



Following the Traces of Teachers' Talk-Moves in Their Students' Verbal and Written Responses

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Abstract

We focus on teachers' ways of leading whole class discussions (WCDs) in mathematics, with the goal of uncovering their traces (if any) in their students' responses (a) while participating in the WCDs and (b) in the written responses in a final test. For this purpose, two 8th-grade probability classes learning a 10-lesson unit with different teachers were observed. Our data sources include (1) video-recordings of the WCDs and (2) the responses of students to final test items. We analyzed the teachers' talk-moves, students' accountable participation, and students' reasoning in the final test items. Interweaving the findings from all analyses we found differences between the classes in students' ways of participation in WCDs and in their corresponding final test responses. The teachers' ways of leading the WCDs contribute to the explanation of these differences.

Keywords Classroom study · Students' participation · Students' written responses · Teachers' talk-moves

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Introduction

In the course of the last few years, we have investigated shifts of knowledge among the individuals, the small groups, and the whole class discussion (WCD) in an inquiry-based mathematics classroom (Hershkowitz, Tabach, Rasmussen, & Dreyfus, 2014; Tabach, Hershkowitz, Rasmussen, & Dreyfus, 2014). The term WCD refers to everything the teacher and the students openly express during the plenary discussion, as opposed to small group work or individual work. The role of the teacher in these classroom settings is central:

In an inquiry classroom the role of the teacher is compound and has many faces: ... She kept an equilibrium between the need of teaching a certain content on one hand, and the strategy of affording opportunities for students to construct their knowledge in argumentative autonomic ways on the other. (Hershkowitz et al., 2014, p. 385)

Prior research has examined various teacher discursive moves (e.g. Chapin & O'Connor, 2007; Mercer & Sams, 2006; Stein, Engle, Smith, & Hughes, 2008) that are believed to support students' learning. However, as O'Connor, Michaels, and Chapin (2015) point out, little strong empirical evidence for the link between high-quality classroom discourse and students' learning exists. The goal of the present study is to contribute to this link, by focusing on the teacher and her talk moves, and to look for her teaching traces in verbal responses of her students in the WCDs and also in students' written responses as individuals in relevant items of a final test. By the term talk moves we mean "families of utterances that help teachers in the moment-to-moment micro-interactive challenges of orchestrating student's discussions" (O'Connor et al., 2015, p. 112). Hence, we continue our line of research on the teachers' central role in the classroom.

Theoretical Background

In this section, we discuss theoretical issues, which are relevant to our main research goal: first, we discuss classroom studies that compare teaching by various teachers in a few classes; second, we discuss studies of the way teachers lead WCDs and their ways of encouraging students to participate in these discussions. Finally, we discuss studies, which link the teacher's way of leading WCDs and students' ways of participation in WCDs. We also intended to discuss studies, which link teachers' ways of leading WCDs to characteristics of their students' written responses in corresponding individual work after an interval of time, but unfortunately, we did not find any such studies.

Classroom Studies: Teaching the Same Content by Different Teachers

In recent years, mathematics education researchers have been considering and exploring the role of the teacher as the leader of the learning process in her classroom. For example, Even and Kvatinsky (2010) studied two different high school teachers with different teaching approaches—one teacher emphasized understanding

while the other focused on procedural fluency, each teaching a probability unit based on the same textbook. Even and Kvatinsky showed similarities and differences between the mathematics presented by the two teachers in their classes: while both teachers addressed the fundamental characteristic of probability, like, uncertainty, they differ for example in the use of un-conventional notations in addition to the classical ones. In another study, Pinto (2013) found that two calculus lessons based on the same lesson plan, given by two university instructors with different backgrounds and teaching agendas, were substantially different in terms of their conceptual focus. Similarly, Sharma and Anderson (2006) studied the nature of classroom learning environments created by each from three secondary teacher-candidates in their science classes. They described and compared different patterns of practice when constructing a classroom learning environment and developing teaching strategies.

Each of the above studies investigated a few teachers in order to highlight the unique behavior of each teacher.

Teachers' Ways of Leading WCDs

Research has accumulated on how teachers lead WCDs in classrooms and why such discussions may support learning as well as the process of reasoned participation (Asterhan & Schwarz, 2009; Conner, Singletary, Smith, Wagner, & Francisco, 2014; Pontecorvo & Girardet, 1993). Opportunities for students to reflect and communicate about their mathematical work have been identified as essential for learning mathematics with understanding, and effectively apply high-level reasoning (e.g. Hershkowitz & Schwarz, 1999). The classroom's discourse led by the teacher presents a powerful opportunity for providing learning with a meaningful context (Cobb & Bauersfeld, 1995; Mercer, 1995; Wood, 1994). That is, the ways by which the teacher leads the WCD, is an important key to learning opportunities provided for the students. During discussion, students can realize how others approach a task and can gain insight into solution strategies and reasoning processes that they may not have considered themselves. By engaging in whole-class teacher-guided reflective discussions, students can explain their reasoning; make mathematical generalizations and connections among concepts, strategies, or representations; and benefit from the collective mathematical work of the class for a given lesson or task (Bauersfeld, 1988; Conner et al., 2014; Hershkowitz & Schwarz, 1999; Schwarz & Linchevski, 2007; Wood, 1994). However, leading this kind of discussions in classrooms is challenging for teachers.

Researchers have observed that different patterns may emerge in classes, as each teacher has his or her own way of leading the classroom discussion (e.g. Wood, 1994). Classroom research has shown that the leading WCDs consists of Initiation (by the teacher)–Response (given by students)–Evaluation (given to students by the teacher) (IRE) (Cazden, 2001). Opting for IRE patterns partly originates from the difficulty teachers have to support the reasoning of individuals and groups in the class. A different way of leading WCDs is that teachers use funneling patterns of questions—starting with a general question, which is immediately followed by a second more focused or narrower question (Wood, 1994). Researchers and teacher educators have tried to foster teacher-student classroom interactions in which students' ideas are at the center of teacher-led discussions (Mercer, 1995; Mercer & Sams, 2006; van Zee & Minstrell, 1997).

Michaels, O'Connor, Hall, and Resnick (2002) and Sohmer, Michaels, O'Connor, and Resnick (2009) provide some evidence of the effect of teachers' talk on their students' learning and understanding by creating a theoretical framework and a methodological tool for this kind of research. They speak about key features of academically productive talk (by both teachers and students) which have been cataloged, characterized, and subsumed by the term *Accountable Talk* (Michaels et al., 2002), which has been defined operationally as follows:

- *Accountable to the learning community*: This is talk that attends seriously to and builds on the ideas of others; participants listen to and learn from each other, grappling with ideas together
- *Accountable to knowledge*: This is talk that is explicitly based on a body of knowledge that is public or accessible to the group as a whole. Speakers try to get their facts right and make explicit the evidence behind their claims and explanations
- *Accountable to rigorous thinking*: This is talk that emphasizes logical connections and the drawing of reasonable conclusions. It is talk that involves explanation and self-correction (Sohmer et al., 2009, p. 106)

Accountable talk is aimed at creating a class community in which the students learn how to solve problems correctly and to justify their solutions in an argumentative way. In addition, Michaels et al. (2002) conceptualized *teachers' talk moves* as tools to enhance productive discussions in class, leading towards an inquiry class community. Operationalization of teachers' talk moves functions as a methodological tool in our research (see the "Methodology" section).

Teachers' Talk Moves and Students' Learning

The development of accountable talk in the classroom based on argumentation requires cooperation among members of the class community—students and their teacher. Teachers have to challenge, to prompt, and to afford opportunities for their students to raise claims and to justify them. In an inquiry classroom, where genuine argumentation is a norm, students are routinely explaining their reasoning, listening to and indicating agreement or disagreement with each other's reasoning.

Prior research has examined various teachers' talk moves that are believed to support student learning (e.g. Chapin & O'Connor, 2007; van Zee & Minstrell, 1997; Stein et al., 2008). Hufferd-Ackles, Fuson, and Sherin (2004) followed one mathematics teacher along several months and were able to show a change in the level of classroom discussions. While at the beginning of the school year a traditional (IRE) pattern of instruction was found, during the following months a shift towards argumentative WCD was noted. The researchers attributed the changes in students' participation to the change in the way the teacher led the WCDs. Boston and Smith's (2009) study is unique as it looked not only at the teacher talk moves but also at students' written work in class. They showed a link between the levels of cognitive demand embedded in tasks that were chosen by middle school mathematics teachers and the level of reasoning expressed in students' responses.

Sohmer et al. (2009) studied science teachers' talk moves while leading whole class discussions. Their list of teachers' talk moves included re-voicing, asking students to restate someone else's reasoning, asking students to apply their own reasoning to someone else's reasoning, prompting students for further participation, asking students to explicate their reasoning and provide evidence, challenging or providing a counter example, and using a wait time. Their work was adapted to mathematics lessons by Heyd-Metzuyanim, Smith, Bill, and Resnick (2016). Yet, empirical studies showing the relationship between teacher ways of leading WCDs and students learning are still needed (O'Connor et al., 2015).

Methodology

Research Aims and Research Questions

Our aims in this paper are to expand the above-mentioned lines of research: by means of the exploration of differences and similarities between two mathematics teachers' talk moves, and in our efforts to trace connections between what teachers say and what students learn, we investigated to what extent and how differences between teachers' talk moves explain, at least partially, differences in students' responses at two points in time and levels: (a) within the learning process in WCDs led by the teachers and (b) in the students' responses to test items that correspond closely to these WCDs. The test responses include correctness of the answers as well as the justifications given by the students to explain the answers.

These aims will be achieved by answering the following research questions:

1. What characteristics of each teacher's talk moves while leading her WCDs can be identified?
2. What characteristics of students' participation along the WCDs in each class can be identified?
3. What are the characteristics of students' responses in each class to the corresponding items in a final test?

Links between questions 1 and 2, and 1 and 3, will serve to address our aim.

Our arguments in answering the above questions are built up through analysis and exemplification via micro-analysis of selected lesson fragments.

Context and Participants

A 10-lesson learning unit in probability was designed by the researchers. The unit deals with concepts and problem-solving aspects of empirical versus theoretical probability, and with one- and two-dimensional sample spaces (for details see Hershkowitz, Hadas, Dreyfus, & Schwarz, 2007). The lessons were designed to be implemented in each class in three different settings: (a) WCDs, in which the teacher and the students publicly interact in plenary discussion; (b) small group work in which the students in pairs or threes privately discuss and the teacher circulates among the groups and responds to questions privately or as she sees fit; and (c) individual student work. The probability unit was implemented, observed, and video-recorded in seven eighth-grade classes.

The researchers who designed the learning unit did not instruct the participating teachers as to how to use the learning materials in their class, nor did they provide professional development. Two classes, those of teachers D and M, were selected as extreme cases of teacher talk moves, for demonstrating the range of ways of leading WCDs and their possible links to students' participation in these WCDs and their responses to the final test items.

The two teachers, D and M, were experienced teachers (more than 10 years of teaching experience) and each one was considered a good teacher in her school. The students of both eighth-grade classes had a high socio-economic background and were considered high-level students in mathematics in their schools. Both classes had learned mathematics from the same textbooks from the beginning of their seventh grade.

Data Sources and Their Analyses

As mentioned above, the study had two main data sources, WCDs and final tests:

Analyses of the Data From WCDs. Two WCD's in each class (four all together) were selected so as to have the same mathematical content in both classes. Our analysis comprised several stages:

First, we checked that the WCDs in the two parallel lessons of each pair were about the same length. **Second**, we coded and counted each teacher's talk moves while leading the WCDs in her classroom. Following Heyd-Metzuyanin et al. (2016), we categorized the teachers' utterances into four categories: (1) Help individual students share, expand, and clarify their own thinking; this goal is captured by two codes: (i) Say more (asking students to expand their ideas) and (ii) Re-voice student reasoning; (2) Help students listen carefully to one another; this goal is captured by one code—(iii) Re-state or explain others; (3) Help students deepen their reasoning; this goal is captured by two codes: (iv) Press for reasoning and (v) Challenge; and (4) Help students think with others; this goal is captured by three codes: (vi) Agree or disagree, as a way to lead to consensus in the class; (vii) Add-on; and (viii) Soliciting additional viewpoints. **Third**, students' participation in the WCDs was quantified by coding and counting based on students' accountable talk developed by the same researchers (Heyd-Metzuyanin et al., 2016). The two suggested categories to capture students' accountable talk in class are Justifications and Agree/disagree with other students or the teacher.

To give the flavor of the teacher's talk moves and students' participation, we selected one episode from each WCD (a total of four episodes) for detailed presentation below.

Analyses of the Final Test Responses. The data is taken from the final test of the probability unit which was administered in the participating classes about a week after the classes finished the unit. We choose to analyze for correctness and justification of students' final test responses to the two test items, which related to the probability notions that were discussed in the two WCDs we have chosen for analysis:

Item A. The situation in Fig. 1 presents a probability game

In this item the simple events are two-dimensional (two spins of the clock), with the one-dimensional simple events having equal probabilities. This kind of item may give

The hand in the clock is spun twice.
 (The partitions in the clock are equal.)

What has greater chance, or are the chances equal:

- The hand will stop twice in the region of 1, or
- the hand will stop once in the region of 3 and once in the region of 4?

Explain!

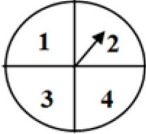


Fig. 1 Final test item A—2D events where the simple events are equi-probable

rise to a very common mistake: considering the events (3, 4) and (4, 3) as the same event. Students were taught to use a two-dimensional table (see Fig. 2) as a tool to list the probabilities of the relevant simple events with the aim to avoid such mistakes. It can be concluded from the table that the chances of having the hand stop twice in the region of 1 are smaller than that of having the hand stop once in the region of 3 and once in the region of 4.

We categorized the answers and justifications separately as correct, incorrect, or missing, and in addition created categories from the students' justifications such as probabilistic versus non-probabilistic justification and one- versus two-dimensional justification, as detailed in the finding section.

Item B. Item B (see Fig. 3) presents a problem where each dimension has only two possible simple events (binomial sample space), but these are not equi-probable. The item presents two contradictory (one correct, one incorrect) replies of two virtual students about the probability of a simple event in a binomial two-dimensional sample space. Note that the probability unit, being designed for beginning students, assumed throughout that all events are independent. This item emphasizes the dilemma of either multiplying the given probabilities or adding them. The numbers were chosen so as to make choosing addition absurd. Students were taught to use an area model (see Fig. 4) as a tool to describe and analyze the probabilities in such two-dimensional binomial sample spaces

We categorized the answers and justifications separately as correct, incorrect, or missing, and in addition created categories from the students' justifications, as detailed in the finding section (Azmon, 2010).

Finally, we interpreted the differences between the findings of the parallel items in the final test of the two classes (the second data source) in light of the findings

	1	2	3	4
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)

Fig. 2 The sample space table for item A


The ‘Tel-Aviv’ school offers a variety of extracurricular programs.
 The probability of encountering a student who is in the drama program is 0.9.
 The probability of encountering a student who is in the philosophy program is 0.2.
 Gal claims that the probability of encountering a student who is in both, the drama program and the philosophy program is $0.2+0.9$. 
 Yam claims that the probability is 0.2×0.9 .
 Which of them do you think is correct? Explain!

Fig. 3 Final test item B—binomial sample space

concerning the teachers’ moves in the teacher-led WCDs and the finding from the students’ participation in these WCDs in each class (the first data source).

Findings

In this section, we first present two pairs of corresponding episodes from the WCDs, to demonstrate the differences between the teachers’ and the students’ talk moves in the two classes. Next, we present the coding and counting of teacher talk moves in the same WCDs, followed by an analysis of students’ participation in these WCDs. Finally, we present the findings from the two final test items. For each item, we present data from students’ responses and analyze differences between students’ responses from the two classes.

The Episodes and Their Analyses

The first pair of episodes (one in each class) focused on the use of a table for calculating the probability of two-dimensional simple events, where the probability of each simple

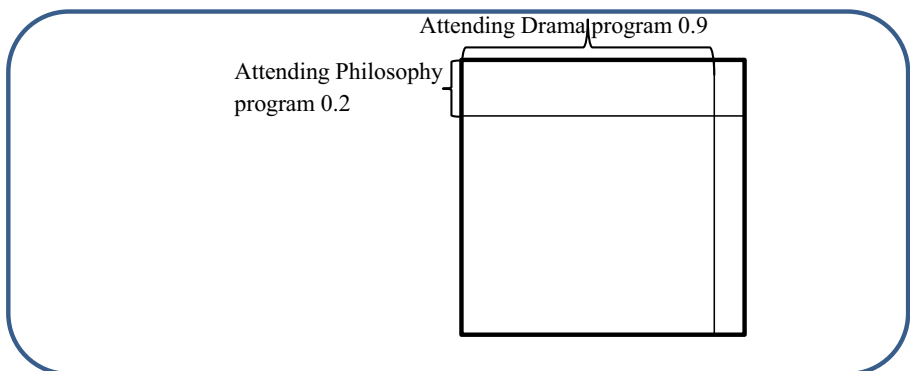


Fig. 4 Area model describing the probabilities for item B’s events

event in each dimension is equal—a similar mathematical content as test-item A (Fig. 1). The second pair of WCDs focused on the use of the area model to compute the probabilities of the simple events in two-dimensional binomial sample spaces—the same mathematical content as test-item B (Fig. 3).

Analysis of Two Pairs of Whole Class Episodes

The First Pair of Episodes. Generally, the tasks in the learning unit were presented in the lessons as probability games where the students were asked to determine if the games are fair and to explain their claims. A table (similar to Fig. 2) was introduced as a tool to calculate the probabilities of two-dimensional events (relevant to test-item A). The first pair of episodes (1 and 2) are taken from the first pair of lessons. Both episodes took place towards the end of the WCD concerning a judgment of fairness of a “two-dimensional clock game,” when the relevant table was already on the board and had been discussed:

-
- 154 Shay The game is not fair
 155 D Why do you think the game is not fair? How did you check?
 156 Shay I made a table
 157 D You made a table and multiplied? Can you tell me what the sample space is? How many possibilities are there?
 158 Shay 12
-

Episode 1. D's class discussing whether the clock game is fair.

In Episode 1, teacher D responded [155] to Shay [154] who claimed that the game is not fair, by asking two questions in the same turn. The two questions [155] are coded as press for reasoning (see the “[Methodology](#)” section Category 3, code iv). However, the second question “How did you check?” might be a call for numerical-algorithmic clarification. Shay, quite naturally, answered the second question only [156], relating to the tool (table) rather than explaining why he thinks the game is not fair. We coded this turn as justification by Shay. The teacher's utterance in turn 157 included three questions. Each of the three questions points to a different stage in the solution algorithm. The first revoiced and elaborated the procedure, the second asked for place values expressed in numbers for numerical detail, and the third asked for a numerical detail. None of the three questions called for deepening the explanation or reasoning. Once more, the student naturally answered only the last question, and even this briefly, giving one specific numerical answer, and thus narrowing down the answer that the teacher might have expected. In 155 and in 157, D demonstrates an extreme case of funneling: She does not allow the students to answer the more general questions and replaces them by narrower ones. So students in the class are exposed to questions, which call for reasoning. However, the teacher's further questions are mainly about procedures and computations. Hence, students made claims without being exposed to the norms of justifying them.

The game discussed in the parallel episode in M's class was different from the one in D's class.

52	M	Is the game fair?
53	Students	The game is not fair!
54	M	Not fair because?
55	Student	Because 12 out of 24 is more than 11 out of 24.
56	M	So, it is really not fair?
57	Itamar	I think the game is fair!
58	M	Because?
59	Itamar	Because 1 out of 24 does not change much, 1 out of 24.

Episode 2. M's class discussing whether the game is fair.

M [52] asked almost the same question as D [155], but she did not follow it with a second question calling for a computational response. Rather, in turns 54, 56, and 58, M pressed her students to express their reasoning: claims and their justifications. By asking "because?," M is fostering students' justifications. Also, she affords the opportunity to have a variety of responses from her students [56]. Her talk moves [54, 58] are similar whether the student's claim is correct [53] or incorrect [57]. M seems to be aware of Itamar's [57] inaccurate claim but encourages him to justify it [58], providing the whole class an opportunity to listen and consider his reasoning. And indeed, we see in this short episode two instances of justification provided by students in response [55, 59]. We coded Itamar [57] as be Agree/disagree.

The Second Pair of Episodes. The WCDs in the second pair of lessons were about the area model, the same mathematical content as test item B (Fig. 3). Here again the classes discussed situations in which the area model was introduced and the two episodes are from the last part of the discussions. Episodes 3 and 4 were initiated by each teacher in her class in D [64] and M [62] by asking how one can check the correctness of the probability computations by the area model. We present the two episodes followed by an analysis, starting again with D's class.

64	D	How can we check that it is correct what we wrote here?
65	Yaad	You add and get 1
66	D	Everybody added? It adds up to?
67	Students	Yes!

Episode 3. D's class discussing how to check the correctness of the area model computations.

62	M	Now what kind of control can we do to see that we do not have a mistake?
63	Yael	$15\% + 15\% + 35\% + 35\% = 100\%$
64	M	Why does it have to be 100% when adding all these?
65	Itamar	Because 100% is the whole
66	M	Because this is the whole, and here we describe all 4 cases that can happen when two people together shoot an arrow.

Episode 4. M's class discussing how to check the correctness of the area model computations.

A similar talk move, pressing for reasoning, was made by both teachers. And indeed, Yaad [65] in D's class and Yael [63] in M's class each provided a correct claim and justification (both turns have been coded as Justification). Yet, Yael's justification is more elaborate. At this point the two teachers' moves are quite different. D [66] followed Yaad's justification with a call for computation. She asked all her students if they got the final numerical answer correctly. Her talk move prevents further elaboration and justification. The talk move of M [64], on the other hand, pressed her class to further elaborate the argument proposed by Yael. And indeed, while in D's class, students' collective "yes!" answer (code: Agree/disagree) closed the episode, Itamar in M's class provided a further justification, which allowed M to elaborate.

Considering the two pairs of episodes together, it seems that there is an important difference between the talk moves of the two teachers while leading the WCDs, as well as in the students' participation. In the next section, we demonstrate these differences by presenting the findings of coding and counting of all the turns in these four WCDs.

Quantification of Teachers' Talk Moves. Table 1 presents the quantification of the teachers' talk moves during the four WCDs.

As can be seen in Table 1 (columns i, ii, iv) the talk moves of both teachers were either helping students to expand their ideas or encouraging the students to deepen their reasoning. That is, the talk moves were directed at the individual student rather than helping students think with others (columns vi, vii, and viii) or explain what others have said (column iii). We *shaded in gray* the two dominant talk move categories for each WCD and teacher. For both teachers, re-voicing students' reasoning is dominant. However, while D couples this talk move with Say more, M couples this talk move with Press for reasoning. This was noted also in the micro-analysis of the episodes from each WCD. Also, while D's talk moves for helping students to deepen their reasoning are almost non-existent, in M's WCDs these talk moves appear to some degree (columns v and vi).

These findings strengthen the findings from the analysis of the episodes in the previous section and show the different combinations of the teachers' talk moves while leading WCDs.

Indications for Student Participation. We now turn to quantify the students' participation in the four WCDs. As mentioned in the "Methodology" section, we categorized those student turns that were justifications or expressing agreement or disagreement. Table 2 presents the distribution of the students' contributions in each WCD.

Table 2 shows that the students in M's class contributed many more justifications than the students in D's class. The findings from Tables 1 and 2 are complementary concerning justifications: M pressed her students to provide justifications, and indeed, almost half of the students' turns in her WCDs are justifications. As for expressing their agreement or disagreement, all numbers are not large, but they show the same ratio M:D as the justifications: The ratio is about 3; more precisely, it is 2.7 for (dis)agree and 2.5 for justification. Since the total number of student turns was about the same for both teachers, one could say that in M's class over 2/3 (51 out of 74) of the student turns fall

Table 1 Quantification of teachers' talk moves while leading WCDs

Teachers' talk moves: categories and codes	Help individual students share, expand, and clarify their own thinking		Help students listen to one another		Help students deepen their reasoning		Help students think with others	
	Say more (i)	Re-voice reasoning (ii)	Re-state or explain others (iii)	Press for reasoning (iv)	Challenge (v)	Agree or disagree (vi)	Add-on others (vii)	Additional viewpoint (viii)
First pair of WCDs	D ($N^a = 44$) 10 (23%)	17 (39%)	1 (2%)	8 (18%)	1 (2%)	—	—	—
	M ($N^a = 37$) 2 (5%)	16 (43%)	—	11 (30%)	4 (11%)	6 (16%)	—	2 (5%)
Second pair of WCDs	D ($N^a = 28$) 9 (32%)	10 (36%)	—	4 (14%)	—	—	—	—
	M ($N^a = 34$) 3 (9%)	18 (53%)	—	21 (62%)	2 (6%)	1 (3%)	—	—

Notes. (1) The percentages are calculated from the total number of teacher's turns in each WCD. (2) The numbers in each row do not sum up to N ; (a) the numbers sum up to less than N in D's class, because some turns could not be coded according to the talk moves categories and codes (e.g. organizational turns), and (b) the numbers sum up to more than N in M's class, because in some cases the same turn was coded twice (e.g. Re-voice student reasoning and Say more). (3) The italic numbers are of the dominant talk move categories for each WCD and teacher

^a N = total number of teacher turns in the WCD

Table 2 Distribution of students' participation turns in each WCD

	Class	Justification	Agree/disagree
First pair of WCDs	D ($N^a = 43$)	10 (23%)	3 (7%)
	M ($N^a = 43$)	18 (42%)	15 (35%)
Second pair of WCDs	D ($N^a = 37$)	3 (8%)	4 (11%)
	M ($N^a = 31$)	14 (45%)	4 (13%)

Note. The percentages in each row do not add up to 100% because some student turns did not belong to either category. For example in the first WCD in D's class the total number of students turns was 43 but only 13 of them could be coded as either Justification or Agree/disagree. The others were unjustified claims, numerical answers, and the like

^a N = total number of student turns in each WCD

into the categories (dis)agree/justify, whereas in D's class, only 1/4 (20 out of 80) falls into these categories, the remainder being numerical answers, claims without justification, and the like. Note that the WCD in the second pair of lessons in both classes had a more technical flavor, as the focus was on introducing the area model; hence, students' participation was lower.

The Final Test

As mentioned above, students from both classes participating in this study took the same final test. In the following, we present some data concerning the responses of students from both classes to items A and B (see Figs. 1 and 3). As mentioned above, item A is on the same topic as the first pair of WCDs (from which episodes 1 and 2 were taken) and item B is on the same topic as the second pair of WCDs (episodes 3 and 4).

Students Responses to Test Item A. We first analyzed students' responses concerning claims as Correct, Wrong, or None (missing). Then we categorized students' justifications in the same way. The findings from both classes to this item are given in Table 3.

As can be seen from Table 3, there are differences between the classes in students' responses to item A in the final test. About two thirds of the students in class M provided correct claims as compared to only about one third in class D. As for justifications, we note that in class D 24 students of the 29 (83%) who provided claims (either correct or wrong) also justified them. In class M, all 23 students (100%) who

Table 3 Students' responses to the final test item A (Fig. 1)

No. (%)	Students' claims			Students' justifications		
	Correct	Wrong	None	Correct	Wrong	None
D ($N^a = 36$)	13 (36%)	16 (45%)	7 (19%)	4 (11%)	20 (55%)	12 (34%)
M ($N^a = 24$)	16 (66%)	7 (29%)	1 (5%)	11 (45%)	12 (50%)	1 (5%)

^a N = the number of students in the class

provided claims (either correct or wrong) also justified them. That is, in both classes the vast majority of the students, who wrote claims, were aware to the need for justifying them. But, in class D, one out of six justifications was correct; in class M, approximately one out of two justifications was correct.

Justifications are at the heart of talk that is *accountable to rigorous thinking*. Therefore, we refined the categorization of students' justifications, both the correct and incorrect ones, into four categories. These categories were created on the basis of students' responses:

- 1) *Non-probabilistic justifications*—justifications that are not connected to probability principles. For example, “it depends how strong the hand is spun,” or “the hand also needs some variations.” We found such justifications accompanying correct and incorrect claims.
- 2) *The equal division of the clock*—Justifications based on the fact that each number on the clock has the same region. For example, “there are four equal regions on the clock hence the hand can stop on any of the four numbers,” or “the chances for 1, 2, 3, or 4 are 1/4; hence, each has equal probability.” We found such justifications accompanying only incorrect claims.
- 3) *Relating to only one dimension*—Justifications based on additive reasoning, viewing the situation as one clock with eight regions. This is a case of an incorrect justification for a correct claim. Examples: “the chances for the hand to stop on 1 is 2/8 while on 3 or 4 it is 4/8,” or “the chances that the hand stops twice on 1 is 2/8 because together we have 8 parts and 1 is on two of them. The chances of getting 4 or 3 are 4/8 because 4 parts have the numbers 3 or 4.”
- 4) *Relating to a two-dimensional table*—We found such justifications in correct and incorrect responses. For example, a two-dimensional table was built. The number “2” was not part of the table, maybe because the item did not relate to it. Such justification, although partial, led to a correct claim. An example of such a table can be seen in Fig. 5. Other justifications used a table in a complete way. For example, “the chances for the hand to stop on 3 and 4 are 2/16 and are larger than that of stopping twice on 1.” Some justifications were only qualitative “the chances for the dial to stop once on 3 and once on 4 are higher since for the first option the dial must stop twice on 1 while for the second option it can stop both on 4 or 3.”

The distribution of the justifications into the four categories is presented in Table 4.

The four categories can be thought of as portraying levels in students' understanding of events in a two-dimensional sample space. A non-probabilistic justification

	1	3	4
1	<u>(1,1)</u>	(1,3)	(1,4)
3	(3,1)	(3,3)	<u>(3,4)</u>
4	(4,1)	<u>(4,3)</u>	(4,4)

Fig. 5 Incomplete table for item A

Table 4 Distribution of justifications according to the four categories

No. (%) Class	Non-probabilistic justifications	The equal division of the clock	Relating to only one dimension	Relating to two dimensions (correct and incorrect)
D ($N^a = 36$)	9 (25%)	8 (22%)	3 (8%)	4 (11%)
M ($N^a = 24$)	2 (8%)	3 (13%)	7 (29%)	11 (46%)

Note. As can be seen in Table 3, 24 (67%) students from D's class and 23 (96%) students from M's class provided justifications. All justifications could be categorized into one of the four categories. The percentages in Table 4 are calculated from the total number of students in each class

^a N = total number of students in the class

represents students who fail to express an understanding of the very nature of probability, namely that one can state chances of events, even without knowing what the outcome of the next trial will be. Students in the second category show understanding that the sizes of the parts of the clock are an important factor in calculating the probability. Students in the third category show an ability to calculate the probabilities of events in one-dimensional sample space, and finally, students in the fourth category show understanding of ideas about two-dimensional sample spaces and how to calculate probabilities of events in such spaces. Only about a fifth of the justifications in D's class reflect understanding of probability calculations (categories 3 and 4), while three quarters of M's students exhibit such understanding. In fact, of the four categories, the fourth (rightmost) column represents the learning goal of this part in the unit. We can see that almost half of the students who provided justifications in M's class wrote such justifications. However, only one sixth of D's students, who provided justifications, provided such justifications.

Student Responses to Item B. The students' responses to item B are presented in Table 5.

Table 5 shows that very high percentages of students from both classes answered the item correctly and provided correct justifications. In order to get a deeper understanding of students' justifications, we further analyzed and categorized students' correct justifications (item B; Fig. 3). All justifications could be categorized into one of the categories:

- 1) *Justifications relying on the multiplication principle.* These include the following sub-categories:

Table 5 Students' responses to the final test item B (Fig. 3)

No. (%) Class	Students' claims			Students' justifications		
	Yam is right	Gal is right	No claim	Correct	Incorrect	None
D ($N^a = 36$)	34 (94%)	1 (3%)	1 (3%)	32 (89%)	0	4 (11%)
M ($N^a = 24$)	22 (92%)	1 (4%)	1 (4%)	21 (88%)	1 (4%)	2 (8%)

^a N = number of students in class

- i. *Justifications relying only on the multiplication procedure* which indicate that the students are aware of the principle that “in calculating the probability that two events happen together we have to multiply their probabilities.” Such a justification shows no evidence of the student’s understanding why a multiplication is required. The student is focusing only on the description of the solution procedure.
 - ii. *Justifications using the area model* (see Fig. 4) by providing a diagram with partition lines according to the given probabilities and calculating the probabilities according to the relevant rectangle area. A student who uses this justification shows an understanding of the multiplication procedure. Calculating the area of the representing rectangle justifies the multiplication principle.
 - iii. *Justifications using “part of”* by calculating the required probability as part of a part (e.g. “cause to find a part of something we have to multiply”).
 - iv. *Justifications using both part of as well as calculating the corresponding rectangle area.*
- 2) *Justifications according to the “probability cannot be greater than 1” principle.* Many students chose to support the claim that “Yam is correct” by the claim that “Gal is wrong”, for example “ $0.2 + 0.9$, the sum of probabilities, will lead to a probability that is greater than 1, an impossible situation,” or “the square area cannot be more than 100%.”
 - 3) *Justifications combining both principles 1 and 2.* For example: “the result of multiplying 0.9 by 0.2 is 0.18, a probability that is smaller than 1.”

Table 6 shows the frequencies and percentages of students’ justifications in each category and sub-category. We can see that, while there were essentially no differences between the classes in terms of the total percentages of correct justifications, there are differences in the ways the multiplication principle was related to within the justifications: While in M’s class 42% provided justifications based on the procedure and its explanation, and only 13% focused on the procedure, in D’s class only 12% provided justifications based on the procedure and its explanation, and 42% focused on the procedure alone.

Table 6 Distribution of students’ justifications into categories for item B

No. (%)	1. Justifications according to the multiplication principle				2. Justifications according to “probability cannot be greater than 1” principle	3. Justifications combining both principles 1 and 2
	(i) multiplication principle only	(ii) area model	(iii) “part of”	(iv) both “part of” and area model		
D ($N^a = 36$)	15 (42%)	1 (3%)	2 (6%)	1 (3%)	10 (30%)	3 (8%)
M ($N^a = 24$)	3 (13%)	9 (38%)	1 (4%)	0	5 (21%)	3 (13%)

The italic numbers are of main categories for each class

^a N = number of students in class

Summary and Discussion

In the current study, we compared the talk-moves of two teachers and their students, in order to follow possible traces of the teachers' talk-moves in their students' verbal and written responses. The analyses were based on four WCDs (two in each teacher's class) and on the students' final test responses. The findings section was organized around three research questions. In this final section, we will provide answers to the research questions and use these answers to fulfill the research aims.

Our aims in the current study were to investigate whether traces of the teachers' talk-moves while leading their WCDs can be seen in (a) students' participation in these WCDs and (b) in students' responses to corresponding written final-test items. The following is a summary of the answers to the three research questions:

1. What characteristics of each teacher's talk moves while leading her WCDs can be identified? As we presented in the findings section, teacher D's typical behavior involves re-voicing her students' utterances and asking them to say more. This behavior was characterized by asking two or three questions in the same turn, the last one usually about numerical answers or computational procedures. The typical talk-moves of teacher M also involved re-voicing. However, M accompanied it with *press for reasoning* talk-moves. That is, while the two teachers' dominant talk move was re-voicing, for D it served for stating facts and asking her students to expand on procedures, while for M it served as a basis on which she prompted her students to explain their reasoning.
2. What characteristics of students' participation in the same WCDs in each class can be identified? As we presented in the findings section, only a small number of students' contributions to the WCDs of D's class could be classified as expressing reasoning or referring to agreement or disagreement with their peers. In contrast, a large number of students' contributions in the WCDs of M's class could be so classified. That is, there was a difference in terms of students' accountable talk in the WCDs of the two classes.
3. What are the characteristics of students' responses to corresponding items in the final-test? The findings show that the differences in students' individual written responses had to do mainly with expressing reasoning: the proportion of students who provided justifications to their claims in M's class was much higher than in D's class (Tables 3 and 5). Also, the level of the justifications in terms of understanding probability principles in M's class was higher than in D's class (Tables 4 and 6).

Let us now come back to the research aims. The first aim was to investigate whether characteristics of the way teachers lead their WCDs can be seen in students' participation in these WCDs. Our findings demonstrate that each teacher had her own typical talk-moves she used to propel the WCDs forward. Each teacher's emphases were reflected in the quality of students' participation. In D's class, the main teacher talk-moves were *re-voice student reasoning* and *say-more*; she did not *press for reasoning*. And indeed, the students' participation coding included a low proportion of justifications. In contrast, in M's class, the main teacher talk-moves were *re-voice student*

reasoning and *press for reasoning*, and indeed, a high proportion of her students' participation consisted of justifications for their claims. This means that the students' contributions echo the emphases of the teacher's talk-moves: the WCDs in the two classes were qualitatively different; in M's WCDs, reasoning became a major component of the students' mathematical discourse.

Our second aim was to investigate whether characteristics of the way teachers lead their WCDs can be seen in students' written responses to the relevant final-test items. Considering the analyses done on students' responses to the two final test items, we found different learning outcomes. The differences were expressed mainly in students' justifications: more students in M's class provided justifications to their answers, and the quality of the justifications provided by students from M's class was more sophisticated in terms of probability reasoning. In light of the findings from analyzing the two teachers' talk moves and students' participation in these WCDs, the gap in terms of justification in the test items is not surprising. The accountability to mathematical knowledge and to rigorous thinking was present in M's students' contributions not only during the WCDs but also in the way they responded individually on the test relevant items. It is surprising that after several weeks and in a test situation, students in M's class who can be expected to give the minimal answer that will grant them a maximal grade, did provide full justifications; in other words, they are likely to give the simplest justification, which in item B, for example, is "probability cannot be greater than 1." We surmise that the final test responses have been influenced by the participation in the class community as a whole. That is, they were influenced both by the teacher talk moves in the WCDs and by the students' participation in these WCDs.

The common ground for the teaching and learning in the two classes were the same mathematical themes and textbooks they had learned from since the beginning of their seventh grade, and the purposefully designed probability unit. That is, the two teachers based their lessons on the same learning materials. Yet, as was found also by others, each teacher enacted the lessons in quite different ways (e.g. Eisenmann & Even, 2011; Ozruso-Haggiag & Tabach, 2018). Based on our study of M's class, we can say that the learning activities in her class allowed for developing accountability to mathematical reasoning. Yet, based on our study of D's class, we can say that those same learning materials could be handled by the teacher in ways that are less in line with accountable talk. That is, if we believe that accountable talk is preferable for learning mathematics in terms of student's learning gains, our findings suggest that designing good learning materials is just one element. Professional development for teachers seems to be needed to help teachers get closer to accountable talk norms in their awareness and practice.

The current study took us beyond the question posed by Even and Kvatinsky (2010): *what mathematics do teachers address in probability lessons*, as we also inquired into students' participation and learning outcomes. The use of two data sources and multiple analyses allowed us to not only portray the differences in the talk-moves each teacher perform while leading her WCDs but also to point at the influence of these moves on students' participation and students' knowledge.

As was mentioned in the "[Introduction](#)" section, little strong empirical evidence for the link between high-quality classroom discourse and student learning exists (O'Connor et al., 2015). We consider the current study as a key contribution to this highly important theme. Specifically, this is a contribution for understanding the role of the teacher in the mechanisms by which knowledge spreads in the classroom.

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