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Students' Creative Process in Mathematics: Insights from Eye-Tracking-Stimulated Recall Interview on Students' Work on Multiple Solution Tasks

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Abstract
Students' creative process in mathematics is increasingly gaining significance in mathematics education research. Researchers often use Multiple Solution Tasks (MSTs) to foster and evaluate students' mathematical creativity. Yet, research so far predominantly had a product-view and focused on solutions rather than the process leading to creative insights. The question remains unclear how students' process solving MSTs looks like—and if existing models to describe (creative) problem solving can capture this process adequately. This article presents an explorative, qualitative case study, which investigates the creative process of a school student, David. Using eye-tracking technology and a stimulated recall interview, we trace David's creative process. Our findings indicate what phases his creative process in the MST involves, how new ideas emerge, and in particular where illumination is situated in this process. Our case study illustrates that neither existing models on the creative process, nor on problem solving capture David's creative process fully, indicating the need to partially rethink students' creative process in MSTs.

Keywords Creative process · Eye tracking (ET) · Mathematical creativity · Multiple solution tasks (MSTs) · Stimulated recall interview (SRI)

Introduction

Creativity is significant for innovation, working out original ideas, and finding new paths of thinking. Creativity has always been of major importance for the domain of mathematics. Mathematicians such as Hadamard and Poincaré have pointed this out,

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referring to moments of sudden illumination, so-called eureka moments that appeared when solving fundamental mathematical problems. Also, mathematics education research focuses increasingly on mathematical creativity (see, e.g. Leikin & Pitta-Pantazi, 2013; Sheffield, 2013; Singer, 2018) with the aim to prepare students for their current and future lives in our increasingly automated and interconnected high-technology-based societies and economies (Organization for Economic Co-operation and Development [OECD], 2014). It appears no longer sufficient for students to solve problems only with routine schemes or familiar heuristics.[1] Educators aim to teach for creativity and to give students the opportunity to work creatively with mathematics (Silver, 1997). Students shall become able to think "out of the box", to connect different topics while solving problems, to have Aha! experiences.

Given the need and interest to foster students' creativity (Sheffield, 2013, 2009), it is necessary to understand students' mathematical creativity process (Leikin & Pitta-Pantazi, 2013). Mathematics education research has predominantly focused on students' creative products and used, for instance, students' written solutions in MSTs to evaluate students' mathematical creativity (e.g. Levav-Waynberg & Leikin, 2012). The available body of research predominantly takes such a *product view*, investigating written accounts of (potentially) creative processes: In this view, the analysis of the creative product is "the starting point of all research effort about creativity (…), given that the creative product is the public face of creativity, the tangible form of the whole process" (Pitta-Pantazi, Kattou & Christou, 2018, p. 30). On the other hand, research about students' creative *processes*—a *process view*—is still rare. Following Pitta-Pantazi, Kattou, and Christou (2018) such research on creative processes predominantly aims "to describe stages, actions and behaviors that are active during the generation of an idea" (p. 39). Creative processes among students are, if studied at all, mostly regarded against the backdrop of stage models (see Sitorus & Masrayati, 2016 for an example, see Pitta-Pantazi, Kattou & Christou, 2018 for an overview). Such models draw on—or are related to—Wallas' (1926) four-stage model of the creative process, which was originally developed to describe professional mathematicians' creative process (Sriraman, 2009). This model, which incorporates the stages preparation-incubation-illumination-verification, has successfully been applied in mathematics education (e.g. Sriraman, 2004, who modeled mathematicians' creative processes), and linear stage models similar to Wallas' model have earned acceptance in the field of mathematics education (see Pitta-Pantazi, Kattou & Christou, 2018 for an overview). Yet, the questions arise whether models developed to capture professional mathematicians' creativity can likewise capture creative processes among *school students* (Haavold & Birkeland, 2017) and how students' creative process looks like after all. Whereas Wallas' model describes mathematicians' long-term creative processes (including phases of unconscious incubation), many creative processes in the mathematics classroom are short-term and less sophisticated. The question of whether school students' creative process includes phases[2] similar to those in Wallas' model is largely unexplored and constitutes a research gap.

[1] In this paper, we use the term *heuristic* to refer to persons' generic rules that often help in solving a range of non-routine problems (see Mousoulides & Sriraman, 2014), such as looking for symmetry, or similar.

[2] Please note that we use the term *phases* to account for elements of students' work on the MST. We omit the term *stages* since this term usually implies some order, as in stage one, stage two, etc. However, we cannot assume that the temporal elements in students' work on MSTs adhere to a certain order.

This article aims to explore what is involved in the creative process of a school student working on an MST. We ask the research question, *What phases does the creative process working on an MST include?* and, in particular, *How do new ideas to tackle the MST come up?* Through pursuing these questions, we want to trace and illustrate the student's creative process and see to what extent his process meets existing models, or whether it does not ("counterexample"). We use a qualitative single-case study of an upper secondary school student—18-year-old David (pseudonym) who worked on a geometry MST—to explore whether existing models may capture the creative process in MSTs in settings like ours adequately. David had proven to be resourceful in finding different solutions when working with MSTs, which provided a rich data set for our study.

Yet, the volatile and partially recursive process of coming up with new ideas and having creative insights is difficult to study. Pitta-Pantazi, Kattou, and Christou (2018) in their overview of current research on mathematical creativity point out, "[w]e feel that demystifying the creative process is not an easy target, due to the fact that it is an internalized procedure that is obvious through the actions and descriptions provided by the solver. Nevertheless, investigating the creative process allows us to identify ways and methods for its improvement." (p. 39) Mere observation of students' actions may give only little insight into students' thinking in the creative process when working on an MST. Verbal reports and thinking aloud protocols may influence and distort students' train of thought, and eventually suffer from anxieties or language issues of the students (Schindler & Lilienthal, 2018). Furthermore, creative thinking is not necessarily linear and may not always be fully conscious (Sheffield, 2013). Thus, it may be difficult to report verbally. Not least for these reasons, we use eye tracking (ET) to gain deeper insights into students' creative process. ET is a promising tool for investigating mathematical creativity (see Schindler, Lilienthal, Chadalavada, & Ögren, 2016, Schindler & Lilienthal, 2017a, 2017b) and it does not influence students' processes in the way thinking aloud protocols or interviews do (Schindler & Lilienthal, 2018).

First, we conducted an ET study, where ET glasses recorded David's gaze when he solved the MST (with a pen on paper). Based on this recording, we created a gaze-overlaid video, which displayed his work on the MST including his gaze wandering around in the figure as a dot (see link for an example). Based on this gaze-overlaid video, we conducted an ET Stimulated Recall Interview (SRI), where David explained his creative process based on the strong stimulus of the gaze-overlaid video.

Mathematical Creativity

Mathematics education research focuses increasingly on mathematical creativity (Leikin & Pitta-Pantazi, 2013; Sheffield, 2013; Singer, 2018). However, perspectives on creativity in mathematics education research are diverse and there is no single shared definition or even a shared conceptualization of mathematical creativity (Leikin, 2009; Mann, 2005; Sriraman, 2005). For sorting the existing views on mathematical creativity, we use the distinction of "big-C" and "little-c", as commonly used in mathematics education (see, e.g. Schindler, Joklitschke, & Rott, 2018, Sheffield, 2018, Sriraman, Haavold & Lee, 2014). These represent "two definitions of the creative process on which many investigations are based" (Prabhu & Czarnocha, 2014, p. 35): "big-C" refers to extraordinary, absolute creativity; and "little-c" to relative creativity of non-experts and students in particular (Schindler et al., 2018; Sheffield, 2018).

Creativity Among Professional Mathematicians: Big-C and Wallas' Model

Big-C conceptualizes the creativity of eminent individuals, serving the growth of the field of mathematics (Sriraman, 2009). It addresses outstanding creative contributions. A prototypical example of big-C-creativity in mathematics is Poincaré's (1948) work on Fuchsian functions (Mann, 2005; Sriraman, 2009). After having worked on these for a considerable amount of time, Poincaré suddenly, when stepping on a bus, had an illumination as a consequence of his long, unconscious work. Hadamard (1954), as influenced by Gestalt psychology, theorized the creative process leading to eminent contributions in the field of mathematics drawing on Wallas' (1926) Gestalt model (Sriraman, 2009). The creative process is modeled as a four-stage model, consisting of a *preparational stage*, where researchers get an insight into the given problem through conscious work; an *incubation stage*, where the problem is not consciously focused on; the moment of *illumination*, where an insight suddenly appears; and a *verification stage*, where the results are expressed and verified and possibly specified or extended. Creativity in this perspective is a long-term process, comprising a considerable amount of conscious and unconscious work (Hadamard, 1954; Mann, 2005).

Creativity Among School Students: Little-C and Multiple Solution Tasks

MSTs. Research addressing students' creativity normally addresses little-c, i.e. relative creativity. Student solutions are considered creative if they "are unique and novel to the students in their particular environments" (Sheffield, 2018, p. 408). Research addressing little-c largely follows a psychometric or behavioral approach (Prabhu & Czarnocha, 2014; Sriraman, 2009). Creativity is characterized as a key component of the ability to find unique and manifold ideas (Guilford, 1967). This ability comprises four aspects: *fluency*, the number of solutions, *flexibility*, the diversity of produced solutions, *originality*, the uniqueness of produced solutions, and *elaboration*, the level of detail of the descriptions. The Torrance Tests of Creative Thinking (Torrance, 1974) are predominant for the little-c perspective, where creativity is aimed to be investigated through paper and pencil tests. This approach has been transferred to mathematics education research by Leikin (2009), who introduced the concept of Multiple Solution Tasks (MSTs) within the domain of mathematics education. MSTs are mathematical tasks that are to be solved in different ways. Students are invited to solve MSTs in as many ways as possible, based on the theoretical assumption that "solving mathematical problems in multiple ways is closely related to personal mathematical creativity" (Leikin & Lev, 2013, p. 185). Creativity is evaluated based on the number, variety, and originality of students' solutions (Leikin & Lev, 2013). Naturally, this approach has a product perspective, as it focuses on results in students' written solutions.

Although the product view on students' work on MSTs has been fruitful, the *process* leading to creative insights, the "act of creativity" (Prabhu & Czarnocha, 2014, p. 35) is seldom investigated. Research in mathematics education does focus, however, on students' *moments of insights*: Czarnocha and Baker (2015) address bisociation, i.e. students' flashes of insights where they connect previously not connected aspects or experiences, Haylock (1987) focuses on overcoming fixations, and Liljedahl (2013) emphasizes that Aha! moments come along with illumination as affective experience. Yet, the whole process of students' creative endeavor is rarely addressed.

Models to describe students' creative process. For describing students' creative processes, some researchers, for instance, Sitorus and Masrayati (2016), use Wallas' four-stages (see above). Yet, the question arises whether this model, which was developed to capture professional mathematicians' creativity, is appropriate for school students as well (Haavold & Birkeland, 2017). Haavold and Birkeland (2017) summarize, "it would be natural to ask if the characteristics of the creative process are the same for different individuals. It might be different for, say, a professional mathematician than a student of mathematics." (p. 184)

Further, the question arises if Wallas' model is suitable for MSTs, where persons are asked to produce different solutions in a limited amount of time. It is questionable whether persons may have illuminations when working on MSTs when the time for incubation is limited.

On the other hand, if MSTs constitute real problems to students (like in Leikin's works) that cannot be solved routinely, then students' work on MSTs might also be considered (a special kind of) problem solving: Students solve a certain problem—yet, in different and various ways. Following Dewey (1910), students' problem solving involves (1) the occurrence of a perceived difficulty, (2) locating and defining the difficulty, (3) the occurrence of a suggestion and idea, (4) reasoning behind the suggestion/idea and inferences, and (5) corroboration or verification of the suggestion/idea. Note that stage three in this model—the suggestion—may be understood to be close to what others call "illumination" or "creative". Dewey (1910) explains: "Suggestion is the very heart of the inference; it involves going from what is present to something absent. Hence, it is more or less speculative, adventurous. Since inference goes beyond what is actually present, it involves a leap, a jump." (p. 75)[3] In the case of MSTs, it is conceivable that students may have such suggestions or ideas about how to solve the MST (in another way). And they may undergo phases similar to the ones described by Dewey while working on an MST—yet, several times: Once one solution is found and verified, students may start over and work towards another suggestion or idea. In this manner, Dewey (1910) emphasizes that "cultivation of a variety of alternative suggestions is an important factor in good thinking." (p. 75) Whether problem-solving models as offered by Dewey (1910) or Pólya (1945)[4] may—to a certain extent—be suitable to capture students' phases in MSTs, or whether Wallas' model might capture students' processes more closely is unclear. The

[3] Note that Dewey, as well as Pólya, emphasize the role of "guesses": Students' tentative ideas about how to solve a problem, which are first only considered before they are proven to work out or verified.

[4] Note that Pólya developed his stage model of problem solving based on Dewey's model together with his experiences as a mathematician and with students (Rott, 2014).

question of how creative processes of school students in mathematics look like has rarely been studied empirically. Empirical studies on the creative process of school students in mathematics still constitute a significant research gap.

Eye-Tracking for Studying Students' Creative Process in Mathematics

Even though existing research using well-established methods such as thinking aloud protocols (Ericsson & Simon, 1980) or written solutions in MSTs has produced a considerable body of knowledge on mathematical creativity (e.g. Levav-Waynberg & Leikin, 2012), we see that it may be difficult for students to reflect and report on their thoughts. Students may not have the metacognitive skills to reflect on their creative process as well as professional mathematicians. Also, environmental factors (called "press" by Rhodes, 1961) play an important role, since students may lack the self-efficacy or self-confidence to take a risk and share their thinking. Or, they may not recognize creativity in their own work that was intuitive to them and just "felt right." In fact, students may be anxious or shy to utter their thoughts for different reasons, for instance, because they think that their problem solving is not straight forward, they perceive certain expectations (e.g. due to prior school experiences), or want to make a good impression. We see that ET may be beneficial to reduce the influence of social expectations, certain perceived norms, and didactical contracts in the data collection process. Besides, ET data may is less affected by issues with memory retrieval, introspection or meta-cognitive reflection, or language and verbalization issues that may affect students' explanations in thinking aloud protocols (see Schindler & Lilienthal, 2018).

The rich data set that ET produces is, however, anything but self-explanatory. ET data must be interpreted based on theoretical foundations and assumptions. Typically, the "eye-mind" hypothesis is assumed (Just & Carpenter, 1976). Expressed in the reductionist view of a brain-computer metaphor, it posits the following: "The primary proposal is that the eye fixates the referent of the symbol currently being processed if the referent is in view. That is, the fixation may reflect what is at the 'top of the stack.' If several symbols are processed in a particular sequence, then their referents should be fixated in the same sequence, and the duration of fixation on each referent may be related to the duration that the corresponding symbol is operated on" (p. 441 f.). It is important to note that the eye-mind hypothesis does not hold in general (Holmqvist, Nyström, Andersson, Dewhurst, Jarodzka & Van de Weijer, 2011; Schindler & Lilienthal, 2019). ET can only cast a fleeting view on some mental processes—in particular only on those processes that relate and reveal themselves through a visual "referent." Accordingly, not all mental processes can be observed with ET. Further, different mental processes can relate to the same visual referent, or a fixated area can be misinterpreted to be a referent, which means that interpretation of ET data is generally not bijective: There is no one-to-one-correspondence between eye movement patterns and mental processes (see Schindler & Lilienthal, 2019).

Despite these challenges, ET is a promising research tool, especially with "visually presented cognitive tasks." (Obersteiner & Tumpek, 2016, p. 257) For investigating the creative process, in particular, Schindler et al. (2016) found that gaze-overlaid videos may offer a "fine-grained access to what students were paying attention to and focusing on." (p. 167) They summarize that "[The analysis of gaze-overlaid videos] especially contributed to reconstructing how new, creative ideas evolved, to reconstructing approaches that were complex and whose written/drawn descriptions did not allow to clearly reconstruct them, and to evaluating the degree of elaboration of students' approaches." (p. 168)[5] These results hint at the potential that ET may provide for investigating students' creative processes in mathematics (see Schindler & Lilienthal, 2017b). ET may help to grasp the creative process in its volatile nature, where students compare information, jump back and forward, or jump to another idea or approach (Schindler & Lilienthal, 2017a). In this article, we use gaze-overlaid videos for a Stimulated Recall Interview (SRI).

Method

Setting the Scene

To explore students' creative process in their work on MSTs, we used data from the research project KMT ("kreativa matteträffar") in Sweden. With the aim to foster mathematical creativity among interested students, four upper secondary school students met every second week and worked on rich mathematical problems, among others, MSTs. When working on MSTs, they always first worked individually and subsequently in groups, where a focus was on discussions of their solutions. The MSTs we used (e.g. Novotná, 2017; Schindler et al., 2018) underwent an a priori analysis, which assessed the MSTs' potential for finding manifold and unique solutions.

For the investigation presented in this article, we chose an MST in planar Euclidean geometry because previous research has shown geometry to be particularly suitable for ET studies (e.g. Muldner & Burleston, 2015). In the selected MST, the student was asked to infer, in as many different ways as possible, the angle size of $\mathcal{E}$ (see Fig. 1) in a regular hexagon. Multiple solutions in this context do not mean that the students were asked to find different angles sizes, but different ways to show the actual angle of 30°. David was acquainted with working on MSTs in this way and had prior experience working with the geometry concepts involved in the hexagon MST. He was, however, not familiar with the geometry MST used in our study. We selected this particular MST since we knew from previous studies that it is rich, motivates students to find manifold approaches, offers multiple affordances (e.g. Schindler et al., 2018), is solvable without extensive background knowledge[6] (which allows to study creativity rather than the level of prior mathematical knowledge), and does not depend heavily on information from text (since we did not aim to study information extraction from text).

Eye-Tracking Device

Even though remote ET devices attached to a computer screen can be more accurate (see, e.g. Epelboim & Suppes, 2001; Muldner & Burleston, 2015), we used ET glasses

[5] Please note that in this paper, we use the word *approach* to designate students' way of doing something, in particular, their way of solving the MST. During their work on MSTs, students may have different approaches.

[6] To minimize the dependency on prior knowledge, the text accompanying the MST in Fig. 1 explained that all angles in a regular hexagon are 120°.

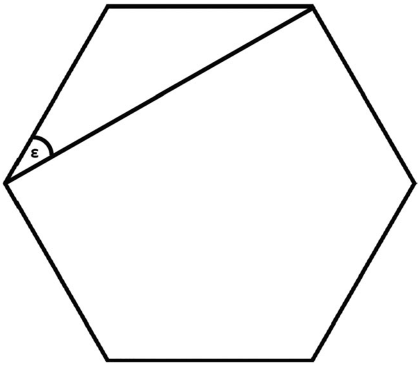

Fig. 1 Geometry MST used in this study

for our study. The use of ET glasses allowed David to work on the geometry MST with paper and pen in a way he was familiar with from prior experiences, and it permitted relatively unrestricted head and body movement. For this study, we chose the ET glasses Pupil Pro (Kassner, Patera & Bulling, 2014). These glasses are comparatively easy to set up and operate. They are relatively unobtrusive (weight < 100 g), and we experienced that students soon "forgot" that they wore the ET glasses and accordingly worked on the MSTs in a natural way. The gaze estimation accuracy of the glasses used in this study is—under ideal conditions—expected to be 0.6° (Kassner, Patera & Bulling, 2014). In our study, the average accuracy was 1.5°.

Stimulated Recall Interview Using Gaze-Overlaid Video

SRI is a method with "which cognitive processes can be investigated by inviting subjects to recall, when prompted by a video sequence, their concurrent thinking during that event." (Lyle, 2003, p. 861) In this study, we explore the creative process by carrying out an SRI with David using the previously recorded *gaze-overlaid video*, which displays David's gestures, writing, drawing, and eye movements, as stimulus. As a special type of "regular" video, gaze-overlaid videos are used to aid reflection (Stickler & Shi, 2017). Since they add the visual focus on actually examined areas, gaze-overlaid videos provide a particularly strong stimulus. Making the students' eye movements visible—especially eye movements that were possibly neither conscious at the recording time to the student nor visible to the naked eye of the researchers—also allows for a deeper level of recall (Stickler & Shi, 2017). It is important to note that the overlaid eye movements, even if they were not conscious, of course nevertheless happened during the experiment and, through conducting them, the student essentially "saw" the corresponding areas—even if unconsciously.

While SRIs avoid important disadvantages of, for example, thinking aloud methods—which may come with high requirements on interaction, strict time constraints, and emotive contexts (Lyle, 2003)—they also have their pitfalls that need to be taken into account when conducting SRI studies (Lyle, 2003). We strived to counterbalance the known detrimental factors as follows. First, to reduce anxiety during the SRI, we chose an environmental context that was familiar to the student. Further, the interviewer and student had established a trustful relationship during the project span. Second, we minimized the time between the session in which the student worked on the

MST and the SRI. Given the constraints of the research project, the SRI took place in the following regular session of the biweekly "math club." Accordingly, the SRI has to be called "delayed recall" (Gass & Mackey, 2000, p. 50). As pointed out by Lyle (2003), in delayed recalls—when the time gap exceeds three days—there is an increased risk that participants, due to problems with retrieving memories, rather (re-)invent mental processes than recall them properly. In our study, however, there are reasons which lead us to believe that David indeed recalled his original thoughts. First of all, we observed the gaze-overlaid video to be a strong stimulus to recall mental processes as pointed out by Stickler and Shi (2017). Second, David was not only strong in mathematics but also exceptionally good in communicating: The descriptions he gave in the SRI indicated clearly that he was able to recall the original situation easily—an impression which is further supported by David using predominantly present tense when describing his approaches to work on the MST. He was also frank about parts of the video sequence he did *not* clearly remember, and said four times "I don't really remember" or "I'm not really sure". Our confidence that David was very well able to recall the original situation and his corresponding thoughts is in line with findings, which indicate that persons are well capable of remembering mental processes related to their eye movements (e.g. Hansen, 1991).

Even though David appeared to recall his thoughts, one may wonder whether the time delay between the actual problem solving and the SRI might have led to incubation in the sense of Wallas (1926) for some of David's approaches. Based on the engagement that David showed when working on the MST, it is likely that he went on thinking about the problem in various ways before the SRI. However, in the SRI, David was asked to describe his thoughts based on the strong stimulus of the gaze-overlaid videos, which he commented on. He was not asked to elaborate on his ideas freely, decoupled from the eye movements. Thus, it was very unlikely for him to talk about approaches or ideas that he did not come up with in the first place. Therefore, we think that a possible incubation of ideas did not have an impact on David's ET stimulated recall of his original ideas.

During the SRI, David (and the interviewer) perused the gaze-overlaid video, produced from the recorded gaze data using the Pupil software Player (Kassner, Patera & Bulling, 2014). The gaze points are shown with a green dot, and subsequent points are connected with magenta lines. Before the actual SRI started, the interviewer explained how the student could identify his gaze in the video and that both the student and the interviewer could pause, wind back or forward at their leisure. During the interview, David smoothly explained his thoughts based on the eye movements shown. We taped the SRI with two external cameras.

Data Analysis

We carried out the data analysis as follows (see example in Table 1). First, we transcribed the 76 min of video of the SRI interview, including the utterances of the student and the interviewer. Since our research questions are explorative and descriptive, we chose to analyze the data inductively following Mayring's content analysis (2014), carrying out the following specific data analysis steps. In a first *paraphrasing step*, we paraphrased content-bearing semantic elements in the transcript that were

Table 1 Example of data analysis steps

Transcript	Paraphrase	Transpose	Category
(D. looks to an upper triangle,back and forth between two triangles and within these triangles) I: So, what are you doing there? D: I'm trying to come up with something that does not quite involve what I, I mean the things I've already done here. Because then it would just be another derivation of those methods. So I want to try to find a new method.	Focusing on aspects in the figure that he has not used beforehand because he wants to find a new approach	Finding a new idea/approach via focusing on aspects in the figure that have not been used beforehand (ambition: find new approach)	Finding a new idea/approach

relevant to our research questions. The obtained paraphrases include the interpretations and explanations as given by David. The aim of the second step, the *transposing step*, was to generalize the paraphrases obtained in the previous step to a common, predefined abstraction level and to re-phrase to obtain a homogenous stylistic level (see Table 1). With respect to David's process of solving the MST, we worked out the phases that could be identified (e.g. "finding a new idea") and we noted what he did in particular in this phase (e.g. "focusing on aspects in the figure that have not been used beforehand"). This enabled us to compare David's phases to two different models: Wallace's creativity stages and Dewey's problem solving stages (see Fig. 4). Finally, the *category development step* assigns descriptions of categories to the transposes: These categories correspond to David's work on the MST and its phases. After we conducted the steps exemplified in Table 1, we carried out a *category revision* (perusing all categories and the corresponding data, revising and partially re-ordering the category system) and *subsumption* (collecting all instances belonging to the possibly refined categories).

Results

This section presents results following our research questions, *How do new ideas to tackle the MST come up?* and, *What phases does his creative process working on an MST include?*

For the reader to follow and understand what happened in David's work on the MST, we first report how many different correct solutions were found, and give a summary of David's observed actions. In sum, David put to paper three solutions based on three approaches (red, green, and blue, Fig. 2). The first, red approach started with the angle sum of 720°, divided by six, and the assumption that 120° equals to $2\varepsilon+60°$, leading to $\varepsilon=30°$ (see highlighted angle). The second, green approach focused on the

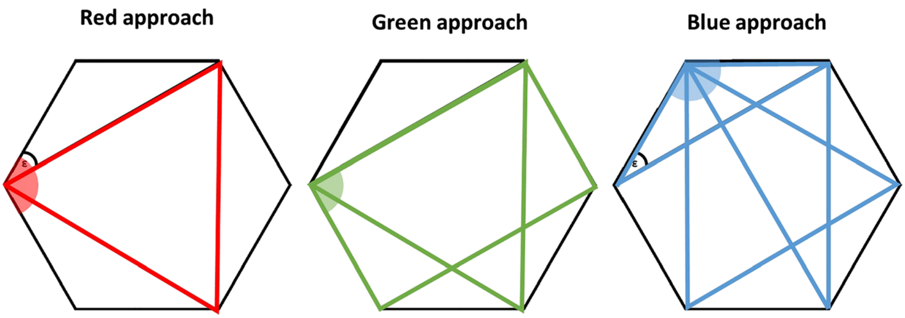

Fig. 2 Abstracted illustration of David's approaches

inscribed rectangle and started from 60°+ ε=90° (focusing on the highlighted angle). The third, blue approach, started from 180°−2ε=4 ε for the upper left angle. David furthermore started to work on four more approaches, which he, however, discarded before putting them to paper (see below). Thus, all in all David undertook seven approaches to solve the given MST, three of which turned out in a final solution.

The Emergence of New Ideas in MSTs

One particular question in creativity research is how students come up with new ideas, which is at the core of the creative process. Based on the SRI, we found that in three of his approaches, David was intentionally searching for symmetry in the diagram. When initially working on the MST and on the first approach to the problem, David stated that he got the inspiration to use symmetry considerations from the figure itself. After having finished the first approach successfully, he drew on this heuristic in later approaches: He intentionally searched for some more symmetry in the figure, which would be the starting point for other approaches. He explained that "I wanted a bit of symmetry" and added, "I started there and then I just wanted to like go further and see what I could find from that." This appeared to be a heuristic that David developed and then used consciously for finding further ideas.

After the green approach, he realized that his approaches sometimes involved facts that he had used in previous approaches. Thus, he started to focus on elements in the figure that he had not used beforehand. In two approaches, he—in such way—focused on aspects in the figure that he had not used in former approaches. David described, "I just looked around in the different parts of the figure and was thinking right, where should I start so that it… Because I, my goal at this point was to make a solution that is a bit different from the other ones. And so I was thinking, where should I start?" This indicated his intended flexibility: He wanted to use aspects of the diagram that he had not used in former approaches.

In a further approach, when his eyes were wandering around in the figure, he described that he was looking around, broadening up his view to find new aspects that could be worth pursuing. This connects to Muldner and Burleston's (2015) finding that highly creative students tend to have longer saccades (i.e. rapid eye movements between two fixations) on average—possibly because they "may have been more divergent in their thinking, by 'branching out' more when they were looking at the figure and as a result saw more opportunities in the geometric forms – in a sense they may have been more holistic." (p. 135)

In one approach (the blue approach), David described that while looking around in the figure, he was getting "a big idea" for solving the MST: "Now I'm getting like a big idea. How am I… Is this even possible? Is it possible to do it in this way? And then I just start strengthening." This indicates his excitement about his new idea, which he—in turn—pursued. This moment of sudden clarity and insight about a possible angle on the problem together with affective components can be considered illuminative (Liljedahl, 2013, see Discussion Section on illumination).

David described that he did not have certainty from the start that his ideas would turn out well and would lead to correct solutions. Initially, his approaches were rather intuitions[7] than clear pictures: He had a feeling about a global idea being useful, without having worked it out analytically yet. He, for instance, described, "It's… Yeah, I just saw. I just saw that it's... (…) … I mean it was just quite obvious to me (…). I started there and then I just wanted to like go further and see what I could find from that." Also, he explained that based on these intuitions, he took one step at a time: He explained, for instance, "I was taking one step at a time. Seeing, can I simplify this to anything? Or can I reduce this to something else? Or can I reduce THAT to something else? And it ended up working out *(chuckles)*!" These processes appear to be close to Dewey's (1910) and Pólya's (1945) assumptions that people first guess or have assumptions, which develop through inferences and reasoning. In David's case, the creative process was intuition-guided, which was then followed by an analytical step-by-step process.

In addition, David described in several instances that when *being stuck*, he tried to get inspiration from different aspects in the figure. He, for instance, described "Yeah, I think here, I'm a little bit stuck because I don't really know how I could… (…) I don't really know how I can show that this is equal to epsilon. (…) Now I'm just trying to go through in my head, like how… what other ways could I possibly use to come up with the fact that there is epsilon." He consciously looked around in the diagram for brainstorming purposes: to find "facts" in the diagram that could be helpful for his line of thought or help him out of a dead end. Such brainstorming and opening-up for new opportunities again may relate to longer average saccade lengths of students who are highly creative (see Muldner & Burleston, 2015).

Phases of David's Creative Process Working on MST

In the following, we present the phases that we identified for David's process of working towards solutions in the MST. The phases showed apparent similarity among all seven approaches. Based on the phases we identified, we developed a tentative model of David's creative process (see Fig. 3).

Looking for a Start. Initially, David always looked for a symmetry or something in the diagram that he had not used in previous approaches (see Section emergence of new

[7] Intuitions, in this sense, can be considered "preliminary, global view[s] of a solution to a problem, which precede […] the analytical, fully developed solution" (Fischbein, 1987, p. 202). Fischbein calls these kinds of intuitions anticipatory.

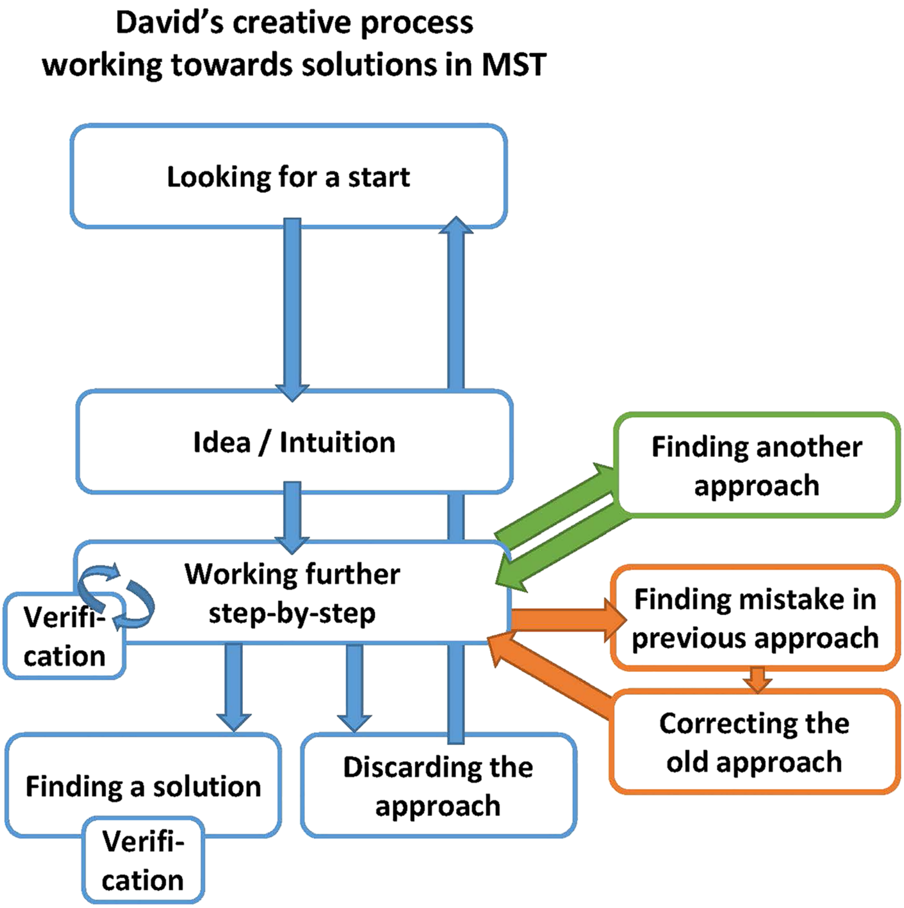

Fig. 3 Illustration of David's creative process

ideas, for a detailed description). We call this phase "looking for a start" in our tentative model of David's process (Fig. 3).

Idea or Intuition. Of course, in the transition from looking for a start to getting an idea or intuition about how to solve the problem, the solution did not appear to David without any transition (see Section emergence of new ideas). In the process to search for an angle on how to approach the problem, David got an intuition in that he, for instance, "just saw" a symmetry and that it was "quite obvious" to him. His descriptions indicate that he was excited about these discoveries. He described, for instance, "That's where I start to actually realize something. And so I get excited *(chuckles)*. (…) It is like: Oh THIS is it!" He stated that he was wondering if his ideas were even possible and whether they would work out. In this moment, he appeared to have an Aha! moment, accompanied by sudden certainty/clarification and affective responses (see Liljedahl, 2013, see Discussion Section on illumination). We call this phase "idea/ intuition" in the model of David's process (Fig. 3).

Working Further Step by Step. After his initial intuition, David went on working out the idea. However, in this phase, he did not devise and carry out a complete plan. Starting from his initial intuition, he rather took one step at a time to work towards a solution of the problem—similar to Dewey's idea about guessing first and then elaborating one's idea through reasoning and inferences. David stated that in the very beginning, he was not sure whether the ideas would work out. And, in fact, only three out of seven approaches he started with actually *turned* into a final solution. In this phase, he also *verified* whether his steps in his reasoning process were correct and would lead to a final approach, "double-checking" facts and reasoning steps (e.g. "I wanted to check many times. Like, is… Like, does this make sense?"). This led to corrections in his calculations several times. After he arrived at final solutions, he again verified, checking whether the solution (30°) matched previous ones. This indicates *verification processes*. We integrate this phase as "working further step-by-step" in the tentative model of David's process and integrate a "verification" connected to this phase (Fig. 3).

Finding Another Approach. During the process of working out a particular approach, it appeared that David discovered another idea to approach the problem. When discovering this opportunity, he held back the idea in the drawing/writing process until the current approach was finished. He explained that he wanted to have one solution correct before writing down a new one. He then returned to this idea after he had finished the current approach. We incorporate this in the tentative model of David's process as "finding another approach." (Fig. 3)

Finding a Mistake in a Previous Approach. While working on a certain approach, David once noticed a fundamental mistake in a previous one. He stated that "I realized that I had made a mistake" and described his emotional response to noticing the mistake: "I panicked a little bit because I realized that I had made a quite big mistake." He directly went on correcting this mistake and the previous approach. Only after having corrected his mistake, he went on with the new approach. In the tentative model of David's creative process, we incorporate this as "finding mistake in previous approach" and "correcting the old approach." (Fig. 3)

Discarding of Approaches. In four instances, David commented that he discarded approaches that he had pursued. He, for instance, dropped an approach stating "I was searching for some more symmetry in the figure. (…) But then I realized that this wouldn't get me very far, so I just started thinking of something else…" In a second case, he realized that it would not bring him further in the process than an approach that he had already started with beforehand. In a third case, he discarded an approach when he noticed that this approach would involve lines that he had already used in another approach. He explained, "I'm trying to come up with something that doesn't quite involve what I, I mean the things I've already done here. Because then it would just be another derivation of those methods. So I want to try to find a new method." In a fourth case, he discarded an approach because he felt it was "too similar" to a previous one and "too easy" and he "wanted to use the symmetry more in the solution." He discarded the approach because it did not meet his expectations. In the tentative model of David's process, we incorporate "discarding the approach", which is connected to the beginning of a new approach: looking for a start (see Fig. 3).

The Order of Approaches and the Role of Writing

Beyond the findings to our research questions, the study gave insights into the sequence of approaches that David came up with in the MST, where he was asked to find more than one solution. The emergence of approaches in the MST was not as serial as it might be assumed. It was *not* the case that David first had a certain idea, then followed it until the very end, finally solved it, and then went on with a new one. Instead, the SRI indicated that the process was volatile and jumpy. This volatility was seldom visible in David's gestures, writing, or drawing while working on the MST. The order of writing down or drawing the approaches did not necessarily coincide with the order they were found. David described his creative process as, first, having a thought, second, getting convinced through conducting a step-by-step proceeding until he saw if the solution turned out to work out (verification), and, only finally, putting the approach on paper by writing it down. This last step can be related to what Sheffield (2013) describes as communicating (here: in written form), where ideas and strategies are described and explained. In one instance, David described "It is now that I've realized that I've actually solved the problem. Like before I wanted to. *(chuckles)* So, I just put that down, that I've actually solved it." Writing down the approach was only the very last step—after he had conducted all steps following his initial idea.

In many instances in the SRI, David stated that he wrote down his thoughts only after he, for instance, was sure about his approach, was sure that the approach matched his ambition, or had checked multiple times whether his approach would turn out well. David explained that he even double-checked before he *drew* lines—in order to avoid mistakes: "Because I didn't want to have to, like, erase this line over here. Because it would be very difficult to see what was going on if I had made a mistake there. So I wanted to check many times."

Still, in some instances, David made use of writing intentionally. He stated that he "wanted to put down the, like all the facts I had about this. Because at this point, I had definitely chosen to use this whole triangle up here. And then I just put down, what information do I have about this triangle now?" It is noteworthy that, again, he only started writing down the steps of this "proof" after he had chosen to pursue an idea.

Discussion and Outlook

This article aimed to explore what is involved in the creative process of a school student working on an MST. We used a new method for investigating the creative process in mathematics: ET SRI. Results of our case study indicated how new ideas and insights came up and what phases this process included. We would like to point out that we present a case study of a single student working on one particular MST in one particular mathematical domain, geometry. Thus, we do not intend to generalize our tentative findings to all students, all problems, or all mathematical areas. Our study illustrates, by counterexample, that neither stage models on the creative process (e.g. Wallas' four-stage creativity model), nor stage models on problem solving (e.g. Dewey's five-stage model of problem solving) do fully capture the creative process of school students in MSTs. In the following, we elaborate on this finding.

Comparing David's Process to Existing Models

Given that there are existing stage theories in mathematics education modeling the creative process (Hadamard, 1954; Wallas, 1926) and problem solving (Dewey, 1910; Pólya, 1945), the question arises how the observed phases of David's work on the MST relate to these models. Were the phases in David's case similar to the stages described for creative mathematicians? Or are they possibly similar to those described in problem solving stage models? Comparing David's creative process to Wallas' (1926) model of the creative process and to problem solving models such as Dewey's (1910), indicates certain resemblances, but also certain differences (Fig. 4).

Comparison of David's Process with the Model of the Creative Process. Both David's observed creative process as well as Wallas' model include an initial or *preparation phase*. Hadamard (1954) describes the preparation phase as a phase of "deliberate labor" (p. 44), where the person thinks about the problem fully consciously, tries to find different angles, recalls prior knowledge, conducts different steps, etc. This phase relates to the phase of *looking for a start* in our tentative model. Next, Hadamard describes an *incubation phase* where "the problem is put aside for a period of time and the mind is occupied with other problems." (Sriraman, 2009, p. 14) This relates to the rest-hypothesis proposed by Poincaré indicating that the mind needs certain freshness and the absence of interferences by putting the problem aside for a certain time. Hadamard (1954) points out that "[i]n this period of incubation, no work of the mind is consciously perceived." (p. 33) However, David's work on the MST did not include phases where he put the problem aside and thought about something else instead. Thus, a typical phase of "incubation" as described in Wallas' and Hadamard's works cannot be identified for David. Of course, this difference lies in the nature of the problems to be solved and the environmental situation given: Whereas mathematicians work on

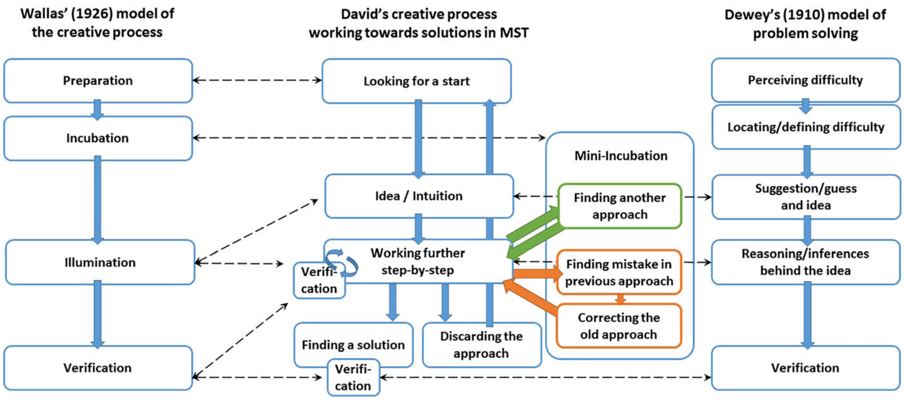

Fig. 4 David's creative process in the MST—compared to existing models

open problems over longer periods, the purpose with MSTs is to solve them in multiple ways within a relatively short time. In our study, the time on the task was less than 20 min.

However, incubation is not only about time (Sriraman, 2009): It is also about setting aside a problem, not specifically thinking about, and when doing something else, it seems that the mind has reached some insight into the previous works on a problem. We suggest that David possibly had such an experience when he—when he was working on a certain approach—realized he had made a mistake in a previous approach. In other words, although he was done with the previous approach, something in his mind triggered him to go back to this approach. This might be evidence of a "mini-incubation" happening while David was working on several approaches. Also, the fact that David found new approaches while he still worked on previous ones might be an indication of "mini-incubation" (see Fig. 4).

Illumination moments as affective experiences and moments of sudden clarification (see Liljedahl, 2013; Sriraman, 2009) appear to play an important role—both for mathematicians and David. David described in the SRI that he got new ideas about how to solve the MST and got very excited and certain about it. This excitement was not an Aha! experience in the sense of an "experience of an untimely and unanticipated presentation of an idea or solution" (Liljedahl, 2004, p. iii). Still, David's discoveries offered excitement for him, together with a feeling of sudden certainty/clarification. The creative process appeared to be an affective experience for David. He did not only get excited about discoveries, insights, and big ideas, but also "panicked" when he realized he had made an error or when the time was running out.

Wallas' model ends with a *verification* stage. In our case study, verification took place both during the *working further step-by-step phase* (where David "double-checked" thoroughly throughout the problem solving process whether his steps were correct) as well as after finishing the respective approach—comparing the solution with previous ones.

Comparison of David's Process with the Model of Problem Solving. First stages in problem solving models address perceiving, locating, and defining a difficulty (Dewey, 1910), or, in Pólya's (1945) words, "understanding the problem." (p. 6) Here, students understand what the difficulty or the problem-to-solve is. In David's case, the SRI revealed no activities related to understanding the problem or the difficulty. Before the first approach, he read the text quickly so that this activity had a subordinate role. For other students, it may take longer to understand the problem. Yet, in MSTs students work towards several solutions. Even if understanding the problem takes some time initially, the understanding-stage is subordinate when students work on further approaches.

The next stage in problem solving models concerns developing a suggestion or idea (Dewey, 1910) or "devising a plan." (Pólya, 1945, p. 8) Dewey, as well as Pólya, points out that plans usually do not come fully-fledged. Pólya (1945) explains, "[t]he way from understanding the problem to conceiving a plan may be long and torturous. In fact, the main achievement in the solution of a problem is to conceive the idea of a plan. This idea may emerge gradually. Or, after apparently unsuccessful trials and a period of hesitation, it may occur suddenly, in a flash or 'bright idea'." (p. 8) In David's case, such "guesses" (Dewey, 1910, p. 75) emerged similarly: He suddenly saw possible solutions, had ideas about how to work on the problem, which was also an affective experience for him, as described above. These moments appeared to play a big role in David's work on the MST.

The next stage in problem solving comprises the reasoning behind the idea or "carrying out the plan." (Pólya, 1945, p. 12) In our case study, David worked further step-by-step, verifying the appropriateness and correctness of these steps. Pólya (1945) refers to such verification as well: "We may convince ourselves of the correctness of a step in our reasoning either 'intuitively' or 'formally'. We may concentrate upon the point in question till we see it so clearly and distinctly that we have no doubt that the step is correct." (p. 13) In this way, David's way of working further from his initial idea or intuition step by step, verifying the respective steps, was similar to what problem solving models describe in this stage.

Finally, problem solving models mention a verification stage or "looking back." (Pólya, 1945, p. 14) Whereas Pólya emphasizes the opportunities of looking back for consolidation of knowledge and development of problem solving abilities, Dewey (1910) emphasizes the verification of the initially conjectural idea. In our study, David compared the solutions with the previous ones he had found, verifying whether his approaches led to a correction solution.

Reflections on the Creative Process of Solving an MST

In previous work, students' creative process on MSTs was—to our knowledge—never systematically investigated in its phases. This is in contrast to the growing need to study the creative processes among students and to better understand how to foster creativity of learners in preparation for an increasingly automated and interconnected high-technology based society and economy (OECD, 2014). Our article addresses this research gap and presents a qualitative case study, which explores the creative process of a school student working on an MST—one of the dominant kinds of tasks in mathematical creativity research.

The student in our case study, David, was partially guided by his intuitions but also consciously applying heuristics to find new solutions. In our study, we identified heuristics that were used in the work on an MST: looking for symmetry or glancing over areas that have not been used before. It is an interesting question for future research, which other such heuristics can be identified, how they relate to the competence levels of school students, and how they indeed afford truly creative moments.

We perceive a doubt or at least precaution among some researchers in the domain of mathematics education about whether MSTs actually capture mathematical creativity, whether they actually do afford illumination, affective experiences, or connecting ideas. One could think of solving MSTs like work on the assembly line, where students *only* produce one idea after another, with heuristics that are always similar—rattling off the different approaches until work time is over. In this perspective, the question arises whether MSTs actually *do* touch creativity in the sense that students have intuitions, or illuminations, certain excitement about having figured something out, and sudden certainty and insight. Our case study showed that this was indeed the case for David. He appeared to have intuitions about how to solve the problem. Intuitions are, as Liljedahl (2004) points out, strongly linked to creativity in mathematics, as they, for instance, "may provide a direction to look in" (p. 22) and, thus, may open up for new, innovative ideas and insights. Further, David in some instances realized that his ideas would work out successfully. In the ET SRI, David described impressively his excitement about such discoveries and satisfaction when he solved the problem. There was a eureka moment ("THIS is it!"), excitement, and also emotional arousal when David then found a solution. Such affective aspects characterize illumination (Liljedahl, 2013) and make David's process creative at heart. For David, solving the MST in multiple ways was not only machining off different solutions, but it was an affective experience that was intuition-led and comprised moments of illumination. One interesting area of further research—relating to the "sensory/cognitive dichotomy" (Sinclair, Bartolini Bussi, de Villiers, Jones, Kortenkamp, Leung & Owens, 2016, p. 694) in geometry education—would be to investigate a possible distinction between student ideas that originate from features of the problem (e.g. salient symmetric shapes) and ideas that originate from dispositions on behalf of the learner (e.g. pre-knowledge about geometric problems) (see Levav-Waynberg & Leikin, 2012). ET research might be valuable for this question in particular (Schindler & Lilienthal, 2019).

David's creative process working on the MST also had similarities to problem solving as described in stage models. In this sense, David's work on the MST can be understood as problem solving with much emphasis on intuitions, guesses, and ideas, with many affective experiences connected to Aha! moments, and a subordinate role of initial stages, such as understanding the problem (Pólya, 1945) or locating/defining a difficulty (Dewey, 1910).

However, even though such existing stage models appear to capture David's creative process on the MST to a certain extent, they do not account for the volatility of David's process. For David, we found that new approaches emerged while he still worked on previous ones, that he held them back for a while with certain ambitions, that he found mistakes in previous approaches and corrected them, and then went on with current ones. We call the process of finding a mistake in previous approaches (or coming up with new ones) while working on a certain approach *mini-incubation*: a shortened incubation period that occurs while the student is working on another approach to the same MST. The volatility in David's process was not described in previous studies, but was instead possible to explore in our study through the special setup, involving ET and SRI. Especially the combination of ET with an SRI (using the gaze-overlaid video as stimulus) seemed to be a powerful method to help "demystifying the creative process" (Pitta-Pantazi, Kattou & Christou, 2018, p. 39) in David's case. Even though ET SRI has certain risks to be taken into account, our study illustrated that this method may help researchers to get closer insights into complex reasoning processes and thoughts. These thoughts can be quick, sophisticated, complex, affective, or intuitive, which may make them difficult to express by school students in thinking aloud protocols only.

It has to be noted that David was a highly engaged student with a passion for mathematics. The question arises if the creative process would be similar for students with less interest in or passion for mathematics. It is probable that "the characteristics of the creative process are" *not* "the same for different individuals" (Haavold & Birkeland, 2017, p. 184). This, and eventually what the different characteristics of the creative process are and how frequently they occur among school students (especially with different competence levels) should be investigated in future research—to which our work lays the foundation.

Finally, we hope that this study, our considerations, and findings can fuel the discussion and further empirical research on school students' creative process in mathematics. It is only through investigating the creative process that one can identify ways and means to better foster creativity in students, and to prepare today's students for their future lives in societies that will require an increasing number of creative thinkers and problem solvers.

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