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Numbers and Space Intervals in Length Measurements in the Spanish Context: Proposals for the Transition to Measuring with the Ruler

Ariadna Gómezescobar[1] ·
Raquel Fernández-Cézar[1] · **Silvia Guerrero**[2]

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Abstract The present work shows how 4–6-year-old Spanish children interpret numbers and space intervals in the ruler when measuring length. To determine it, 4 ad hoc rulers are designed and used with a sample of 103 children from two schools of Toledo province (Spain). The sample is characterized respecting conservation and measurement with the standard ruler confirming that these children mostly neither conserve nor use the standard ruler correctly, regardless their time exposure to instruction. With the use of our rulers, we confirm that numbers hinder in measuring length, and discrete units imbedded in the ruler help children to measure correctly. A good scaffold is found to help children conceptualize space intervals as iterating objects consisting on the use of rulers with discrete units on them. Its use is recommended preceding the one of standard rulers.

Keywords Length measurements · Numbers · Preschoolers · Space intervals

✉ Ariadna Gómezescobar
ariadna.tic@gmail.com

Raquel Fernández-Cézar
Raquel.fcezar@uclm.es

Silvia Guerrero
Silvia.guerrero@uclm.es

[1] Dpto de Matemáticas, Universidad de Castilla La Mancha, Facultad de Educación, Toledo 45071 Toledo, Spain

[2] Dpto de Psicología, Universidad de Castilla La Mancha, Facultad de Educación, Toledo 45071, Spain

Introduction

Measurement is a crucial aspect of the applied branches of science, technology, engineering, and mathematics (STEM), and it is widely present in everyday life (Lehrer, 2003; Levine, Kwon, Huttenlocher, Ratliff, & Dietz, 2009; Sophian, 2007; Wilson & Rowland, 1993). Particularly, measuring length is one of the earliest measurement activities children face at schools. It could provide a route for learners into mathematics, acting as a link between real life and the abstract world of numbers. For instance, measuring length can provide space intuition to develop models that support number sense. But it is not typically used this way in instruction at school (Chamorro & Belmonte, 1991; Ryan & Williams, 2007). Rather teachers first introduce numbers and then measurements, contributing to an *arithmetization* of measurement (Chamorro & Belmonte, 1991), that is, provoking that the focus of measurement is on numbers and not on measuring itself. If you ask children if they know how to measure length, in most cases, their answer is "yes." The reason can be that the word is familiar for them. Even if you ask their parents or teachers about whether children know how to measure, their answer is very often positive. Therefore, measuring is socially considered a simple task for children. Nonetheless, the wide research on this field does not show it (Battista, 2006; Boulton-Lewis, Wilss, & Mutch, 1996; Bragg & Outhred, 2004; Castle & Needham, 2007; Clements, 1999; Clements & Stephan, 2004; Cullen & Barrett, 2010; Kamii & Clark, 1997; Kamii, 2006; Levine et al., 2009; MacDonald, 2012; Nunes, Light, & Mason, 1993; Piaget, Inhelder, & Szeminska, 1960; Stephan & Clements, 2003; Solomon, Vasilyeva, Huttenlocher, & Levine, 2015; Zöllner & Benz, 2013).

Length Measurement

Most of the literature in measuring length is aligned with the fact that a child's early understanding of a measurement is mainly perceptual starting from direct comparisons ("this is longer than…, and so it is bigger"). But they show also perceptual errors in indirect measures (comparisons through the use of rulers or other instruments). About the particular ages, Piaget's theory states that until 7 years, children had no use of sticks or rulers, and that 4- and 5-year-old children attempted to make comparisons (Piaget et al., 1960). Therefore, it seems that to use the ruler is harder for children because it involves not only direct comparison (perception) but also the inferential use of transitivity.

Conservation, transitivity, length perception, partitioning, unit iteration, accumulation of distance, relation to number, tiling, identical units, standardization, proportionality, additivity, and origin (zero point) are considered as crucial aspects in the development of mental models for length measurements (Battista, 2006; Belmonte, 2005; Boulton-Lewis, 1987; Chamorro & Belmonte, 1991; Clements, 1999; Hiebert, 1981; Lehrer, 2003; Nunes et al., 1993; Piaget, 1978; Piaget et al., 1960; Stephan & Clements, 2003). Besides that, Piagetian theory entails that these stages are not fulfilled for ages below 7. Therefore, assuming this order in the learning stages for length measurements, it would not be expected that children younger than 7 succeed in measuring length with standard tools. This is one of the reasons why our sample consists of children younger than this crucial age.

Those stages should be taken into account by instruction. But, in most cases, they are not (Kamii, 2006): instruction make children familiar with the vocabulary involved and with the standard and non-standard tools (ruler, rods, discrete pieces), but it does not help them construct a mental model for measurement.

Discrete Units and Continuous Tools for Measuring

Research on the analysis of measurement is split in two branches: measurements with discrete units and with the standard ruler. The measurement with discrete pieces seems to be an affordable task for early age children, kindergarteners, and preschoolers (Bush, 2009; Carpenter & Lewis, 1976; Levine et al., 2009; Solomon et al., 2015; Stephan, Cobb, Gravemeijer, & Estes, 2001). Some other researchers say that it is useless because we want children to be ready for everyday life, and so we want them ready to measure with standard tools like the ruler. The research in this line shows that it is not a so simple task for them (Barrett, Jones, Thornton, & Dickson, 2003; Castle & Needham, 2007; Cullen & Barrett, 2010; Kamii, 1995; Lehrer, 2003; Nührenbörger, 2001; Nunes et al., 1993; Zöllner & Benz, 2013). It seems that to measure with discrete units or pieces is seen as a task completely different from measuring with the ruler. The former is a less demanding task for children. Therefore, learning to measure implies to connect in children's minds these two processes and make them converge. For this purpose, it would be pertinent some research in the transition from discrete units to the ruler, like the present study.

Many authors (Boulton-Lewis, 1987; Clements, 1999; Hiebert, 1981; Piaget et al., 1960) agree about categorizing length measurement as a highly demanding task in mathematics education. They also consider conservation and transitivity as the first two stages towards the understanding of measurement. Nevertheless, there is no consensus about the age or the order of acquisition of these two stages.

Intending to clarify these points, in the last decade, Kamii (2006) performed a study with 383 children from grades 1–5 (6 to 11 years) in two public schools in the USA. The trial involved different length measurement tasks, but none of them involved real objects: they used graphical representations to be measured. She concluded that 14% of children in grade 3 (age 8–9) gave a correct answer when measuring lines with a real ruler, while 85% gave the right result when using strips or blocks as discrete units. After analysis of the answers given by children, the misconceptions observed in those who failed were "to mark or say the end point" in 30% of the cases and "to count marks" in 31% of the cases. Even, a high 15% say "I don't know." The youngest children analyzed in this study are grade 1 (6–7 years) children, and among them, 29% did measure correctly either with strips or blocks (discrete and countable units). It is not reported how they did with the ruler. Since Kamii's study, most of the wrong results or strategies followed by children in measuring are related to conceptualization of the role of numbers and the role of intervals/marks, as it is also reported by Cullen and Barrett (2010).

Conceptualization of Spatial Intervals

The difficulties of conceptualization of spatial intervals have been analyzed by Solomon et al. (2015). They worked with small samples (< 20) of kindergarteners (5–6 years) and second graders (about 8 years). The proposed tasks involved a picture including a crayon to be measured and a ruler and aligned coins as discrete units. As in the previously reported works, they used graphical tasks without an actual measurement situation. To give the answer, children just clicked on any of the proposed numbers without any action required. In addition to that, they conclude that the presence of drawn discrete units works for children to measure objects when the measurement comprises a whole number of the discrete units (toothbrushes and paper clips in this work). Other conclusion of their research is the difficulty to inhibit the misleading numerical information because of the pupil's tendency to read the ruler number aligned with the end of the object, which authors call the read-off strategy. As a consequence, they succeed with not numerated discrete units, but fail with numerated ones.

Aligned with the previously described works, to discover the interpretation that children do of numbers and space intervals in measuring length is crucial in our research. But differently from them, in this case, children are offered real measurement situations. This kind of settings for learning makes children use all their background knowledge and intuition, explicitly and implicitly, about measuring length, as the National Council of Teachers of Mathematics currently reports (Wickstrom & Jurczak, 2016). Therefore, we did not propose children to measure graphical representations, but real objects.

Trying to analyze strategies applied by children when using discrete units and the ruler, Cullen and Barrett (2010) and Levine et al. (2009) work with kindergartners and second graders. They place discrete units on the ruler space intervals to help in the understanding of accumulation of distance and focus the attention on the correspondence between starting point of object and measuring tool. The former authors find that to relate intervals to discrete iterating objects, or virtual representations, can motivate the use of a successful strategy that counts the intervals and not the ticks.

New Challenges: Early Ages and Real Measurement Situations

In all the reviewed literature, children of different ages were faced to graphical representations of objects and not to real measurement situations. We consider children can develop cognitive skills and abilities when they are offered real situation in which they can work hands on and develop measurement intuition. Therefore, by taking into account Piaget's theory about early ages for measurements, and the fact that in Spain the transition between infant (preschool) and primary (elementary) education takes place at 5–6 years old, what entails in most cases a difference in the school instruction (Kamii & Clark, 1997), we chose 4–7-year age range to work with. On the other hand, we make children face real measurement situations with objects and rulers in order to determine the strategy they implement to measure successfully. Nonetheless, in a standard ruler, numbers, ticks, and intervals are embedded, making difficult to identify what causes children to fail in measuring: the presence of numbers or the lack of identification of space intervals as iterating objects. By putting them apart in different rulers, the conditions in which children succeed can be determined and the strategies that they apply (Cullen & Barrett, 2010) when they succeed can be identified. Consequently, the question about whether numbers and space intervals help or hinder length measurements in the Spanish context can be answered.

Objectives

In the present work, the following objectives are pursued:

- To determine the use children make of numbers in rulers
- To examine the interpretation children make of space intervals in rulers
- To identify the strategies children apply when they succeed in measuring length
- To provide a scaffold for instruction that could precede the use of the standard ruler

Method

Participants

The participants were 103 pupils (50 girls) of two state-funded schools that serve a low-to-middle area in Toledo province, Castilla La Mancha, a region at the center of Spain. The immigrant population (non-Spanish) in these schools is not negligible, amounting 13 children in our whole sample. There were three age groups: second (N = 38) and third grade (N = 30) of preschool education (4–5 and 5–6 years old) and first grade of primary education (6–7 years old; N = 35). The average age in each group was 4.73 years for the 4–5 years group, 5.66 for the 5–6, and 6.75 for the 6–7. Besides age, two factors were considered in the sample: sex and family origin.

The age groups had been chosen for two reasons: firstly, because in Spain, this age elapse includes the transition between infant (preschool) and primary (elementary) education; secondly, because in the reviewed literature (Boulton-Lewis, 1987; Clements, 1999; Hiebert, 1981; Piaget et al., 1960), it was considered that to succeed in measuring with the ruler, children must manage transitivity, and that transitivity was reached after conservation. Since conservation is assumed not to be achieved until 7 years, and younger children had no use of sticks or rulers, this age range was particularly interesting for our study.

Materials

In the characterization of this sample respecting conservation, we used two same length cardboard strips (see “Procedure” section).

To carry out our experiment, four ad hoc rulers with arbitrary units were designed. These instruments have been designed by mainly taking into account Solomon et al. (2015) and Levine et al. (2009) preceding studies. In the former study, the authors use rulers and numerated and non-numerated discrete units to measure, but without embedding them in the rulers; besides, the discrete units have circular shape, which may not allow children being as precise as with rectangular discrete units fitted along the object. In the latter one, the authors use discrete movable units placed along the ruler, but not a specific ruler with fixed and integrated discrete units. Our ad hoc rulers (see Fig. 1) combine numerated and non-numerated rectangular units embedded on the ruler as well as standard rulers with and without numbers. The use of the standard ruler with arbitrary units but without numbers, that is, with only marks, is not utilized in any of the above studies, but is important for our purpose of determining the understanding of space intervals.

Rulers were numbered from 1 to 4 as follows:

- 1NUM: ruler with numerated marks
- 2MAR: ruler with marks
- 3DUNUM: ruler with numerated discrete units
- 4DU: ruler with discrete units

Procedure

The children were individually interviewed and recorded. Once confirmed with a pilot sample that their difficulties to measure length were detected in shifted objects situations, that is, when they face the measuring of objects that are not aligned at zero with the ruler, we focused on this kind of measurements.

The sample was characterized regarding conservation and measurement with the standard ruler. To avoid interferences with the use of ad hoc rulers, these two points are relegated to the end of the interview. To study conservation, children were offered two same length strips. The researcher asked whether they were same length with the two aligned and then she asked again with them shifted (Piaget et al., 1960).

To analyze children performance on measurement with the standard ruler, children were proposed to measure an 8-cm-long cardboard. The researcher posed the question: *Can you tell me how long it is?* The child freely measured and answered the question. When he/she gave the answer, the researcher provoked "incidentally" the alignment with zero and repeated the question.

When working with our ad hoc rulers, children were proposed to measure two cardboard blocks 4 and 5 units long, L4 and L5. In the proposals analyzed, the object was always shifted. The researcher asked: *How long is the object?* Then, she asked: *How do you know it?* The answer to the last question was very important because it let us determine the strategy followed by the children. It was provided orally in some cases and pointing out or marking with fingers on the ruler in other cases.

In order to avoid possible learning through previous trials during the interview, four alternative sequences were designed and randomly offered to children (see Table 1).

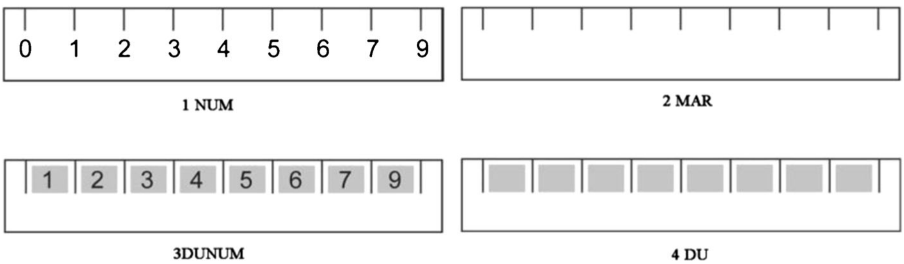

Fig. 1 Ad hoc rulers

Table 1 Proposed ruler sequences

Sequences				
A	1NUM	2MAR	3DUNUM	4DU
B	2MAR	1RMA	4DU	3DUNUM
C	3DUNUM	4DU	1NUM	2MAR
D	4DU	3DUNUM	2MAR	1NUM

Statistical Analysis

The data were analyzed with Statistical Package for Social Sciences, SPSS, v. 22. The percentage of correct answers was quantified as a measure of success. The possible association with correct answers of factors such as the ruler sequence offered, sex, or family origin was analyzed through non-parametric tests. At this early ages, family influences could affect to manage length measurement.

To analyze the strategy used by children, Cullen and Barrett (2010) ideas were considered. In our particular case, the meaningful categories were the following:

- End point (EP): children read the number close to the endpoint of the object
- Count ticks (CT): children counted the tick marks
- Count intervals (INT): children pointed to the middle of the interval situated between two consecutive marks and count
- Others (OTH): children who said/did anything different from the previous categories or said/did nothing

Regarding the designed rulers, we discriminated correct answers per ruler attempting to identify if any ruler or set of rulers was more convenient to scaffold instruction.

Results

Characterization of the Sample

We characterized the children sample regarding conservation by using aligned and shifted cardboard strips. A majority of children did not reach this target: 0% out of the 4-year-olds, 3.3% out of the 5-year-olds, and 14.3% out of the 6-year-olds conserved. Therefore, our finding confirmed the association of conservation and age ($\chi^2 = 7186$; $p = .03$).

The sample was also characterized regarding the use of the standard ruler. The 15.8% of the 4-year-olds, the 26.7% of the 5-year-olds, and the 22.9% of the 6-year-olds measured correctly with the standard ruler. It represented the 21.4% of the sample. Although the percentages were different for the age groups, the Kruskal-Wallis test showed no association between giving a correct answer and age (χ^2 (2, $N = 103$) = 5.46, $p = .07$).

The possible association of conservation and measuring correctly with the ruler was carried out through Mann-Whitney test (U = 216.500, $p = .26$), providing null association.

Measuring with Ad Hoc Rulers

Every ruler was used to measure the L4 and L5 object in the sequences provided in Table 1. No association of the sequence was observed on the percentage of correct answers (Kruskal-Wallis test, χ^2 (3, N = 103) = 1.842, p = .61). Table 2 shows the correct answer percentage split by age group. By using Kruskal-Wallis test for the association of age and percentage of correct answers, no association was found (χ^2 (3, N = 103) = 2.363, p = .50). Comparing the rulers, a considerable higher percentage was obtained with the 4DU ruler.

In the sample composition, two social factors were taken into account: sex and family origin. The possible association of right answers and these factors was analyzed through U-Mann Whitney test, and the results showed no association at all (U = 1306.0, p = .90 for sex and U = 576.5, p = .93 for family origin).

Strategies

To go deeper in the previous results, we analyzed the justification provided by pupils on the reasons for the given answers when they succeeded, compiled in Table 3. We used the categories indicated in the procedure and investigated the possible association with correct answer percentage through Kruskal-Wallis test. This analysis shed no association for rulers 1NUM (χ^2 (2, N = 103) = 5.58, p = .06) and 2MAR (χ^2 (2, N = 103) = 7.32, p = .03), while it showed for rulers 3DUNUM and 4DU (χ^2 (3, N = 103) = 20.10, p = .00; χ^2 (2, N = 103) = 29.93, p = .00).

Discussion

Our results on the characterization of the sample respecting conservation are that age is a factor that associates to conservation, while is not associated with measuring correctly with the standard ruler. On the other hand, for this age range in Spain, our findings do not confirm the positive association achieved by other authors (Nunes et al., 1993) between the success in measuring with the ruler and age. The 4-year-olds and 5-year-olds are children at preschool grades in Spain, while the 6-year-olds are in the first grade of primary education. They have been exposed to instruction for 1, 2, and 3 school years, respectively. Our results reveal that the exposure to education at school is not associated in any way with measuring correctly with the standard ruler. From that, it can be inferred a poor effect of instruction on children of these ages in our context, which should be known and discussed with teachers. As a result, the transition between

Table 2 Percentage of correct answers with ad hoc rules

	1NUM	2MAR	3DUNUM	4DU	Total
4–5-year-olds	4.0	6.6	6.6	26.3	10.9
5–6-year-olds	8.4	16.7	13.4	35.0	18.4
6–7-year-olds	7.2	8.6	28.6	42.9	21.8
Total	6.4	8.8	16.0	34.5	16.4

Table 3 Percentages of strategies used in correct answers when measuring with ad hoc rulers

% correct answers	EP	INT	CT	OTH
1NUM	16.3	0	32.5	51.3
2MAR	0	16.1	67.9	16.1
3DUNUM	0	75.3	5.9	24.5
4DU	0	77.5	4.5	18.0

preschool and elementary education is not remarkable regarding length measurement with the standard ruler.

Regarding the use of the ad hoc rulers, Table 2 shows that there is no association between age and success in measuring, as indicated in the "Results" section, neither is between conservation and correct answer percentage. Therefore, the low level of conservation does not entail a low success in measuring correctly with these rulers.

Discriminating the results by ruler, the highest success, 34.5%, is achieved with 4DU. This ruler includes discrete units on the space intervals, what can be interpreted as a good scaffold for facilitating children to build the mental idea on the space interval as an object entity.

The next ruler in decreasing order of success is 3DUNUM. This is the one children achieve 16% out of correct answers with, slightly below half the percentage obtained with 4DU. This ruler includes not only the discrete units but also the numbers in the middle of the interval. The use of this ruler reveals that the presence of numbers obscures the interpretation of the space interval as mental objects in these ages. Children use numbers as labels rather than as mathematical symbols that indicate the iteration or numerosity of a unit, depriving them of cardinal meaning. Consequently, children just pay attention to the one that fits with the extremities of the object.

For the other two rulers, the percentage of correct answers decreases from 2MAR to 1NUM, the first one including only marks and the second one including marks and numbers. It supports the findings on the hindering effect of numbers in length measurement inferred from our results and expressed in the previous paragraph. All the success percentages are well below 10% except for 5-year-olds with 2MAR ruler and much lower than with 3DUNUM and 4DU rulers. These two rulers do not include objects on the discrete units. In these situations, it seems to be more difficult for children to assign conceptual entity as iterating objects to space intervals.

Since our results confirm that the majority of children does not manage measuring with the ruler, and the null effect of instruction on children in our sample, the results obtained with the ad hoc rulers on real measurement situations could constitute a good scaffold to current instruction. The instruction should take into account that numbers are to be introduced once the entity to space intervals is mentally assigned by children. Therefore, preceding the use of the ruler, the proposed sequence would be 4DU-3DUNUM-2MAR-1NUM.

The association of two social factors, sex and family origin, with success in measuring length is analyzed. The conclusion is that in this set of data there is no any association, so sex or family origin are not factors determining the success in measuring length.

Regarding the strategies, Table 3 shows those used by children when they succeed in measuring. Particularly EP, CT, and INT are mostly indicated. For the rulers that include the discrete units on the intervals, 4DU and 3DUNUM, there is association between success and the indicated strategy, which mostly is counting intervals, INT, for both, and being used by 77.5 and 75.3% of children, respectively. Besides that, these rulers facilitate children to get the highest success (Table 2). For 2MAR, the ruler that includes space intervals with ticks, there is association with the pointed out strategy, and 67.9% of children used CT in this case. Contrary to this trend, the ruler 1NUM, the one with ticks and numbers and so the most similar to the standard ruler, does not show any association with strategies provided by children. Our interpretation of this fact is that with this kind of ruler, children are providing answers in a more random and less reflective way. It is important to remember that the success achieved by children with this ruler was really low, well below 10%, which means that a few children are in this set. Therefore, it is not worth to discuss this issue due to the low percentage of children who succeeded with this ruler. On the other hand, a remarkable aspect is the high percentage of children that provide mostly non-mathematical arguments that appear in the column named other. It is particularly high in 4 years olds and can be related with the step considered by Mialaret (1984) in the learning of mathematical concepts, called *action with verbalization*: at this stage, they just act and do not know yet exactly how to orally describe this action.

The strategy that entails a deeper understanding of what is measuring with our rulers seems to be counting intervals (INT). Children using this strategy seem to assign entity to space intervals, which could be related to the idea of unit iteration, one of the key aspects considered by experts (Chamorro & Belmonte, 1991; Kamii, 2006; Wilson & Rowland, 1993) in the understanding of measuring length.

Typical instruction travels from observable and manipulative discrete units to the ruler without any transition, and many children are not provided with neither the appropriate pace nor the mental scaffold that could support this developmental transit.

With our ad hoc-designed rulers, where numbers and space intervals are put apart, we create a convenient material to scaffold instruction at a stage preceding the use of standard ruler: firstly, the use of a ruler with discrete units on the space intervals (4DU) that empower the identification of the space interval by coordination with the discrete unit and, secondly, the use of this ruler with numbers at the center of the discrete units, like 3DUNUM, to favor the identification of number as a cardinal of units. The third step can be the removal of the discrete units keeping numbers on the center of the intervals, before using the standard ruler.

To conclude, we can say that most of the children in our sample, Spanish school children aged from 4 to 6 years, do not realize the importance of the relative position of the object extremities on the standard ruler. As a result, they had no correct use of it. In addition to that, they do not conserve length. Both findings converge in our sample, regardless the exposure to instruction. We determine that for these children, numbers hinder in measuring length (Solomon et al., 2015) and that a good scaffold to conceptualize space intervals as iterating objects can be achieved through the use of rulers with discrete units on them.

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References

Barrett, J. E., Jones, G., Thornton, C., & Dickson, S. (2003). Understanding children's developing strategies and concepts for length. In D. H. Clements, & G. Bright (Eds.), *Learning and teaching measurement: 2003 yearbook* (pp.17–30). Reston, VA: National Council of Teachers of Mathematics.

Battista, M. T. (2006). Understanding the development of students' thinking about length. *Teaching Children Mathematics, 13*(3), 140–146.

Belmonte, J. M. (2005). La construcción de magnitudes lineales en educación infantil [The building of length magnitudes in infant education]. In M. C. Chamorro (Ed.) *Didáctica de las matemáticas para educación infantil* (pp. 315–345). Madrid, Spain: Pearson Educación.

Boulton-Lewis, G. (1987). Recent cognitive theories applied to sequential length measuring knowledge in young children. *British Journal of Educational Psychology, 57*(3), 330–342.

Boulton-Lewis, G. M., Wilss, L. A., & Mutch, S. L. (1996). An analysis of young children's strategies and use of devices for length measurement. *Journal of Mathematical Behavior, 15*(3), 329–347.

Bragg, P., & Outhred, L. (2004). A measure of rulers—The importance of units in a measure. In M. J. Hoines & A. B. Fuglestad (Eds.), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* (pp. 159–166). Bergen, Norway: Bergen University College.

Bush, H. (2009). Assessing children's understanding of length measurement: A focus on three key concepts. *Australian Primary Mathematics Classroom, 14*(4), 29–32.

Carpenter, T. P., & Lewis, R. (1976). The development of the concept of a standard unit of measure in young children. *Journal for Research in Mathematics Education, 7*, 53–58.

Castle, K., & Needham, J. (2007). First graders' understanding of measurement. *Early Childhood Education Journal, 35*(3), 215–221.

Chamorro, M. C., & Belmonte, J. M. (1991). *El problema de la medida: Didáctica de las magnitudes lineales* [The problem of measurement: Didactics of linear magnitudes]. Madrid, Spain: Síntesis.

Clements, D. H. (1999). Teaching length measurement: Research challenges. *School Science and Mathematics, 99*(1), 5–11.

Clements, D. H., & Stephan, M. (2004). Measurement in pre-K to grade 2 mathematics. In D. H. Clements, J. Sarama, & A.-M. DiBiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 299–317). Mahwah, NJ: Lawrence Erlbaum.

Cullen, C., & Barrett, J. E. (2010). *Strategy use indicative of an understanding of units of length*. Paper presented at the 34th Annual Conference of the International Group for the Psychology of Mathematics in Education, Belo Horizonte.

Hiebert, J. (1981). Cognitive development and learning linear measurement. *Journal for Research in Mathematics Education, 12*(3), 197–211.

Kamii, C. (1995). *Why is the use of the ruler so hard?* Paper presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Columbus, OH.

Kamii, C. (2006). Measurement of length: How can we teach it better? *Teaching Children Mathematics, 13*(3), 154–158.

Kamii, C., & Clark, F. B. (1997). Measurement of length: The need for a better approach to teaching. *School Science and Mathematics, 97*(3), 116–121.

Lehrer, R. (2003). Developing understanding of measurement. In J. Kilpatrick, W. G. Martin, & D. E. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 179–192). Reston, VA: National Council of Teachers of Mathematics.

Levine, S. C., Kwon, M., Huttenlocher, J., Ratliff, K. R., & Dietz, K. (2009). Children's understanding of ruler measurement and units of measure: A training study. In N. A. Taatgen & H. van Rijn (Eds.), *Proceedings of the 31st Annual Conference of the Cognitive Science Society* (pp. 2391–2395). Austin, TX: Cognitive Science Society.

MacDonald, A. (2012). Young children's photographs of measurement in the home. *Early Years, 32*(1), 71–85.

Mialaret, G. (1984). *Las matemáticas: Cómo se aprenden, cómo se enseñan: Un texto base para psicólogos, enseñantes y padres* [Mathematics: How they are learned and taught: A basic text for psychologist, teachers and parents]. Ed: Visor.

Nührenbörger, M. (2001). *Children's measurement thinking in the context of length.* Paper presented at Annual Conference on Didactics of Mathematics, Ludwigsburg, Germany.

Nunes, T., Light, P., & Mason, J. (1993). Tools for thought: The measurement of length and area. *Learning and Instruction, 3*(1), 39–54.

Piaget, J. (1978). *Psicología del niño* [Child psychology]. Madrid, Spain: Morata.
Piaget, J., Inhelder, B., & Szeminska, A. (1960). *The child's conception of geometry*. New York, NY: Basic Books.
Ryan, J., & Williams, J. (2007). *Children's mathematics 4–15: Learning from errors and misconceptions*. New York, NY: McGraw-Hill.
Solomon, T. L., Vasilyeva, M., Huttenlocher, J., & Levine, S. C. (2015). Minding the gap: Children's difficulty conceptualizing spatial intervals as linear measurement units. *Developmental Psychology, 51*(11), 1564–1573.
Sophian, C. (2007). Measuring spatial factors in comparative judgments about large numerosities. In D. D. Schmorrow & L. M. Reeves (Eds). *Proceedings of the 3rd International Conference on Foundations of Augmented Cognition* (Vol. 4565, pp. 157-165). Berlin, Germany: Springer.
Stephan, M., & Clements, D. H. (2003). Linear and area measurement in prekindergarten to grade 2. In D. H. Clements & G. Bright (Eds.), *Learning and teaching measurement: 2003 yearbook* (pp. 3–16). Reston, VA: National Council of Teachers of Mathematics.
Stephan, M., Cobb, P., Gravemeijer, K., & Estes, B. (2001). The role of tools in supporting students' development of measuring conceptions. In A. A. Cuoco (Ed.), *The roles of representation in school mathematics: 2001 yearbook* (pp. 63–76). Reston, VA: National Council of Teachers of Mathematics.
Wickstrom, M. H., & Jurczak, L. M. (2016). Inch by inch, we measure: Reflect and discuss. *Teaching Children Mathematics, 22*(8), 468–475.
Wilson, P. S., & Rowland, R. (1993). Teaching measurement. In Jensen & J. Robert (Eds.), *Research ideas for the classroom: Early childhood mathematics* (pp. 171–191). Reston, VA: National Council of Teachers of Mathematics, Inc.,: Macmillan Publishing Company.
Zöllner, J., & Benz, C. (2013). *How four to six year old children compare lengths indirectly*. Paper presented at the Eight Congress of European Research in Mathematics Education (Cerme 8), Antalya, Turkey.