming overwhelming and losing focus. We agree with Hiebert and Morris (2009) and Ball et al. (2009) that a lesson plan could be overburdened with information and thereby rendered confusing and unusable. This implies that the iterative cycle of revising lessons plans will likely prove fruitful only up to a point. In addition, this is a study based on one novice MTE over one semester as part of a labor-intensive process. Despite these limitations, we believe that using problems of practice as an analytic lens to determine lesson plan revisions, has offered unique insights into the journey of a novice MTE. We submit that the proposed improvements and the process described here may have applicability to a broader MTE community. At the very least, we propose that our work might stimulate further discussion about how to support novice MTEs in developing skills and practices relevant to their work of teaching PTs.

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